Intelligent Data Management - Exercise 2

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Assignment 1: Bonferroni's Principle

- a. The number of days of observation was raised to 2000.
 - The number of pairs of days is:

$$\binom{2000}{2} = 2 \times 10^6$$

• Final result:

$$5 \times 10^{17} \times 2 \times 10^{6} \times 10^{-18} = 1,000,000$$

- b. The number of people observed was raised to 2 billion (and 200,000 hotels).
 - The chance that 2 people visit the same hotel on one given day is:

$$\frac{10^{-4}}{2 \times 10^5} = \frac{1}{2} \times 10^{-9}$$

• The chance that 2 people visit the same hotel on 2 different days:

$$\left(\frac{1}{2} \times 10^{-9}\right)^2 = \frac{1}{4} \times 10^{-18}$$

• The number of pairs of people:

$$\binom{2 \times 10^9}{2} = \frac{(2 \times 10^9)^2}{2} = 2 \times 10^{18}$$

• Final result:

$$2 \times 10^{18} \times 5 \times 10^5 \times \frac{1}{4} \times 10^{-18} = 250,000$$

- c. A pair is suspected if they stay on the same hotel for 3 different days.
 - The chance that 2 people visit the same hotel on 3 different days:

$$\left(10^{-9}\right)^3 = 10^{-27}$$

• Choose 3 days in 1000 days:

$$\binom{1000}{3} = \frac{1000!}{3! \times (1000 - 3)!} = 166, 167, 000$$

• Final result:

$$5 \times 10^{17} \times 166, 167,000 \times 10^{-27} = 0.083$$

Assignment 2: Base of the natural logarithm

- a. From the theory, we have: $(1+a)^b \approx e^{ab}$ and $(1-a)^b \approx e^{-ab}$, when a is small and b is large. In term of e, give approximation to:
 - $(1.01)^{500}$

$$(1.01)^{500} = (1+10^{-1})^{500} \approx e^{10^{-1} \times 500} \approx e^{50}$$

• $(1.05)^{1000}$

$$(1.05)^{1000} = (1 + 5 \times 10^{-2})^{1000} \approx e^{5 \times 10^{-2} \times 1000} \approx e^{50}$$

• $(0.9)^{40}$

$$(0.9)^{40} = (1 - 0.1)^{40} \approx e^{-0.1 \times 40} \approx e^{-4}$$

- b. Taylor expansion: $e^x \approx \sum_{i=0}^{\infty} x^i/i!$. Use the Taylor expansion of e^x to compute:
 - $e^{1/10}$

$$e^{1/10} \approx \sum_{i=0}^{\infty} \frac{\left(\frac{1}{10}\right)^i}{i!} \approx 1 + \frac{1}{10} + \frac{1}{200} + \frac{1}{6000} + \frac{1}{240,000} + \dots \approx 1.105$$

• $e^{-1/10}$

$$e^{-1/10} \approx \sum_{i=0}^{\infty} \frac{\left(\frac{-1}{10}\right)^i}{i!} \approx 1 - \frac{1}{10} + \frac{1}{200} - \frac{1}{6000} + \frac{1}{240,000} + \dots \approx 0.905$$

 \bullet e^2

$$e^2 \approx \sum_{i=0}^{\infty} \frac{2^i}{i!} \approx 1 + \frac{2}{1} + \frac{4}{2} + \frac{8}{6} + \frac{16}{24} + \dots \approx 7$$