Intelligent Data Management - Exercise 2

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Assignment 1: Bonferroni's Principle

a. The number of days of observation was raised to 2000.

- The number of pairs of days is:

$$\binom{2000}{2} = 2 \times 10^6$$

- Final result:

$$5 \times 10^{17} \times 2 \times 10^6 \times 10^{-18} = 1,000,000$$

b. The number of people observed was raised to 2 billion (and 200,000 hotels).

- The chance that 2 people visit the same hotel on one given day is:

$$\frac{10^{-4}}{2 \times 10^5} = \frac{1}{2} \times 10^{-9}$$

- The chance that 2 people visit the same hotel on 2 different days:

$$\left(\frac{1}{2} \times 10^{-9}\right)^2 = \frac{1}{4} \times 10^{-18}$$

- The number of pairs of people:

$$\binom{2 \times 10^9}{2} = \frac{\left(2 \times 10^9\right)^2}{2} = 2 \times 10^{18}$$

- Final result:

$$2 \times 10^{18} \times 5 \times 10^5 \times \frac{1}{4} \times 10^{-18} = 250,000$$

c. A pair is suspected if they stay on the same hotel for 3 different days.

- The chance that 2 people visit the same hotel on 3 different days:

$$\left(10^{-9}\right)^3 = 10^{-27}$$

- Choose 3 days in 1000 days:

$$\binom{1000}{3} = \frac{1000!}{3! \times (1000-3)!} = 166,167,000$$

- Final result:

$$5 \times 10^{17} \times 166,167,000 \times 10^{-27} = 0.083$$

Assignment 2: Base of the natural logarithm

a. From the theory, we have: $(1+a)^b \approx e^{ab}$ and $(1-a)^b \approx e^{-ab}$, when a is small and b is large. In term of e, give approximation to:

- $(1.01)^{500}$

$$(1.01)^{500} = \left(1 + 10^{-1}\right)^{500} \approx e^{10^{-1}\times 500} \approx e^{50}$$

- $(1.05)^{1000}$

$$(1.05)^{1000} = \left(1 + 5 \times 10^{-2}\right)^{1000} \approx e^{5\times10^{-2}\times 1000} \approx e^{50}$$

- $(0.9)^{40}$

$$(0.9)^{40} = (1 - 0.1)^{40} \approx e^{-0.1\times 40} \approx e^{-4}$$

b. Taylor expansion: $e^x \approx \sum_{i=0}^{\infty} x^i/i!$. Use the Taylor expansion of e^x to compute:

- $e^{1/10}$

$$e^{1/10} \approx \sum_{i=0}^{\infty} \frac{\left(\frac{1}{10}\right)^i}{i!} \approx 1 + \frac{1}{10} + \frac{1}{200} + \frac{1}{6000} + \frac{1}{240,000} + ... \approx 1.105$$

- $e^{-1/10}$

$$e^{-1/10} \approx \sum_{i=0}^{\infty} \frac{\left(\frac{-1}{10}\right)^i}{i!} \approx 1 - \frac{1}{10} + \frac{1}{200} - \frac{1}{6000} + \frac{1}{240,000} + ... \approx 0.905$$

- e^2

$$e^2 \approx \sum_{i=0}^{\infty} \frac{2^i}{i!} \approx 1 + \frac{2}{1} + \frac{4}{2} + \frac{8}{6} + \frac{16}{24} + ... \approx 6.333$$