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ORIGINAL RESEARCH PAPER

Optimization of Crop Harvesting Schedules and Land Allocation Through Linear Programming

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Abstract
Food security is a major global challenge due to population growth, resource constraints, and the threat of climate change. In addition, the agricultural sector offers low profitability for the farmers that plant and harvest the crops. The net worth of a region's agricultural output depends on the constant fluctuations of farmgate prices and costs associated with planting and harvesting. In order to maximize the profit and crop value of an area, it follows that production should be profit-oriented and ensure harvest and selling when prices are high. To optimize this scheduling problem, mathematical problems are discussed. In this study, a linear programming model is designed to identify profitable seasons for harvesting crops to aid in the scheduling of farmers and agricultural offices. To demonstrate, a case study on the rice and maize output of the Philippines' Cagayan Region is conducted, as the region is a major producer of these two crops. The resulting solution translates to a 13.8% increase in the region's profitability. With the flexible structure of the developed model, researchers and agricultural planners can identify optimal harvesting strategies for any location.

Keywords Linear modeling · Food production · Crop selection · Income maximization · Food security · Harvest planning

Introduction

The elimination of world hunger is the second of the United Nations Sustainable Development Goals (SDGs). This requires nations to achieve food security, defined as universal access to healthy food, regardless of physical, social, and economic status (Poudel and Gopinath 2021). The goal of food security must also consider regional preferences and diets. One aspect of the challenge to improving food security is the need to meet projected demands brought by population increases, estimating a 70 to 110% increase in the current production rate by the year 2050 (Foley et al. 2011; Wu et al. 2018). To meet this goal, agriculture is an important sector of any nation's supply chain, as it provides sustenance for its people. Part of this involves the production of staple cereal crops, a major part of people's diets.

Because of the large scale of operation in agriculture, it is necessary to scale up production efficiently to make better use of resources. Massive land areas are solely for crops and animals, which conflicts with other purposes such as urbanization, ecological conservation, and greenhouse gas and climate management (Wu et al. 2018). This introduces significant tradeoffs for future agricultural expansions, as the land could serve other interests to reduce its environmental footprint (Foley et al. 2011). In addition, agricultural irrigation alone represents about 70% of global freshwater usage (Ma et al. 2021). This would further strain water supplies as production is expected to increase. To minimize the stress that agriculture has on the world's resources, it is desired to produce better yields with the current input rather than expanding further with inefficient production rates and greenhouse gas production (Wu et al. 2022).

Climate change adds strain to the goal of food security, as climate conditions such as temperature, rainfall, and carbon

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dioxide content fluctuate. This unprecedented change in global variables has been noted to negatively impact the growth and production of crops (Liu et al. 2020; Rao et al. 2022). Changes in suitable agricultural sites are another issue, lessening viable cultivation areas as soils dry up and temperatures rise (Li et al. 2022). The rising sea levels brought about by climate change threaten low-lying land areas with flooding. Overcoming climate change's effects on agriculture requires forecasting and adaptation in farming strategies to minimize the impacts on crop productivity (Genua-Olmedo et al. 2022).

The task of mitigating the effects of large-scale agriculture and climate change while increasing production is a daunting task that requires collaborative efforts from all parties involved. Efforts from scientists and researchers are needed to update agricultural practices and eliminate techniques that are deemed inefficient (Senthilkumar 2022). There is also value in partnerships for agriculture-based research, as these create new solutions, technologies, and plant variants that are more accustomed to tackling the issue of food security (Smyth et al. 2021). Another aspect of the global food supply is trade. Commerce across nations has been identified as the leading cause that increases the crop supply diversity of each country (Aguiar et al. 2020). This is due to crop production having specialization and diversification trends at the national level. The importance of international trade enables food security for more countries and peoples.

Despite feeding the masses and the importance of agriculture, the income generated from farming is minimal. Statistical reports show a farmer's investment return on some crops is low. Investments are made to provide better seed varieties and irrigation networks to aid in this endeavor, among others (Department of Agriculture - Cagayan Valley Region 2021).

For some regions around the world, there is a significant difference between actual and potential yields, amounting to a 50% yield gap (West et al. 2014). These production inefficiencies are brought about by uncertain conditions such as the weather, climate change, soil, and cropping systems (Schils et al. 2022). The trending push for organic farming has also worsened the yield gap. Crops grown with organic methods yield, on average, only 80% of conventional strategies (de Ponti et al. 2012). To overcome these uncertainties and improve agriculture's nutritional and economic benefits, more efficient use of resources is needed to bridge the yield gap.

Mathematical modeling has been tapped in various fields to provide insight for solutions that minimize costs or maximize benefits. Similarly, it has been explored in agriculture to make efficient use of natural resources and ensure food security (Mellaku and Sebsibe 2022). The best value of an objective function(s) can be solved by linking variables through mathematical relations. Linear programming (LP) is a widely used method due to its capability to simultaneously operate with large amounts of variables and constraints (Jain et al. 2018).

To assist in the decision-making for profit-oriented agricultural production planning, the study developed an optimization model for allocating arable areas for target crops. This linear programming model would provide goals for a region's seasonal harvest areas, improving the scheduling of planting seasons and profits. The benefits of a simpler model are twofold: firstly, a light optimization model may attain a higher utilization by farmers as a decision-guiding tool compared to models of higher complexity (Ruoff 2015). Secondly, the model can be used as a complementary tool alongside more sophisticated models to observe the impact of additional variables and constraints on the optimized results.

While various LP models have been developed for agricultural planning, each with different complexities and variables used, there is a gap for a model that develops harvest schedules based solely on farm area and farmgate prices, requiring other data points to formulate a crop plan such as water and labor requirements, capital, and soil conditions (Jain et al. 2018). In contrast, this study aims at a simpler model utilizing only two forms of data that are readily accessible from government statistical databases. This enables easier implementation of the model compared to others.

Readily available agricultural data guided the model's design. The case study will consider data collected from published government statistics, including amounts harvested, farmland area, farmgate prices, and costs required to grow the crops. The model's use will be tested through a case study with data from the Cagayan Region in the Philippines. This region is selected for its high contribution to the nation's food security, which can be representative of regions in other nations that are also dedicated to agricultural production. Due to gaps in the available statistics, the model and the case study will not involve the use and cost of storage, government subsidies, production quotas, and seed varieties. The model would also not consider the investment and construction of irrigation facilities, as these costs can wildly vary depending on the location. The model will not integrate considerations on favorable weather conditions and consecutive monthly planting and harvesting.

Review of Related Literature

Linear Programming in an Agricultural Context

Model optimization and LP are used in various fields to solve logistic problems. It has been explored for electrical grid planning to identify optimal power allocation across the different sectors and categories of users (Balcita et al. 2021). It is also possible to minimize manufacturing costs through LP by properly selecting suppliers, production facilities, and transportation plans (Galindo et al. 2021). LP was also used in humanitarian operations that distribute goods, such as maximizing the nutritional value of food aid programs (Valencia et al. 2021).

Due to the versatility of this technique, it has also been explored in the field of agriculture. This is due to the myriad of constraints that could potentially impact production; it becomes important to account for whatever factors are relevant in each situation. Indeed, a literature review paper shows that LP is ahead of other mathematical model–based optimization techniques in terms of the number of agricultural studies that utilize the method (Mellaku and Sebsibe 2022).

LP has been used to develop agricultural models around the globe. It is often used to find the optimal ratio of crops to produce for maximum returns, as shown in Bhatia (2019) for a farm model in India's Jaipur district. There is also the potential to introduce new LP models to create better decision-making tools by adding complexities to the model. As such, it is possible to convert LP into a multi-stage process to optimize the same objective function over the course of several time periods, which may assist Spanish farmers with long-term planning (Galán-Martín et al. 2015). Another study includes data interval and probability to address uncertainty in its solutions. The study related food security requirements, farmer income, land coverage, and environmental impact to optimize crop production and selection in China's grain-producing areas (Lu et al. 2013). LP can also be used in small-scale optimization, such as a study that sought to optimize the land use of small farms in Uganda for food security and profit, based on their logbooks (Ruoff 2015). This shows the versatility and adaptability of LP as an optimization strategy.

Mixed integer LP finds usage in system optimization studies. This has been applied to find optimal expansion strategies for value chains. One model was developed to add or remove plantations based on forecasted demand of agricultural product while minimizing cost (Rajakal et al. 2019). Multi-objective LP allows for optimizing more than one function, letting one model meet multiple goals. This is the case with another value chain expansion model. Focused on sustainability, the objectives aim to maximize a facility's profit while minimizing the carbon footprint and water footprint (Rajakal et al. 2021a). Another study conducted a multi-objective LP with fuzzy programming to maximize the economic gains of water irrigated, and the agricultural benefit per acre used (Yang et al. 2020). This led to the solution prioritizing both water-efficient and land-efficient crops.

Another topic of interest is the optimization of water supply and distribution. Some studies present LP solutions for water allocation with a factor of uncertainty in the risk of shortage, such as two-stage mixed integer LP (Zhang and Guo 2018) and Copula function–based probability interval LP (Yue et al. 2022). A case study for Northern Chile presents a mixed integer LP to optimize the operation of a solar-powered water supply system to meet the agricultural region's needs while minimizing the cost of purchasing additional energy from the power grid (Vergara-Fernandez Luis et al. 2022).

Aside from crop selection, production profit, and water allocation, LP can also be used to optimize other things in an agricultural context. Probability-based LP was used in the supply chain distribution of perishable goods to maximize total profit, minimize the risk of lower profit, and maximize the chance of higher profit (Jarernsuk and Phruksaphanrat 2019). This was done by managing distribution to retailers based on the sale price and freshness. A study utilized weighted goal programming to achieve two separate LP and stochastic models to minimize the cost and maximize the shelf life of dairy cattle feed (Saxena and Khanna 2015). Another study utilizes integer LP and mixed integer LP to optimize the routing of an unmanned ground vehicle (UGV). To minimize the complexity of the decision variables required in each step, a branch and bound algorithm assists in finding initial feasible solutions faster than commercially available solutions. This routing solution was deemed beneficial to agriculture as UGVs have been explored to automate tasks such as spraying and sample collection (Fotio Tiotsop et al. 2020). LP has been used to determine optimal planting strategies for plantations, as perennial crops have optimal maturities for maximum productivity (Rajakal et al. 2021b). The strategies developed by the model enable the minimization of cost and/or carbon impact of a plantation while meeting the production demands.

While LP is widely used to optimize challenges in agricultural planning and resource management, there may be situations that cannot be described linearly. In this circumstance, other models can be considered instead such as dynamic programming, multiple stage linear programming, quadratic programming, and target MOTAD (minimization of total absolute deviations) (Jain et al. 2018). Optimization strategies such as fuzzy goal programming are useful when there are multiple objectives to meet and when the data collected is not precise. Metaheuristics such as genetic algorithm and particle swarm optimization are algorithmic methods that search for solutions and are capable of optimizing nonlinear problems (Memmah et al. 2015).

Rice and Maize Production in Cagayan Region of the Philippines

This section discusses the importance of rice and maize, the two crops used in the computational case study. This is followed by presenting the relevant statistics and data that will be used in the formulation of the case study, which will focus on a single region in the Philippines.

Maize and rice are the two most important grain crops, with wheat as the third (Erenstein et al. 2013). These three staple cereal crops contribute to 35% of caloric intake for people around the world (Soto-Gómez and Pérez-Rodríguez 2022). Cereal crops also hold an additional purpose as feeds for livestock, which consume about 31% of the global cereal production (Mottet et al. 2017). Their twofold importance to the human diet necessitates the large-scale production dedicated to each crop. Around the world, each year, 1137 and 757 million tons of maize and rice are produced, respectively (Erenstein et al. 2021). Produced over a cumulative area of 361 million hectares, these two cereal crops occupy large portions of land, and yields are needed to rise to maintain global food security. Despite this forecasted demand, it has also been predicted that farm numbers will slowly decrease (Erenstein et al. 2021). With other negative factors that threaten farm yields, such as global warming, it becomes crucial to improve the appeal of farming as a profession through policies and technological advancements (Liu et al. 2020).

The Philippines is an archipelagic country in Southeast Asia. About 42% of its land area is dedicated to agricultural use (World Bank 2017). Rice is the staple crop of the Filipino diet and it is estimated that 118.81 kg of rice is consumed per individual each year (Department of Agriculture Communications Group 2020). With a population of 108 million, this equates to a national consumption of 12.9 million metric tons a year. Yellow maize also holds importance in the country as a major component of animal livestock feeds and as a raw material for the manufacturing sector. On the other hand, white maize acts as a budget substitute grain when rice is scarce in supply (Gerpacio et al. 2004).

Among the regions of the Philippine nation, the Cagayan Valley boasts a high percentage share of rice and yellow maize production. In 2020, the region produced 13.7% of the nation's rice for a total of 2.65 million metric tons (MMT), ranking it the second most productive region for rice. The same year, it ranked the highest in overall maize production, accounting for 22.9% of the nation's total maize output (Philippine Statistics Authority 2021a). This is equal to an overall volume of 1.86 MMT. The region's self-sufficiency levels for the two crops are exceedingly high, at 273% for rice and 509% for maize. This highlights the region's role as a supplier of major cereal crops to other parts of the Philippines (Department of Agriculture - Cagayan Valley Region 2021). The region's contribution to the nation's food security justifies it being the focus of the case study. Table 1 shows the annual yields of the region from 2017 to 2021 as it maintains its prominence in cereal production (Philippine Statistics Authority 2021a, 2022a). Table 2 lists the quarterly amounts produced for the cereal crops (Philippine Statistics Authority 2022a).

The region also maintains promise in the development of its agricultural production. The region's total crop value showed a comparatively high 5.9% growth rate from 2020 to 2021, accounting for 9.4% of the nation's total crop value. This makes it third among the Philippine regions regarding the net worth of its crops. For the year 2021, the region produced 51.2 billion and 26.3 billion Philippine Pesos (PHP) worth of rice and maize, respectively (Philippine Statistics Authority 2022b). This was computed through the region's average farmgate value of crops, which were 17.42 PHP for rice and 14.16 PHP for maize. Using the average conversion rate of 49.26 PHP:1 USD, the region's rice and maize production had a value of 1.57 billion USD for the year 2021. Despite this, the returns for the farmers are low. National statistics note that the ROI of rice is only 46% for the year 2020 (Philippine Statistics Authority 2021b). Income and job creation are also part of the main concerns of the Department of Agriculture (Department of Agriculture - Cagayan Valley Region 2021). Due to the fluctuations in price throughout the year, it is also important to investigate monthly price variations. Monthly farmgate prices per region cannot be acquired, so instead, Table 3 presents the national averages (Philippine Statistics Authority 2022c).

Table 1 Annual yield (in MMT) and percent contribution of Cagayan Region to Philippine rice and maize production from 2017 to 2021 (Philippine Statistics Authority 2021a, 2022a)

Year	Rice yield	Percent	Maize yield	Percent
2021	2.91	14.6%	1.89	22.8%
2020	2.65	13.7%	1.86	22.9%
2019	2.64	14.1%	1.87	23.4%
2018	2.38	12.5%	1.63	20.9%
2017	2.66	13.8%	1.84	23.2%

Table 2 Quarterly amount of rice, yellow maize, and white maize produced by Cagayan Region for 2021 (in metric tons) (Philippine Statistics Authority 2022a)

Quarter	Irrigated rice	Rainfed rice	Yellow maize	White maize
Q1	562,063	68,630	556,692	8888
Q2	884,540	71,961	432,020	4949
Q3	387,500	3404	341,270	3271
Q4	846,556	85,297	539,375	4705

Cagayan Region's high crop output is made possible by several factors. It allocates a large amount of land for growing crops. For rice and maize, respectively, the region allocates 52% and 35% of its total farming area, for a total of 281.2 and 189.3 thousand hectares (kha), based on the latest report from 2020 (Department of Agriculture - Cagayan Valley Region 2021). Using double and triple cropping practices, it was able to multiply the land's use and able to harvest from what is effectively 613.1 and 432.9 thousand hectares of land in the year 2021 (Philippine Statistics Authority 2022d).

For the year 2021, the region was ranked highest in land allocation for maize, but only third in land allocation for rice, overtaken by the Western Visayas region. This suggests that Cagayan Valley has better cultivation practices and seeds than Western Visayas. Indeed, it has a higher average harvest per hectare at 4.48 tons/hectare compared to the other region's 3.49. This could be linked to the higher yields

Table 3 Monthly national farmgate prices of rice and maize for the year 2021 (in Philippine Peso per kilogram)[1] (Philippine Statistics Authority 2022c)

Month	Rice (palay)	Yellow corn	White corn
January	16.40	12.20	13.00
February	16.81	12.67	14.43
March	17.10	12.55	14.11
April	17.07	12.87	13.56
May	16.97	13.41	13.91
June	17.18	13.68	14.95
July	17.34	13.84	16.08
August	17.23	14.33	16.65
September	16.18	15.48	16.35
October	15.78	15.39	14.42
November	16.35	15.87	14.70
December	16.56	15.43	15.06

[1]The average currency conversion rate for 2021 is 1 USD = 49.26 PHP

Table 4 Computed average 2021 yield per hectare of Cagayan Valley for rice and maize (in tons per hectare) (Philippine Statistics Authority 2022a, d)

Crop type	Total yield (t)	Farm area (ha)	Mean tons per hectare (t/ha)
Rice (irrigated)	2,680,658.5	544,477.0	4.9233
Rice (rainfed)	229,291.9	68,644.4	3.3403
Yellow maize	1,869,356.3	425,273.1	4.3957
White maize	21,813.4	7651.3	2.8509

of irrigated rice over rainfed rice, as most of the rice plantations in Cagayan are irrigated compared to less than half in Western Visayas (Philippine Statistics Authority 2021a). A breakdown of Cagayan Region's average yield per hectare can be seen in Table 4, taken from 2021 data.

A quarterly breakdown region's land use for the year 2021 for four different kinds of rice and maize is presented in Table 5 (Philippine Statistics Authority 2022d). It shows that most farmers harvest more in certain quarters of the year than others. The third quarter consistently has the least harvested area, while the last quarter shows the most area harvested for rainfed rice, irrigated rice, and yellow maize. White maize is most harvested in the first quarter of the year. This difference in quarterly harvests may be dictated by weather.

In terms of seasons, most of the region is labeled as Type III climate, with some seaside and westmost sections as Types II and I, respectively (Basconcillo et al. 2018). These climate conditions provide ample rain for most of the year, allowing rainfed agricultural strategies to coexist in the region with irrigation methods. These types are based on their monthly rainfall trends and are summarized in Table 6.

Table 5 Quarterly area harvested per crop type in Cagayan Region for 2021 (in thousand hectares) (Philippine Statistics Authority 2022d)

Crop type	Q1	Q2	Q3	Q4
Rice (irrigated)	112.3	163.9	80.3	188.0
Rice (rainfed)	19.5	20.3	1.1	27.7
Yellow maize	120.8	95.8	82.8	125.9
White maize	2.8	1.7	1.3	1.8

Table 6 Climate types in the Philippines (Basconcillo et al. 1983, 2018)

Climate type	Wet season	Dry season
I	July–September	December–April
II	Maximum: November–April	N/A
III	April–December	January–March
IV	Entire year	N/A

Due to the practice of double cropping and the growth period of rice and maize taking up about 120 days or 4 months for most varieties, crops would be occupying farmlands for about 8 months of the year (USDA Foreign Agricultural Service 2016). Under the broader topic of multi-cropping, double cropping is the practice of two planting and harvest periods per year, either of the same crop or the same kind (Moura and Goldsmith 2020). This is done to increase the annual productivity of agricultural lands or to address increases in demand.

Aside from farm area and growth time, costs are another important factor in the farming process. Various expenses are needed before farmers can sell their harvests. This will also affect the overall return of investment of the farmers. The average prices for 2020 are presented, as there is insufficient data for quarterly or monthly price variations.

The Philippine Statistic Database provides a comprehensive breakdown of the total computed costs required for a hectare of maize and/or corn. The database of production costs includes, but is not limited to, the purchase of seeds, fertilizer, and pesticides, salaries for labor, sacks and tying materials, land tax and landowner shares, irrigation fees, rental fees, transport costs, repair costs, and utility costs. This is further broken down into payments settled in cash, payments settled in kind, and opportunity costs given up due to a lack of personally owned resources and tools (Philippine Statistics Authority 2021b). Table 7 presents the cost of production per hectare, along with the revenue gained. Due to limitations in the database, the data is limited to seasonal costs for rice and average annual costs for maize, not monthly breakdowns. Notably, maize offers farmers a minimal ROI, and rice is more profitable in dry seasons.

Methodology

The methodology will go over the description of the developed LP model and its components. After this, a case study solution is demonstrated using data from the Philippines' Cagayan Region. Forecasting was done to acquire future data to match the case study's goals. The procedure of codifying the LP model is then performed and solved through MATLAB. Computational experiments are performed afterward to identify the model's sensitivity to changes.

Introduction of the Developed LP Model

This section presents an LP model for optimizing a location's harvest area for multiple crops over multiple time periods to maximize overall profit and crop value. It will be composed of the relevant decision variables, objective function, and constraints. By adding decision variables per time period, the LP gains the capacity to consider time periods and the fluctuating value of goods.

Decision Variables The model utilizes several decision variables to represent the target area to harvest. The values of these will allow farmers and government agencies to better plan the planting and harvest seasons of crops, in order to sell when market prices are more profitable. These variables, represented by $x_{i,j}$, have two indexes: i for the type of crop and j for the time period. Thus, a single variable represents the amount of land to harvest the crop i during the time period j. If there are n crops to consider across m time periods, this results in a total number of nm decision variables.

Objective Function This equation is the LP model's target to optimize. In the case of this model, profit maximization is desired and is represented by:

$$\max\ z = \sum_{i=1}^{n} \sum_{j=1}^{m} \left(x_{i,j} * \left(G_{i,j} - \sum C_{i,j} \right) \right) \tag{1}$$

where

$G_{i,j}$ = gross return per hectare for crop i during time j

$\sum C_{i,j}$ = total cost per hectare for crop i during time j.

Both G and $\sum C$ use the same indexes of i and j as the decision variables and have currency/hectare as its units. The total cost involved represents the various expenses associated with growing, harvesting, and processing the crop.

Constraints These equations restrict the potential values of the decision variables through relations with relevant data.

Land Allocation Constraints. Plants have a growth period before harvest, preventing the use of a plot of land for multiple consecutive harvests. For each time period m, this can be represented by:

$$\sum_{j=m-t}^{m} x_{i,j} \leq \mathrm{TLA}_i \tag{2}$$

where

t = growth period of crop in number of time periods.

TLA_i = total land allocated for crop(s) i.

This is created for each time period and for each crop type. It is also assumed that the time periods are cyclical in nature, such that in a monthly model (12 time periods) and a growth period of 3 months, (t = 3), the land allocation constraint for January (m = 1) would be:

$$x_{i,10} + x_{i,11} + x_{i,12} + x_{i,1} \leq \mathrm{TLA}_i \tag{3}$$

It is important that each crop type i is given its own land allocation constraint. Allocating a farm space across multiple crops may lead to the LP model selecting only the most profitable crop and not growing the other crops.

Double Cropping Constraint. It is most common in agricultural practices to have two cropping periods and this constraint ensures that the usage of land does not exceed this. Repeated for each crop and land allocation scheme, the constraint is represented by:

$$\sum_{j=1}^{m} x_{i,j} \leq 2 * \mathrm{TLA}_i \tag{4}$$

Table 7 Seasonal total production cost per hectare and financial statistics of rice and maize in Cagayan Region for 2020 (in Philippine Peso per hectare) (Philippine Statistics Authority 2021b)

	Season	Irrigated rice	Rainfed rice	Yellow maize	White maize
Cost per hectare	Dry	59,173	43,195	51,082	34,569
	Wet	65,971	43,057		
Gross return per hectare	Dry	85,018	57,708	53,010	36,260
	Wet	74,879	47,720		
Net return	Dry	25,845	14,513	1928	1691
	Wet	8908	4663		
Return of interest	Dry	43.677%	33.599%	3.774%	4.892%
	Wet	13.503%	10.830%		

Maximum Monthly Harvest Constraint. To scale the model for large areas, the speed of harvesting must be accounted for. This ensures that the target plot of land to harvest for a given time period is a feasible size and is represented by:

$$x_{i,j} \leq \mathrm{MHA}_i \tag{5}$$

where

MHA_i = maximum harvest area for crop i over a time period.

Non-negativity Constraint. The model does not allow for negative values for its decision variables. The constraint is equated as:

$$x_{i,j} \geq 0 \tag{6}$$

Case Study on Cagayan's Rice and Maize Production

The model presented will be implemented with Cagayan Valley's data on its rice and maize production, with a goal of identifying an optimal solution for 2022's season. As the data for 2022 is incomplete, basic forecasting methods are utilized to adjust necessary values.

Decision Variables A total of 48 decision variables was used, for the planning of four crop types: irrigated rice (i = 1), rainfed rice (i = 2), yellow maize (i = 3), and white maize (i = 4) over the span of 12 months.

Data Forecasting and Preparation This section compiled the necessary values in the objective function and constraints. The model required the monthly prices and costs of the four crops for the 12 months of 2022. These were forecasted using the least-squares method for the best fit line, adjusted with seasonal indexing. Equations 7 and 8 were used to compute the least-squares line's slope and intercept, respectively. The seasonal indices found through Eq. 9 are then multiplied to their associated values predicted from the least-squares method to adjust for seasonal variation. The overhead bars found in these three equations refer to the mean value of these variables. The price and cost history from 2017 to 2021 was analyzed for this purpose (Philippine Statistics Authority 2019, 2020, 2022c). Notably, the forecasts show a reduction in farmgate prices for all four crop types.

$$b = \frac{\sum xy - n\overline{x}\,\overline{y}}{\sum x^2 - n\overline{x}^2} \tag{7}$$

$$a = \overline{y} - b\overline{x} \tag{8}$$

$$\text{seasonal index}_i = \frac{\text{average price of month } i}{\text{average price}} \tag{9}$$

Table 8 compiles the gross return per hectare, $G_{i,j}$, which was obtained by multiplying monthly farmgate prices per kilogram, expected tons per hectare (Table 4), and 1000 kg per ton. The cost per hectare, $\sum C_{i,j}$, can be found in Table 9 and was assumed to be identical per month for both white and yellow maize, due to the lack of seasonal data. The $\sum C_{i,j}$ of irrigated and rainfed rice is differentiated by seasonal data, where a j index of 1 to 5 uses dry season costs, while wet season costs are used by 6 to 12.

The model's constraints also required identifying the TLA_i and MHA_i. The land areas harvested per crop type are also forecasted with annual data from 2017 to 2021 (Philippine Statistics Authority 2022d). The forecasted values are listed in Table 10. This value of total area harvested is assumed to be the $2{*}\mathrm{TLA}_i$, due to the conventional standard of double cropping. The values for MHA_i were also estimated by the quarterly harvested area from Table 5 and dividing the highest entry per row by three, as a quarter has 3 months. In this way, it represents an approximation of how much is a known possible harvest limit per month.

Sensitivity Analysis of the Case Study Model

A sensitivity analysis was performed after the initial computation of the case study. This was done to assess the change in the maximum profit when variables in the model are adjusted. These include increases in a crop's MHA or TLA. The objective function's sensitivity will also be tweaked by decreasing the cost per hectare associated with each crop. The sensitivity analysis would provide further

Table 8 Forecasted 2022 monthly gross returns per hectare for rice and maize (in Philippine Peso)

$G_{i,j}$ forecast	Irrigated rice $i = 1$	Rainfed rice $i = 2$	Yellow maize $i = 3$	White maize $i = 4$
$j = 1$	77,962.42	52,894.98	58,460.21	38,541.92
$j = 2$	78,805.62	53,467.07	59,800.65	40,904.33
$j = 3$	79,292.36	53,797.30	60,233.01	41,623.03
$j = 4$	80,626.89	54,702.74	61,279.71	42,637.26
$j = 5$	81,328.51	55,178.77	61,904.85	42,889.50
$j = 6$	80,985.61	54,946.12	62,200.57	42,647.12
$j = 7$	80,744.99	54,782.87	62,838.59	42,808.36
$j = 8$	80,511.99	54,624.78	62,936.69	42,486.46
$j = 9$	77,294.77	52,442.00	63,425.32	41,295.06
$j = 10$	72,865.19	49,436.68	62,445.19	40,211.83
$j = 11$	72,098.91	48,916.78	63,125.09	40,731.21
$j = 12$	72,921.61	49,474.95	63,481.32	40,767.55

Table 9 Forecasted 2022 costs per hectare for rice and maize (in Philippine Peso)

	Season	Irrigated rice $i = 1$	Rainfed rice $i = 2$	Yellow maize $i = 3$	White maize $i = 4$
Forecasted cost per hectare	Dry	61,398.90	40,536.30	54,418.50	36,159.50
	Wet	69,804.69	40,317.01		

Table 10 Estimated values for 2022 total land allocation and monthly harvest area for rice and maize

	Irrigated rice $i = 1$	Rainfed rice $i = 2$	Yellow maize $i = 3$	White maize $i = 4$
2*TLA	548,451.43	62,440.22	434,376.06	7,420.68
TLA	274,225.71	31,220.11	217,188.03	3,710.34
MHA	62,659.00	9,247.33	41,952.23	945.00

Table 11 Optimal yield, net value, and profit of the case study

Crop type	Optimal yield (t)	Net value (PHP)	Profit (PHP)
Rice (irrigated)	2,700,190.9	43,764,672,877	8,113,684,592
Rice (rainfed)	208,569.1	3,393,490,335	870,003,944
Total rice	2,908,760	47,158,163,212	8,983,688,536
Yellow maize	1,909,386.8	27,061,016,497	3,422,922,876
White maize	21,155.6	312,919,914	44,591,836
Total maize	1,930,542.4	27,373,936,411	3,467,514,712

insight on effectively improving the region's productivity for future years.

Each variable's value was incrementally increased to note the rate of change in the objective function's maximum value. The range acquired lists what value the rate of change is constant in.

Implementation in MATLAB and Code Availability

The case study's LP model was solved using MATLAB version R2022a. The Optimization Toolbox add-on provided the commands and packages for LP model solving. This was performed on a Windows 10 computer with a Ryzen 5 3500× processor (6 cores at 3.6GHz) and 16 GB memory.

The entire LP model of the case study was initialized and solved in a singular .m script file. A second script file was used for the succeeding sensitivity analysis. A GitHub repository of the .m script files used in this paper is available at the web address https://github.com/jiegocustodio/Crop-Harvest-Linear-Model.

Results and Discussion

Optimal Solution of Case Study

MATLAB was able to run the scripts without any error, delay, or discernable computation time. The summary log stated that the process took twelve iterations to arrive at the optimal solution through the dual simplex method. With an exit flag value of 1, the Optimization Toolkit identified an optimal solution and terminated it successfully. Thus, the developed model formulated a solution to the case study.

The objective function, which was to maximize Cagayan Region's profit for 2022, had an optimal value of 12,449,676,598.76 or 12.4 billion PHP. The optimal yield and value of the harvests are listed in Table 11. This is compared to 2021's estimated profit of 10.9 billion PHP or a 13.8% increase. It is forecasted that the value of maize will drop while rice increases. When comparing the net worth of the products, the optimal solution is worth 74.53 billion PHP in farmgate prices, a 3.86% drop from the 77.52 billion PHP net worth produced in 2021. This may be attributed to the overall forecasted drop in farmgate prices.

The solution array of the 48 decision variables in Table 12 lists the target harvest area per month to break down further how the LP model arrived at this computed profit. The values of the decision variables can be seen in Table 12. In summary, irrigated rice is best harvested from January to September; rainfed rice from January to August; yellow maize from February to December; and white maize from February to September.

New planting seasons can be derived to take advantage of this optimal harvest schedule, as shown in Fig. 1. The planting periods were set to accommodate their corresponding harvest periods, and the two harvest seasons were split as evenly as possible. Thus, the schedule shows the earliest and latest periods a farm can begin planting and harvesting to meet the two harvest batches. A caveat is that the second planting season must immediately occur after the first harvest. This may or may not be feasible with the resources and workforce available. As such, the optimal harvest seasons may be difficult to achieve. Improving the maximum allowable harvest is recommended to condense the planting and harvest periods further.

Table 12 Optimal decision variable values of the case study, in hectares to harvest

Optimum values (hA)	Irrigated rice $i=1$	Rainfed rice $i=2$	Yellow maize $i=3$	White maize $i=4$
$j=1$	62659.00	3478.12	0	0
$j=2$	62659.00	9247.33	14853.76	875.34
$j=3$	62659.00	9247.33	41952.23	945.00
$j=4$	62659.00	9247.33	41952.23	945.00
$j=5$	62659.00	3478.12	41952.23	945.00
$j=6$	62659.00	9247.33	41952.23	875.34
$j=7$	62659.00	9247.33	41952.23	945.00
$j=8$	62659.00	9247.33	41952.23	945.00
$j=9$	47179.42	0	41952.23	945.00
$j=10$	0	0	41952.23	0
$j=11$	0	0	41952.23	0
$j=12$	0	0	41952.23	0

Table 13 Rate of change of objective function per unit increase in cost per hectare per crop

	Rate of change (PHP/(PHP/ha))	Current value (PHP/ha)	Range (PHP/ha)
Irrigated rice (dry)	−313,295	61,398.90	0–70,486
Irrigated rice (wet)	−235,156	69,804.69	60,734–77,295
Rainfed rice (dry)	−34,698	40,536.30	39,976–40,767
Rainfed rice (wet)	−27,741	40,317.01	40,086–40,878
Yellow maize	−434,376	54,415.50	0–59,798
White maize	−7420	36,159.50	0–40,904

Table 14 Rate of change of objective function per unit increase in total land area per crop

	Rate of change (PHP/ha)	Current value (ha)	Range (ha)
Irrigated rice	14,982	274,225.71	250,636–281,964
Rainfed rice	26,999	31,220.11	27,742–36,990
Yellow maize	10,761	217,188.03	209,762–230,737
White maize	11,232	3710.34	2833–3779

Unlike the past data that suggests harvests spread quarterly, the optimal solution instead concentrates as much harvest into the months where most profit can be gained. This is to gain as much income from periods when the prices benefit the farmer. As the model assumes that products are sold on the same month as harvested, this does not consider the possibility of stocking grains to sell at favorable prices.

Succeeding Sensitivity Analysis

The sensitivity analysis was performed to assess the changes in the objective function as variables changed. This provided important information on the most impactful values to modify and improve profits. Tables 13, 14, and 15 list down the unit prices of the changed values, as well as the ranges to which this holds true. A summary of these rates of change and values can be found in Fig. 2 a and b.

Among the 14 variables modified, the ones with the most impact are the costs per hectare of irrigated rice and yellow maize. The reduction of these costs by just one can result in significant increases in the profit objective function due to the large coverage of land that these two crops occupy. This also suggests that increases in the cost of the two crops can cause drops in profitability. Policies that act to make crops cheaper to plant will improve the value of harvests. The economic benefits are less for the area-based constraints, TLA_i and MHA_i. While these may be the easiest metrics to increase in the model, there are the previously discussed drawbacks of land used in agriculture which is the land that could have been used for ecological or urban purposes.

Conclusion

In this study, a linear programming model was developed to identify optimal harvest periods and areas to maximize the income and profit of a region. The model focuses on

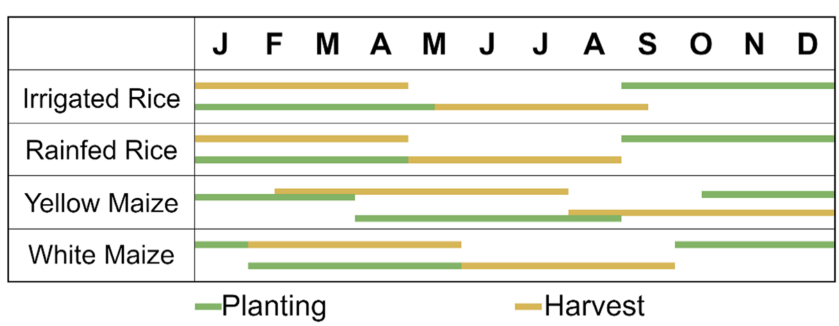

Fig. 1 Proposed planting and harvest periods for rice and maize in Cagayan Valley

Table 15 Rate of change of objective function per unit increase in monthly harvest area per crop

	Rate of change (PHP/ha)	Current value (ha)	Range (ha)
Irrigated rice	63,922	62,659	60,940–68,556
Rainfed rice	2765	9247	7805–10,407
Yellow maize	25,881	41,952	39,489–43,437
White maize	3087	945	928–1236

data related to the profits and costs of crops, as well as area-based constraints. The model can also incorporate multiple crops simultaneously. Sensitivity analysis of the case study revealed that decreasing the cost per hectare is the most valuable factor when determining the economic value of a region's agricultural sector, especially when reducing the cost of the widely planted varieties.

With food security becoming an increasingly pressing issue as the years pass, mitigation strategies are needed at all levels to prevent widespread hunger. Of special interest are the cereal crops as these provide high amounts of calories for people worldwide. At the regional level, better planning of harvests and improving crop production for profit can provide agricultural regions the income to invest in better technologies and uplift the value of farmers.

The developed model can serve as a supplementary model alongside sophisticated LP models for planning harvest schedules in other regions across the globe. As it is a relatively simple model that requires fewer data inputs, it can be easier to implement as a decision-guiding model for farmers, compared to models that need more information.

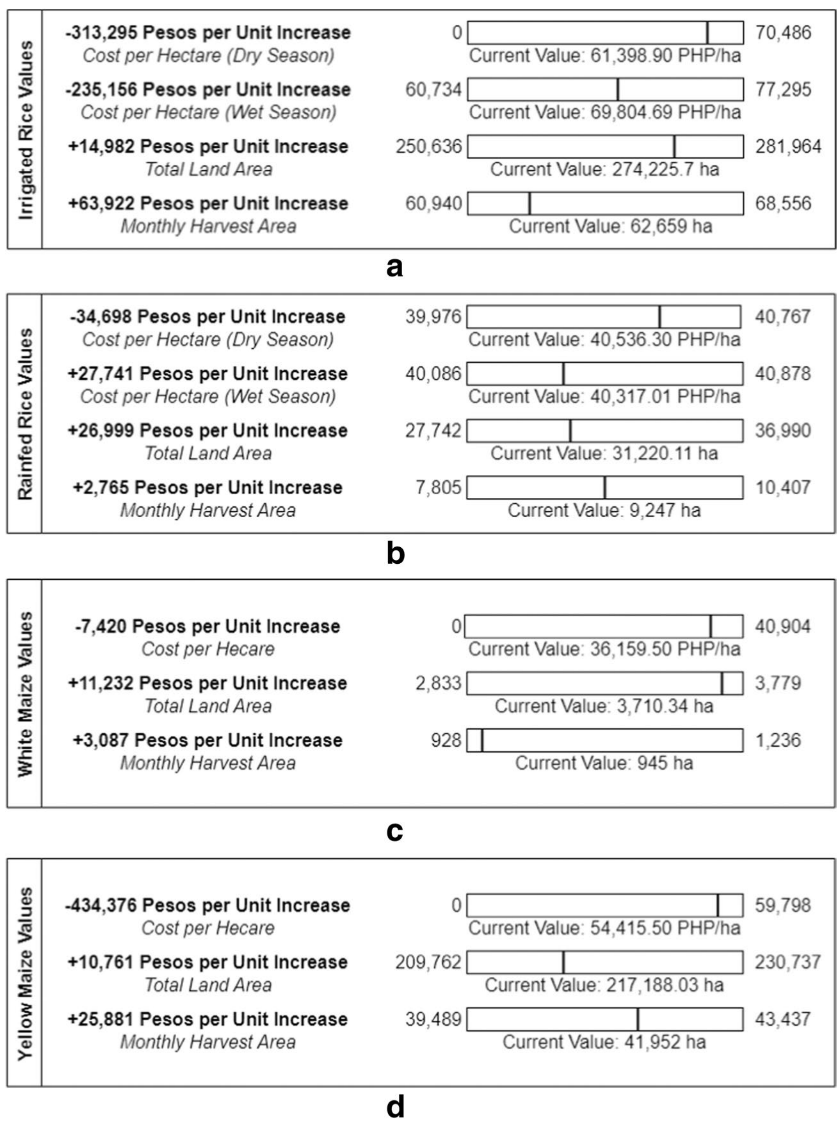

Fig. 2 **a** Sensitivity analysis summary of rates of change in objective function by varying irrigated rice values. **b** Sensitivity analysis summary of rates of change in objective function by varying rainfed rice values. **c** Sensitivity analysis summary of rates of change in objective function by varying white maize values. **d** Sensitivity analysis summary of rates of change in objective function by varying yellow maize values

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Data Availability All data that were analyzed in this study are included and discussed in the "Rice and Maize Production in Cagayan Region of the Philippines" section of this research article. These datasets were taken from statistic reports made by the Department of Agriculture of the Philippines.

Declarations

Competing Interests The authors declare no competing interests.

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