Roman Numerals in Idris

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Roman Numerals

- The ancient Romans used the letters

  I, V, X, L,

  C, D, and M

to write numbers, such as 1, 2 or 4999

Roman Numerals

- Each letter represented a value, ie.

  I=1, V=5, X=10, L=50

  C=100, D=500, M=1000

irrespective of their position in a number

Roman Numerals

- Those values were simply added to obtain the value of the number, eg.

MLXVI = 1000 + 50 + 10 + 5 + 1 = 1066 or

XXXXII = 10 + 10 + 10 + 10 + 1 + 1 = 42

Roman Numerals

- Ordering the letters by their values allowed for the use of substractive notation, eg.
  - IIII = 4 became IV
  - LXXXX = 90 became XC

Motivation for this Talk

- Roman numerals allow us to demonstrate a couple of interesting techniques when implementing conversion functions in Idris, such as using ...

Motivation for this Talk

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- Category Theory

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- Homotopy Type Theory

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- Set Theory

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- Logic and Proof Theory

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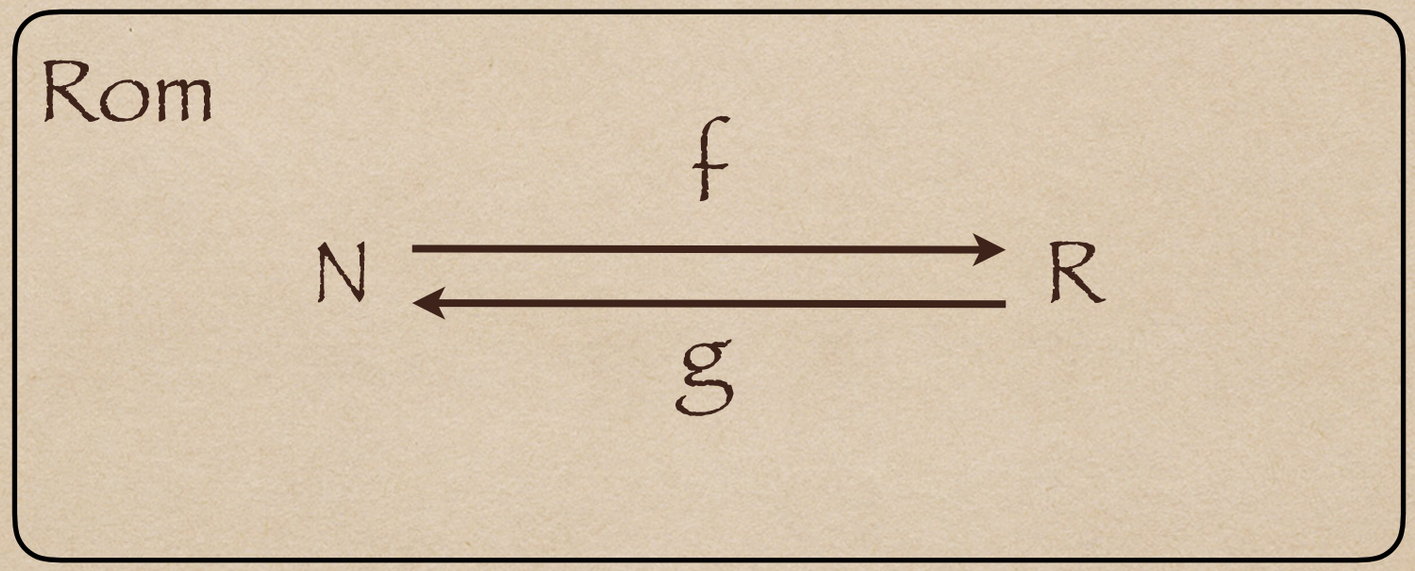
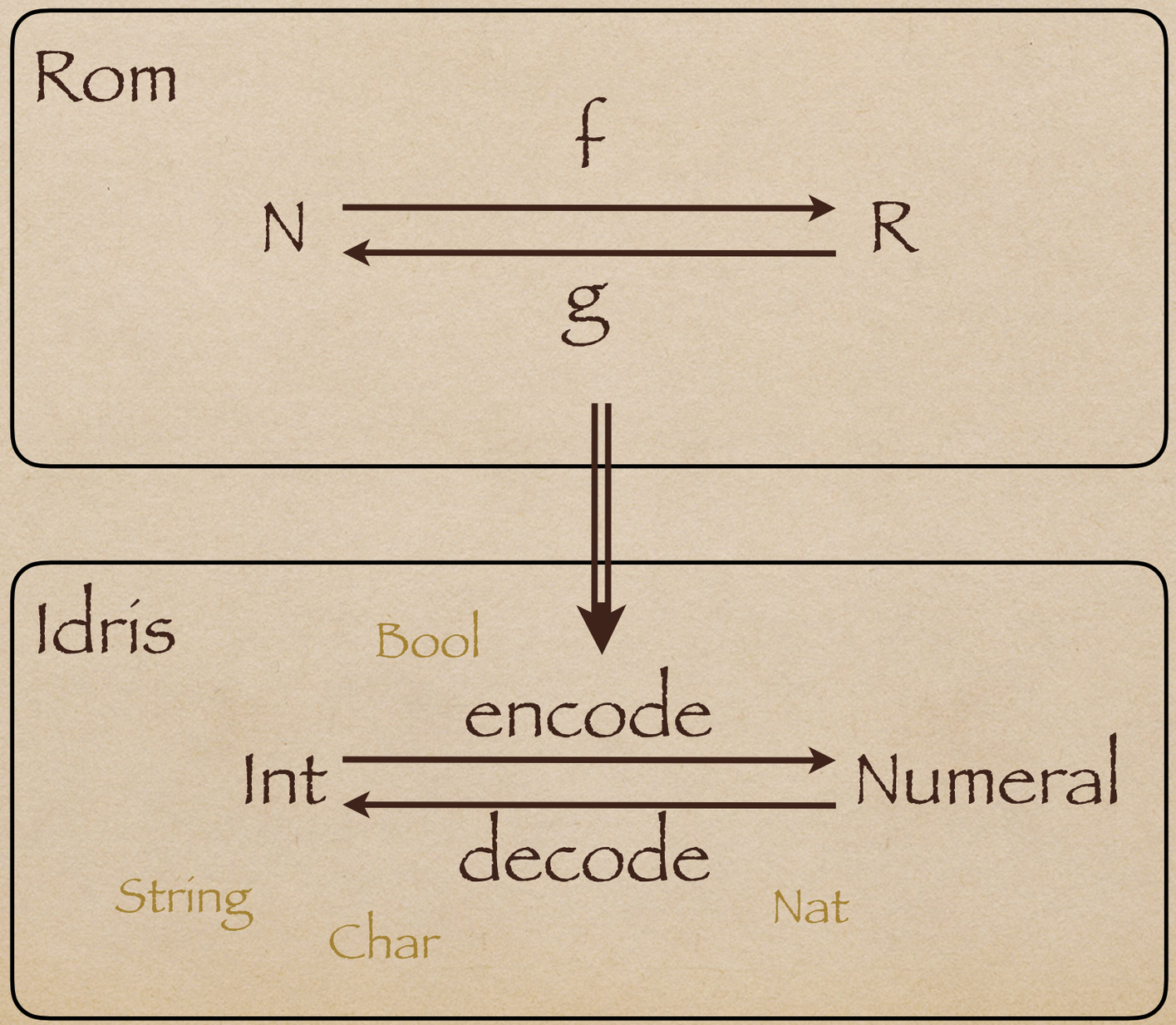
- Category Theory
- Homotopy Type Theory
- Set Theory
- Logic and Proof Theory
- Intuistic Type Theory (ie. Dependent Types)

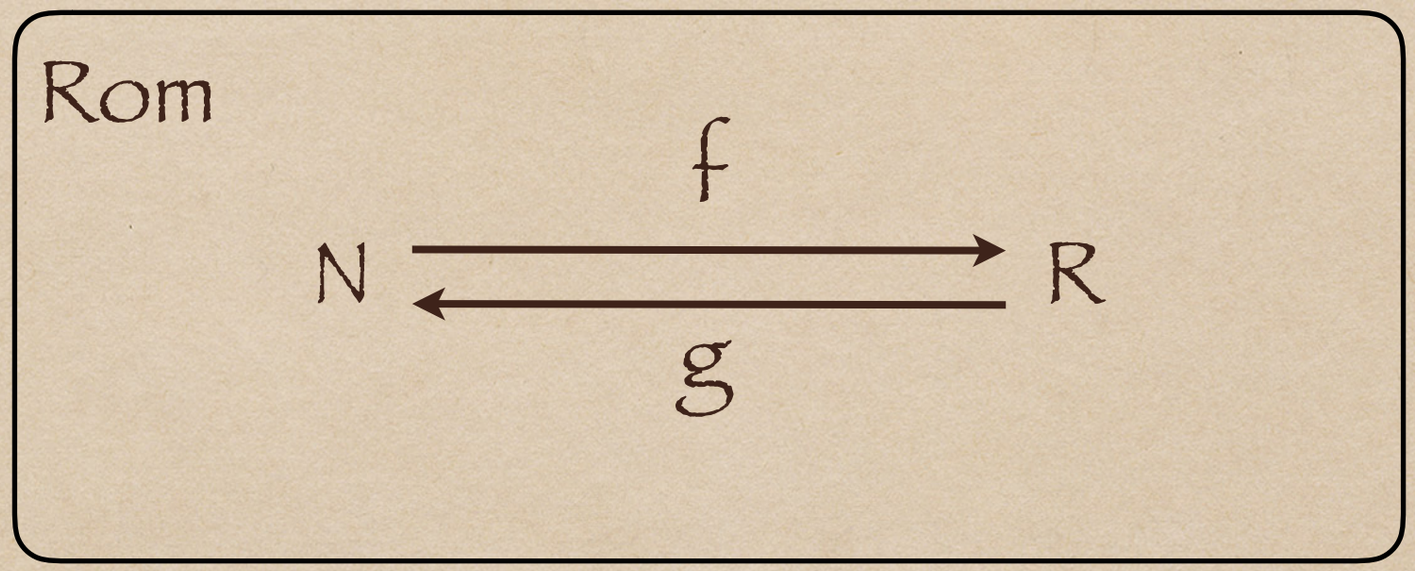
Category Theory



Category Theory



Category Theory



- It would be nice if we could establish a bijection between N and R, ie.

  $g \circ f = id_N$ and $f \circ g = id_R$

  because, well ...

Homotopy Type Theory

Homotopy Type Theory

- Just kidding ...

Set Theory

- ... but we can have ourselves be inspired

Set Theory

- Bijections allow us to establish equivalences and transport proofs along "discrete paths"
- Example: {true, false} <-> {false, true}

Set Theory

- So we want to establish - and prove! - a bijection between (a subset of) the integers and the Roman numerals

Set Theory

- What about 0, -2, 299'792'458 ?
- What about IIII and IV ?
- How do we prove a bijection anyway?

Set Theory

- A function f is bijective iff it is both injective and surjective, ie.

  $\forall a,b.\ f(a) = f(b) \Rightarrow a = b$

  $\forall y\ .\ \exists x\ .\ f(x) = y$

Set Theory

- Ok, we will need some rules ...

Set Theory

- We will only consider the integers {1, ..., 4999}
- We will not allow subtractive notation (ie. IV)
- I, X, C, and M may only appear 4 times
  V, L, and D may only appear once
- The letters have to appear in order

Logic and Proof Theory

- How do we encode those rules?

Logic and Proof Theory

- We use a Logical Relation!

Logic and Proof Theory

- ... or kind of - we are still in inspiration mode

Logic and Proof Theory

- We conjecture that we will get unique Roman numerals with values {1, ..., 4999}
(and hence a nice bijection)
if we ...

Logic and Proof Theory

- Allow all letters I, V, X, L, C, D, and M to create single-letter numerals

- Allow those letters to be prepended to a valid numeral iff the value of the resulting numeral is less than the value of the next higher letter

- For M, we set this value to 5000

Intuistic Type Theory

- data Digit = I | V | X | L | C | D | M

Intuistic Type Theory

- value : Digit -> Int
- value I = 1

  value V = 5

  value X = 10

  ...

  value M = 1000

Intuistic Type Theory

- limit : Digit -> Int
- limit I = value V

  limit V = value X

  limit X = value L

  ...

  limit M = 5000

Intuistic Type Theory

- data Numeral : List Digit -> Int -> Type where

- SimpleNumeral :
  (d: Digit) -> Numeral [d] (value d)

- ComplexNumeral :
  (d: Digit) -> (n: Numeral ds v) ->
  True = (limit d) > (value d + v) ->
  Numeral (d::ds) (value d + v)

Intuistic Type Theory

- ... but let's start coding!

Roman Numerals in Idris

Thank you!