A. Let $p \equiv 1 \bmod 3$. Let $F(X) = X^{\frac{p+1}{2}} + aX^{\frac{p+5}{6}} + bX$ be a polynomial over $\mathbb{F}_p$. In the case $p = 31$, we have that $F(X) = X^{16} + aX^6 + bX$. We have that

$$F^5(X) = (13\,a^2 + 8\,a^2b^2 + 23\,a^5)X^{30} + \cdots .$$

$$F^{10}(X) = (5\,a{+}25\,ab^2{+}12\,a^4{+}10\,ab^4{+}25\,a^4b^2{+}29\,a^7{+}17\,ab^6{+}25\,a^4b^4{+}25\,a^7b^2{+}14\,ab^8{+}$$
$$16\,a^{10} + 12\,a^4b^6)X^{30} + \cdots +$$

$$F^{15}(X) = (1{+}b^4{+}18\,b^8{+}12\,b^2{+}27\,b^{10}{+}a^{15}{+}15\,b^{14}{+}2\,a^6b^8{+}a^{12}b^2{+}22\,a^3b^{10}{+}$$
$$24\,a^9b^4{+}29\,a^6b^6{+}10\,a^3b^8{+}24\,a^9b^2{+}28\,a^6b^4{+}29\,a^3b^6{+}8\,a^6b^2{+}10\,a^3b^4{+}22\,a^3b^2{+}21\,a^3b^{12}{+}$$
$$14\,a^9b^6 + 14\,a^9 + 14\,a^6 + 21\,a^{12} + 21\,a^3 + 21\,b^{12} + 14\,b^6)X^{30} + \cdots$$

For the following $[a, b]$ the polynomial $F(X) = X^{16} + aX^6 + bX$ is a permutation of $\mathbb{F}_{31}$. [2, 7], [2, 24]=[2,-7], [10, 7] [10, 24]=[10,-7], [16, 13], [16, 18]=[16,-13], [17, 5], [17, 26]=[17,-5], [18, 13], [18, 18]=[18,-13], [19, 7], [19, 24]=[19,-7], [22, 5], [22, 26]=[22,-5], [23, 5], [23, 26]=[23,-5], [28, 13], [28, 18]=[28,-13].
Note that $F^5(a, b) = F^{10}(a, b) = F^{15}(a, b) = 0$.

B. In the case $p = 37$, we have that $F(X) = X^{19} + aX^7 + bX$. We have that

$$F(X)^6 = (34\,a^2b + 34\,a^2b^3 + 33\,a^5b)X^{36} + \cdots .$$

$$F(X)^{12} = (7\,a{+}15\,ab^2{+}2\,a^4{+}16\,ab^4{+}19\,a^4b^2{+}18\,a^7{+}15\,ab^6{+}29\,a^4b^4{+}32\,a^7b^2{+}8\,ab^8{+}$$
$$20\,x^{10} + 19\,x^4y^6 + 16\,x^7y^4 + 3\,xy^{10} + 20\,x^{10}y^2 + 2\,x^4y^8)X^{36} + \cdots$$

$$F(X)^{18} = (34\,b{+}12\,a^3b^{15}{+}12\,a^9b^9{+}12\,a^{15}b^3{+}20\,a^6b^{11}{+}10\,a^{12}b^5{+}12\,a^{15}{+}15\,b^{13}{+}$$
$$2\,a^3b^{13}{+}25\,a^9b^7{+}36\,a^{15}b{+}9\,a^6b^9{+}21\,a^{12}b^3{+}26\,a^3b^{11}{+}32\,a^9b^5{+}25\,a^6b^7{+}10\,a^{12}b{+}9\,a^3b^9{+}9\,a^9b^3{+}$$
$$25\,a^6b^5{+}a^3b^7{+}34\,a^9b{+}9\,a^6b^3{+}12\,a^9{+}24\,b^7{+}34\,a^{17}{+}35\,a^3b^5{+}20\,a^6b{+}21\,a^3b^3{+}32\,a^3b{+}24\,b^{11}{+}15$$
$$12\,b^3)X^{36} + \cdots$$

For the following $[a, b]$ the polynomial $F(X) = X^{19} + aX^7 + bX$ is a permutation of $\mathbb{F}_{37}$. [11, 5], [11, 32]=[11,-5], [18, 17], [18, 20]=[18,-17], [24, 17], [24, 20]=[24,-17], [27, 5], [27, 32]=[27,-5], [32, 17], [32, 20]=[32,-17], [36, 5], [36, 32]=[32,-5] .

The coeficient of X^{p-1} in $F(X)^{l(\frac{p-1}{6})}$ is

$$P_l(a, b) = \sum_{\substack{3i_1+i_2+l\equiv 0 \bmod 6\\ i_1+i_2+i_3=l(\frac{p-1}{6})}} \binom{l(\frac{p-1}{6})}{i_1, i_2, i_3} a^{i_2} b^{i_3}$$

for $l = 1, 2, 3$.

Si $p = 1327$, $\frac{p+1}{2} = 664$ y $\frac{p-4}{3} = 443$, el número de polinomios de permutacion de la forma: $F(X) = X^{\frac{p+1}{2}} + aX^{\frac{p-1}{3}+1} + bX$ es 26478

Cosideramos $F(X) = X^{\frac{p-1}{2}+1} + aX^{\frac{p-1}{3}+1} + bX$ sobre $\mathbb{F}_p$. Sea $N_p(a,b)$ el número de PP de $\mathbb{F}_p$ de la forma $F(X) = X^{\frac{p-1}{2}+1} + aX^{\frac{p-1}{3}+1} + bX$. Sea $N_p(a)$ el número de PP de $\mathbb{F}_p$ de la forma $F(X) = X^{\frac{p-1}{2}+1} + aX$. Sea $N_p(b)$ el número de PP de $\mathbb{F}_p$ de la forma $F(X) = X^{\frac{p-1}{3}+1} + aX$.

p	$N_p(a,b)$	$N_p(a)$	$N_p(b)$
19	0	8	7
31	0	14	6
37	12	16	3
43	48	20	6
61	60	28	12
67	78	32	12
73	54	34	15
79	96	38	12
97	174	46	24
103	162	50	24