**ATOC7500 – Application Lab #3**

**Empirical Orthogonal Function (EOF) Analysis**

**in class October 5 and October 7, 2020**

**Note: This application lab requires netcdf4 and cartopy packages.**

**A reminder of the EOF/PCA Analysis Recipe – 5 steps**

**1) Prepare your data for analysis. Examples might include:**

**a) subsetting the global data to a smaller domain**

**b) subtract the mean**

**b) standardizing the data (divide by the standard deviation)**

**d) cosine weighting (Account for the decrease in grid-box area as one approaches the pole (i.e. weight your data by the cosine of latitude)**

**e) detrend the data**

**f) remove the seasonal or diurnal cycle**

**g) remove NaN – EOF analysis does not work with missing data.**

**2) Calculate the EOFs and PCs using one of the two methods discussed in class: a) Eigenanalysis of the covariance matrix**

**b) Singular Value Decomposition (SVD).**

**3) Plot the first 10 eigenvalues (scaled as the percent variance explained) in order of variance explained. Add error bars following North et al. 1982. Describe how you determined the effective degrees of freedom N\*. How many statistically significant EOFs are there?**

**4) Plot EOF patterns and PC timeseries (usually just the first three or so unless you want to look at more).**

**5) Regress the data (unweighted data if applicable) onto standardize values of the 3 leading PCs. In other words, project the standardized principal component onto the original anomaly data X to get the EOF in pjysical units. You should have one regression pattern for each PC – i.e., the EOF pattern associated with a 1 standard deviation anomaly of the PC. *Note: The resulting patterns will be similar to the EOFs but not identical.***

**Notebook #1 – EOF analysis using images of people**

**ATOC7500\_applicationlab3\_eigenfaces.ipynb**

**LEARNING GOALS:**

1) Complete an EOF analysis using Singular Value Decomposition (SVD).

2) Provide a qualitative description of the results. What are the eigenvalues, the eigenvectors, and the principal components? What do you learn from each one about the space-time structure of your underlying dataset?

**DATA and UNDERLYING SCIENCE:**

In this notebook, you apply EOF analysis to a standard database for facial recognition: the At&t database.

<https://www.cl.cam.ac.uk/research/dtg/attarchive/facedatabase.html>

*“Our Database of Faces, (formerly 'The ORL Database of Faces'), contains a set of face images taken between April 1992 and April 1994 at the lab. The database was used in the context of a face recognition project carried out in collaboration with the Speech, Vision and Robotics Group of the Cambridge University Engineering Department.*

*There are ten different images of each of 40 distinct subjects. For some subjects, the images were taken at different times, varying the lighting, facial expressions (open / closed eyes, smiling / not smiling) and facial details (glasses / no glasses). All the images were taken against a dark homogeneous background with the subjects in an upright, frontal position (with tolerance for some side movement).”*

The goal is to think a bit “out of the box” of Atmospheric and Oceanic Sciences about potential applications for the methods you are learning in this class for other applications.

**Questions to guide your analysis of Notebook #1:**

1. **Execute all code without making any modifications. What do the EOFs (spatial patterns) tell you? What do the PCs tell you? How do you interpret what you are finding?**

**The EOFs tell you which parts of the collection of 400 faces are most different from each other and therefore are most useful in identifying the unique aspects of the faces.**

1. **Reconstruct a face. How many EOFs do you need to reconstruct a face from the database? Does it depend on the face that it used?**

**With ~25-100 EOFs used the face becomes recognizable or distinct from the other faces, but the difference between original and constructed looks more similar to the original than the constructed. It is only at around 250-500 EOFs that the face is clear and there are no features that are really obscured (the difference of the two pictures looks like noise). Face 129 is easy to reconstruct from only ~75 EOFs, this is likely because there are some distinticve features such as a beard and glasses. These are not present in most of the faces, so there is a**

**If 10304 EOFs are used then the reconstructed face is almost exactly the same as the original face.**

1. **Food for thought: The database contains 75% white men (**[**https://www.cl.cam.ac.uk/research/dtg/attarchive/facesataglance.html**](https://www.cl.cam.ac.uk/research/dtg/attarchive/facesataglance.html)**). How do you think this database limitation impacts the utility of the database for subjects who are not white men? What are some parallels that you might draw when analyzing atmospheric and oceanic sciences datasets? *Hint: Think about the limitations of extrapolation beyond the domain where you have data.***

**Using historical data for EOFs and then analyzing RCP8.5 data for a climate variable such as sea ice would not be very good. For example there would not be any training data for the EOFs where the Central Arctic was ice-free but most of the RCP8.5 realizations would have this.**

**Notebook #2 – EOF analysis of Observed North Pacific Sea Surface Temperatures**

**ATOC7500\_applicationlab3\_eof\_analysis\_cosineweighting\_cartopy.ipynb**

**LEARNING GOALS:**

1) Complete an EOF analysis using the two methods discussed in class: eigenanalysis of the covariance matrix, Singular Value Decomposition (SVD).

2) Assess the statistical significance of the results, including estimating the effective sample size.

3) Provide a qualitative description of the results. What are the eigenvalue, the eigenvector, and the principal component? What do you learn from each one about the space-time structure of your underlying dataset?

4) Assess influence of data preparation on EOF results. What happens when you remove the seasonal cycle? What happens when you detrend? What happens when you cosine weight by latitude? What happens when you standardize your data (divide by standard deviation)? What happens when you compute anomalies?

**DATA and UNDERLYING SCIENCE:**

In this notebook, you will analyze observed monthly sea surface temperatures from HadISST (http://www.metoffice.gov.uk/hadobs/hadisst/data/download.html). The data are in netcdf format in a file called HadISST\_sst.nc. *Note that this file is ~500 MB so it might take a bit of time to download.* You will subset the data to only look at the North Pacific. Depending on how you prepare your data for analysis – you might expect to see different spatial patterns (eigenvectors) and different time series (principal components). Some things you might look for in your results are the Pacific Decadal Oscillation, “global warming”, the seasonal cycle, …. Depending on your data preparation – your hypothesis for what you should see in your EOF analysis should change. Note: In this dataset - land is NaN, sea ice is -999 – the notebook sets all values over land and sea ice to 0 for the EOF analysis.

**Questions to guide your analysis of Notebook #1:**

1. **Your first time through the notebook – Execute all code without making any modifications. Provide a physical interpretation for at least the first two EOFs and principal components (PC). What do the EOFs (spatial patterns) tell you? What do the PC time series for the EOFs tell you? What do you think of the method for estimating the effective sample size (Nstar)? Can you propose an alternative way to estimate Nstar? Do you get the same results using eigenanalysis and SVD? If you got a different sign do you think that is meaningful?.**

**EOFs look like ENSO or PDO.**

**We could use Wilks or Leith to calculate N\* instead. This could use the standardized dataset with the 804 time steps for the 7248 data points, or could do each of the individual points and take the mean or min. which would yield a value < or equal to the value of N\* used which is just that all points are not independent.**

**SVD and eigenanalysis yield different signs, but this is no meaningful, as the PC is the opposite sign too. So the true value (EOF x PC) is still the same. Below is eigenanalysis and -1x SVDA picture containing object

Description automatically generated**

**A picture containing graphical user interface

Description automatically generatedShape

Description automatically generated**

**A picture containing room

Description automatically generated**

**Graphical user interface

Description automatically generated**

**2) Save a copy of the notebook, rename it. Repeat the analysis but this time do not remove the seasonal cycle. What do you think you will see? Discus your results with your neighbor. How do the EOFs and PC change? Was removing the seasonal cycle from the data useful? What impacts does removing the seasonal cycle have on your analysis?**

**Text

Description automatically generated**

**The first EOF is dominated by the seasonal cycle, this is immediately obvious as the frequency of the shifting from positive to negative PC is seasonal.**

**A picture containing icon

Description automatically generated**

**The second EOF is much more similar to the PDO, which can be found when removing the seasonal cycle.**

**Graphical user interface

Description automatically generated**

**3) Save a copy of the notebook, rename it. Repeat the analysis but this time detrend the data. Discus your results. How do the EOFs and PC change? Was detrending the data useful? What impacts does detrending have on your analysis?**

**A picture containing graphical user interface

Description automatically generated**

**Detrending seems to make little difference, just make the strength of the EOF1 smaller.Graphical user interface

Description automatically generated**

**The second EOF however is still similar to the first EOF unilke what is seen in the original analysis. Detrending the data appears to remove some of the signal. This might just be because there are an odd number of PDO cycles so one phase is now dominant by detrending.** Graphical user interface

Description automatically generated

**4) Save a copy of the notebook, rename it. Repeat the analysis but this time do not apply the cosine weighting. Discus your results. How do the EOFs and PC change? Was cosine weighting the data useful? What impacts does cosine weighting have on your analysis? What are examples of analyses where cosine weighting would be more/less important to do?**

**A picture containing graphical user interface

Description automatically generated**

**Now the waters near Alaska have much higher EOF weightings than before**

**A picture containing room

Description automatically generated**

**The second EOF doesn’t really show any physical pattern. Not cosine weighting seems to distort the effectiveness of defining the physical pattern as too much weighting is put on smaller changes in the high latitudes.**

**Graphical user interface

Description automatically generated**

**5) Save a copy of the notebook, rename it. Repeat the analysis but this time do not standardize the data (i.e., comment out dividing by standard deviation). Discus your results. How do the EOFs and PC change? Was standardizing the data useful? What impacts does standardizing the data have on your analysis?**

**A picture containing graphical user interface

Description automatically generated**

**Graphical user interface, website

Description automatically generated**

**Without standardizing, it seems the pattern especially in the second EOF is not as defined. This is probably because those areas with high standard deviations (high variability) are weighted too strongly. When comparing regions with different variability it is very important to standardize.Graphical user interface, website

Description automatically generated**