**ATOC7500 – Application Lab #4**

**Spectral Analysis of Timeseries**

**in class Monday October 19 and Wednesday October 21**

ASK IF YOU HAVE QUESTIONS ☺

**Notebook #1 – Spectral analysis of hourly surface air temperatures from Fort Collins, Colorado at Christman Field**

**ATOC7500\_applicationlab4\_fft\_christman.ipynb**

**LEARNING GOALS:**

1) Complete a spectral analysis using two different functions in Python (direct FFT from numpy and using scipy which has more options). Describe the results including an interpretation of the spectral peaks and an assessment of their statistical significance.

2) Contrast applying a Boxcar and a Hanning Window when calculating the power spectra. What are the advantages/disadvantages of these two window types? What are the implications for the resulting power spectra?

**DATA and UNDERLYING SCIENCE:**

In this notebook, you analyze two years (January 1, 2013 through December 31, 2014) of hourly surface temperature observations from Christman Field in Fort Collins, Colorado. Missing data have been already treated. The data are in .csv format and are called Christman\_data\_nomissing.csv.

**Questions to guide your analysis of Notebook #1:**

1) Look at your data. What are the autocorrelation and e-folding time of your data? What spectral peaks do you expect to find in your analysis and how much power do you think they will have?

**AR1 = 0.9901, e-folding time = 100.91 hours.**

**The data is for 2 years with a sampling interval of t=1 hour, so I would expect there to be diurnal and seasonal cycles picked up. This should be picked up by the analysis as the lowest frequency resolved is k=1 or 1 hour-1, the Nyquist frequency is k=N/2 = (T-1)/2 1 year. Because the Nyquist frequency is so close to the seasonal cycle the power may be slightly reduced from what would be found from a longer time series.**

2) Calculate the power spectra using the Numpy method, which assumes a Boxcar window that is the length of your entire dataset. Graph the power spectra, the red noise fit to the data, and the 99% confidence interval. What statistically significant spectral peaks did you find? What do they represent? How did you assess the statistical significance (what is the null hypothesis that you are trying to reject)? Compare back to Barnes and Hartman notes to make sure all of the equations and functions in the notebook are working as you expect them too.

**Statistically significant peaks at:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Hr-1** | **Hr** | **Days** | **Normalized P** | **x1e6** |
| **0.00012** | **8474.58** | **353.11** | **0.620000** | **620000** |
| **0.041** | **24.39** | **1.02** | **0.120000** | **120000** |
| **0.083** | **12.05** | **0.50** | **0.004000** | **4000** |
| **0.124** | **8.06** | **0.34** | **0.001000** | **1000** |
| **0.166** | **6.02** | **0.25** | **0.000100** | **100** |
| **0.205** | **4.88** | **0.20** | **0.000050** | **50** |
| **0.250** | **4.00** | **0.17** | **0.000010** | **10** |
| **0.292** | **3.42** | **0.14** | **0.000008** | **8** |
| **0.334** | **2.99** | **0.12** | **0.000008** | **8** |

**The main peaks are at yearly and diurnal cycles (1 year, 1 day). All of the other peaks are much much less powerful and appear to just be at multiples of the sampling distribution.**

**The null hypothesis is that the peaks are there by random chance. With the 99% confidence interval calculated the peaks which exceed that threshold are defined as significant.**

**Chart

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3) Calculate the power spectra using the scipy method. Check that you get the same result as you got using the Numpy method. Next – compare the power spectra obtained using both a Boxcar window and a Hanning window. Assume a window length that is the entire length of the dataset. Do you get the same statistically significant peaks when applying the Hanning window and the Boxcar window? How do they differ? Can you explain why?

**There is more noise with the Boxcar method due to the side lobes, e.g. at a frequency ~0.0085 hr-1. Boxcar therefore makes it harder to identify the frequency of specific peaks. Hanning is generally more conservative as it doesn’t have as much noise meaning the significant peaks by that metric are more likely to be signal not noise. However, both methods smooth and distort the power spectra or Fourier transform as it is impossible to use an infinite spectrum analysis.**

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*4) If time – take a look at other surface meteorological variables in the dataset. Do you obtain similar spectral peaks?*

**Notebook #2 – FFT analysis using Dome-C Ice Core Data**

**ATOC7500\_applicationlab4\_fft\_EPICA.ipynb**

**LEARNING GOALS:**

1) Calculate power spectra of a dataset available on a non-uniform temporal grid. Describe the results including an interpretation of the spectral peaks and an assessment of their statistical significance.

2) Contrast applying a Boxcar and a Hanning Window when calculating the power spectra. What are the advantages/disadvantages of these two window types? What are the implications for the resulting power spectra?

3) Apply a Hanning Window with various window lengths - What are the advantages/disadvantages of changing the window length and the implications for the resulting power spectra in terms of their statistical significance and temporal precision?

4) Apply a Hanning Window with various window lengths and use Welch’s method (Welch’s Overlapping Segment Analysis, WOSA). How does WOSA change the results and why?

**DATA and UNDERLYING SCIENCE:**

In this notebook, you will perform a power spectral analysis of the temperature record from the Dome-C Ice Core, taken at 75 South and 123 East (Jouzel et al. 2007). The temperature data go back ~800,000 years before present. They are unevenly spaced in time. The data are available on-line here, courtesy of the NOAA Paleoclimatology Program and World Data Center for Paleoclimatology:

ftp://ftp.ncdc.noaa.gov/pub/data/paleo/icecore/antarctica/epica\_domec/edc3deuttemp2007.txt More information on the data is available at:

https://www.ncdc.noaa.gov/paleo-search/study/6080

**Questions to guide your analysis of Notebook #2:**

1. Look at your data and pre-process for FFT analysis: Power spectra analysis assumes that input data are on an evenly spaced grid. The Dome-C temperature data are not uniformly sampled in time. Regrid the Dome-C temperature data to a uniform temporal grid in time. Plot the data before and after re-gridding to make sure the re-gridding worked as expected.
2. Signal and Noise: What is the autocorrelation and e-folding time of your data? What spectral peaks do you expect to find in your analysis and how much power do you think they will have? *Hint: Think back to the Petit 1999 Vostok ice core dataset discussed in class.*

**Temperature lag-1 autocorrelation = 0.96 and Te = 25.0 units of time intervals ~1003 years, so ~25 ky.**

**E-folding time is setting the null hypothesis for red noise. Autocorrelation typically decays as an exponential function. High autocorrelation and short e-folding time would be a steeper slope of the 99% CI line.**

1. Use Boxcar Window to calculate power spectra: Calculate the power spectra using the Numpy method, which assumes a Boxcar window that is the length of your entire dataset. Graph the power spectrum, the red noise fit to the data, and the 99% confidence interval. What statistically significant spectral peaks did you find? What do they represent?

**Frequencies correspond to ~100.328, 40.131, 23.607, 22.932 ka. These correspond to eccentricity, obliquity, and precession respectively with the last two showing the range of the oscillation period for obliquity.**

1. Compare Boxcar Window vs. Hanning Window: Calculate the power spectra using the SciPy method. Compare the results obtained using a Boxcar window that is the length of your entire dataset to those obtained using a Hanning window that is the length of your entire dataset. Graph the power spectrum, the red noise fit to the data, and the 99% confidence interval. What statistically significant spectral peaks did you find? What do they represent? What are the differences between the results obtained using the Boxcar window and the Hanning window? Is the intuition that you gained by looking at Fort Collins temperatures the same as what you are seeing here with Dome-C temperature records? Why or Why not?

**Boxcar**

frequency 0.01000 or ~ 100.3284 ka

frequency 0.02500 or ~ 40.13135 ka

frequency 0.04250 or ~ 23.60667 ka

frequency 0.04375 or ~ 22.93220 ka

**Hanning**

frequency 0.01000 or ~ 100.3284 ka

frequency 0.01125 or ~ 89.18077 ka

frequency 0.02375 or ~ 42.24352 ka

frequency 0.02500 or ~ 40.13135 ka

**Hanning smooths everything, boxcar has side lobes which can give you false peaks. However with the Hanning approach there are more significant peaks than Boxcar in the low frequencies, but it does not detect the higher frequencies. We find that the Boxcar method actually does better in identifying the known modes of variability than Hanning.**

**Yes they are similar to the Fort Collins data results. There is a similar smoothing effect on the power spectra, the peaks are lower etc.**

1. Hanning Window with different window lengths: Using the SciPy method, compare the power spectra obtained using Hanning window with different window lengths. Graph the power spectra, the red noise fit to the data, and the 99% confidence interval. Did you find any statistically significant spectral peaks? How does decreasing the window length affect the temporal precision of the spectral peaks and their statistical significance? Did you find the classic tradeoff between 1) high spectral/temporal resolution but low quality statistics, and 2) high quality statistics but low spectral/temporal resolution?

**As you decrease the window length you increase the degrees of freedom so make the CI less certain, thus more peaks are significant. If the window is too large you get better resolution of the peaks but you’re averaging a large range of frequencies so are suppressing the amplitudes and will make fewer significant.**

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1. Add WOSA (Welch Overlapping Segment Averaging): Having found what you think is a good balance between precision in the identification of the spectral peaks and statistical significance – Try applying WOSA (Welch Overlapping Segment Averaging) in addition to using the Hanning Window with different window lengths. How does this change your results?

**WOSA enables you to increase DOF to make it more statistically robust, but still maintaining temporal resolution for the lower frequencies.**

**The difference with overlapping is that there is weighting at the edges of the windows. Without overlapping, regions at the edge of the windows will have zero weighting so will be ignored. Peaks which are significant using WOSA should be trusted more than without using it.**

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