Math 320 Homework 5.4

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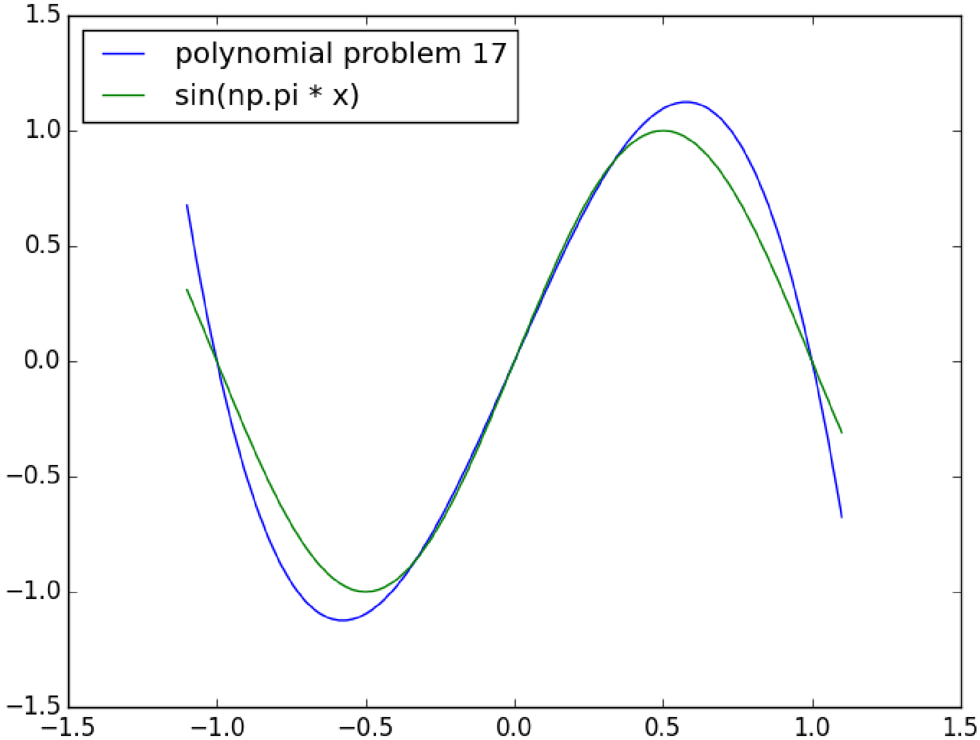
5.16

For the first part of the piecewise function, where $n = 0$, we have the inner product given by

$$\begin{aligned}
\langle T_0, T_0 \rangle &= \int_{-1}^{1} \frac{T_0^2}{\sqrt{1-x^2}} dx \\
&= \int_{-1}^{1} \frac{1}{\sqrt{1-x^2}} \frac{(2^{n-1})^2}{(2^{n-1})^2} \cos(0 \cdot \cos^{-1}(x)) dx \\
&= \int_{-1}^{1} \frac{1}{\sqrt{1-x^2}} \frac{(2^{n-1})^2}{(2^{n-1})^2} \cos(0) dx \\
&= \int_{-1}^{1} \frac{1}{\sqrt{1-x^2}} 1 dx \\
&= \int_{-1}^{1} \frac{1}{\sqrt{1-x^2}} dx \\
&= \pi
\end{aligned}$$

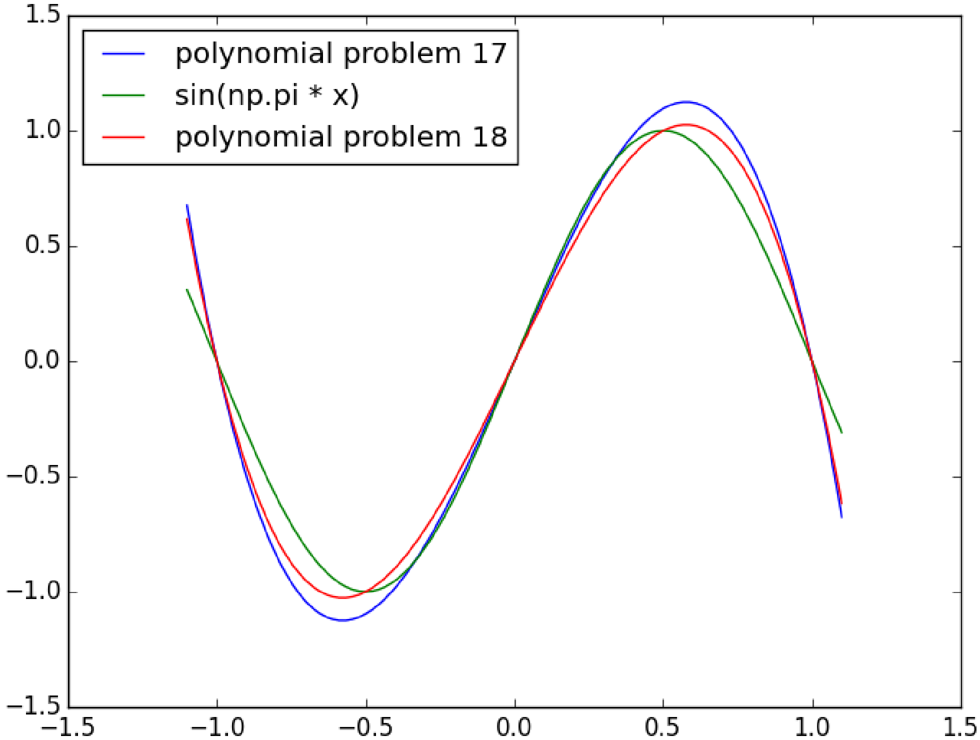
As for the second part, where $n \neq 0$, we have the result both by integration and by remark 5.4.2.

5.17



polynomial problem 17
sin(np.pi * x)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

5.18



5.19

Linear change of variables function $\tilde{x} : [-1, 1] \to [a, b]$ given by

$$\tilde{x}(x) = a\left(1 - \left(\frac{x}{2} + .5\right)\right) + b\left(\frac{x}{2} + .5\right)$$

Points of $[a, b]$ corresponding to Chebyshev roots under this map:

$$\tilde{x}(z_j) = a\left(1 - \left(\frac{\cos(\left[\frac{\pi}{n}\left(j + \frac{1}{2}\right)\right)]}{2} + .5\right)\right) + b\left(\frac{\cos(\left[\frac{\pi}{n}\left(j + \frac{1}{2}\right)\right)]}{2} + .5\right) \quad j = 0, 1, 2, \cdots, n-1$$

Analogue of Prop. 5.4.6:

If $|f^{(n+1)}(x)|$ is bounded by M on $[a, b]$, and if $p(x)$ is the interpolating polynomial $p(x)$ of the function $f(x)$ at the Chebyshev zeros $\tilde{x}(z_0), x(z_1), \cdots, x(z_n)$, then

$$||f - p||_{L^\infty} \leq \frac{M}{2^n(n+1)!}$$