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3.7

We want to show that p' is, in fact, a probability measure. To do so, we need to show that

(i) $p'(F) = 1$

(ii) Additivity: If$\{E_i\}_{i\in I} \subset \mathscr{F}'$ is a collection of pairwise-disjoint events, indexed by a countable set I, then

$$p'\Big(\cup_{i\in I} E_i\Big) = \sum_{i\in I} p'(E_i)$$

(i) is obvious, as noted in the remark, that p' is the probability metric defined on $\mathscr{F}'$ such that $p'(F) = 1$, since $\mathscr{F}'$ is the power set of F.

As for (ii), we note that this condition was fulfilled for Ω when it was our sample space. Since $E' \subset E$, we know that the same condition will apply when F is our new sample space.

3.8

We know, using DeMorgan's Law and definition 3.2.3, that

$$P\Big(\cup_{k=1}^n E_k\Big) = 1-P\Big(\Big(\cup_{k=1}^n E_k\Big)^c\Big) = 1-P\Big(\cap_{k=1}^n E_k^c\Big) = 1-\Pi_{k=1}^n P(E_k^c) = 1-\Pi_{i=1}^n (1-P(E_k))$$

Which is the desired result.

3.9

Using Bayes formula, we have that

$$.004 \text{ prevalence} \quad .95 \text{ sensitivity} \quad .95 \text{ specificity} \implies P(C|T^+) = .070896$$

$$.004 \text{ prevalence} \quad .95 \text{ sensitivity} \quad .90 \text{ specificity} \implies P(C|T^+) = .03675$$

$$.004 \text{ prevalence} \quad .95 \text{ sensitivity} \quad .999 \text{ specificity} \implies P(C|T^+) = .79233$$

$$
\begin{aligned}
.004 \text{ prevalence} \quad & .90 \text{ sensitivity} \quad .95 \text{ specificity} \implies P(C|T^+) = .07116 \\
.004 \text{ prevalence} \quad & .999 \text{ sensitivity} \quad .95 \text{ specificity} \implies P(C|T^+) = .07063 \\
.001 \text{ prevalence} \quad & .95 \text{ sensitivity} \quad .95 \text{ specificity} \implies P(C|T^+) = .01866 \\
.05 \text{ prevalence} \quad & .95 \text{ sensitivity} \quad .95 \text{ specificity} \implies P(C|T^+) = .5
\end{aligned}
$$

3.10

Where R implies that the witness was right, and W implies that the witness was wrong. Then we have that

$$
\begin{aligned}
P(Blue) &= P(R|Blue) - P(W|Red) \\
&= \frac{P(Blue|R)P(R)}{P(Blue|W)P(W) + P(Blue|R)P(R)} - \frac{P(Red|W)P(W)}{P(Red|W)P(W) + P(Red|R)P(R)} \\
&= \frac{(.1)(.8)}{(.1)(.2) + (.1)(.8)} - \frac{(.9)(.2)}{(.9)(.2) + (.9)(.1)} = .1333333
\end{aligned}
$$

3.11

$$
\frac{1\left(\frac{1}{250000000}\right)}{1\left(1\frac{1}{250000000}\right) + \frac{1}{3000000}\left(1 - \frac{1}{250000000}\right)} = .01186
$$

3.12

Let the doors be denoted D_1, D_2, D_3. The probability that the car is behind any one of these doors is $\frac{1}{3}$, and the probability that a goat is behind any one of these doors is $\frac{2}{3}$. Say we pick D_1. This means that there is a $\frac{1}{3}$ chance that the door picked has the car, meaning that there is a $\frac{2}{3}$ chance that one of the other doors has the car. Once Monty opens one of the other doors, say D_2, the probability that the car lies behind D_3 is now $\frac{2}{3}$ because D_2 had a goat, and so the $\frac{2}{3}$ probability concentrates into D_3.

Ultimately, if you switch doors at this point, you double your odds of getting the car.
By the same logic, but with doors $D_1, D_2, \cdots, D_{10}$, the likelihood that any given door has the car is initially $\frac{1}{10}$, meaning that the likelihood that one of the other doors has the car is $\frac{9}{10}$. So if Monty opens 8 of the other doors, your probability is now $\frac{9}{10}$ (if you switch doors) that the new door you pick will have the car behind it, increasing your odds by 9.