Math 320 Homework 4.5

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4.25

We have that

$$
\begin{aligned}
(\mathbf{f} * \mathbf{g})_0 &= 1 \cdot 0 + 4 \cdot 0 + 3 \cdot 1 + 2 \cdot 0 = 3 \\
(\mathbf{f} * \mathbf{g})_2 &= 2 \cdot 0 + 1 \cdot 0 + 4 \cdot 1 + 3 \cdot 0 = 4 \\
(\mathbf{f} * \mathbf{g})_3 &= 3 \cdot 0 + 2 \cdot 0 + 1 \cdot 1 + 4 \cdot 0 = 1 \\
(\mathbf{f} * \mathbf{g})_4 &= 4 \cdot 0 + 3 \cdot 0 + 2 \cdot 1 + 1 \cdot 0 = 2
\end{aligned}
$$

$$
\begin{aligned}
(\mathbf{g} * \mathbf{h})_0 &= 0 \cdot 1 + 0 \cdot i + 1 \cdot -1 + 0 \cdot -i = -1 \\
(\mathbf{g} * \mathbf{h})_2 &= 0 \cdot 1 + 0 \cdot i + 0 \cdot -1 + 1 \cdot -i = -i \\
(\mathbf{g} * \mathbf{h})_3 &= 1 \cdot 1 + 0 \cdot i + 0 \cdot -1 + 0 \cdot -i = 1 \\
(\mathbf{g} * \mathbf{h})_4 &= 0 \cdot 1 + 1 \cdot i + 0 \cdot -1 + 0 \cdot -i = i
\end{aligned}
$$

$$
\begin{aligned}
(\mathbf{g} * \mathbf{g})_0 &= 0 \cdot 0 + 0 \cdot 0 + 1 \cdot 1 + 0 \cdot 0 = 1 \\
(\mathbf{g} * \mathbf{g})_2 &= 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 1 + 1 \cdot 0 = 0 \\
(\mathbf{g} * \mathbf{g})_3 &= 1 \cdot 0 + 0 \cdot 0 + 0 \cdot 1 + 0 \cdot 0 = 0 \\
(\mathbf{g} * \mathbf{g})_4 &= 0 \cdot 0 + 1 \cdot 0 + 0 \cdot 1 + 0 \cdot 0 = 0
\end{aligned}
$$

$$
\begin{aligned}
(\mathbf{h} * \mathbf{k})_0 &= 1 \cdot 1 + -i \cdot -1 + -1 \cdot 1 + i \cdot -1 = 0 \\
(\mathbf{h} * \mathbf{k})_2 &= i \cdot 1 + 1 \cdot -1 + -i \cdot 1 + -1 \cdot -1 = 0 \\
(\mathbf{h} * \mathbf{k})_3 &= -1 \cdot 1 + i \cdot -1 + 1 \cdot 1 + -i \cdot -1 = 0 \\
(\mathbf{h} * \mathbf{k})_4 &= -i \cdot 1 + -1 \cdot -1 + i \cdot 1 + 1 \cdot -1 = 0
\end{aligned}
$$

$$
\begin{aligned}
(\mathbf{h} * \mathbf{h})_0 &= 1 \cdot 1 + -i \cdot i + -1 \cdot -1 + i \cdot -i = 4 \\
(\mathbf{h} * \mathbf{h})_2 &= i \cdot 1 + 1 \cdot i + -i \cdot -1 + -1 \cdot -i = 4i \\
(\mathbf{h} * \mathbf{h})_3 &= -1 \cdot 1 + i \cdot i + 1 \cdot -1 + -i \cdot -i = -4 \\
(\mathbf{h} * \mathbf{h})_4 &= -i \cdot 1 + -1 \cdot i + 1 \cdot -1 + 1 \cdot -i = -4i
\end{aligned}
$$

And thus we have that

$$
\begin{aligned}
(\mathbf{f} * \mathbf{g}) &= \begin{bmatrix} 3 & 4 & 1 & 2 \end{bmatrix}^T \\
(\mathbf{g} * \mathbf{h}) &= \begin{bmatrix} -1 & -i & 1 & i \end{bmatrix}^T \\
(\mathbf{g} * \mathbf{g}) &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T \\
(\mathbf{h} * \mathbf{k}) &= \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T \\
(\mathbf{h} * \mathbf{h}) &= \begin{bmatrix} 4 & 4i & -4 & -4i \end{bmatrix}^T
\end{aligned}
$$

4.26

$$
\begin{aligned}
F_4\mathbf{f} \odot F_4\mathbf{g} &= \begin{bmatrix} 2.5 & .5 - .5i & -.5 & .5 + .5i \end{bmatrix}^T \\
F_4\mathbf{g} \odot F_4\mathbf{h} &= \begin{bmatrix} 0 & -1 & 0 & 0 \end{bmatrix}^T \\
F_4\mathbf{g} \odot F_4\mathbf{g} &= \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}^T \\
F_4\mathbf{h} \odot F_4\mathbf{k} &= \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T \\
F_4\mathbf{h} \odot F_4\mathbf{h} &= \begin{bmatrix} 0 & 4 & 0 & 0 \end{bmatrix}^T
\end{aligned}
$$

$$
F_4^{-1} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}
$$

By multiplying this matrix by each hadamard product, we get the vectors from 4.25.

4.27

Using zeroes as the coefficients of certain polynomials to get the polynomials to be 2^n long, we can use FFT to calculate the fourier transform with a temporal complexity of $O(n)$. Splitting them then into evens and odds ω_n.

We then have two Discrete fourier transforms that are $n/2$ long. Then we take the hadamard product of these polynomials. Then we can apply the inverse of the

FFT to this vector of coefficents with $O(n)$ temporal complexity. Total temporal compexity is given, then, by

$$T(n) \leq 4T(n) + cn$$

which by the master theorem is $O(n \log(n))$

4.28

Let

$$A = \begin{bmatrix} a_0 & a_{n-1} & \dots & a_2 & a_1 \\ a_1 & a_0 & \dots & a_3 & a_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{n-2} & a_{n-3} & \dots & a_0 & a_{n-1} \\ a_{n-1} & a_{n-2} & \dots & a_1 & a_0 \end{bmatrix} \quad \mathbf{g} = \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_{n-1} \end{bmatrix} \quad \mathbf{a} = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{bmatrix}$$

Note,

$$A\mathbf{g} = \begin{bmatrix} a_0g_0 & a_{n-1}g_1 & \dots & a_2g_{n-1} & a_1g_n \\ a_1g_0 & a_0g_1 & \dots & a_3g_{n-1} & a_2g_n \\ \vdots & \vdots & & \vdots & \vdots \\ a_{n-2}g_0 & a_{n-3}g_1 & \dots & a_0g_{n-1} & a_{n-1}g_n \\ a_{n-1}g_0 & a_{n-2}g_1 & \dots & a_1g_{n-1} & a_0g_n \end{bmatrix}$$

and, by definition of the convolution, we have

$$\mathbf{f} * \mathbf{g} = \begin{bmatrix} a_0g_0 & a_{n-1}g_1 & \dots & a_2g_{n-1} & a_1g_n \\ a_1g_0 & a_0g_1 & \dots & a_3g_{n-1} & a_2g_n \\ \vdots & \vdots & & \vdots & \vdots \\ a_{n-2}g_0 & a_{n-3}g_1 & \dots & a_0g_{n-1} & a_{n-1}g_n \\ a_{n-1}g_0 & a_{n-2}g_1 & \dots & a_1g_{n-1} & a_0g_n \end{bmatrix}$$

Thus, they are identical.

4.29

We have that

$$p(\lambda) = \lambda^4 + 8\lambda^3 + 20\lambda^2 + 16\lambda = 0$$

yielding

$$\lambda_1 = -4, \lambda_2 = -2 = \lambda_3, \lambda_4 = 0$$

with eigenvectors

$$v_1 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \quad v_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad v_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$