Econ 580 General Equilibrium Homework

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15.B.6

Upon imposing $p_1 = 1, p_2 = p$, we have

$$w_1 = p_1 w_1 + p_2 w_2 = 1$$

$$w_2 = p_1 w_1 + p_2 w_2 = p$$

$$MRS^1_{12} = \frac{1}{p}$$

$$MRS^2_{12} = \frac{1}{p}$$

Let $p_L = 1$ and $q = p_1^{1/3}$, then

$$q = 1, \frac{3}{4}, \frac{4}{3}$$

and

$$\frac{p_1}{p_2} = 1, \left(\frac{3}{4}\right)^{1/3}, \left(\frac{4}{3}\right)^{1/3}$$

15.B.10 (a)

If consumer one only takes into account his utility from one of the goods, and the other good's quantity increases, he could see a decrease in wealth and utility. This applies to a monopoly since it will, unimpeded, refrain from supplying the maximum quantity of the good so as to avoid a decrease in price.

15.B.10 (b)

Let (p, x) be equilibrium prices and allocations after original endowment (ω_1, ω_2). Let (p', x') be equilibrium prices and allocations after a new endowment (ω_1', ω_2'). Then x, x' are both elements of the interior of the edgeworth box. By the first FTWE, these allocations are both pareto optimal, thus

$$x_{11} = x'_{12} \quad x_{12} = x'_{21}$$

Now, letting $p = p'$ we know that $p >> 0$ and by strong monotonicity, $\omega_1' \geq \omega_1$. Therefore,

$$p \cdot \omega_1' > p \cdot \omega_1$$

and since

$$p \cdot x_1' = p \cdot \omega_1' > p \cdot \omega_1 = p \cdot x_1$$

we have that

$$x_1' > x_1$$

15.B.10 (c)

Upon a wealth transfer from individual 2 to individual 1 where price ratio does not change, then there will be excess demand for one of the goods. Then price ratio will change to as to maintain equilibrium, implying a decrease in wealth for individual 1 since he alone supplies good 1.
Figure:

15.B.10 (d)

Figure:

16.C.2

Suppose, to the contrary, that for $x_i \preceq x_i^*$, we have that $p \cdot x_i \leq w_i$, then said x_i would be affordable and x_i^* wouldn't be optimal. Therefore we have a contradiction and the desired result.

16.C.4

In order for x to be pareto optimal relative to the u_i's, we must have that upon increasing u_i, we must decrease u_j where $i \neq$. However, we know that this is true of $U_i \forall i$. However, a decrease in U_i must necessarily mean a decrease in some u_i, and we have the desired result.

16.D.2

Local nonsatiation, as continuity and convexity both hold.

16.E.2

Let $(x, y), (x', y')$ be two allocation with $\lambda \in [0, 1]$. If

$$x'' = \lambda x + (1 - \lambda)x' \text{ and } y'' = \lambda y + (1 - \lambda)y'$$

by convexity of production and consumption sets, (x'', y'') is feasible by concavity of utility functions and

$$u_i(x_i'') \geq \lambda u_i(x_i) + (1 - \lambda)u_i(x_i')$$

17.B.2

Because $p >> 0$, there exists a consumer i such that $w_1 > 0$. Since $p \sum w_i > 0$, $\quad p_n^e \to 0$ and consumer i has strictly monotone preferences, it follows that

$$z_{ie} = x_{ie}(p_n^l, p \cdot w_i) - w_i$$

since

$$x_{ie}(p_n^l, p \cdot w_i) \to \infty \text{ as } p_n^e \to 0$$
$$\implies z_{ie}(p) \to \infty$$
$$\implies max\{z_1(p^n), \cdots, z_z(p^n)\} \to \infty$$

17.C.1

Let $p \in \Delta, q' \in f(p), q'' \in f(p), \lambda \in [0, 1]$. Case 1:

$$p \in int\Delta . \forall q \in \Delta(z(p))((1 - \lambda)\, q' + \lambda q'')$$
$$= (1 - \lambda)z(p)q' + \lambda z(p)q''$$
$$\geq (1 - \lambda)z(p)q + \lambda z(p)q$$
$$\implies (1 - \lambda)z(p)q' + \lambda z(p)q'' \in f(p)$$

Case 2:

$$p \in bd(\Delta)$$

If $p_l > 0$, then $q_l' = q_l'' = 0$ and $(1 - \lambda)q_l' + \lambda q_e''$

$$\implies (1 - \lambda)z(p)q' + \lambda z(p)q'' \in f(p)$$

17.C.4

$$p_1 + 2p_2 = w_1$$
$$2p_1 + p_2 = w_2$$
$$\implies \frac{3}{2}(p_1 + p_2) = \overline{w}$$

Post tax, we have

$$w_1 = \frac{3}{2}(p_1 + p_2) + \frac{1}{2}(p_1 + 2p_2 - \frac{3}{2}(p_1 + p_2)) = \frac{5}{4}p_1 + \frac{7}{4}p_2$$

$$w_2 = \frac{7}{4}p_1 + \frac{5}{4}p_2$$