Math 320 Homework 5.5

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5.20

Let

$$j, k, n \in \mathbb{N}, \quad \omega = e^{i\pi/n}$$

Note that

$$\begin{aligned}
\Re(\omega^{k(n+j)}) &= \Re(e^{i\pi/n*k(n+j)} \\
&= \Re(e^{\frac{i\pi kn + i\pi kj}{m}}) \\
&= \cos\left(\frac{\pi kn + \pi kj}{n}\right) \\
&= \cos\left(\pi k + \frac{\pi kj}{n}\right)
\end{aligned}$$

Now let $x = \frac{\pi kj}{n}$. If k is even, we have that

$$\begin{aligned}
\cos(\pi k + x) &= \cos(x) \\
&= \cos(-x) \\
&= \cos(\pi k - x) \\
&= \cos(\pi k - \frac{\pi kj}{n}) \\
&= \cos(\frac{\pi kn - \pi kj}{n} \\
&= \Re(e^{\frac{i\pi kn - i\pi kj}{n}}) \\
&= \Re(e^{\frac{i\pi}{n}*k(n-j)}) \\
&= \omega^{k(n-j)}
\end{aligned}$$

Now if k is odd we get

$$\begin{aligned}\cos(\pi k + x) &= \cos(\pi + x)\\ &= \cos(\pi - x)\\ &= \cos(\pi k - x)\\ &= \cos(\pi k - \frac{\pi k j}{n})\\ &= \cos(\frac{\pi k n - \pi k j}{n})\\ &= \Re(e^{\frac{i\pi k n - i\pi k j}{n}})\\ &= \Re(e^{\frac{i\pi}{n} * k(n-j)})\\ &= \omega^{k(n-j)}\end{aligned}$$

5.21

Given $n \in \mathbb{N}$, let $a_{n+j} = a_{n-j}$, and

$$c_k = \frac{1}{2n} \sum_{j=0}^{2n-1} a_j \omega_{2n}^{-jk}$$

It suffices to show that the imaginary parts of c_k sums to zero, or more precisely:

$$i \sum_{j=0}^{2n-1} a_j (\sin(\frac{-2\pi k j}{n})) = 0$$

Upon plugging in $n + j$ and $n - j$ into j, we get

$$a_{n+l} \sin(\frac{-2\pi(n+l)k}{n}) + a_{n-l} \sin(\frac{-2\pi(n-l)k}{n}) = 0$$

yielding

$$\begin{aligned}a_{n+l} \sin(\frac{-2\pi nk - 2\pi lk}{n}) + a_{n-l} \sin(\frac{-2\pi nk + 2\pi lk}{n}) &= a_{n+l} \sin(\frac{-2\pi lk}{n}) + a_{n-l} \sin(\frac{2\pi lk}{n})\\ &= -a_{n+l} \sin(\frac{2\pi lk}{n}) + a_{n-l} \sin(\frac{2\pi lk}{n})\\ &= -a_{n-l} \sin(\frac{2\pi lk}{n}) + a_{n-l} \sin(\frac{2\pi lk}{n}) = 0\end{aligned}$$

Now as for where $j = 0, n$, we have

$$= a_0 \sin(\frac{2\pi \cdot 0 \cdot j}{n}) = 0$$

and

$$= a_n \sin(2\pi - l) = 0$$

Therefore, imaginary parts of coefficients sum to zero and we have the desired result.

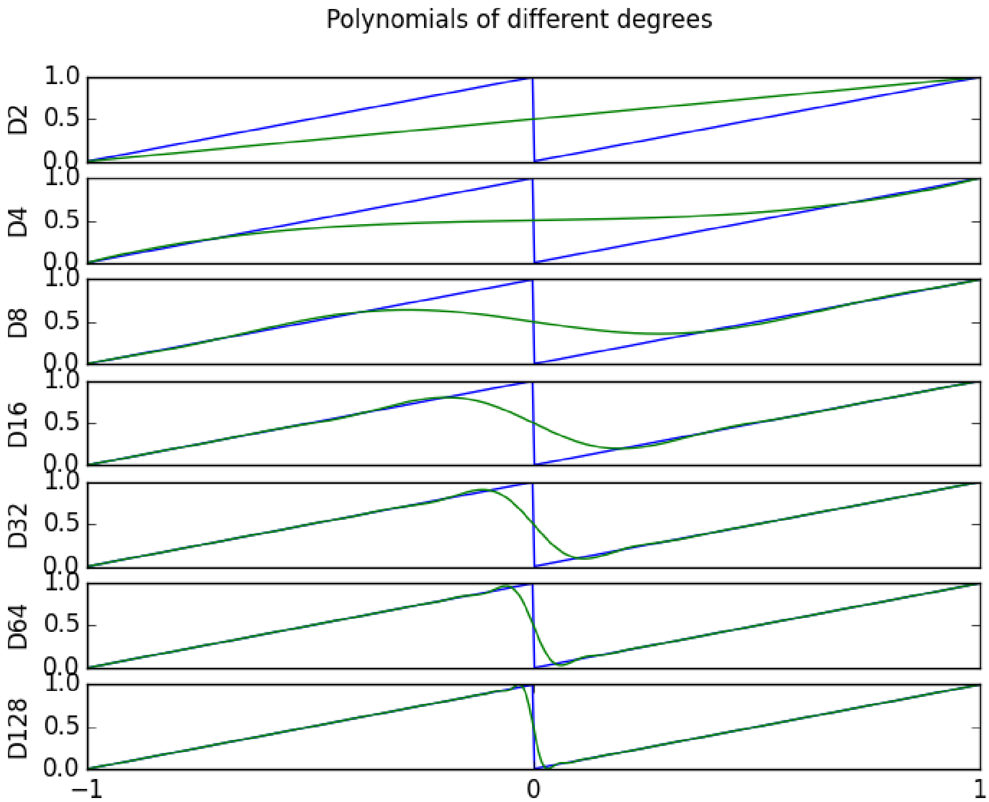
5.22

We have the following:

$$
\begin{aligned}
a_k &= \gamma_k \Re(\mathrm{DFT}(f(x_0), ..., f(x_{2n-1}))_k = \gamma_k \Re(\mathrm{DFT}(0, 1, -1, 0, 1, -1, 0) \\
\implies a_0 &= \frac{1}{2}\Re(\mathrm{DFT}(0, 1, -1, 0, 1, -1, 0))_0 = 0 \cdot \frac{1}{2} = 0 \\
a_1 &= \Re(\mathrm{DFT}(0, 1, -1, 0, 1, -1, 0))_1 = \frac{2}{3} \\
a_2 &= \Re(\mathrm{DFT}(0, 1, -1, 0, 1, -1, 0))_2 = 0 \cdot 1 = 0 \\
a_3 &= \Re(\mathrm{DFT}(0, 1, -1, 0, 1, -1, 0))_3 = \frac{-2}{3} \\
\implies p(x) &= 0T_0 + \frac{2}{3}T_1 + 0T_2 - \frac{2}{3}T_3 \\
&= \frac{2}{3}x - \frac{2}{3}(4x^3 - 3x) \\
&= \frac{2}{3}x - \frac{8}{3}x^3 + 2x \\
&= \frac{8}{3}x - \frac{8}{3}x^3 \\
&= -\frac{8}{3}(x^3 - x)
\end{aligned}
$$

which is the same polynomial as before.

5.23



Polynomials of different degrees
D2
D4
D8
D16
D32
D64
D128
1.0
0.5
0.0
−1
0
1