

8.1

Characteristic equations:

$$\begin{aligned} r_t &= ae^{z_t} k_t^{\alpha-1} \\ c_t &= e^{z_t} k_t^\alpha - k_{t+1} \\ c_t^{-1} &= \beta E\{c_{t+1}^{-1} r_{t+1}\} \end{aligned}$$

Steady states:

$$\begin{aligned} \bar{r} &= \alpha \bar{k}^{\alpha-1} \\ 1 &= \beta \bar{r} \\ \bar{c} &= \bar{k}^\alpha - \bar{k} \end{aligned}$$

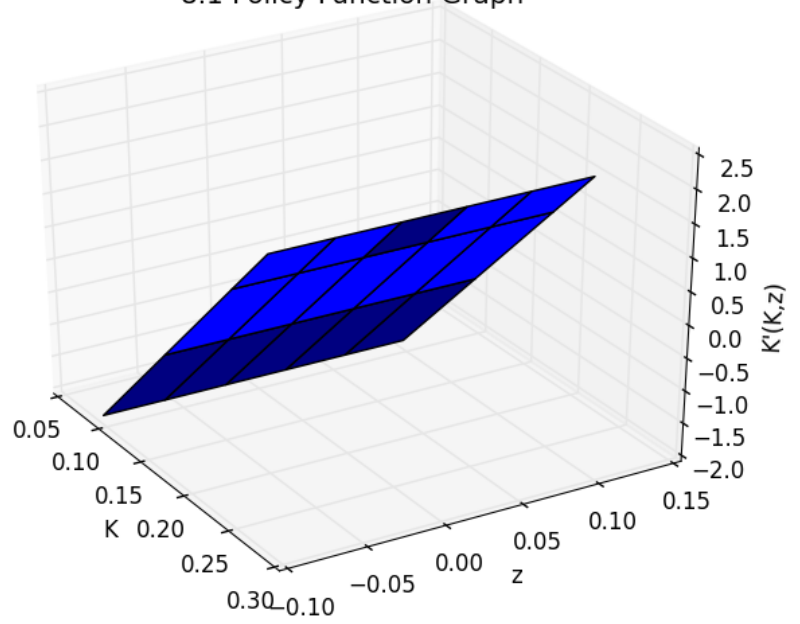
Linearization and solving of $F, G, H, L, \&M$:

$$\begin{aligned} F &= \frac{\alpha \bar{K}^{\alpha-1}}{\bar{K}^\alpha - \bar{K}} \\ H &= \frac{\alpha^2 \bar{K}^{2(\alpha-1)}}{\bar{K}^\alpha - \bar{K}} \\ G &= -\frac{\alpha \bar{K}^{\alpha-1} (\alpha + \bar{K}^{\alpha-1})}{\bar{K}^\alpha - \bar{K}} \\ L &= -\frac{\alpha \bar{K}^{2\alpha-1}}{\bar{K}^\alpha - \bar{K}} \\ P &= \frac{-G \pm \sqrt{G^2 - 4FH}}{2F} \\ M &= \frac{\alpha^2 \bar{K}^{2(\alpha-1)}}{\bar{K}^\alpha - \bar{K}} \\ Q &= -\frac{LN + M}{FN + FP + G} \end{aligned}$$

and we get:

$$\begin{aligned} F &= 2.38051966287 \\ H &= 1.68371931464 \\ G &= -5.16150615175 \\ L &= -1.6279703344 \\ M &= 1.68371931464 \\ P &= 0.83409605 \\ Q &= 0.14998134 \end{aligned}$$

8.1 Policy Function Graph



Policy function:

Note that this is similar to the closed form solution.

8.2

Characteristic equations:

$$\begin{aligned} c_t &= e^{z_t} k_t^\alpha - k_{t+1} \\ c_t^{-1} &= \beta E\{c_{t+1}^{-1} r_{t+1}\} \\ r_t &= a e^{z_t} k_t^{\alpha-1} \end{aligned}$$

Steady states:

$$\begin{aligned} \bar{c} &= \bar{k}^\alpha - \bar{k} \\ 1 &= \beta \bar{r} \\ \bar{r} &= \alpha \bar{k}^{\alpha-1} \end{aligned}$$

After linearizing,

$$\begin{aligned} \tilde{r}_t &= z_t + (\alpha - 1) \tilde{k}_t \\ \tilde{c}_t &= -\frac{\bar{k}}{\bar{c}} \tilde{k}_{t+1} + \frac{\bar{k}^\alpha}{\bar{c}} \alpha \tilde{k}_t + \frac{\bar{k}^\alpha}{\bar{c}} z_t \\ \tilde{c}_t &= E\{\tilde{r}_{t+1}\} - E\{c_{t+1}\} \end{aligned}$$

Then we have that

$$\begin{aligned}
F &= \frac{\bar{k}^2}{\bar{c}} \\
G &= \frac{\alpha \bar{k}^{\alpha+1} - \bar{k}^2}{\bar{c}} + (\alpha - 1)\bar{k} \\
H &= \frac{\alpha \bar{k}^{1+\alpha}}{\bar{c}} \\
L &= \frac{\bar{k} - \bar{k}^{\alpha+1}}{\bar{c}} \\
M &= \frac{\bar{k}^{1+\alpha}}{\bar{c}}
\end{aligned}$$

and that

$$\begin{aligned}
P &= \frac{-G \pm \sqrt{G^2 - 4FH}}{2F} \\
Q &= -\frac{LN + M}{FN + FP + G}
\end{aligned}$$

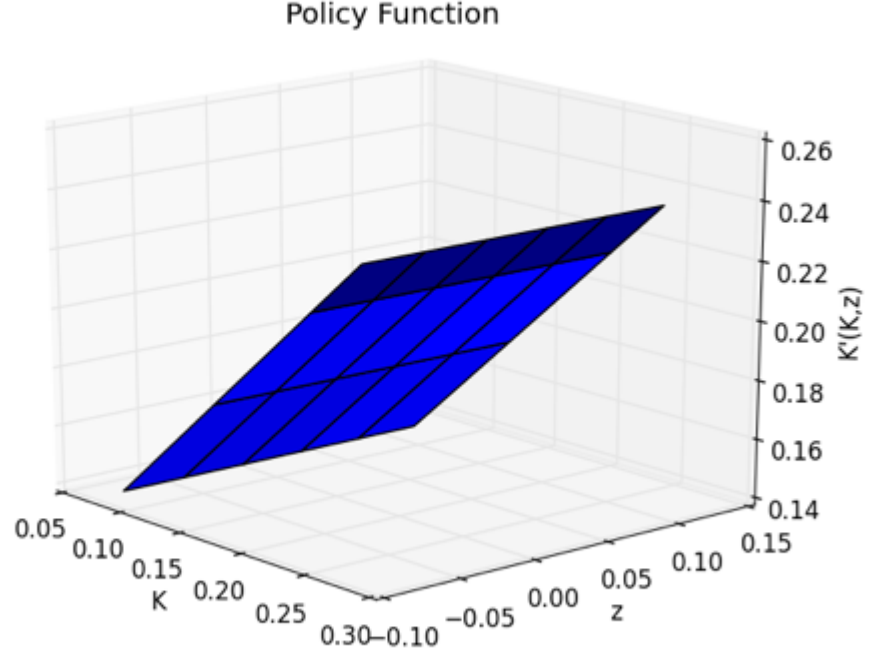
Policy function:

$$\tilde{X}_t = (I_P)\bar{X} + P\tilde{X}_{t-1} + Q\tilde{z}_t$$

Numerical answers:

$$\bar{k} = .19278, \bar{r} = 1.0204, \bar{c} = .36926, F = .100646, G = -4.8408, L = .22864, H = .102700, M = .293428,$$

Policy function graphed:



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$$\begin{aligned}
 E[F\tilde{X}_{t+1} + G\tilde{X}_t + H\tilde{X}_{t-1} + L\tilde{Z}_{t+1} + M\tilde{Z}_t] &= 0 \\
 E[F(P\tilde{X}_t) + G\tilde{Z}_{t+1} + L\tilde{Z}_{t+1} + M\tilde{Z}_t] &= 0 \\
 E[(FP + G)\tilde{X}_t + H\tilde{X}_{t-1} + (FQ + L)\tilde{Z}_{t+1} + M\tilde{Z}_t] &= 0 \\
 E[(FP + G)(P\tilde{X}_{t-1} + Q\tilde{Z}_t) + H\tilde{X}_{t-1} + (FG + L)(N\tilde{Z}_t + \varepsilon_{t+1}) + M\tilde{Z}_t] &= 0 \\
 &= (FP^2 + GP + H)\tilde{X}_{t-1} + (FPQ + GQ + FQN + LN + M)\tilde{Z}_t = 0 \\
 [(FP + G)P + H]\tilde{X}_{t-1} + [(FQ + L)N + (FP + G)Q + M]\tilde{Z}_t &= 0
 \end{aligned}$$

8.4

Steady state values:

$k = 7.2875, c = 1.4845, r = 0.1215, w = 1.3280, l = 0.5797, T = 0.0742, y = 2.2133$
and $i = 0.7287$

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	\bar{k}	\bar{c}	\bar{r}	\bar{w}	\bar{l}	\bar{T}	\bar{y}	\bar{i}
δ	-48.345	-3.511	1	-7.287	1.32	-0.176	-4.121	-0.61
τ	-2.323	-0.234	0.023	-0.165	-0.139	0.849	-0.467	-0.232
\bar{z}	0	0	0	0	0	0	0.77	0
α	25.986	2.085	0	4.396	-0.769	0.104	4.684	2.599
γ	0.139	0.028	0	0	0.019	0.001	0.042	0.014
ξ	-0.802	-0.163	0	0	-0.11	-0.008	-0.243	-0.08
β	65.438	1.751	-1.096	7.988	0.26	0.088	8.295	6.544
a	-1.849	-0.377	0	0	-0.254	-0.019	-0.562	-0.185

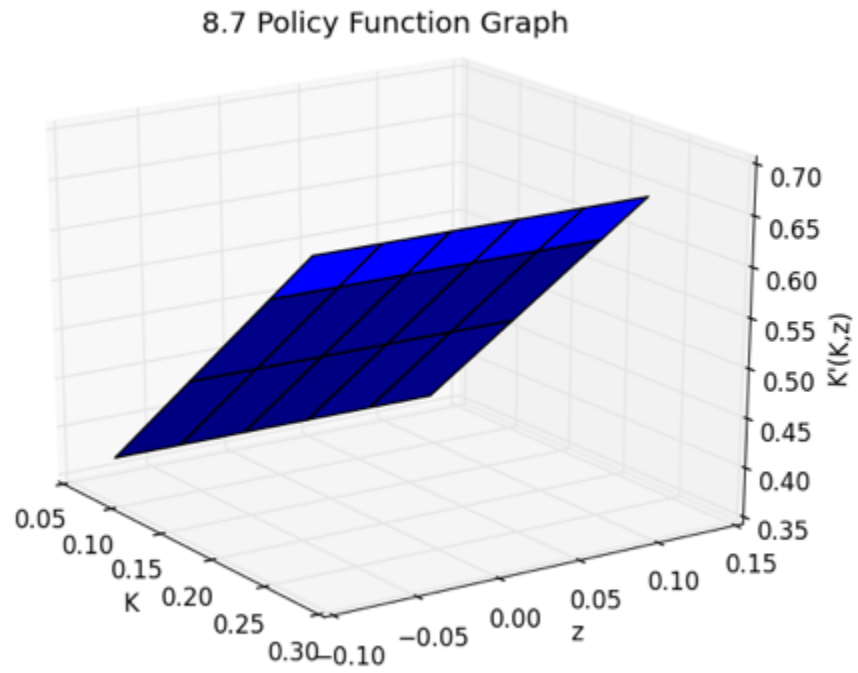
8.6

We have that

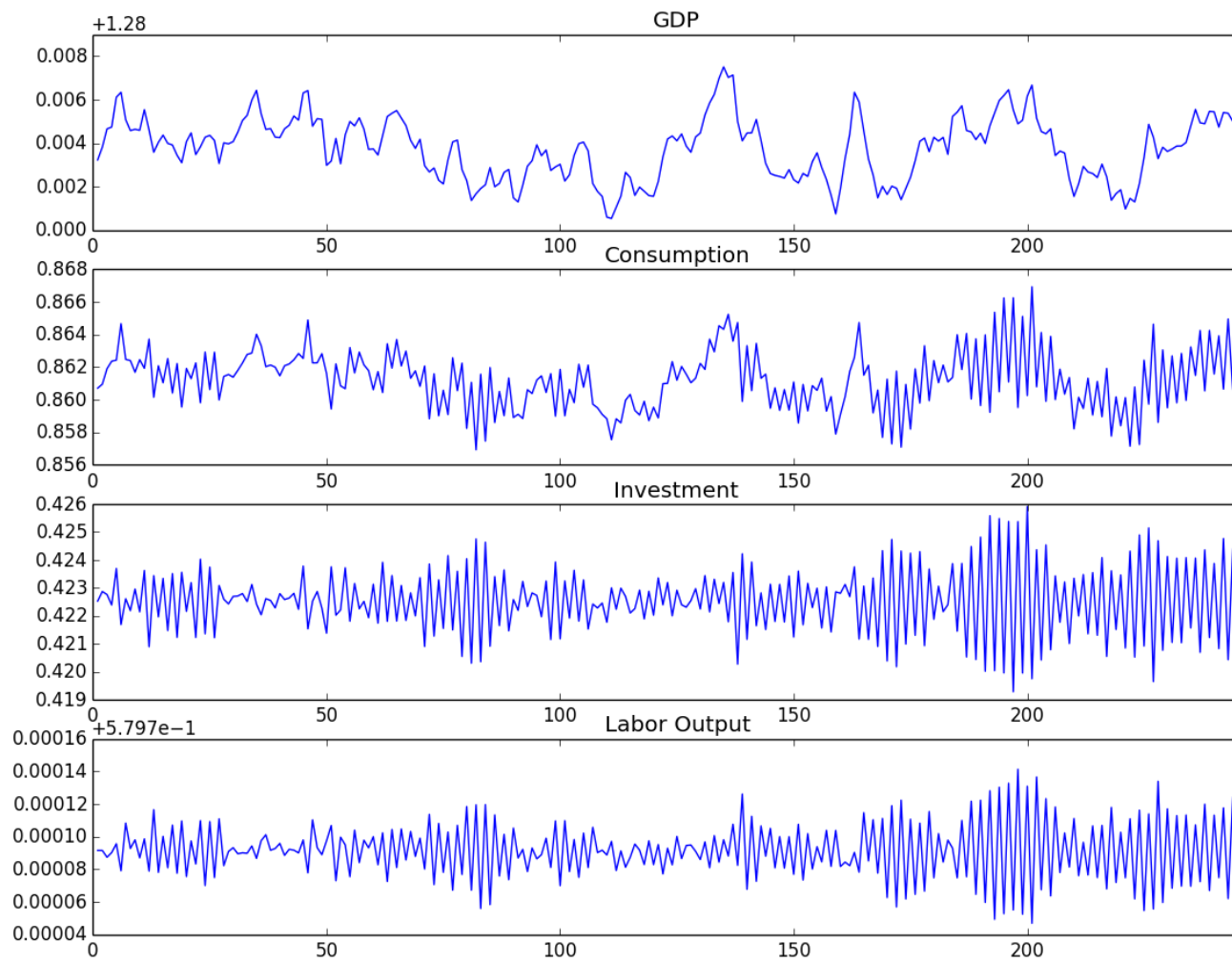
$$\begin{aligned}
 F &= \begin{bmatrix} 2.9046023 & -3.74013067 \\ 0. & 0. \end{bmatrix} \\
 G &= \begin{bmatrix} -5.88765983 & 3.8571742 \\ 2.90460231 & -8.11672985 \end{bmatrix} \\
 H &= \begin{bmatrix} 2.96699945 & 0. \\ -2.87232918 & 0. \end{bmatrix} \\
 L &= \begin{bmatrix} -2.1684962 \\ 0. \end{bmatrix} \\
 M &= \begin{bmatrix} 2.23635669 \\ -1.6363563 \end{bmatrix} \\
 P &= \begin{bmatrix} 0.9153012 & 0. \\ -0.02633366 & 0. \end{bmatrix} \\
 Q &= \begin{bmatrix} 0.54506616 \\ -0.00654893 \end{bmatrix}
 \end{aligned}$$

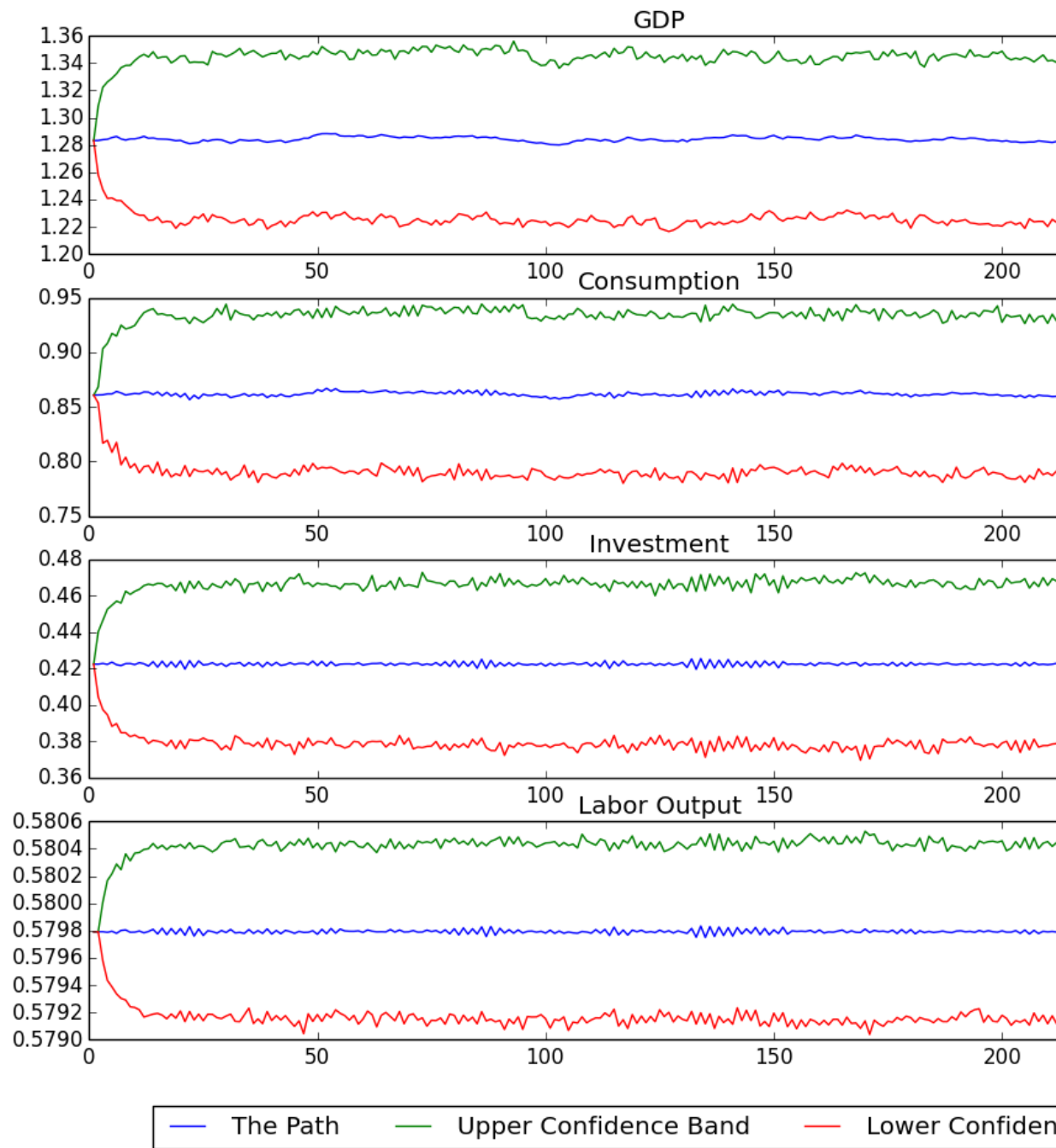
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Linearized policy function graph:



8.8



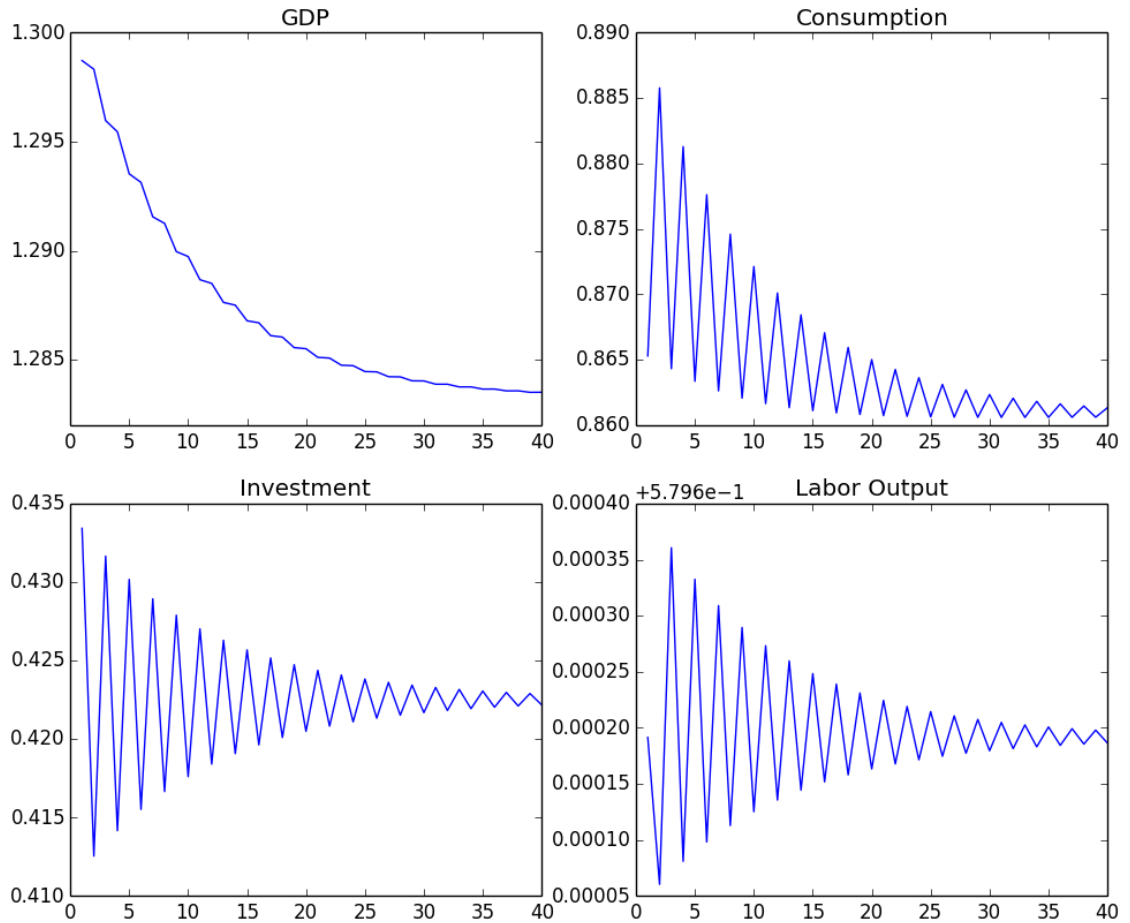


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The Average of Simulations						
Moments	mean	standard deviation	coefficient of variation	relative volatility	persistence	cyclicality
GDP	1.283715524	0.03510534	37.24852981	1	0.891533728	1
Consumption	0.861192975	0.042786171	24.98820726	1.227219641	0.221775606	0.78149153
Investment	0.422522549	0.02636165	12.26032256	0.764775265	-0.905572216	0.062553135
Labor Output	0.579791437	0.000380504	16.82381253	0.011037014	-0.88766256	0.015412588

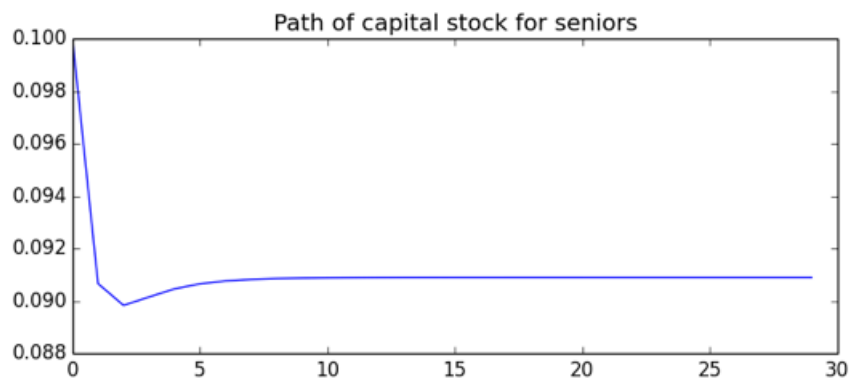
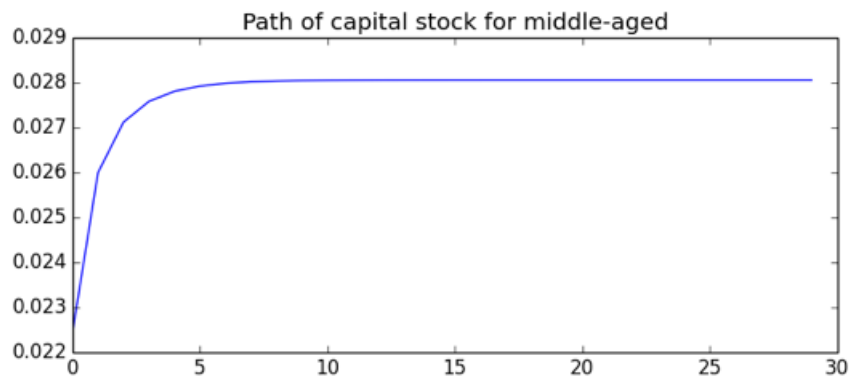
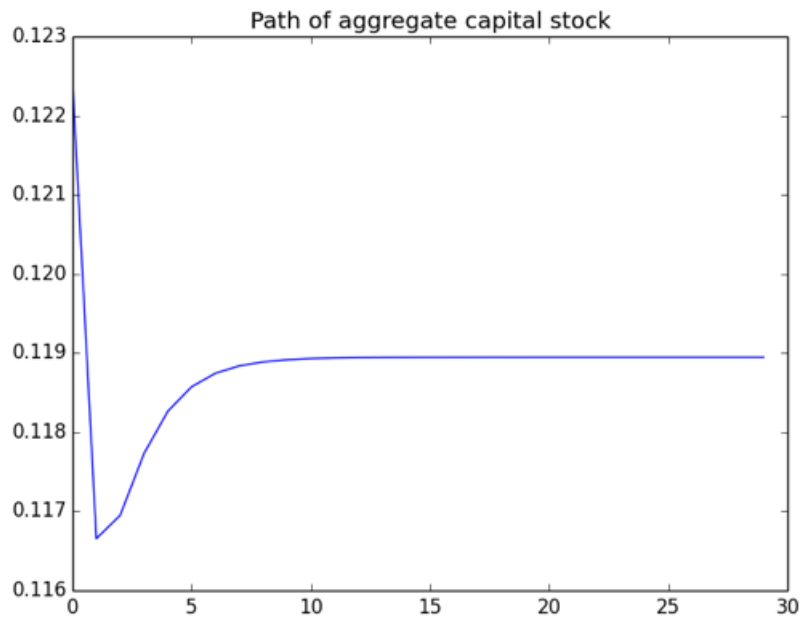
The Standard Errors of Simulations						
Moments	mean standard error	standard deviation	coefficient of variation	relative volatility	persistence	cyclicality
GDP	0.009415936	0.00478594	5.09039541	0	0.029556273	0
Consumption	0.009078493	0.004416592	3.417720146	0.096610274	0.183179171	0.061570432
Investment	0.000345482	0.003954702	1.675552461	0.155610955	0.028189465	0.007479617
Labor Output	1.10E-05	5.67E-05	2.299393347	0.002229548	0.03376364	0.028413377

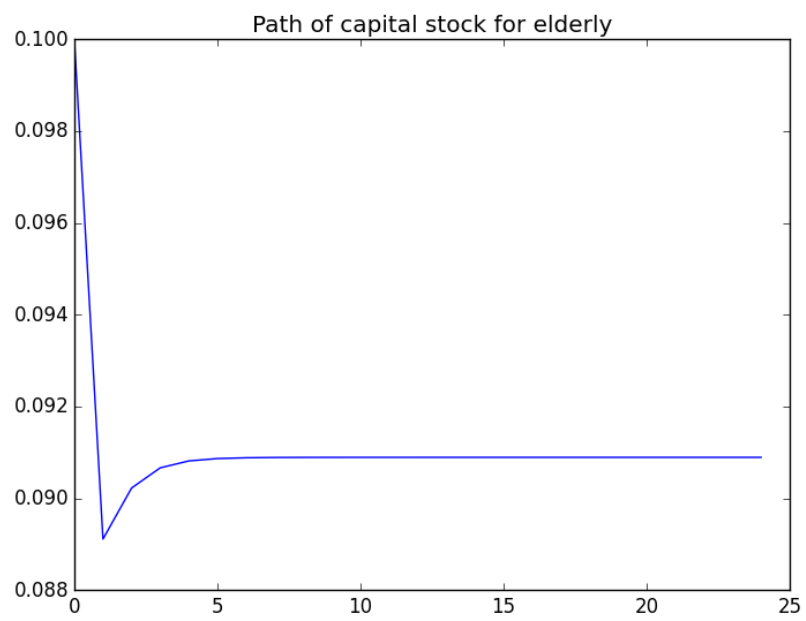
8.10



8.11

Lower computational time leads to a loss in accuracy.





8.12

