Math 320 Homework 3.4

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3.20

$$.5(.5)^3 = .5^4$$

and for the second part we have that

$$\binom{3}{1}(p(T=1)) + \binom{3}{2}(p(T=2)) + \binom{3}{3}(p(T=3)) = 3(\frac{1}{8}) + 3(\frac{1}{8}) + 1(\frac{1}{8}) = \frac{7}{8}$$

3.21

We should use the negative binomial distribution.

$$\binom{19}{17}.9^{17}.1^3 = 171 \cdot .16677 \cdot .001 = .0285$$

3.22

Given the law of the unconscious statistitian, we will have 100 for the first scenario, 200 for the second scenario.

3.23 (i)

We would use the Poisson distribution

3.23 (ii)

Where X occurs in a fixed interval of time (or space) and independently of the time since the last event.

3.23 (iii)

If the space is not fixed or X occurs in a dependent way.

3.23 (iv)

$$\sum_{\lambda=0}^{100} \frac{e^{-\lambda}\lambda^x}{x!} = \sum_{x=0}^{100} \frac{e^{-200/3}(200/3)^x}{x!} = .9999$$

We want the complement of this, though.

$$\implies 1 - .9999 = .0001$$

2.24

Note, $P(Y = 1) = \lambda e^{-\lambda}$ then we have that

$$\begin{aligned} P(X_n = 1) &= \lim_{n\to\infty} \binom{n}{1} \frac{\lambda}{n}(1 - \frac{\lambda}{n})^{n-1} \\ &= \lim_{n\to\infty} \frac{n!}{(n-1)!}(\frac{\lambda}{n}(1 - \frac{\lambda}{n})^{n-1} \\ &= \lambda \lim_{n\to\infty}(1 + \frac{-\lambda}{n})^{n-1} \\ &= \lambda e^{-\lambda} \end{aligned}$$

Therefore what we want to show holds for $k = 1$. Now, for our inductive hypothesis, suppose it holds for $k = j - 1$.
Note that

$$P(Y = k) = \frac{e^{-\lambda}\lambda^k}{k!} = \frac{\lambda}{k}\frac{e^{-\lambda}\lambda^{k-1}}{(k-1)!} = \frac{\lambda}{k}P(Y = k - 1)$$

and we also have the following:

$$\begin{aligned} P(X_n = k) &= \lim_{n\to\infty} \binom{n}{k} \frac{\lambda}{n}^k (1 - \frac{\lambda}{n})^{n-k} \\ &= \lim_{n\to\infty} \frac{\lambda}{n}\frac{n-(k-1)}{k}\frac{n!}{(n-(k-1))!(k-1)!}(1 - \frac{\lambda}{n})^{n-(k-1)}(1 - \frac{\lambda}{n})^{-1} \\ &= \lim_{n\to\infty} \frac{\lambda(n-(k-1)}{nk}(1 - \frac{\lambda}{n})^{-1}P(X_n = k-1) \\ &= \frac{\lambda}{k}\lim_{n\to\infty}\frac{(n-(k-1))n}{(n-\lambda)n}P(X_n = k-1) \\ &= \frac{\lambda}{k}P(Y = k-1) & = P(Y = k) \end{aligned}$$

The result, then, holds for all $k \in \mathbb{N}$, and we have the desired result.

2.25

$$
\begin{aligned}
\mathrm{V}(x) &= E[x^2] - E[x]^2 \\
&= \sum_{k=0}^{\infty} k^2 \cdot P(x = k) - \lambda^2 \\
&= \sum_{k=0}^{\infty} k^2 \cdot \frac{e^{-\lambda}\lambda^k}{k!} - \lambda^2 \\
&= \sum_{k=0}^{\infty} k \cdot \frac{e^{-\lambda}\lambda^k}{(k-1)!} - \lambda^2 \\
&= \lambda e^{-\lambda} \sum_{k=1}^{\infty} \frac{k\lambda^{k-1}}{(k-1)!} - \lambda^2 \\
&= \lambda e^{-\lambda} \sum_{k=1}^{\infty} \left(\frac{\lambda^{k-1}}{(k-2)!} + \frac{\lambda^{k-1}}{(k-2)!} \right) - \lambda^2 \\
&= \lambda e^{-\lambda} \left(\lambda \sum_{k=1}^{\infty} \frac{\lambda^{k-2}}{(k-2)!} + \sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{(k-1)!} \right) - \lambda^2 \\
&= \lambda e^{-\lambda} \left(\lambda e^{\lambda} + e^{\lambda} \right) - \lambda^2 \\
&= \lambda
\end{aligned}
$$

Which is the desired result.