Math 344 Homework 3.3

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Exercise 3.12

If this process is applied to a linearly dependent then this process will yield zero vectors because they are linear combinations of each other.

3.13

Gram-Schmidt yields the basis

$$B = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

By Theorem 3.2.3, we have that each a_i, or in other words, the coefficients that we multiply the vectors x_i(in this case, the vectors of B) by to yield x(in this case $(2e_1 + 3e_2) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$), is given by $\langle x, x_i \rangle = a_i$. This yields $a_1 = \frac{5}{\sqrt{2}}$ and $a_2 = \frac{-1}{\sqrt{2}}$, and we have that

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{5}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Exercise 3.14

The set of orthogonal vectors for this set are:

$$E = \{\frac{1}{\sqrt{\pi}}, \frac{x}{\sqrt{\frac{\pi}{2}}}, \frac{x^2 - \frac{1}{2}}{\sqrt{\frac{\pi}{8}}}, \frac{x^3 - \frac{3}{4}x}{\sqrt{\frac{7\pi}{8}}}\}$$

These are all orthogonal to each other.