Math 320 Homework 5.2

Chris Rytting

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5.6

Given $A = \{(-1,2),(0,4),(1,-6),(2,-16)\}$ we have

$$p(x) = \frac{\sum_{j=0}^{3} \frac{w_j}{x-x_j} y_j}{\sum_{j=0}^{3} \frac{w_j}{x-x_j}} = \frac{\frac{1}{-1}\frac{1}{-2}\frac{1}{-3}}{(x+1)}(2) + \frac{\frac{1}{1}\frac{1}{-1}\frac{1}{-2}}{(x)}(-4) + \frac{\frac{1}{2}\frac{1}{1}\frac{1}{-1}}{(x-1)}(-6) + \frac{\frac{1}{3}\frac{1}{2}\frac{1}{1}}{(x-2)}(-16)$$

$$= \frac{\frac{1}{3(x+1)} - \frac{2}{x} + \frac{3}{x-1} - \frac{16}{6(x-2)}}{\frac{1}{-6(x+1)} + \frac{1}{2x} - \frac{1}{2(x-1)} + \frac{1}{6(x-2)}} = -\frac{29x^3}{56} + \frac{53x^2}{8} - \frac{93x}{4} + \frac{190}{7}$$

5.7

Every time we evaluate

$$L_{n,j}(x) = \frac{w(x)}{(x-x_j)w_j}$$

we have to compute all $w_j \quad \forall n$ n times. Therefore the naive lagrange interpolation has a temporal complexity of $O(n^2)$.

As for the barycentric weights given by

$$w_j = \prod_{k=0, k\neq j}^{n} \frac{1}{(x_j - x_k)}$$

the computation thereof has a temporal complexity $O(n) \subset O(n^2)$. After finding w_j, we have

$$p(x) = \frac{\sum_{j=0}^{n} \frac{w_j}{x-x_j} y_j}{\sum_{j=0}^{n} \frac{w_j}{x-x_j}}$$

which we will do n times, so its temporal complexity is in $O(n)$ in the numerator. The same logic applies for the bottom, and then we divide the two sums so we have a temporal complexity of $O(2n+1) \subset O(n)$.

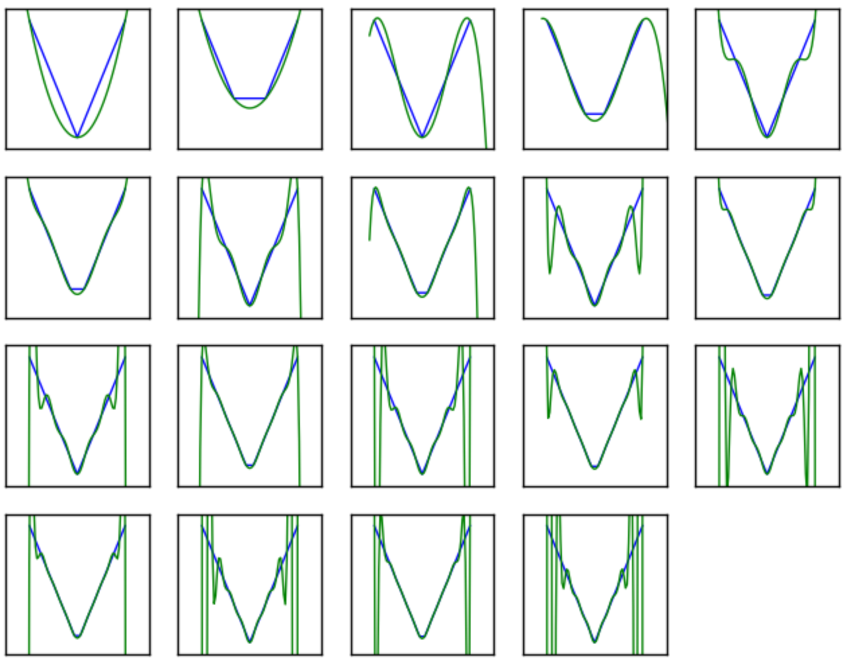
5.8

See code.

5.9

```
import numpy as np
from matplotlib import pyplot as plt

def calculate_weights(j, xvector):
    wj = []
    for k in xrange(len(xvector)):
        if k != j:
            wj.append(1./(xvector[j]-xvector[k]))
    return np.prod(wj)

def polynomial(x, xvector,y,w):
    num = []
    den = []
    for j in xrange(len(x)):
        num.append(w[j]*y[j]/(xvector-x[j]))
        den.append(w[j]/(xvector-x[j]))
    num = sum(num)
    den = sum(den)
    return(num/den)

def f(x):
    return np.abs(x)

def problem8():
    x = [-1., 0., 1., 2.]
    y = [2., -4., -6., -16.]
    w = []
    for j in xrange(len(x)):
        w.append(calculate_weights(j,x))
    xvector = np.linspace(-1.1,2.1,100)
    a = polynomial(x,xvector,y,w)
    plt.plot(xvector, a)
    plt.plot(x,y)
    plt.show()

def problem9():
    v = [-1.5,1.5,-.1,1.1]
    pts = range(2,21)
    xvector = np.linspace(-1.1,2.1,100)
    norm = []
    for n in pts:
        x = np.linspace(-1,1,n+1)
        y = f(x)
```

```
        ax = plt.subplot(4,5,n-1)
        ax.axis(v)
        ax.plot(x,y)
        w = []
        for j in xrange(len(x)):
            w.append(calculate_weights(j,x))
        a = polynomial(x,xvector,y,w)
        ax.plot(xvector,a)
        ax.axes.get_xaxis().set_visible(False)
        ax.axes.get_yaxis().set_visible(False)
        norm.append(np.linalg.norm(y,ord = np.inf)-np.linalg.norm(a,ord=np
    plt.show()
    print norm
```



Finding the L infinity norm in python, we have that $n = 3$ (4 points) is closest.