Math 320 Homework 4.6

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By exercise 4.10 and example 4.2.6, we have that

$$\mathscr{F}^{-1}(\text{sinc}(at)) = (\text{rect}_a(\xi)) = \frac{1}{2\pi}\hat{f}(-t) = \frac{1}{2\pi}\mathscr{F}(\text{sinc}(-at)$$

but since sinc is even, we have

$$\text{rect}_a(\xi) = \frac{1}{2\pi}\hat{f}(-t) = \frac{1}{2\pi}\mathscr{F}(\text{sinc}(-at) = \frac{1}{2\pi}\mathscr{F}(\text{sinc}(at))$$

$$\implies 2\pi\text{rect}_a(\xi) = \mathscr{F}(\text{sinc}(at)$$

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We need to shift f by $\frac{b+a}{2}$. To do so, we let

$$g(t) = e^{-it(\frac{b+a}{2}}f(t)$$

where g is a symmetric version of f.

Thus, we have that

$$g(t) = \sum_{-\infty}^{\infty} g(t_k)\text{sinc}(\frac{b-a}{2}t - k\pi)$$

Therefore,

$$f(t) = \sum_{-\infty}^{\infty} e^{it(\frac{b+a}{2}}e^{-it_k(\frac{b+a}{2}}f(t_k)\text{sinc}(\frac{b-a}{2}t - k\pi)$$

$$\implies f(t) = \sum_{-\infty}^{\infty} e^{i(t-t_k)\frac{b+a}{2}}f(t_k)\text{sinc}(\frac{b-a}{2}t - k\pi)$$

which is the desired result.

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Suppose to the contrary that this holds for $N = 1$. Then we have that

$$\begin{aligned}
\sin(t) &= \sum_{-\infty}^{\infty} \sin(t_k)\mathrm{sinc}(Nt - \pi k) \\
&= \sum_{-\infty}^{\infty} \sin(\frac{k\pi}{N})\mathrm{sinc}(Nt - \pi k) \\
&= \sum_{-\infty}^{\infty} \sin(k\pi)\mathrm{sinc}(Nt - \pi k)
\end{aligned}$$

However, since $k \in \mathbb{Z}$, we have that $\sin(k\pi) = 0 \quad \forall k$, yielding that

$$\sin(t) \sum_{-\infty}^{\infty} \sin(k\pi)\mathrm{sinc}(Nt - \pi k) = 0 \forall t$$

this is a contradiction since for

$$\sin(\frac{\pi}{2}) \neq 0$$