

11.1

Consider

$$\begin{aligned}
\hat{X} + \hat{Y} &= \{\hat{X}_1 + \hat{Y}_1, \dots, \hat{X}_T + \hat{Y}_T\} \\
S_{X+Y}(w) &= \lim_{T \rightarrow \infty} E\{|(X + Y)^T(w)|^2\} \\
&= \lim_{T \rightarrow \infty} E\{|(\hat{X}^T(w) + \hat{Y}^T(w))|^2\} \\
&= \lim_{T \rightarrow \infty} E\{|(\hat{X}^T(w))^2 + (\hat{Y}^T(w))^2 + 2\hat{X}^T(w)\hat{Y}^T(w)|\} \\
&\leq \lim_{T \rightarrow \infty} E\{|(\hat{X}^T(w))^2|\} + \lim_{T \rightarrow \infty} E\{|(\hat{Y}^T(w))^2|\} + \lim_{T \rightarrow \infty} E\{|2\hat{X}^T(w)\hat{Y}^T(w)|\} \\
S_{X+Y}(w) &= S_X(w) + S_Y(w) + 2Re(S_{XY}(w))
\end{aligned}$$

11.2

By attempting to minimize left side of the equation, we have the first order conditions

$$\begin{aligned}
g_1 : -2(y_1 - g_1) + 2\lambda(g_2 - 2g_1 + g_0)(-2) &= 0 \\
-2y_1 + 2g_1 - 4\lambda(g_2 - 2g_1 - g_0) &= 0
\end{aligned}$$

We also have the second order conditions

$$\begin{aligned}
g_2 : -2(y_2 - g_2) - 4\lambda(g_3 - 2g_2 + g_1) - 4\lambda(g_2 - 2g_1 + g_0) &= 0 \\
-2(y_2) + 2(g_2) - 4\lambda(g_3 - g_2 - g_1 + g_0) &= 0
\end{aligned}$$

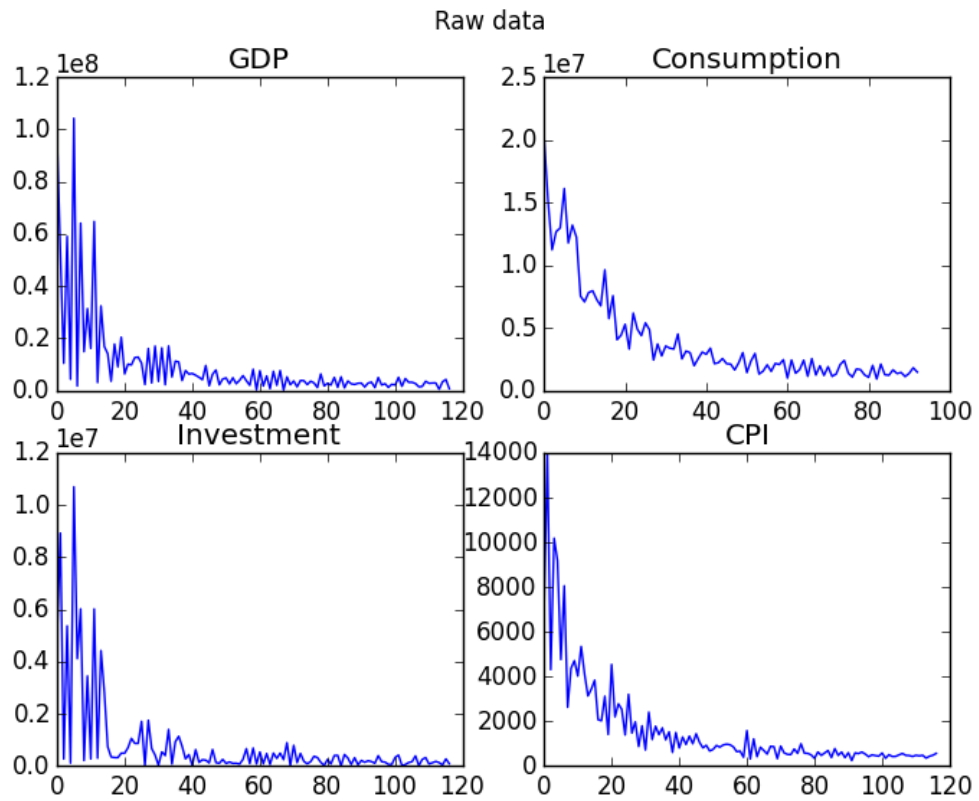
and thereby we have the n^{th} order condition:

$$g_n : -2(y_n - g_n)^2 + \lambda(2(g_{n+2} - 2g_{n+1} + g_n) - 4(g_{n+1} - 2g_n + g_{n-1}) + 2(g_n - 2g_{n-1} + g_{n-2})) = 0$$

which yields the linear

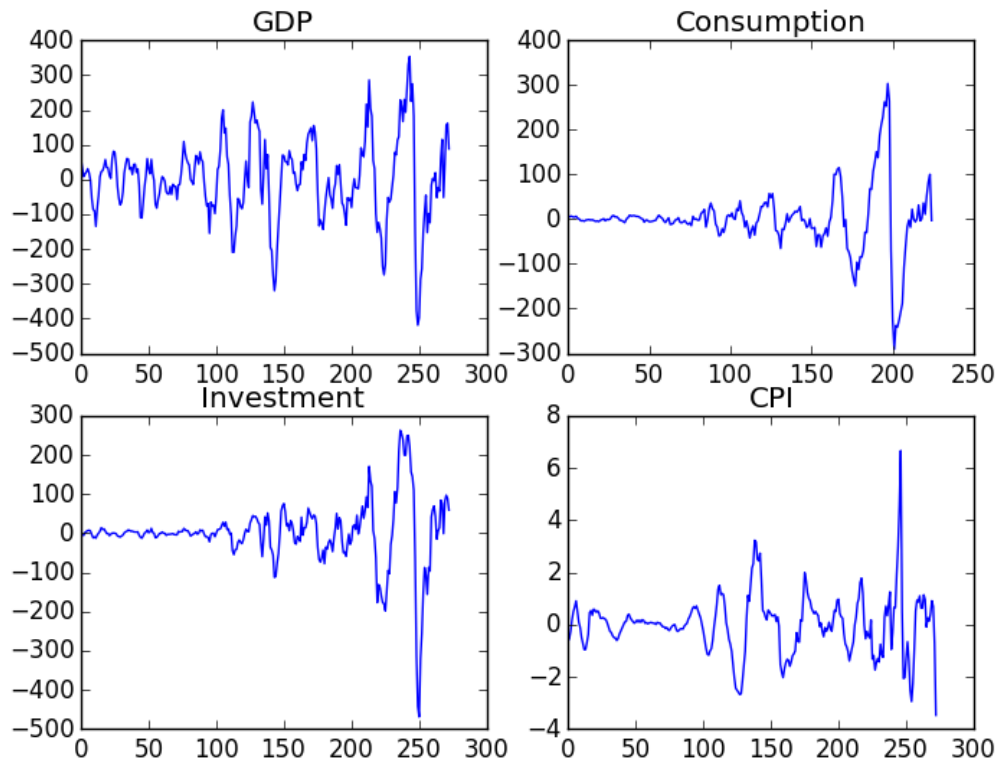
$$-2y_n + 2g_n + 2\lambda(g_{n+2} - 4g_{n+1} + 6g_n - 4g_{n-1} + g_{n-2}) = 0$$

11.3

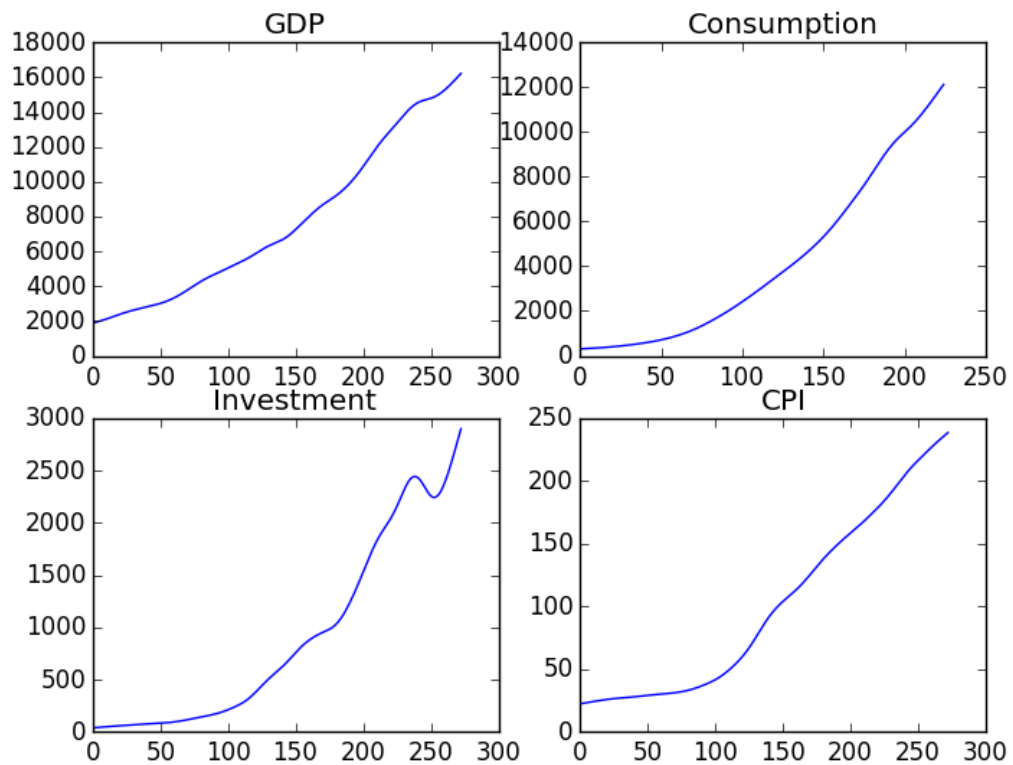


11.4

HP Filter Cyclical

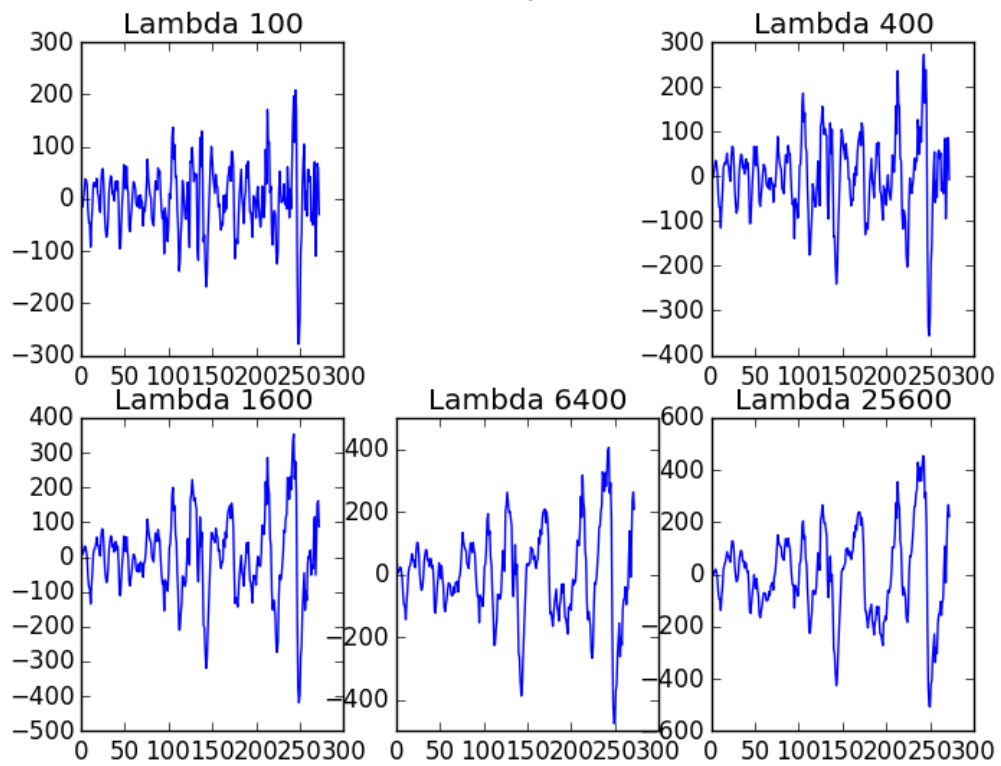


HP Filter Trend

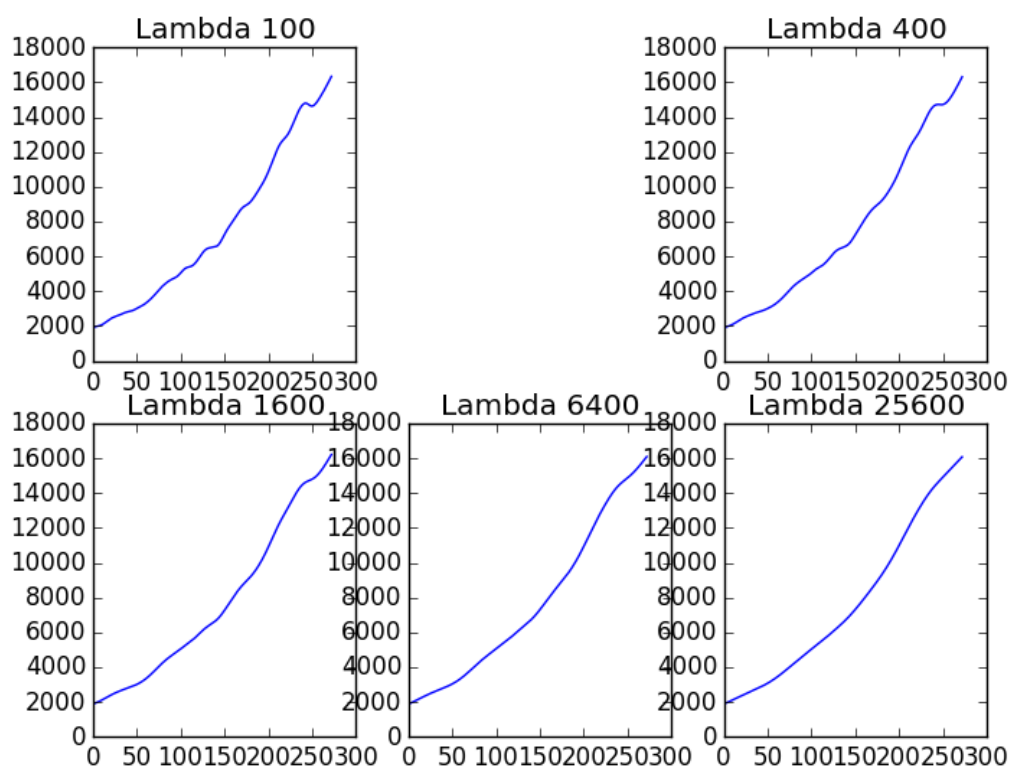


11.5

GDP Cycle



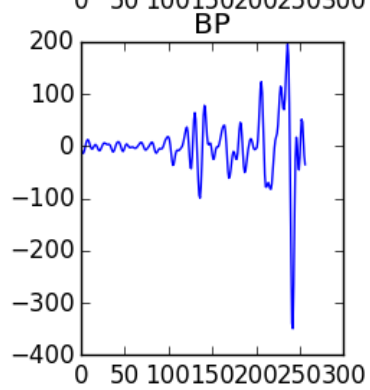
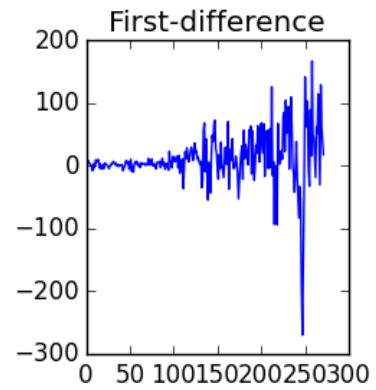
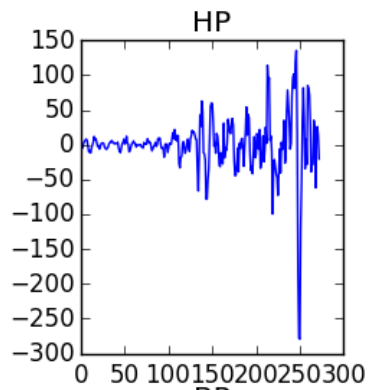
GDP Trend



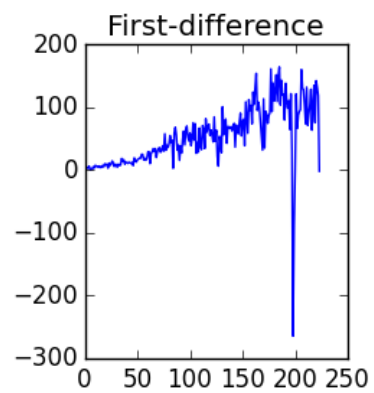
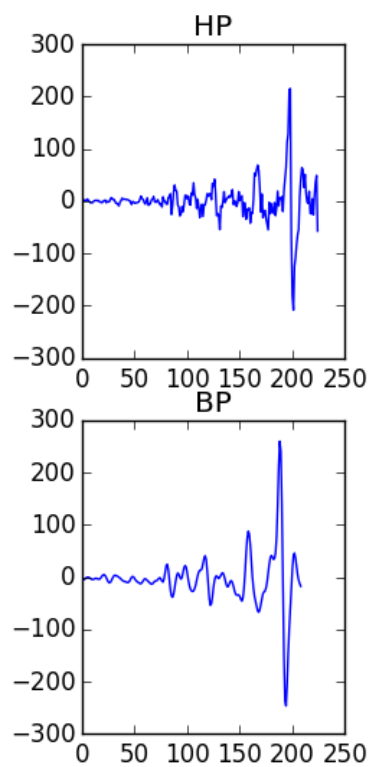
GDP	Cyclical					Trend			
Lambda	Mean	Standard Deviation	Correlation	Autocorrelation	Mean2	Standard Deviation3	Correlation4	Autocorrelation5	
100	-2.25E-11	64.9777218	0.02474946	0.681620681	7649.7075	4415.035884	0.99989176	0.999977552	
400	-2.64E-11	87.84670189	0.031436888	0.804520518	7649.7075	4414.27786	0.99980216	0.999984778	
1600	1.39E-11	116.5468319	0.037666307	0.877563091	7649.7075	4413.313245	0.999651743	0.999990012	
6400	-8.26E-10	145.5417096	0.043848027	0.915385281	7649.7075	4412.180927	0.999456849	0.999992692	
25600	1.39E-09	171.5745965	0.052489829	0.936542422	7649.7075	4410.489558	0.999245137	0.999994353	
Consumption	Cyclical					Trend			
Lambda	Mean	Standard Deviation	Correlation	Autocorrelation	Mean2	Standard Deviation3	Correlation4	Autocorrelation5	
100	-1.47E-11	38.34148846	0.02122168	0.717238161	4159.8418	3649.831	0.986200926	0.999984776	
400	-3.68E-11	53.96746161	0.027211618	0.835089597	4159.8418	3649.479288	0.986116527	0.999989001	
1600	1.11E-10	71.77907623	0.03276305	0.895408162	4159.8418	3649.046656	0.985991416	0.999991442	
6400	-5.32E-10	87.44154449	0.038266455	0.923853481	4159.8418	3648.399732	0.985893698	0.999992414	
25600	2.19E-09	101.4530205	0.046194678	0.94114046	4159.8418	3646.855913	0.985943475	0.999993483	
Investment	Cyclical					Trend			
Lambda	Mean	Standard Deviation	Correlation	Autocorrelation	Mean2	Standard Deviation3	Correlation4	Autocorrelation5	
100	-4.42E-12	41.47887197	0.021513135	0.699055834	902.01832	890.1851451	0.984277908	0.999743072	
400	-1.37E-11	59.33724149	0.027261117	0.832248311	902.01832	888.0531543	0.985824226	0.999841328	
1600	1.70E-11	83.33898231	0.031350602	0.903170111	902.01832	885.5702668	0.987464483	0.999922864	
6400	-4.34E-11	106.189414	0.034415855	0.933585745	902.01832	883.5374086	0.988557253	0.999965977	
25600	1.94E-10	122.501866	0.038351789	0.946860878	902.01832	881.9094212	0.989198807	0.999979969	
CPI	Cyclical					Trend			
Lambda	Mean	Standard Deviation	Correlation	Autocorrelation	Mean2	Standard Deviation3	Correlation4	Autocorrelation5	
100	-1.47E-11	38.34148846	0.02122168	0.717238161	4159.8418	3649.831	0.986200926	0.999984776	
400	-3.68E-11	53.96746161	0.027211618	0.835089597	4159.8418	3649.479288	0.986116527	0.999989001	
1600	1.11E-10	71.77907623	0.03276305	0.895408162	4159.8418	3649.046656	0.985991416	0.999991442	
6400	-5.32E-10	87.44154449	0.038266455	0.923853481	4159.8418	3648.399732	0.985893698	0.999992414	
25600	2.19E-09	101.4530205	0.046194678	0.94114046	4159.8418	3646.855913	0.985943475	0.999993483	

11.6

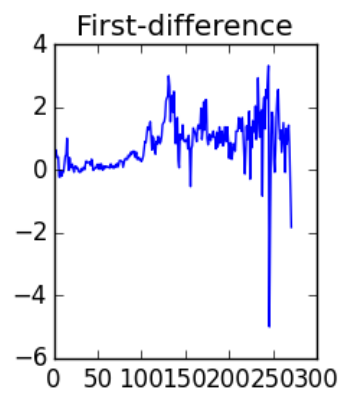
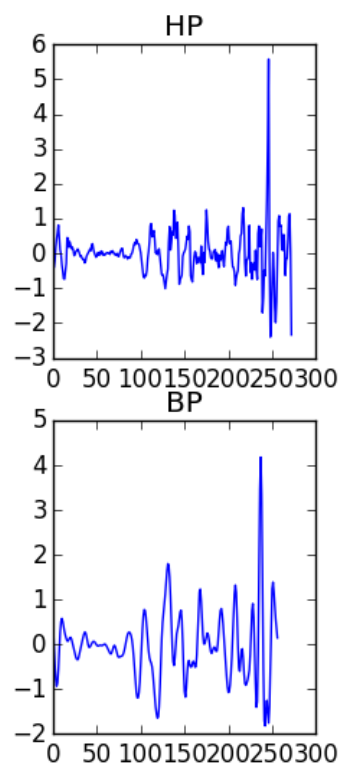
Investment

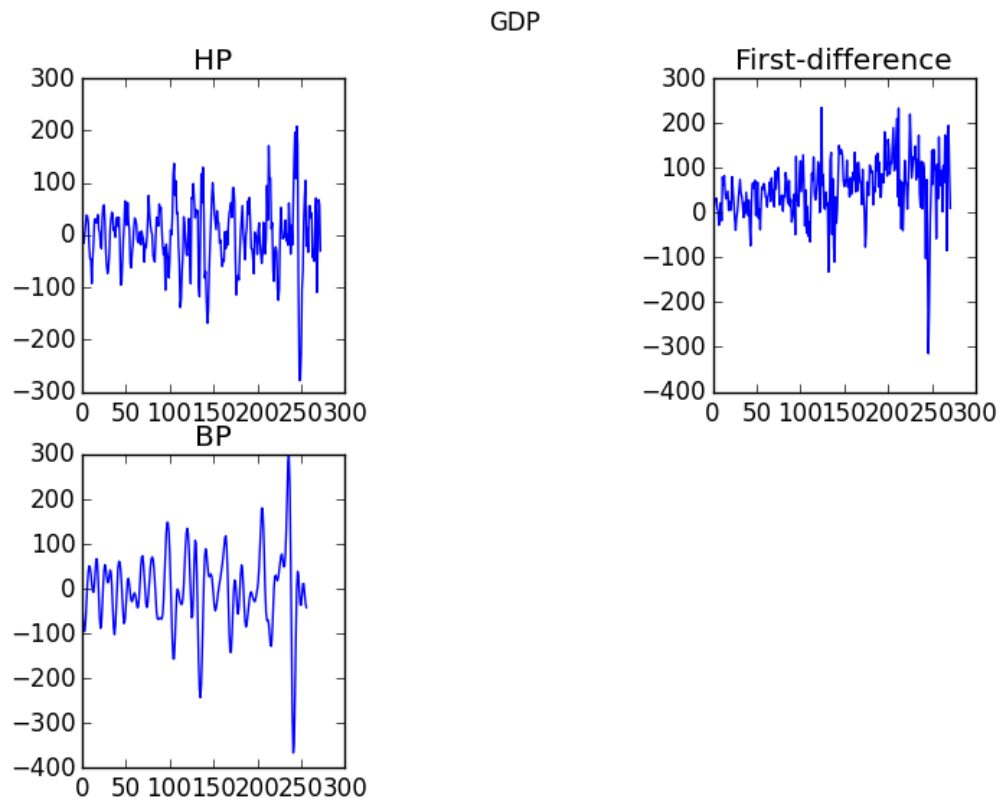


Consumption



CPI





Here are the different values requested for each series:

GDP:

First-difference

Mean = 0.784930147059

Std = 0.831049857977

Correlation = 0.384323520588

Autocorrelation = 0.54495746495

HP

Mean = 3.35184871311e-12

Std = 1.13268427423

Correlation = -0.00469118734317

Autocorrelation = 0.836207709801

BP

Mean = -0.0299699997469

Std = 0.780953878382

Correlation = 0.0518286280981

Autocorrelation = 0.892077087025

Consumption:

First-difference

Mean = 52.7147321429

Std = 48.1384249078

Correlation = 0.663184463727

Autocorrelation = 0.689913577226

HP

Mean = 1.75837464247e-11

Std = 83.3389823065

Correlation = 0.0313506020433

Autocorrelation = 0.903170110974

BP

Mean = -4.18832038676

Std = 50.9369790265

Correlation = -0.00499167776043

Autocorrelation = 0.906662966667

Investment:

First-difference

Mean = 10.7522058824

Std = 41.2467110398

Correlation = 0.181971196678

Autocorrelation = 0.455339024967

HP

Mean = 1.02860819122e-10

Std = 71.779076232

Correlation = 0.0327630499623

Autocorrelation = 0.895408162229

BP

Mean = -1.64783219425

Std = 55.6126386658

Correlation = -0.0275753430097

Autocorrelation = 0.895068261543

CPI:

First-difference

Mean = 0.784930147059

Std = 0.831049857977

Correlation = 0.384323520588
Autocorrelation = 0.54495746495

HP

Mean = 3.35184871311e-12
Std = 1.13268427423
Correlation = -0.00469118734317
Autocorrelation = 0.836207709801

BP

Mean = -0.0299699997469
Std = 0.780953878382
Correlation = 0.0518286280981
Autocorrelation = 0.892077087025

11.7

S&P Index:

OLS

Mean = 7.20602060925
Std = 0.115802246374
Autocorrelation = 1.0

HP

Mean = 3.24617825498e-13
Std = 0.115746507918
Autocorrelation = 0.806375713294

BP

Mean = -0.0181075102815
Std = 0.121154250719
Autocorrelation = 0.874542562966

First-difference

Mean = 0.0137429626107
Std = 0.0758607763258
Autocorrelation = 0.419822311381