8.1

Characteristic equations:

$$r_t = ae^{z_t}k_t^{\alpha-1}$$

$$c_t = e^{z_t}k_t^{\alpha} - k_{t+1}$$

$$c_t^{-1} = \beta E\{c_{t+1}^{-1}r_{t+1}\}$$

Steady states:

$$\bar{r} = \alpha\bar{k}^{\alpha-1}$$

$$1 = \beta\bar{r}$$

$$\bar{c} = \bar{k}^{\alpha} - \bar{k}$$

Linearization and solving of $F, G, H, L, \&M$:

$$F = \frac{\alpha\overline{K}^{\alpha-1}}{\overline{K}^{\alpha} - \overline{K}}$$

$$H = \frac{\alpha^2\overline{K}^{2(\alpha-1)}}{\overline{K}^{\alpha} - \overline{K}}$$

$$G = -\frac{\alpha\overline{K}^{\alpha-1}(\alpha + \overline{K}^{\alpha-1})}{\overline{K}^{\alpha} - \overline{K}}$$

$$L = -\frac{\alpha\overline{K}^{2\alpha-1}}{\overline{K}^{\alpha} - \overline{K}}$$

$$P = \frac{-G \pm \sqrt{G^2 - 4FH}}{2F}$$

$$M = \frac{\alpha^2\overline{K}^{2(\alpha-1)}}{\overline{K}^{\alpha} - \overline{K}}$$

$$Q = -\frac{LN + M}{FN + FP + G}$$

and we get:

$$F = 2.38051966287$$

$$H = 1.68371931464$$

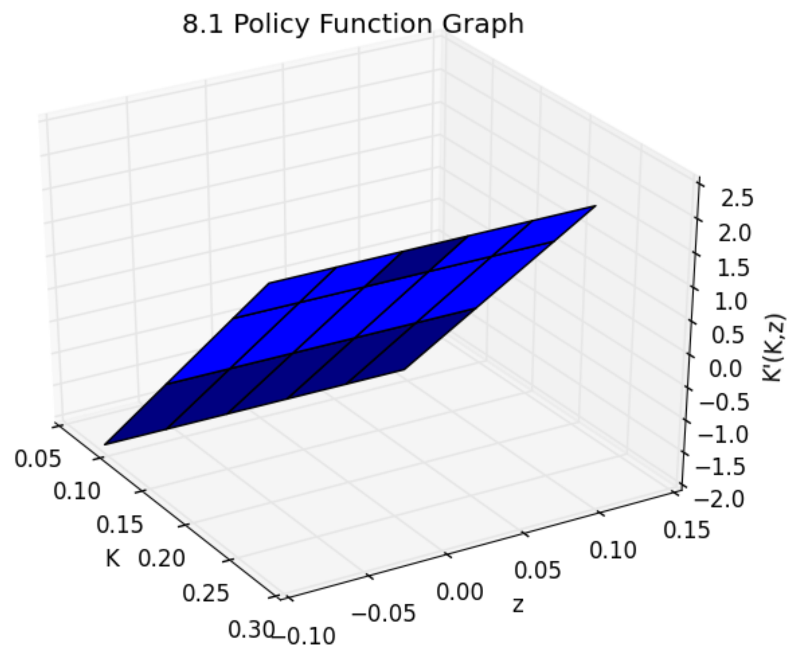
$$G = -5.16150615175$$

$$L = -1.6279703344$$

$$M = 1.68371931464$$

$$P = 0.83409605$$

$$Q = 0.14998134$$



Policy function:
Note that this is similar to the closed form solution.

8.2

Characteristic equations:

$$c_t = e^{z_t} k_t^{\alpha} - k_{t+1}$$
$$c_t^{-1} = \beta E\{c_{t+1}^{-1} r_{t+1}\}$$
$$r_t = a e^{z_t} k_t^{\alpha - 1}$$

Steady states:

$$\bar{c} = \bar{k}^{\alpha} - \bar{k}$$
$$1 = \beta \bar{r}$$
$$\bar{r} = \alpha \bar{k}^{\alpha - 1}$$

After linearizing,

$$\tilde{r}_t = z_t + (\alpha - 1)\tilde{k}_t$$

$$\tilde{c}_t = -\frac{\bar{k}}{\bar{c}}\tilde{k}_{t+1} + \frac{\bar{k}^{\alpha}}{\bar{c}}\alpha \tilde{k}_t + \frac{\bar{k}^{\alpha}}{\bar{c}} z_t$$
$$\tilde{c}_t = E\{\tilde{r}_{t+1}\} - E\{\tilde{c_{t+1}}\}$$

Then we have that

$$F = \frac{\bar{k}^2}{\bar{c}}$$

$$G = \frac{\alpha \bar{k}^{\alpha+1} - \bar{k}^2}{\bar{c}} + (\alpha - 1)\bar{k}$$

$$H = \frac{\alpha \bar{k}^{1+\alpha}}{\bar{c}}$$

$$L = \frac{\bar{k} - \bar{k}^{\alpha+1}}{\bar{c}}$$

$$M = \frac{\bar{k}^{1+\alpha}}{\bar{c}}$$

and that

$$P = \frac{-G \pm \sqrt{G^2 - 4FH}}{2F}$$

$$Q = -\frac{LN + M}{FN + FP + G}$$

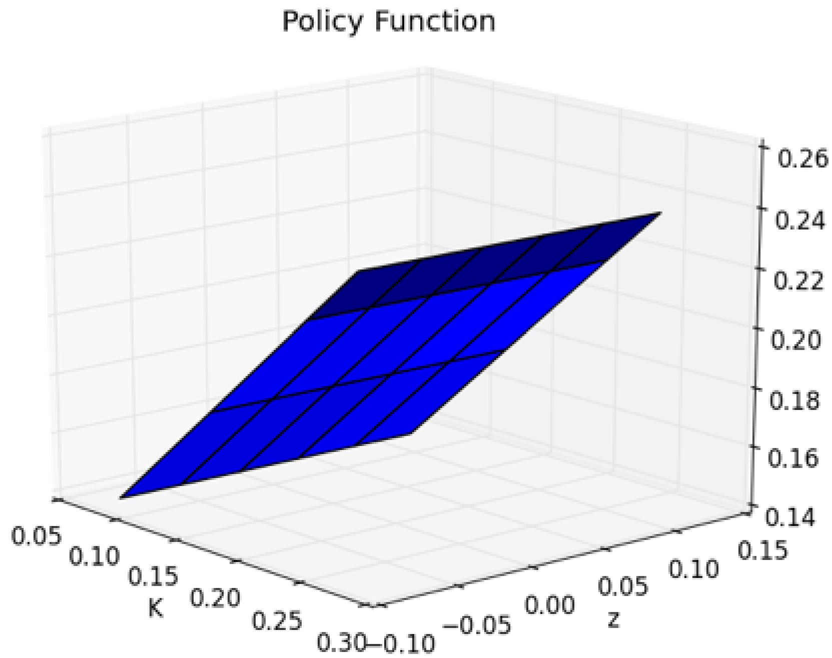
Policy function:

$$\tilde{X}_t = (I_P)\bar{X} + P\tilde{X}_{t-1} + Q\tilde{z}_t$$

Numerical answers:

$\bar{k} = .19278, \bar{r} = 1.0204, \bar{c} = .36926, F = .100646, G = -4.8408, L = .22864, H = .102700, M = .293428,$

Policy function graphed:



8.3

$$E[F\tilde{X}_{t+1} + G\tilde{X}_t + H\tilde{X}_{t-1} + L\tilde{Z}_{t+1} + M\tilde{Z}_t] = 0$$
$$E[F(P\tilde{X}_t) + G\tilde{Z}_{t+1} + L\tilde{Z}_{t+1} + M\tilde{Z}_t] = 0$$
$$E[(FP + G)\tilde{X}_t + H\tilde{X}_{t-1} + (FQ + L)\tilde{Z}_{t+1} + M\tilde{t}] = 0$$
$$E[(FP + G)(P\tilde{X}_{t-1} + Q\tilde{Z}_t) + H\tilde{X}_{t-1} + (FG + L)(N\tilde{Z}_t + \varepsilon_{t+1}) + M\tilde{Z}_t] = 0$$
$$= (FP^2 + GP + H)\tilde{X}_{t-1} + (FPQ + GQ + FQN + LN + M)Z_t = 0$$
$$[(FP + G)P + H]\tilde{X}_{t-1} + [(FQ + L)N + (FP + G)Q + M]\tilde{Z}_t = 0$$

8.4

Steady state values:

$k = 7.2875, c = 1.4845, r = 0.1215, w = 1.3280, l = 0.5797, T = 0.0742, y = 2.2133$ and $i = 0.7287$

8.5

	$\bar{k}$	$\bar{c}$	$\bar{r}$	$\bar{w}$	$\bar{l}$	$\bar{T}$	$\bar{y}$	$\bar{i}$
δ	-48.345	-3.511	1	-7.287	1.32	-0.176	-4.121	-0.61
τ	-2.323	-0.234	0.023	-0.165	-0.139	0.849	-0.467	-0.232
$\bar{z}$	0	0	0	0	0	0	0.77	0
α	25.986	2.085	0	4.396	-0.769	0.104	4.684	2.599
γ	0.139	0.028	0	0	0.019	0.001	0.042	0.014
ξ	-0.802	-0.163	0	0	-0.11	-0.008	-0.243	-0.08
β	65.438	1.751	-1.096	7.988	0.26	0.088	8.295	6.544
a	-1.849	-0.377	0	0	-0.254	-0.019	-0.562	-0.185

8.6

We have that

$$F = \begin{bmatrix} 2.9046023 & -3.74013067 \\ 0. & 0. \end{bmatrix}$$

$$G = \begin{bmatrix} -5.88765983 & 3.8571742 \\ 2.90460231 & -8.11672985 \end{bmatrix}$$

$$H = \begin{bmatrix} 2.96699945 & 0. \\ -2.87232918 & 0. \end{bmatrix}$$

$$L = \begin{bmatrix} -2.1684962 \\ 0. \end{bmatrix}$$

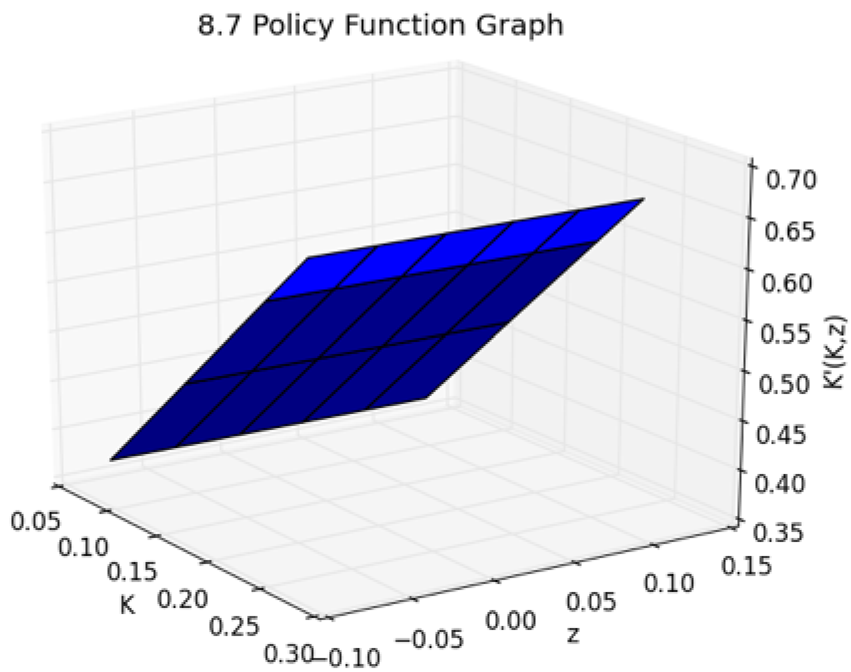
$$M = \begin{bmatrix} 2.23635669 \\ -1.6363563 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.9153012 & 0. \\ -0.02633366 & 0. \end{bmatrix}$$

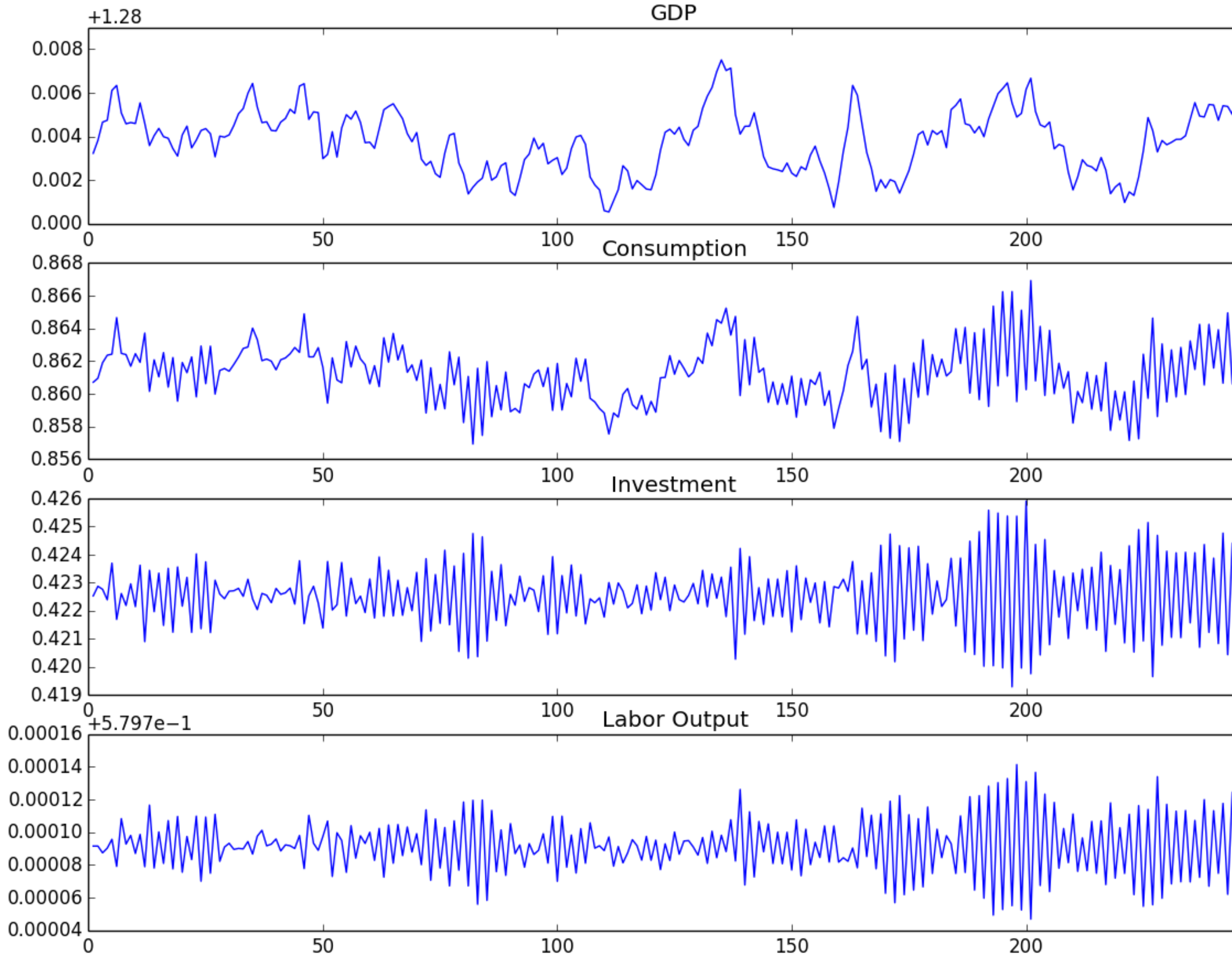
$$Q = \begin{bmatrix} 0.54506616 \\ -0.00654893 \end{bmatrix}$$

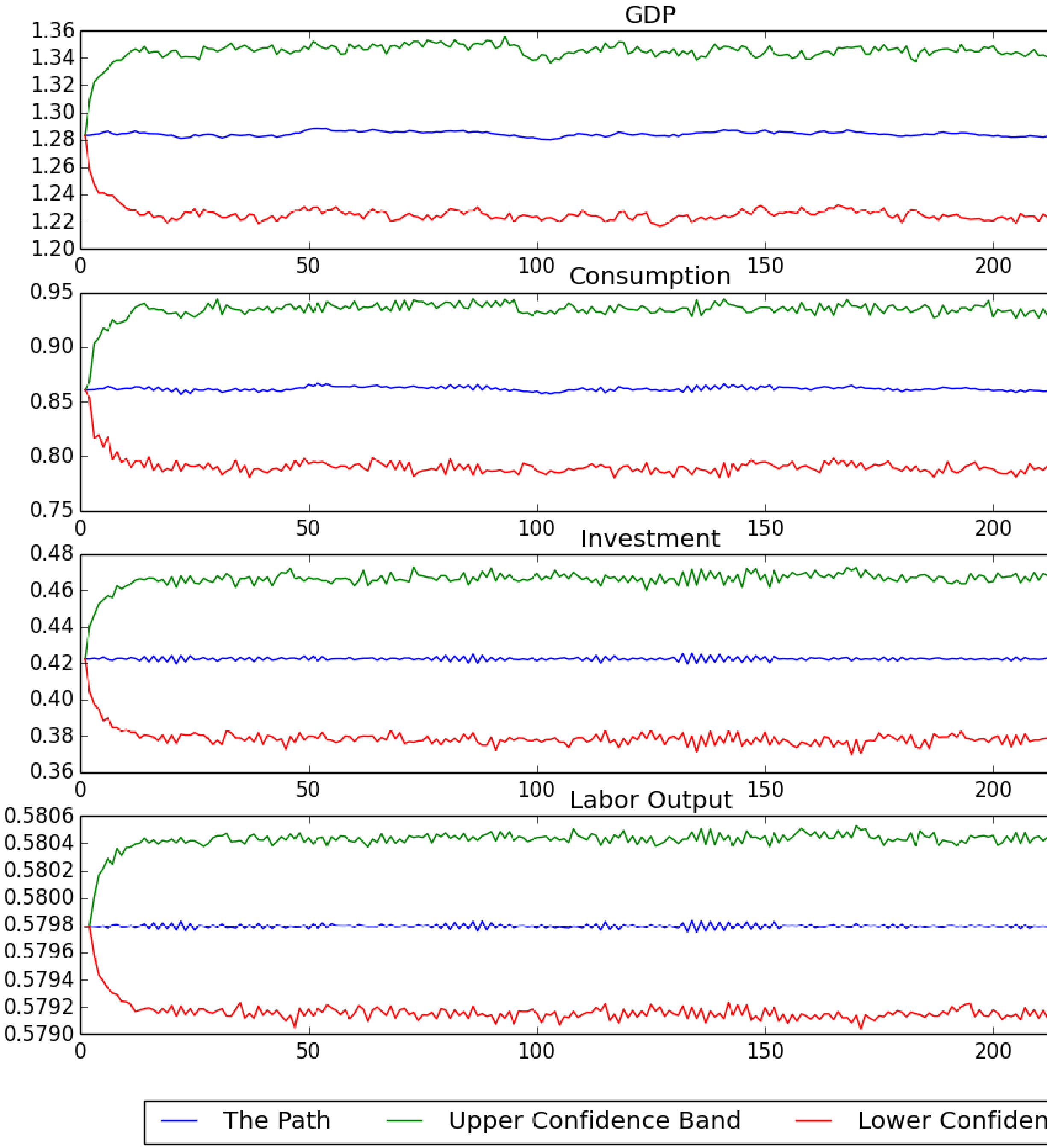
8.7

Linearized policy function graph:



8.8





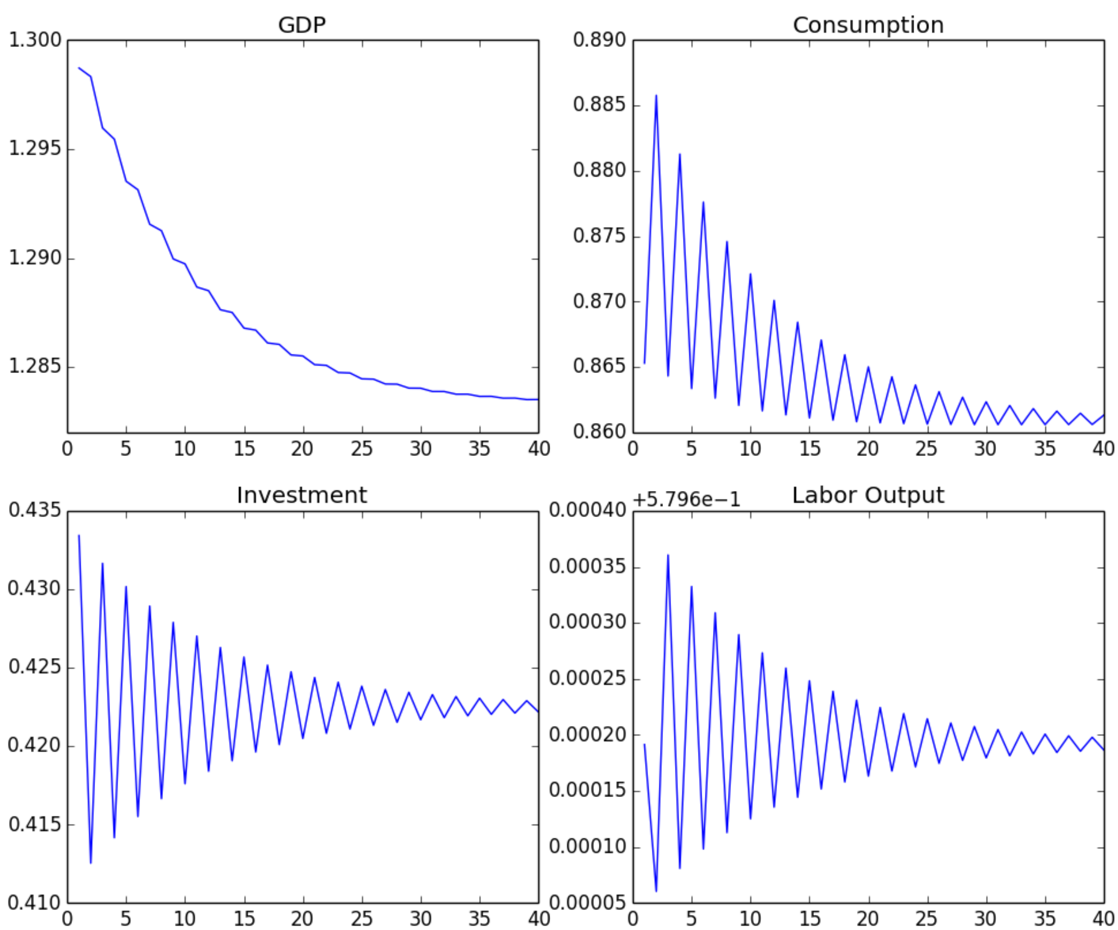
GDP
1.36
1.34
1.32
1.30
1.28
1.26
1.24
1.22
1.20
0
50
100
150
200
Consumption
0.95
0.90
0.85
0.80
0.75
Investment
0.48
0.46
0.44
0.42
0.40
0.38
0.36
Labor Output
0.5806
0.5804
0.5802
0.5800
0.5798
0.5796
0.5794
0.5792
0.5790
The Path
Upper Confidence Band
Lower Confiden

8.9

The Average of Simulations						
Moments	mean	standard deviation	coefficient of variation	relative volatility	persistence	cyclicality
GDP	1.283715524	0.03510534	37.24852981	1	0.891533728	1
Consumption	0.861192975	0.042786171	24.98820726	1.227219641	0.221775606	0.78149153
Investment	0.422522549	0.02636165	12.26032256	0.764775265	-0.905572216	0.062553135
Labor Output	0.579791437	0.000380504	16.82381253	0.011037014	-0.88766256	0.015412588

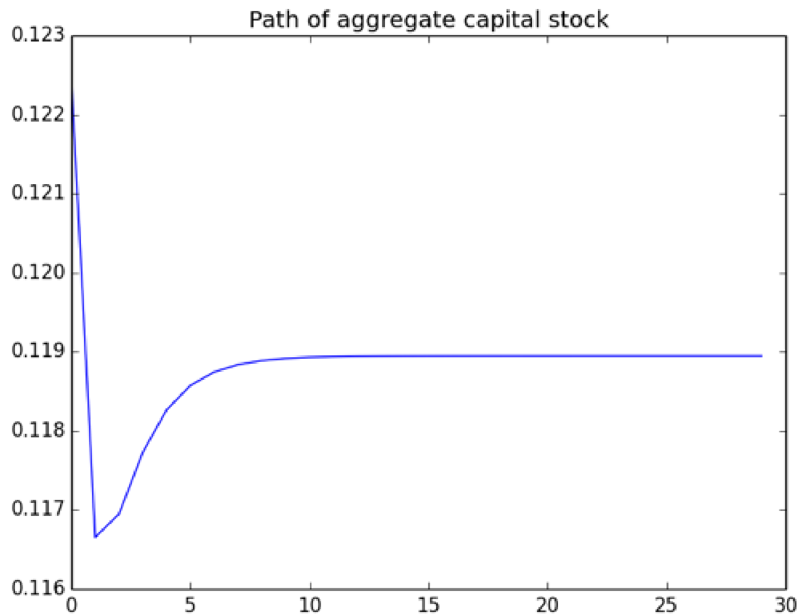
The Standard Errors of Simulations						
Moments	mean standard error	standard deviation	coefficient of variation	relative volatility	persistence	cyclicality
GDP	0.009415936	0.00478594	5.09039541	0	0.029556273	0
Consumption	0.009078493	0.004416592	3.417720146	0.096610274	0.183179171	0.061570432
Investment	0.000345482	0.003954702	1.675552461	0.155610955	0.028189465	0.007479617
Labor Output	1.10E-05	5.67E-05	2.299393347	0.002229548	0.03376364	0.028413377

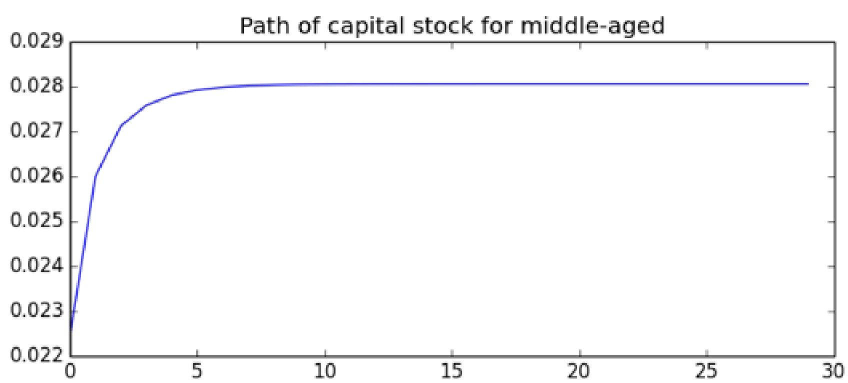
8.10

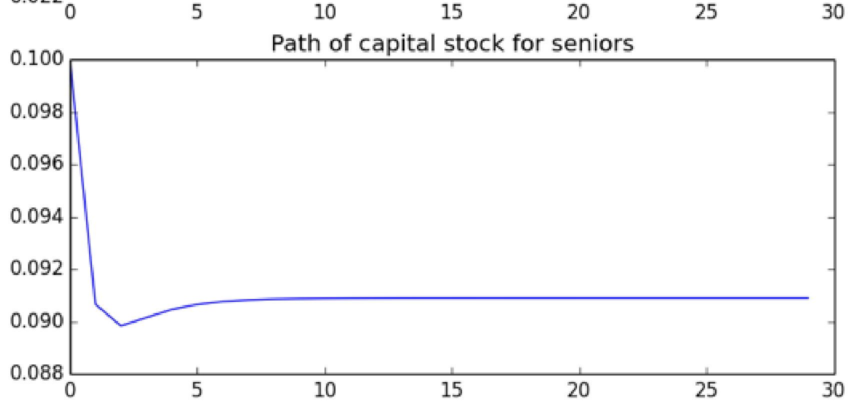


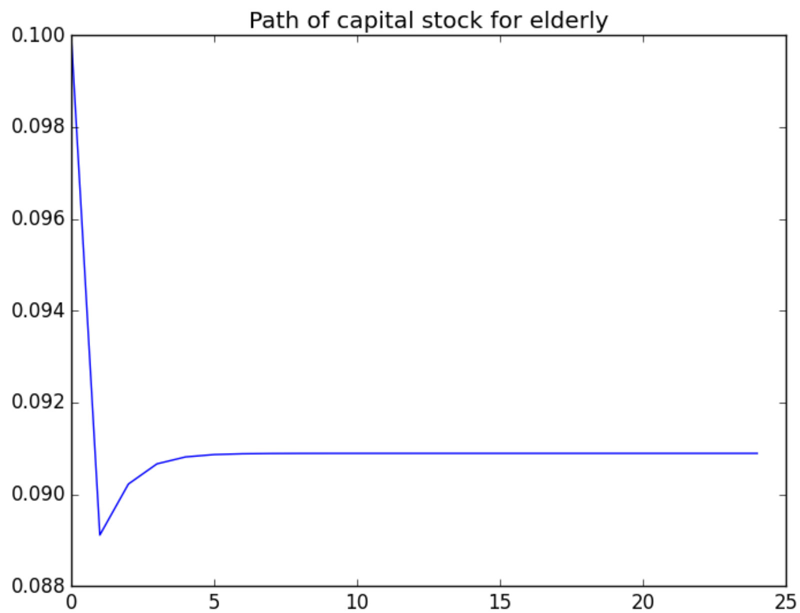
8.11

Lower compuational time leads to a loss in accuracy.

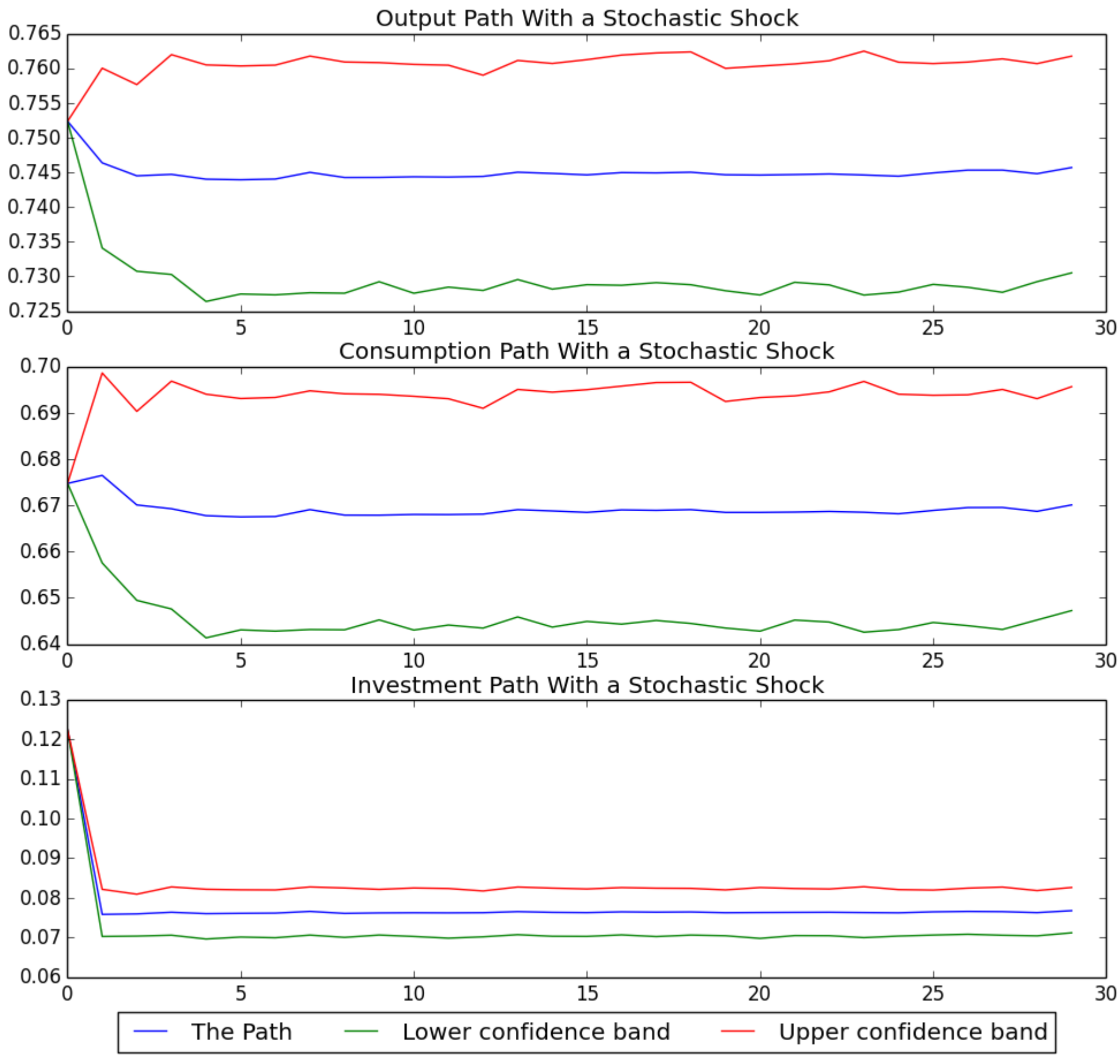


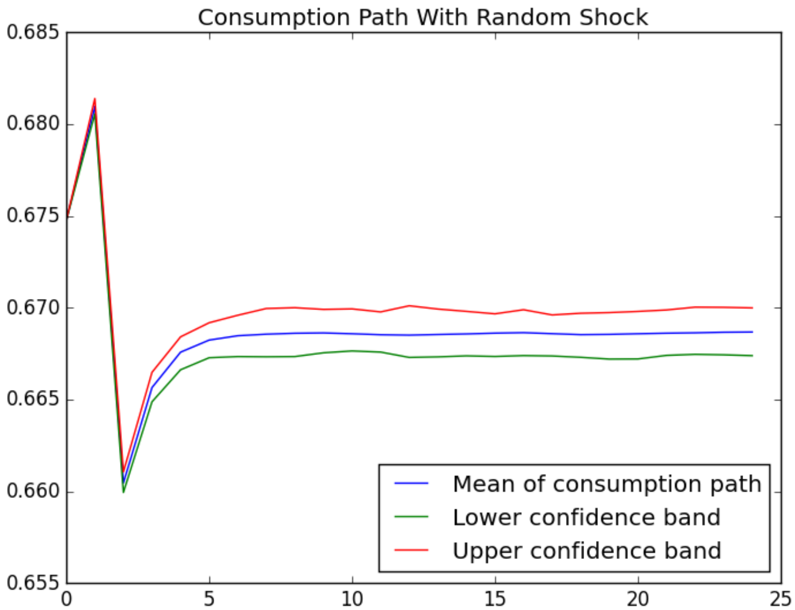


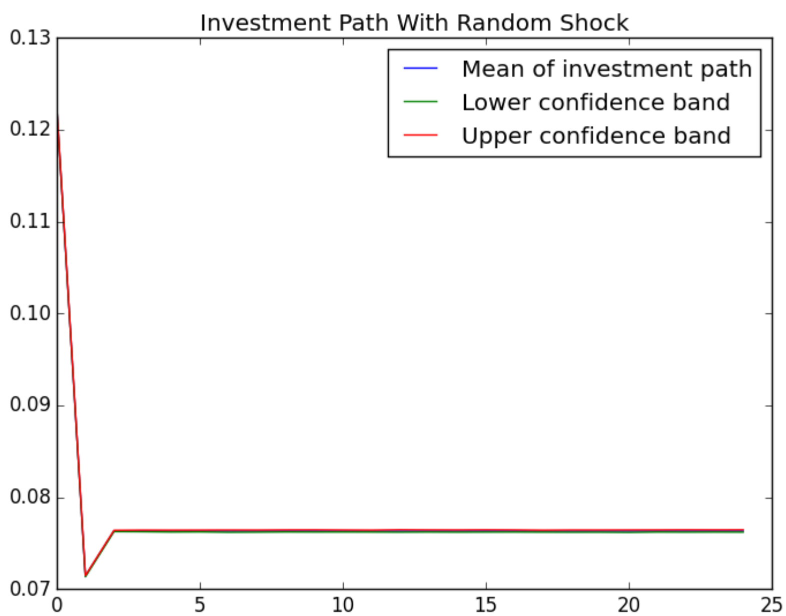



Path of capital stock for elderly
0.100
0.098
0.096
0.094
0.092
0.090
0.088
0
5
10
15
20
25

8.12


Output Path With a Stochastic Shock
0.765
0.760
0.755
0.750
0.745
0.740
0.735
0.730
0.725
0
5
10
15
20
25
30
Consumption Path With a Stochastic Shock
0.70
0.69
0.68
0.67
0.66
0.65
0.64
Investment Path With a Stochastic Shock
0.13
0.12
0.11
0.10
0.09
0.08
0.07
0.06
The Path
Lower confidence band
Upper confidence band


Consumption Path With Random Shock
0.685
0.680
0.675
0.670
0.665
0.660
0.655
0
5
10
15
20
25
Mean of consumption path
Lower confidence band
Upper confidence band


Investment Path With Random Shock
0.13
0.12
0.11
0.10
0.09
0.08
0.07
0
5
10
15
20
25
Mean of investment path
Lower confidence band
Upper confidence band