Math 344 Homework

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5.42

By (i) of 5.7.13,

$$\left\| \int_a^b f(t)dt \right\| \leq (b-a) \sup_{t\in[a,b]} \|f(t)\|$$

Now, $\|f(t)\|$ is a real valued function, so

$$\sup_{t\in[a,b]} \|f(t)\| \leq \|f(t)\|$$

$$\begin{aligned} \implies \int_a^b \|f(t)\|dt &= (b-a)\|f(t)\| \\ \left\| \int_a^b f(t)dt \right\| &\leq (b-a) \sup_{t\in[a,b]} \|f(t)\| \leq (b-a)\|f(t)\| \end{aligned}$$

5.43

$$\int_\alpha^\beta f(t)dt = \int_\alpha^\gamma f(t)f(t)\mathbb{1}_{[\alpha,\gamma]}$$

by (i) of 5.7.13,

$$\int_\alpha^\beta f(t)dt = \int_\alpha^\gamma f(t)dt + \int_\gamma^\beta f(t)dt$$

5.44

Let

$$\delta(x,y) < \delta, \quad \epsilon = \frac{\delta}{M}$$

Now we have that

$$F(x) = \int_a^b f(x)dx$$

is a bounded and linear transformation since it is continuous.

$$\implies \int_a^b f(t)dt < M$$

since $M > 0$. This yields

$$\Big|\int_a^b f(x)dt - \int_a^b f(y)dt\Big| = |F(x) - F(y)| < M\delta < \epsilon$$

5.45

Let

$$d(x,y) < \delta = \frac{\epsilon}{2}$$

Then

$$|x^{1/3} - y^{1/3}| \leq 1$$

$$\implies |x^{1/3} - y^{1/3}| < \frac{|x-y|\epsilon}{2} < \epsilon$$

We know that $x, y \in [-1, 1]$. However,

$$f(x) = \frac{x^{-2/3}}{3}$$

So we have

$$\lim_{x \to 0^-} f(x) = -\infty$$

and

$$\lim_{x \to 0^+} f(x) = \infty$$

derivative is not bounded though, so it is not in the space of continuous functions.

5.46

There exists no finite partition such that

$$f(x) = \begin{cases} 0 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

This function is not in the space of step functions.

$$f \notin S([a,b]; X)$$

and the regulated integral is undefined.