DEFVecSp $: 1.x+y = y+x2.(x+y)+z = x+(y+z)3.Add.Id.0 \in V|0+x = x4.\exists Add.Inv.(-x)|x+(-x) = 0(5.)F.Dis.Lawa(x+y) = ax+ay(6.)S.Dis.Law(a+b)(x) = ax+bx(7.)Mul.Id.1x = x(8.)(ab)x = a(bx)$ **THM1.1.13** If W is a subset of a vector space V s.t. $\mathbf{x}, \mathbf{y} \in$ @ and for any $a, b \in \mathbb{F}$ the vector $a\mathbf{x} + b\mathbf{y} \in W$, then W is a subspace of V. **DEFLinHull** of $S\langle S\rangle$, smallest subspace of V that contains S,i.e. intersection of all subspaces of V that contain S. **THM1.2.6Span**(S) $= \langle S\rangle$. **DEF** $\bigoplus$ Where W_1, W_2 are subspaces of V, then $W_1 + W_2 = W_1 \bigoplus W_2$ if $W_1 \cap W_2 = 0$. **DEFComplementarysubspaces**W_1 and W_2 if $V = W_1 \bigoplus W_2$ **THMReplacement**: V is a vector space spanned by $S = s_1, \cdots, s_m$. If $T = t_1, \cdots, t_n$ is a L.I. subset of V, then $\leq m$ and $\exists S' \subset S$ having $m-n$ elements such that $T \cup S'$ spans V. **THMExtension**: W is a subspace of VIf $T = t_1, \cdots, t_n$ and $S = s_1, \cdots, s_m$ span W and V, respectivley, then $\exists S' \subset S$ having $m-n$ elements such that $T \cup S'$ is a basis for V. **DEFQuotientSpaces**: W subspace of V. The set $x + W|x \in V(orequivalently[[x]]|x \in V)$ of all cosets of W in V is denoted V/W and is called the quotient of V modulo W. **DEF** $\boxplus$ $\boxdot$:Let W be a subspace of V. Define operations $\boxplus : V/W \times V/W \to V/W$ and $\boxdot : \mathbb{F} \times V/W \to V/W$ given by (i) $(x+W) \boxplus (y+W) = (x+y)+W$ and $a \boxdot (x+W) = (ax)+W$. These are the operations of vector addition and scalar multiplication on V/W. **CHAP2 DEFLineartransformation** Let V and V be vector spaces over $\mathbb{F}$. A map $L : V \to W$ is a linear transformation from V into W if $L(ax_1 + bx_2) = aL(x_1) + bL(x_2)$ for $x_1, x_2 \in V$ and $a, b \in \mathbb{F}$ **COR2.1.17** A linear transforamtion is invertible if and only if it is bijective. **Prop.2.1.24** : If $V \cong W$ are isomorphic vector spaces, with isoorphism $L : V \to W$, then: (i) A linear equation holds in V iff it also holds in W: that is $\sum_{i=1} a_i \mathbf{x_i} = \mathbf{0}$ holds in V iff $\sum_{i=1} a_i L_i \mathbf{x_i} = \mathbf{0}$ holds in W. (ii) A set B $= \{\mathbf{v_i}, \ldots, \mathbf{v_n}\}$ is a basis of V iff LB $= \{L\mathbf{v_i}, \ldots, L\mathbf{v_n}\}$ is a basis for W. Moreover, the dimension of V is equal to the dimension of V. (iii) The subspaces of V are in vijective correspondence with the subspaces of W. (iv) If K: W $\to$ U is any linear transformation, then the composition KL:V$\to$ U is also a linear transformation and we have $\mathscr{N}(KL) = L^{-1}\mathscr{N}(K) = \{v|L(\mathbf{v}) \in \mathscr{N}(K)\}$ and $\mathscr{R}(KL) = \mathscr{R}(K)$ **THMF.Iso**. If V and X are vector spaces and $L : V \to X$ is a linear transformation, then $V/\mathscr{N}(L) \cong \mathscr{R}(L)$. in particular, if L is surjective, then $V/N(L) \cong X$. **THM2.2.7** If V is a finite-dimensional vector space and W is a subspace of V, then $\dim(V) = \dim(W) + \dim(V/W)$ **THMRank − Nullity** Let V and W be finite-dimensional vector spaces. If $L : V \to$ is a linear transformation then $\dim(V) = \dim\mathscr{R}(L) + \dim\mathscr{N}(L) = \text{rank}(L) + \text{nullity}(L)$. **CORSec.Iso.Thm**. Assume V_1 and V_2 are subspaces of V. Then $V_1/(V_1 \cap V_2) \cong (V_1 + V_2)/V_2$. **CORDim.Formula** If V_1 and V_2 are finite-dimensional subspaces of a vector space V, then $\dim(V_1) + \dim(V_2) = \dim(V_1 \cap V_2) + \dim(V_1 + V_2)$ **DEFSimilarMatrices** Two square matrices $A, B \in M_n(\mathbb{F})$ are said to be similar if there exists a nonsingular $P \in M_n(\mathbb{F})$ such that $B = P^-1AP$. **DEFBernsteinPolynomials** Given $n \in \mathbb{N}_{\geq 0}J$, the Bernstein polynomials $B_j^n(x)_{j=0}^n$ of degree n are defined as $B_j^n(x) = \binom{n}{j}x^j(1-x)^{n-j}$, where $\binom{n}{j} = \frac{n!}{j!(n-j)!}$ **LEM2.5.3**For $j = 0, 1, \ldots, n \quad B_j^n(x) = \sum_{i=j}^n (-1)^{i-j}\binom{n}{i}\binom{i}{j}x^i$ **THM2.5.4** For any $n \in \mathbb{N}$, the set T_n of degree n Bernstein polynomials $T_n = B_j^n(x)_{j=0}^n$ forms a basis for $\mathbb{F}[x]^n$ **DEFTrace** The trace is the sum of the elements along the main diagonal **PROP2.6.2**All of the elementary matrices are invertible. **DEFRowEquivalence**The B is said to be row equivalent to the matrix A if there exists a finite collection of elementary matrices $E_1, E_2, \ldots, E_n$ such that $B = E_1E_2 \ldots E_n$ **DEFREF** A is REF if (i) leading coefficient of each row is strictly to the right of the previous row's leading coefficient (ii) All nonzero rows are above any zero rows and **RREF** if (iii) the leading coefficient of every row is 1 (iv) The leading coefficient of every row is the only nonzero entry in its column. **DEFPermutation** Different arrangements of a set. Even if it has an even number of inversions, odd if an odd number of inversions. Sign is 1 if even, -1 if odd. **DEFInversion** A pair $(\sigma(i), \sigma(j))$ such that $i < j$ and $\sigma(i) > \sigma(j)$. **THM2.8.7** If $A, B \in M_n(\mathbb{F})$, then $\det(AB) = \det(A)\det(B)$ **COR2.8.8**$\det(A^{-1} = (\det(A))^{-1})$ **Cramer′sRule** If $A \in M_n(\mathbb{F})$ is nonsingular, then the unique solution to $Ax = b$ is $x = A^{-1}b = \frac{\text{adj}(A)}{\det(A)}b$. Moreover, if $A_i(b) \in M_n(\mathbb{F})$ is the matrix A with the i-th column replaced by b, then the i-th coordinate of x is $x_i = \frac{\det(A_i(b))}{\det(A)}$