Math 320 Homework 3.1

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3.1 (i)

$$\Omega = \{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,1),(2,2)\}$$

3.1 (ii)

$$E = \{(1,1),(1,2),(2,1),(2,2)\}$$

3.1 (iii)

Where $E \subset \Omega$.

$$\frac{|E|}{|\Omega|} = \frac{4}{9}$$

3.1 (iv)

Note, the probability of Ω is

$$P = \{\frac{4}{49},\frac{6}{49},\frac{4}{49},\frac{6}{49},\frac{9}{49},\frac{6}{49},\frac{4}{49},\frac{6}{49},\frac{4}{49}\}$$

$\implies$ Probability is $\frac{25}{49}$.

3.2 (i)

We will use $P(E) = 1 - P(E^c)$, where $P(E^c)$ is where we have no pairs of shoes. I.E. we choose eight left shoes, and 0 right shoes, and from eight pairs of shoes we choose one shoe. Then the probability is as follows:

$$1 - \frac{\binom{10}{8}\binom{10}{0}\binom{2}{1}\binom{2}{1}\binom{2}{1}\binom{2}{1}\binom{2}{1}\binom{2}{1}\binom{2}{1}\binom{2}{1}}{\binom{20}{8}}$$

3.2 (ii)

Having exactly one pair of shoes, we will choose 7 left shoes and 1 right shoe. From one pair of shoes we choose two, while from six we choose one. Then the probability is as follows:

$$\frac{\binom{10}{7}\binom{10}{1}\binom{2}{2}\binom{2}{1}\binom{2}{1}\binom{2}{1}\binom{2}{1}\binom{2}{1}\binom{2}{1}}{\binom{20}{8}}$$

3.3 (i)

Three of a kind is three cards that are the same type from 5. Note, total number of possibilites is $C(52,5)$.

$$\text{Prob} = \frac{13 \cdot C(4,3) \cdot C(12,2)4^2}{C(52,5)}$$

3.3 (ii)

Two Pairs in the same hand, given by

$$\text{Prob} = \frac{11 \cdot C(13,2) \cdot C(4,2)^2 \cdot 4}{C(52,5)}$$

3.3 (iii)

Full House:

$$\text{Prob} = \frac{13 \cdot 12 \cdot C(4,3) \cdot C(4,2)}{C(52,5)}$$

3.4 (i)

This is similar to the probability of a classroom with n people having all distinct birthdays, but replacing one of the distinct birthday probabilities with $\frac{1}{365}$ giving us the probability:

$$\frac{365!}{(365-n+1)! \cdot 365^n}$$

3.4 (ii)

This is similar to the part (i), but we replace another distinct birthday with $\frac{1}{365}$, yielding:

$$\frac{365!}{(365-n+2)! \cdot 365^n}$$

3.4 (iii)

This is similar to part (ii), but instead of replacing the second distinct birthday with $\frac{1}{365}$, we replace it with $\frac{1}{364}$ changing our probability to:

$$\frac{365!}{(365-n+2)! \cdot 365^{n-1} \cdot 364}$$

3.5

We have that

$$P(E^c) = 1 - P(E) \implies P(E) = 1 - ap^n$$

where E is the event that $n = 0$. Given the definition of a we have the following:

$$\begin{aligned} 1 - ap^n &\geq 1 - \left(\frac{1-p}{p}\right) p^n \\ &= 1 - (1-p)p^{n-1} \end{aligned}$$

Yielding the final result:

$$P(E) \geq 1 - (1-p)p^{n-1}$$

3.6

We know the following

$$\Omega = \{B_1, B_2, \cdots, B_n\}$$

where Ω has n elements and $\mathscr{F}$ is the power set of Ω. Any $A \in \mathscr{F}$, then, will be a set consisting either of the empty set, a single $B_i \in \Omega$, or multiple $B_i, \cdots, B_j \in \Omega$.

For the empty set, the probability will be 0 since $P(\Omega) = 1$.

For $A = B_i \in \Omega$, the probability of A will obviously just be the probability of B_i happening, implying that $P(A) = \sum_{i \in I} P(A \cap B_i) = P(B_i \cap B_i) + P(B_i \cap B_j) + \cdots + P(B_i \cap B_n) = P(B_i \cap B_i) + 0 + \cdots + 0) = P(B_i \cap B_i) = P(B_i) = P(A)$.

A similar argument follows for the case where $A = \{B_1, B_2, \cdots, B_i\}$, since

$$\begin{aligned} P(B_1) &= P(B_1 \cap A) = P(B_1) \\ P(B_2) &= P(B_2 \cap A) = P(B_2) \\ &\cdots \\ P(B_i) &= P(B_i \cap A) = P(B_i) \end{aligned}$$

And we know then, that the probability of A will be equal to the sum of the probability of all its elements happening, or more precisely,

$$P(A) = P(B_1) + P(B_2) + \cdots + P(B_i)$$

Which is the desired result.