Math 320 Homework 3.6

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3.33

Note that

$$P\left(\left|\frac{B}{n}-p\right|\geq\varepsilon\right)=P\left(\left|\frac{B}{n}-p\right|^2\geq\varepsilon^2\right)$$

By Markov's Inequality, we have

$$\leq\frac{E\left[\left(\frac{B}{n}-p\right)^2\right]}{\varepsilon^2}$$

$$=\frac{E\left[\left(\frac{B-pn}{n}\right)^2\right]}{\varepsilon^2}$$

$$=\frac{\frac{1}{n^2}E\left[(B-pn)^2\right]}{\varepsilon^2}$$

$$=\frac{E\left[(B-pn)^2\right]}{n^2\varepsilon^2}$$

$$=\frac{E\left[B^2-2Bnp+(np)^2\right]}{n^2\varepsilon^2}$$

$$=\frac{E\left[B^2\right]-E\left[2Bnp\right]+E\left[(np)^2\right]}{n^2\varepsilon^2}$$

$$=\frac{E\left[B^2\right]-2npE\left[B\right]+E\left[(np)^2\right]}{n^2\varepsilon^2}$$

$$=\frac{E\left[B^2\right]-2E\left[B\right]E\left[B\right]+(np)^2}{n^2\varepsilon^2}$$

$$=\frac{E\left[B^2\right]-2E\left[B\right]^2+E\left[B\right]^2}{n^2\varepsilon^2}$$

$$=\frac{E\left[B^2\right]-E\left[B\right]^2}{n^2\varepsilon^2}$$

$$=\frac{\sigma_B^2}{n^2\varepsilon^2}$$

and since B is a binomial random variable,

$$=\frac{p(1-p)}{n^2\varepsilon^2}$$

and since $n\geq 1$

$$\leq\frac{p(1-p)}{n\varepsilon^2}$$

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The given equation, as $n \to \infty$,

$$P\left(\left|\frac{1}{n}\sum_{i=1}^{n} X_i\right| < \varepsilon\right) = P\left(\left|\frac{X_1 + X_2 + \cdots + X_n}{n} - 0\right| < \varepsilon\right) = P\left(\left|\frac{X_1 + X_2 + \cdots + X_n}{n} - \mu\right| < \varepsilon\right)$$

is clearly the complement of the equation given for the Weak Law of Large Numbers so we have that, as $n \to \infty$

$$P\left(\left|\frac{X_1 + X_2 + \cdots + X_n}{n} - \mu\right| \geq \varepsilon\right) + P\left(\left|\frac{X_1 + X_2 + \cdots + X_n}{n} - \mu\right| < \varepsilon\right) = 1$$

Rearranging the inequality, and noting that as $n \to \infty$,

$$P\left(\left|\frac{X_1 + X_2 + \cdots + X_n}{n} - \mu\right| \geq \varepsilon\right) = 0$$

we have that

$$0 + P\left(\left|\frac{X_1 + X_2 + \cdots + X_n}{n} - \mu\right| < \varepsilon\right) = 1$$

$$\implies P\left(\left|\frac{X_1, + X_2 + \cdots + X_n}{n} - \mu\right| < \varepsilon\right) = 1$$

3.35

Let X_i be our random variable. By the Central Limit Theorem, we have that

$$\begin{aligned} P(\sum_{i=1}^{n} X_i \leq 2000) &= P(\sum_{i=1}^{n} \frac{X_i - n\mu}{\sqrt{n}\sigma} \leq \frac{2000}{\sqrt{n}\sigma}) \\ &= P(\sum_{i=1}^{n} \frac{X_i - 10 \cdot 176}{\sqrt{10} \cdot 30} \leq \frac{240}{\sqrt{10} \cdot 30}) \\ &= P(\sum_{i=1}^{n} \frac{X_i - 1760}{\sqrt{10} \cdot 30} \leq \frac{240}{\sqrt{10} \cdot 30}) \end{aligned}$$

yielding

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{240}{\sqrt{10}\cdot 30}} e^{-x^2/2} dx \approx .99429$$

3.36

Let X_i be our random variable. By the Central Limit Theorem, we have that

$$\begin{aligned} P(\sum_{i=1}^{n} X_i \geq 5500) &= 1 - P(\sum_{i=1}^{n} \frac{X_i - 6242 \cdot .801}{\sqrt{6242} \cdot .399428} \leq \frac{5500 - 6242 \cdot .801}{\sqrt{6242} \cdot .399428}) \\ &= 1 - P(\sum_{i=1}^{n} \frac{X_i - 5000}{\sqrt{6242} \cdot .399428} \leq \frac{500}{\sqrt{6242} \cdot .399428}) \end{aligned}$$

yielding

$$1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{500}{\sqrt{6242} \cdot .399428}} e^{-x^2/2} dx \approx 2.22045 \times 10^{-16}$$

3.37

Let X_i be our random variable. Define it as follows:

$$\begin{cases} 1 & \text{if a 6 is rolled.} \\ 0 & \text{if otherwise.} \end{cases}$$

p, in this case, is $\frac{1}{6}$, and the variance is given by $p(1-p) = \frac{1}{6} \cdot \frac{5}{6} = \frac{\sqrt{5}}{6}$. By the Central Limit Theorem, we have that

$$\begin{aligned} P\left(150 \leq \sum_{i=1}^{n} X_i \leq 200\right) &= P\left(\frac{150 - \frac{1}{6}900}{6\sqrt{900}\sqrt{5}} \leq \sum_{i=1}^{n} \frac{X_i - \frac{1}{6}900}{6\sqrt{900}\sqrt{5}} \leq \frac{200 - \frac{1}{6}900}{6\sqrt{900}\sqrt{5}}\right) \\ &= P\left(0 \leq \sum_{i=1}^{n} \frac{X_i - 150}{5\sqrt{5}} \leq \frac{50}{5\sqrt{5}}\right) \end{aligned}$$

yielding

$$\frac{1}{\sqrt{2\pi}} \int_{0}^{\frac{50}{5\sqrt{5}}} e^{-x^2/2} dx \approx .499996$$

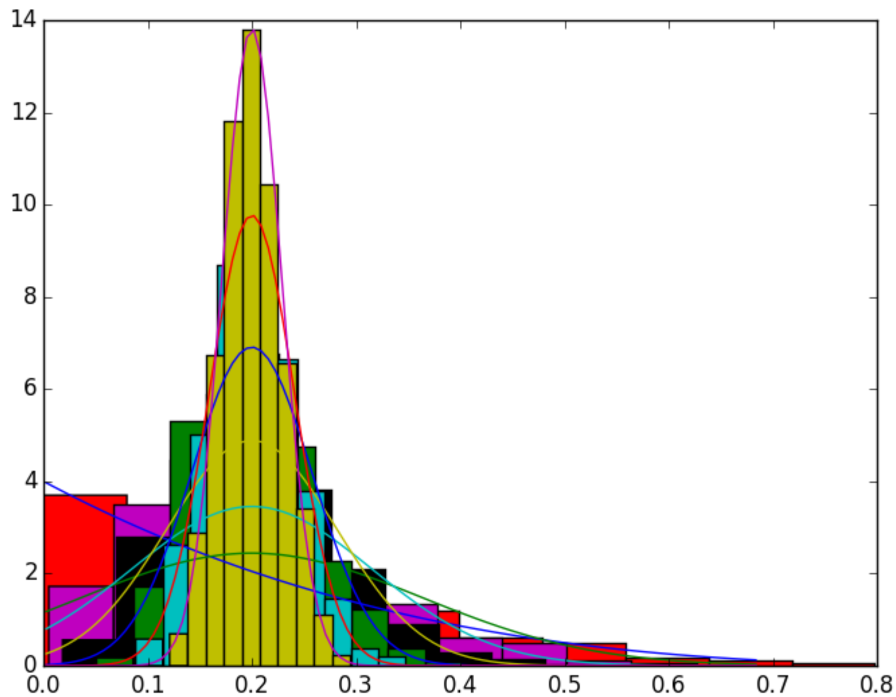
3.38

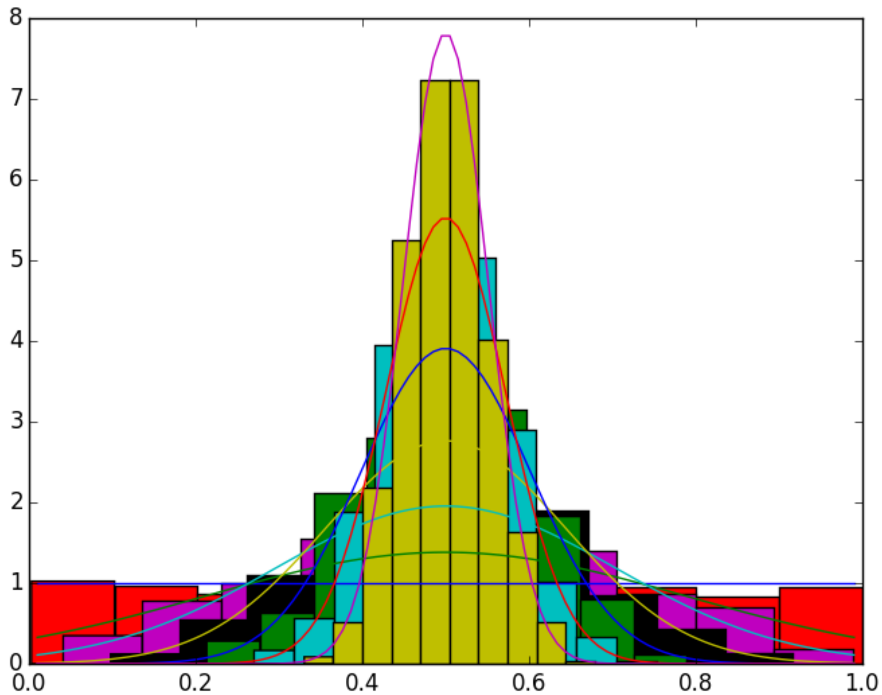
```
import numpy as np
from scipy import stats
import matplotlib.pyplot as plt
import matplotlib.mlab as mlab
import sys

#Problem 38
```

```
def Problem1(a,b):
    mean1, var1 = stats.beta.stats(a,b, moments='mv')
    x = np.linspace(stats.beta.ppf(.01,a,b), stats.beta.ppf(.99, a, b), 10
    plt.plot(x,stats.beta.pdf(x,a,b))
    print "Mean: ", mean1,"\nVariance: ", var1
    vals = np.array([1,2,4,8,16,32])
    for i in vals:
        plt.plot(x,mlab.normpdf(x,mean1,np.sqrt(var1/i)))
        xbar = np.average(stats.beta.rvs(a,b,size=(i,1000)),axis = 0)
        plt.hist(xbar,normed=True)

Problem1(1,4)
plt.show()
Problem1(1,1)
plt.show()
```

OUTPUT

```
Mean: 0.2
Variance: 0.0266666666667
Mean: 0.5
Variance: 0.0833333333333
```




8
7
6
5
4
3
2
1
0
0.0
0.2
0.4
0.6
0.8
1.0