The 3D Interaction Tool: A Pointing Device for Virtual Reality Applications

Lukas Gemar

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Abstract

The 3D Interaction Tool is a hand-held device that enables a user to interact with virtual reality applications. Since both the position and orientation of the tool are tracked, user input with the tool accomplishes tasks such as selecting, translating, and rotating three-dimensional objects, or free-hand drawing in space. The tools position is tracked by a software application that filters observations from a camera attached to the virtual reality display, and the orientation is found by means of observations from an inertial measurement unit attached to the tool itself. Simple computer-aided design and drawing applications demonstrate the capabilities of the 3D Interaction Tool.

1 Introduction

1.1 Nomenclature

Position vectors definitions in the inertial coordinate frame:
$\vec{p}_s \equiv$ position of the spherical tracking target on tool rear
$\vec{p}_r \equiv$ position of the tool rear
$\vec{p}_f \equiv$ position of the tool front
$\vec{p}_c \equiv$ position of the tracking camera
$\vec{p}_d \equiv$ position of the virtual reality display

Rotated position vectors:
$\vec{p}'_s \equiv$ position of the spherical tracking target on tool rear, camera coordinates
$\vec{p}'_r \equiv$ position of the tool rear in display coordinates
$\vec{p}'_f \equiv$ position of the tool front in display coordinates

Attitude matrices:
$\mathbf{A}_c \equiv$ attitude of the tracking camera
$\mathbf{A}_d \equiv$ attitude of the virtual reality display
$\mathbf{A}_t \equiv$ attitude of the interaction tool
$\mathbf{A}'_t \equiv$ attitude of the interaction tool in display coordinates

Body-referenced quaternion representations of attitude:

$\overline{\mathbf{q}}_c \equiv$ attitude of the tracking camera
$\overline{\mathbf{q}}_d \equiv$ attitude of the virtual reality display
$\overline{\mathbf{q}}_t \equiv$ attitude of the interaction tool

Position error covariances in inertial coordinate frame:
$\mathbf{P}_{ps} \equiv$ covariance of spherical tracking target position
$\mathbf{P}_{pr} \equiv$ covariance of the position of the rear of the interaction tool
$\mathbf{P}_{pf} \equiv$ covariance of the position of the front of the interaction tool
$\mathbf{P}_{pc} \equiv$ covariance of the position of the tracking camera
$\mathbf{P}_{pd} \equiv$ covariance of the position of the virtual reality display

Attitude error covariances in the inertial coordinate frame:
$\mathbf{P}_{ac} \equiv$ covariance of the tracking camera attitude quaternion error
$\mathbf{P}_{ad} \equiv$ covariance of the display attitude quaternion error
$\mathbf{P}_{at} \equiv$ covariance of the interaction tool attitude quaternion error

Position error covariances in rotated body coordinates:
$\mathbf{P}'_{ps} \equiv$ covariance of the sphereical tracking target position
$\mathbf{P}'_{pr} \equiv$ covariance of the position of the rear of the interaction tool
$\mathbf{P}'_{pf} \equiv$ covariance of the position of the front of the interaction tool

Attitude error covariances in the rotated body coordinates:
$\mathbf{P}'_{ac} \equiv$ covariance of the tracking camera attitude angular error
$\mathbf{P}'_{ad} \equiv$ covariance of the display attitude angular error
$\mathbf{P}'_{at} \equiv$ covariance of the interaction tool attitude angular error

2 Tool Model

2.1 Tool position and attitude

There are three tracking targets: $\vec{p}_f$, $\vec{p}'_f$, and $\mathbf{A}'_t$. The vector $\vec{p}_f$ tracks the trajectory of the front of the interaction tool in inertial coordinates. Tracking $\vec{p}_f$ is required in order to preserve information about where the tool has been after the tracking camera has rotated. In order to render the position of the tool on the virtual reality application, the vector $\vec{p}'_f$, the position of the tool in display coordinates, is found by the following equation: $\vec{p}'_f = \mathbf{A}_d(\vec{p}_f + \vec{p}_d)$. In order to render the orientation of the tool in the VR environment, the attitude of the tool in display coordinates is found by the product $\mathbf{A}'_t$.

The tracking targets are not measured directly, but they are inferred from measurements of $\vec{p}'_s$, $\mathbf{A}_c$, $\mathbf{A}_d$, $\mathbf{A}_t$, $\vec{p}_c$ and $\vec{p}_d$.

$$\vec{p}_r = \mathbf{A}_c^{-1}\vec{p}'_s - \vec{p}_c$$
$$\vec{p}_f = \vec{p}_r + \mathbf{A}_t^{-1}\vec{l}$$
$$\vec{p}'_f = \mathbf{A}_d(\vec{p}_f + \vec{p}_d)$$
$$\mathbf{A}'_t = \mathbf{A}_d\mathbf{A}_t^{-1}\vec{l}$$

$$\mathbf{P}'_{pr} = \mathbf{P}'_{ps} + \left[\ \vec{p}'^{\,X}_s \ \right] \mathbf{P}'_{ac} \left[\ \vec{p}'^{\,X}_s \ \right]^T$$

$$\mathbf{P}_{pr} = \mathbf{A}_c^{-1}(\mathbf{P}'_{ps} + \left[\ \vec{p}'^{\,X}_s \ \right] \mathbf{P}'_{ac} \left[\ \vec{p}'^{\,X}_s \ \right]^T)(\mathbf{A}_c^{-1})^T + \mathbf{P}_{pc}$$

$$\mathbf{P}_{pf} = \mathbf{P}_{pr} + \mathbf{A}_t^{-1} \left[\ \vec{l}^X \ \right] \mathbf{P}'_{at} \left[\ \vec{l}^X \ \right]^T (\mathbf{A}_t^{-1})^T$$

$$\mathbf{P}'_{pf} = \mathbf{A}_d(\mathbf{P}_{pf} + \mathbf{P}_{pd})\mathbf{A}_d{}^T$$

$$\mathbf{P}'_{pf} = \mathbf{P}'_{ps} + \left[\ \vec{p}'^{\,X}_s \ \right] \mathbf{P}'_{ac} \left[\ \vec{p}'^{\,X}_s \ \right]^T + \mathbf{A}'_t \left[\ \vec{l}^X \ \right] \mathbf{P}'_{at} \left[\ \vec{l}^X \ \right]^T (\mathbf{A}'_t)^T + 2\mathbf{A}_d\mathbf{P}_{pd}\mathbf{A}_d{}^T$$

References

[1] Ern J Lefferts, F Landis Markley, and Malcolm D Shuster. Kalman filtering for spacecraft attitude estimation. *Journal of Guidance, Control, and Dynamics*, 5(5):417–429, 1982.