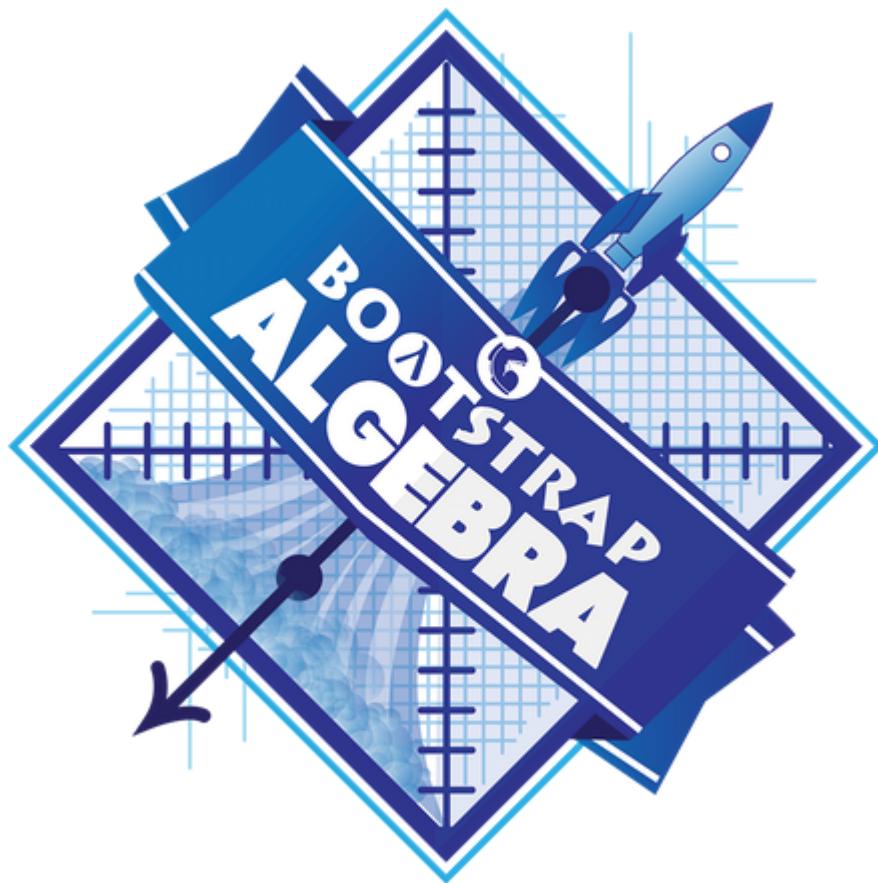


Name: _____



Student Workbook

Fall, 2024 - WeScheme Edition



BOOTSTRAP

Equity • Scale • Rigor

Workbook v3.0

Brought to you by the Bootstrap team:

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- Ben Lerner
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Computing Needs All Voices!

The pioneers pictured below are featured in our Computing Needs All Voices lesson. To learn more about them and their contributions, visit <https://bit.ly/bootstrap-pioneers>.



We are in the process of expanding our collection of pioneers. If there's someone else whose work inspires you, please let us know at <https://bit.ly/pioneer-suggestion>.

Notice and Wonder

Write down what you Notice and Wonder from the [What Most Schools Don't Teach](#) video.

"Notices" should be statements, not questions. What stood out to you? What do you remember? "Wonders" are questions.

What do you Notice?	What do you Wonder?

Windows and Mirrors

Think about the images and stories you've just encountered. Identify something(s) that served as a mirror for you, connecting you with your own identity and experience of the world. Write about who or what you connected with and why.

Identify something(s) from the film or the posters that served as a window for you, giving you insight into other people's experiences or expanding your thinking in some way.

Reflection: Problem Solving Advantages of Diverse Teams

This reflection is designed to follow reading [LA Times Perspective: A solution to tech's lingering diversity problem? Try thinking about ketchup](#)

- 1) The author argues that tech companies with diverse teams have an advantage. Why?

- 2) What suggestions did the article offer for tech companies looking to diversify their teams?

- 3) What is one thing of interest to you in the author's bio?

- 4) Think of a time when you had an idea that felt "out of the box". Did you share your idea? Why or why not?

- 5) Can you think of a time when someone else had a strategy or idea that you would never have thought of, but was interesting to you and/or pushed your thinking to a new level?

- 6) Based on your experience of exceptions to mainstream assumptions, propose another pair of questions that could be used in place of "Where do you keep your ketchup?" and "What would you reach for instead?"

The Math Inside video games

- Video games are all about *change!* How fast is this character moving? How does the score change if the player collects a coin? Where on the screen should we draw a castle?
- We can break down a game into parts, and figure out which parts change and which ones stay the same. For example:
 - Computers use **coordinates** to position a character on the screen. These coordinates specify how far from the left (x-coordinate) and the bottom (y-coordinate) a character should be. Negative values can be used to "hide" a character, by positioning them somewhere off the screen.
 - When a character moves, those coordinates change by some amount. When the score goes up or down, it *also* changes by some amount.
- From the computer's point of view, the whole game is just a bunch of numbers that are changing according to some equations. We might not be able to see those equations, but we can definitely see the effect they have when a character jumps on a mushroom, flies on a dragon, or mines for rocks!
- Modern video games are *incredibly* complex, costing millions of dollars and several years to make, and relying on hundreds of programmers and digital artists to build them. But building even a simple game can give us a good idea of how the complex ones work!

Notice and Wonder

Write down what you Notice and Wonder about the [Ninja Cat Game](#).

"Notices" should be statements, not questions. What stood out to you? What do you remember?

What do you Notice?	What do you Wonder?

Reverse Engineer a video game

This page is designed to be used with the [Ninja Cat Game](#).

What is changing in the game? What variables is the program keeping track of? The first example is filled in for you.



Thing in the Game	What Changes About It?	More Specifically... what variable(s) are being tracked?
Dog	Position	x-coordinate

Estimating Coordinates



The coordinates for the PLAYER (NinjaCat) are: (,)
x y

The coordinates for the DANGER (Dog) are: (,)
x y

The coordinates for the TARGET (Ruby) are: (,)
x y

Brainstorm Your Own Game

Created by: _____

Background

Our game takes place: _____
In space? The desert? A mall?

Player

The Player is a _____
The Player moves only up and down.

Target

Your Player GAINS points when they hit The Target.

The Target is a _____
The Target moves only to the left or right.

Danger

Your Player LOSES points when they hit The Danger.

The Danger is a _____
The Danger moves only to the left or right.

Artwork/Sketches/Proof of Concept

Below is a **640x480 rectangle**, representing your game screen.

- Label the bottom-left corner (0,0).
- Label the other three corners with their corresponding coordinates.
- In the rectangle, sketch a picture of your game!

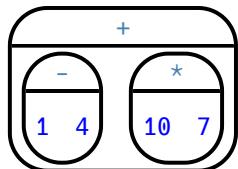


Order of Operations

If you were to write instructions for getting ready for school, it would matter very much which instruction came first: putting on your socks, putting on your shoes, etc.

Sometimes we need multiple expressions in mathematics, and the order matters there, too! Mathematicians didn't always agree on the **Order of Operations**, but at some point it became important to develop rules to help them work together.

To help us organize our math into something we can trust, we can *diagram* a math expression using the **Circles of Evaluation**. For example, the expression $1 - 4 + 10 \times 7$ can be diagrammed as shown below.



Order of Operations is important when programming, too!

To convert a **Circle of Evaluation** into Code, we walk through the circle from outside-in, moving left-to-right. We type an open parenthesis when we *start* a circle, and a close parenthesis when we *end* one. Once we're in a circle, we first write the **function** at the top, then write the inputs from left to right. The circle above, for example, would be programmed as `(+ (- 1 4) (* 10 7))`.

Completing Circles of Evaluation from Arithmetic Expressions

For each expression on the left, finish the Circle of Evaluation on the right by filling in the blanks.

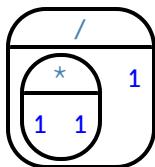
	Arithmetic Expression	Circle of Evaluation
1	$4 + 2 - \frac{10}{5}$	
2	$7 - 1 + 5 \times 8$	
3	$\frac{-15}{5 + -8}$	
4	$(4 + (9 - 8)) \times 5$	
5	$6 \times 4 + \frac{9 -- 6}{5}$	
★	$\frac{20}{6 + 4} - \frac{5 \times 9}{-12 - 3}$	

Matching Circles of Evaluation and Arithmetic Expressions

Draw a line from each Circle of Evaluation on the left to the corresponding arithmetic expression on the right.

Circle of Evaluation

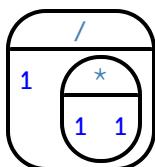
Arithmetic Expression



1

A

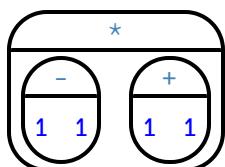
$$1 \div (1 \times 1)$$



2

B

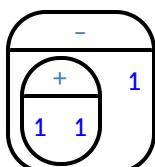
$$(1 + 1) - 1$$



3

C

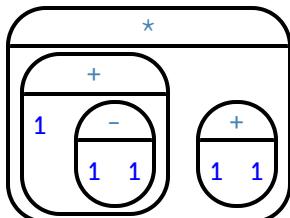
$$(1 \times 1) \div 1$$



4

D

$$(1 + (1 - 1)) \times (1 + 1)$$



5

E

$$(1 - 1) \times (1 + 1)$$

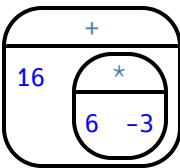
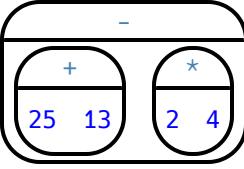
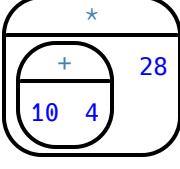
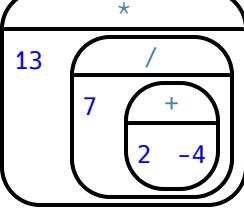
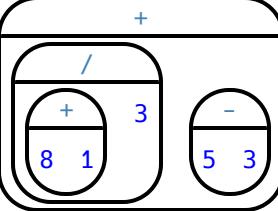
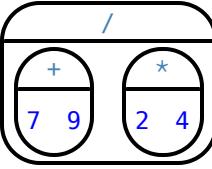
Translate Arithmetic to Circles of Evaluation & Code (Intro)

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code.

	Arithmetic Expression	Circle of Evaluation	Code
1	$(3 \times 7) - (1 + 2)$		
2	$3 - (1 + 2)$		
3	$3 - (1 + (5 \times 6))$		
4	$(1 + (5 \times 6)) - 3$		

Completing Partial Code from Circles of Evaluation

For each Circle of Evaluation on the left, finish the Code on the right by filling in the blanks.

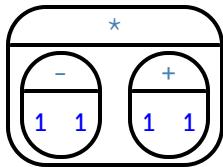
	Circle of Evaluation	Code
1		(+ ____ (* 6 ____))
2		(____ (+ ____ 13) (____ ____ 4))
3		(____ (+ ____ 4) ____)
4		(____ 13 (____ 7 (____ 2 -4)))
5		(____ (____ (____ 8 1) 3) (____ 5 3))
6		(/ (+ ____ ____) (* ____ ____))

Matching Circles of Evaluation & Code

Draw a line from each Circle of Evaluation on the left to the corresponding Code on the right.

Circle of Evaluation

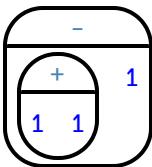
Code



1

A

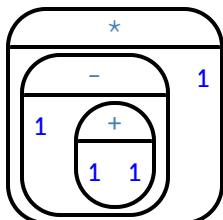
($*$ ($-$ 1 $(+$ 1 1)) 1)



2

B

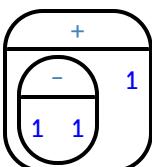
($*$ ($-$ 1 1) $(+$ 1 1))



3

C

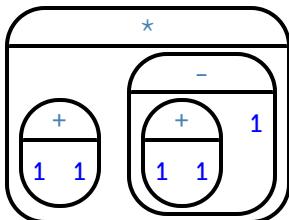
($*$ $(+$ 1 1) $(-$ $(+$ 1 1) 1))



4

D

($-$ $(+$ 1 1) 1)



5

E

($+$ ($-$ 1 1) 1)

Translate Arithmetic to Circles of Evaluation & Code (2)

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code.

	Arithmetic Expression	Circle of Evaluation	Code
1	$6 \times 8 + (7 - 23)$		
2	$18 \div 2 + 24 \times 4 - 2$		
3	$(22 - 7) \div (3 + 2)$		
4	$24 \div 4 \times 2 - 6 + 20 \times 2$		

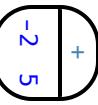
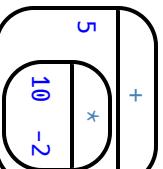
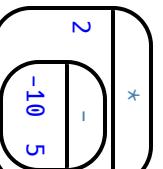
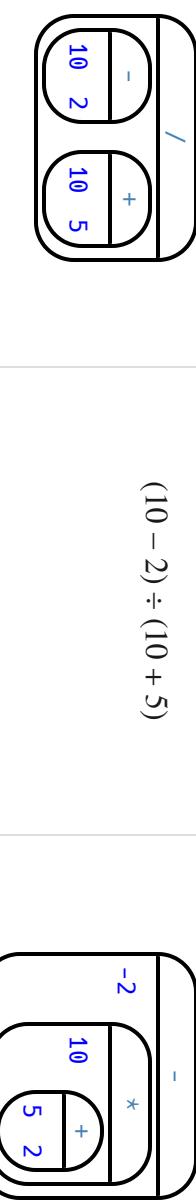
Arithmetic Expressions to Circles of Evaluation & Code - Challenge

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code. Hint: Two useful functions are `sqr` and `sqrt`.

Arithmetic Expression	Circle of Evaluation	Code
1 $\frac{16 + 3^2}{\sqrt{49} - 2}$		
2 $45 - 9 \times (3 + (2 - 4)) - 7$		
3 $50 \div 5 \times 2 - ((3 + 4) \times 2 - 5)$		

Matching Circles of Evaluation & Code

Cut out the cards in the table below, mix them up, and try to match the Circle of Evaluation with the Arithmetic Expression.

1	A $-2 + 5$	2 	B $10 + 2$
3	C $5 + (10 \times -2)$	4 	D $(10 \div -2) + 5$
5	E $2 \times (-10 - 5)$	6 	F $5 \times (-10 \div -2)$
7	G $(10 - 2) \div (10 + 5)$	8 	H $-2 - (10 \times (5 + 2))$

Creating Circles of Evaluation from Arithmetic Expressions

For each arithmetic expression on the left, draw its Circle of Evaluation on the right.

	Arithmetic Expression	Circle of Evaluation
1	$4 - (6 - 17)$	
2	$25 + 14 - 12$	
3	$1 + 15 \times 5$	
4	$15 \div (10 + 4 \times -2)$	

Creating Circles of Evaluation from Arithmetic Expressions 2

For each arithmetic expression on the left, draw its Circle of Evaluation on the right.

	Arithmetic Expression	Circle of Evaluation
1	$6 + 17 - -2$	
2	$(2 + 17) \times (12 - 8)$	
3	$23 \times 14 \times (3 + 20)$	
4	$5 - 17 + 14 \times 5$	

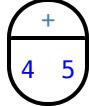
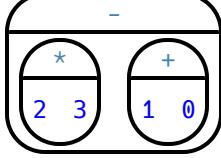
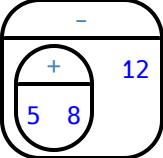
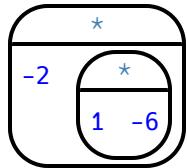
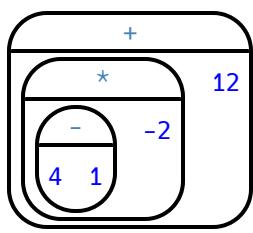
Creating Circles of Evaluation from Arithmetic Expressions 3

For each expression on the left, draw its Circle of Evaluation on the right.

	Arithmetic Expression	Circle of Evaluation
1	$9 \times (17 + 2)$	
2	$(2 + 17) \times (12 - 8)$	
3	$19 - (12 + 11)$	
4	$\frac{7}{7 \times (9 + 8)}$	

Converting Circles of Evaluation to Arithmetic Expressions

For each Circle of Evaluation on left, write the arithmetic expression on the right.

	Circle of Evaluation	Arithmetic Expression
1		
2		
3		
4		
5		

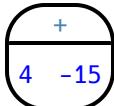
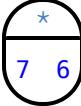
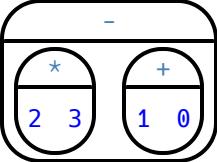
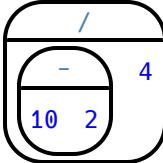
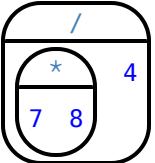
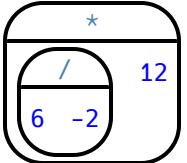
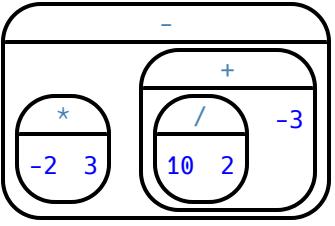
Converting Circles of Evaluation to Arithmetic Expressions 2

For each Circle of Evaluation on left, write the arithmetic expression on the right

	Circle of Evaluation	Arithmetic Expression
1		
2		
3		
4		
5		

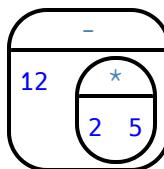
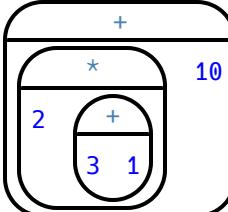
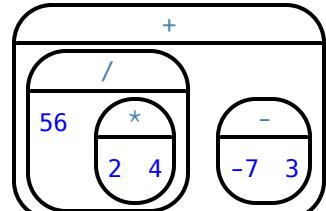
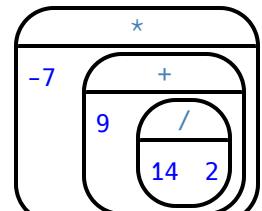
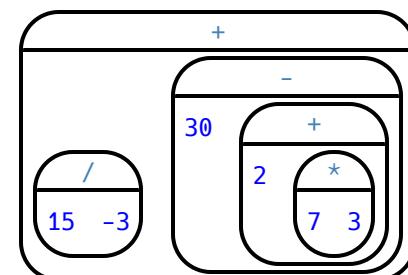
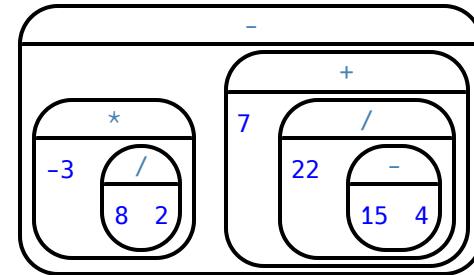
Evaluating Circles of Evaluation

Write each Circle of Evaluation as an arithmetic expression and evaluate it.

	Circle of Evaluation	Arithmetic Expression	Answer
1			
2			
3			
4			
5			
6			
7			

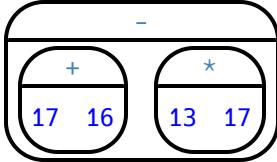
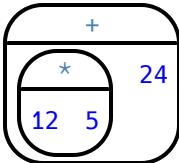
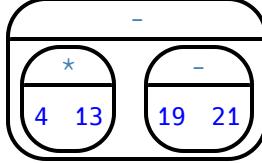
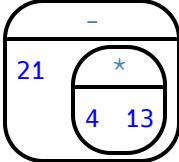
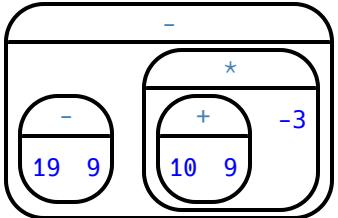
Evaluating Circles of Evaluation 2

Write each Circle of Evaluation as an arithmetic expression and evaluate it.

	Circle of Evaluation	Aithmetic Expression	Answer
1			
2			
3			
4			
5			
6			

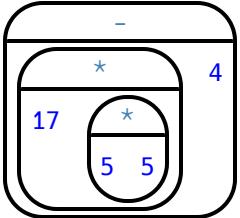
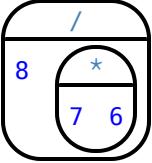
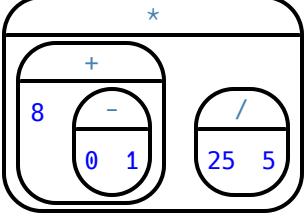
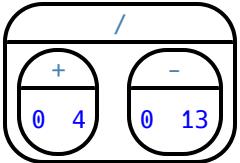
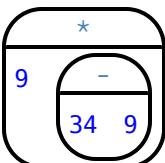
Converting Circles of Evaluation to Code

For each Circle of Evaluation on the left-hand side, write the code for the Circle on the right-hand side

	Circle of Evaluation	Code
1		
2		
3		
4		
5		

Converting Circles of Evaluation to Code 2

For each Circle of Evaluation on the left-hand side, write the code for the Circle on the right-hand side

	Circle of Evaluation	Code
1		
2		
3		
4		
5		

Translate Arithmetic Expressions to Circles of Evaluation & Code 3

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code.

	Arithmetic Expression	Circle of Evaluation	Code
1	$9 \div 3 + 7 - 8 \div 4$		
2	$6 \times (5 + 3) - 2$		
3	$3 - (1 + 5 \times 6)$		
4	$15 \div 3 + (2 + 1)$		

Translate Arithmetic Expressions to Circles of Evaluation & Code 4

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code.

	Arithmetic Expression	Circle of Evaluation	Code
1	$15 - 9 \div (2 + 1)$		
2	$(9 + 6) \times 7 + 8 \div 2$		
3	$7 - (8 \times 3 + 2)$		
4	$5 + 8 \div 2 \times 4$		

Translate Arithmetic Expressions to Circles of Evaluation & Code 5

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code.

	Arithmetic Expression	Circle of Evaluation	Code
1	$6 + (5 - 3) \div 2$		
2	$-15 \div 3 \times (2 + 1)$		
3	$8 - 6 \div (-2 + -1) \times -4$		
4	$10 \div -5 \times 3 --7$		

Translate Arithmetic Expressions to Circles of Evaluation & Code 6

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code.

	Arithmetic Expression	Circle of Evaluation	Code
1	$7 \times -4 + -10 \div 2$		
2	$-5 \div 5 \times 4 - 8$		
3	$9 \times 3 + -6 - 8 \times 4$		
4	$6 + (-5 + 3) \div 2$		

Translating Circles of Evaluation to Code - w/Square Roots

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code.

HINT: The function name is `sqrt`.

	Arithmetic Expression	Circle of Evaluation	Code
1	$\sqrt{9}$		
2	$\sqrt{5 + 1}$		
3	$\sqrt{4} + 1$		
4	$3 \times \sqrt{3} + \sqrt{7}$		

Arithmetic Expressions to Circles of Evaluation & Code - Challenge 2

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code.

Arithmetic Expression	Circle of Evaluation	Code
1 $(10 - (3 + 4)) \times \frac{7 - \sqrt{4}}{5 \times (2 + 4)} + 7$		
2 $8 - (9 + 2 \times (4 - 1))$		
3 $2 \times 4^2 + 8 \div 4 \times 2$		

Arithmetic Expressions to Circles of Evaluation & Code - Challenge 3

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code.

Arithmetic Expression	Circle of Evaluation	Code
1 $27 - 5 \times (4^2 - 16) + \sqrt{9}$		
2 $3 \times 4^2 - 2 \times \sqrt{25 - 4^2}$		
3 $5^2 \times (8 - (3 + 2)) - \frac{\sqrt{100}}{2}$		

Arithmetic Expressions to Circles of Evaluation & Code - Challenge 4

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code.

	Arithmetic Expression	Circle of Evaluation	Code
1	$45 \div 3^2 + 8 \times -2 - \sqrt{16}$		
2	$11 + (5 - 3)^2 \div 5 - 6 \times 2$		
3	$2^3 + \frac{8^2 + 4^2}{9 - 5} \times 2 \times (9 - 4 \times 2)$		

Introduction to Programming

The **Editor** is a software program we use to write Code. Our Editor allows us to experiment with Code on the right-hand side, in the **Interactions Area**. For Code that we want to *keep*, we can put it on the left-hand side in the **Definitions Area**. Clicking the "Run" button causes the computer to re-read everything in the Definitions Area and erase anything that was typed into the Interactions Area.

Data Types

Programming languages involve different **data types**, such as Numbers, Strings, Booleans, and even Images.

- Numbers are values like `1`, `0.4`, `1/3`, and `-8261.003`.
 - Numbers are *usually* used for quantitative data and other values are *usually* used as categorical data.
- Strings are values like `"Emma"`, `"Rosanna"`, `"Jen and Ed"`, or even `"08/28/1980"`.
 - All strings *must* be surrounded in quotation marks.
- Booleans are either `true` or `false`.

All values evaluate to themselves. The program `42` will evaluate to `42`, the String `"Hello"` will evaluate to `"Hello"`, and the Boolean `false` will evaluate to `false`.

Operators

Operators (like `+`, `-`, `*`, `<`, etc.) are treated the same way as functions: after all, they have inputs and outputs and obey the same rules!

Applying Functions

Applying functions (and operators!) works much the way it does in math. Every function has a name, takes some inputs, and produces some output. The function name is written first, followed by a list of **arguments**.

- In math this could look like $f(5)$ or $g(10, 4)$.
- In WeScheme, these examples would be written as `(f 5)` and `(g 10 4)`.
- Applying the operator to the inputs 1 and 2 would look like `(1 2)`.
- Applying a function to make images would look like `(star 50 "solid" "red")`.
- There are many other functions, for example `num-sqr`, `num-sqrt`, `triangle`, `square`, `string-repeat`, etc.

Functions have **contracts**, which help explain how a function should be used. Every Contract has three parts:

- The *Name* of the function - literally, what it's called.
- The *Domain* of the function - what *types of values* the function consumes, and in what order.
- The *Range* of the function - what *type of value* the function produces.

Strings and Numbers

Make sure you've loaded the [WeScheme](#), clicked "Run", and are working in the *Interactions Area*.

Strings

String values are always in quotes.

- Try typing your name (*in quotes!*).
- Try typing a sentence like "I'm excited to learn to code!" (*in quotes!*).
- Try typing your name with the opening quote, but *without the closing quote*. Read the error message!
- Now try typing your name *without any quotes*. Read the error message!

1) Explain what you understand about how strings work in this programming language. _____

Numbers

2) Try typing 42 into the Interactions Area and hitting "Enter".

3) Is 42 the same as "42"? Why or why not? Write your answer below:

4) What is the largest number the editor can handle?

5) Try typing `0 . 5`. Then try typing `. 5`. Then try clicking on the answer. Experiment with other decimals. Explain what you understand about how decimals work in this programming language. _____

6) What happens if you try a fraction like `1 / 3`? _____

7) Try writing **negative** integers, fractions and decimals. What do you learn? _____

Booleans

Boolean-producing expressions are yes-or-no questions and will always evaluate to either **true** ("yes") or **false** ("no"). What will each of the expressions below evaluate to? Write down your prediction in the blanks provided and then type the code into the Interactions Area to see what it returns.

Prediction	Result	Prediction	Result
1) <code>(<= 3 4)</code> _____	2) <code>(string>? "a" "b")</code> _____		
3) <code>(= 3 2)</code> _____	4) <code>(string<? "a" "b")</code> _____		
5) <code>(< 2 4)</code> _____	6) <code>(string=? "a" "b")</code> _____		
7) <code>(>= 5 5)</code> _____	8) <code>(string<>? "a" "a")</code> _____		
9) <code>(>= 4 6)</code> _____	10) <code>(string>=? "a" "a")</code> _____		
11) <code>(<> 3 3)</code> _____	12) <code>(string<>? "a" "b")</code> _____		
13) <code>(<> 4 3)</code> _____	14) <code>(string>=? "a" "b")</code> _____		

15) In your own words, describe what `<` does.

16) In your own words, describe what `>=` does.

17) In your own words, describe what `<>` does.

	Prediction:	Result:
18) <code>(string=? "a tree" "trees")</code>	_____	_____
19) <code>(string=? "tree" "tree")</code>	_____	_____
20) <code>(string-contains? "catnap" "cat")</code>	_____	_____
21) <code>(string-contains? "cat" "catnap")</code>	_____	_____

22) In your own words, describe what `string-contains?` does. Can you generate another expression using `string-contains?` that returns true?

23) There are infinite numbers values out there (...-2,-1,0,-1,2...) and infinite string values ("a", "aa", "aaa"...). But how many different Boolean values are there?

Applying Functions

Make sure you've loaded the [WeScheme](#), clicked "Run", and are working in the *Interactions Area*. Type this line of code into the Interactions Area and hit "Enter":

```
(triangle 50 "solid" "red")
```

- 1) What is the name of this function? _____
- 2) What did the expression evaluate to? _____
- 3) How many arguments does `triangle` expect? _____
- 4) What data type does the `triangle` function produce? _____

Catching Bugs

5) (triangle 20 "solid")
triangle: expects 3 arguments, but given 2: 20 solid at: line 1, column 0, in <interactions>

Can you spot the mistake? _____

6) (triangle "solid" "red" 20)
triangle: expects a non-negative number as 1st argument, but given: solid; other arguments were:
red 20 at: line 1, column 0, in <interactions>

Can you spot the mistake? _____

7) (triangle 20 40 "solid" "red")
triangle: expects 3 arguments, but given 4: 20 40 solid red at: line 1, column 0, in
<interactions>

Can you spot the mistake? _____

8) (triangle 20 solid "red")
solid: this variable is not defined at: line 1, column 0, in <interactions>

Can you spot the mistake? _____

9) (triangle 20 "striped" "red")
triangle: expects a style ("solid" / "outline") or an opacity value [0–255] as 2nd argument,
but given: "striped"; other arguments were: 20 "red" at: line 1, column 0, in <interactions>

Can you spot the mistake? _____

Practicing Contracts: Domain & Range

Consider the following Contract:

```
is-beach-weather :: Number, String -> Boolean
```

Note: The contracts on this page are not defined in Pyret and cannot be tested in the editor.

1) What is the **Name** of this function? _____

2) How many arguments are in this function's **Domain**? _____

3) What is the **Type** of this function's **first argument**? _____

4) What is the **Type** of this function's **second argument**? _____

5) What is the **Range** of this function? _____

6) Circle the expression below that shows the correct application of this function, based on its Contract.

- A. (`is-beach-weather 70 90`)
- B. (`is-beach-weather 80 100 "cloudy"`)
- C. (`is-beach-weather "sunny" 90`)
- D. (`is-beach-weather 90 "stormy weather"`)

Consider the following Contract:

```
cylinder :: Number, Number, String -> Image
```

7) What is the **Name** of this function? _____

8) How many arguments are in this function's **Domain**? _____

9) What is the **Type** of this function's **first argument**? _____

10) What is the **Type** of this function's **second argument**? _____

11) What is the **Type** of this function's **third argument**? _____

12) What is the **Range** of this function? _____

13) Circle the expression below that shows the correct application of this function, based on its Contract.

- A. (`cylinder "red" 10 60`)
- B. (`cylinder 30 "green"`)
- C. (`cylinder 10 25 "blue"`)
- D. (`cylinder 14 "orange" 25`)

Matching Expressions and Contracts

Match the Contract (left) with the expression described by the function being used (right). Note: The contracts on this page are not defined in Pyret and cannot be tested in the editor.

Contract	Expression
<code>; make-id :: String, Number -> Image</code> 1	A <code>(make-id "Savannah" "Lopez" 32)</code>
<code>; make-id :: String, Number, String -> Image</code> 2	B <code>(make-id "Pilar" 17)</code>
<code>; make-id :: String -> Image</code> 3	C <code>(make-id "Akemi" 39 "red")</code>
<code>; make-id :: String, String -> Image</code> 4	D <code>(make-id "Raïssa" "McCracken")</code>
<code>; make-id :: String, String, Number -> Image</code> 5	E <code>(make-id "von Einsiedel")</code>

Contract	Expression
<code>; is-capital :: String, String -> Boolean</code> 6	A <code>(show-pop "Juneau" "AK" 31848)</code>
<code>; is-capital :: String, String, String -> Boolean</code> 7	B <code>(show-pop "San Juan" 395426)</code>
<code>; show-pop :: String, Number -> Image</code> 8	C <code>(is-capital "Accra" "Ghana")</code>
<code>; show-pop :: String, String, Number -> Image</code> 9	D <code>(show-pop 3751351 "Oklahoma")</code>
<code>; show-pop :: Number, String -> Number</code> 10	E <code>(is-capital "Albany" "NY" "USA")</code>

Using Contracts

Use the contracts to write expressions to generate images similar to those pictured. Go to [WeScheme](#) to test your code.

`; ellipse :: Number, Number, String, String -> Image`



Use the Contract to write an expression that generates a similar image:



Use the Contract to write an expression that generates a similar image:

What changes with the first Number?

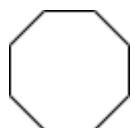
What about the shape changes with the second Number?

Write an expression using `ellipse` to produce a circle.

`; regular-polygon :: Number, Number, String, String -> Image`



Use the Contract to write an expression that generates a similar image:



Use the Contract to write an expression that generates a similar image:

What changes with the first Number?

What about the shape changes with the second Number?

Use `regular-polygon` to write an expression for a square!

How would you describe a **regular polygon** to a friend?

Triangle Contracts

Respond to the questions. Go to [WeScheme](#) to test your code.

1) What kind of triangle does the `triangle` function produce? _____

There are lots of other kinds of triangles! And WeScheme has lots of other functions that make triangles!

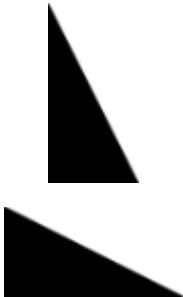
```
triangle :: (size:: Number, style :: String, color :: String) -> Image
```

```
right-triangle :: (base::Number, height::Number, style::String, color::String) -> Image
```

```
isosceles-triangle :: (leg::Number, angle::Number, style::String, color::String) -> Image
```

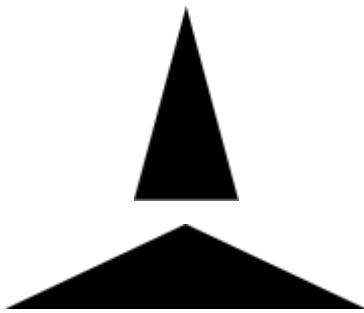
2) Why do you think `triangle` only needs one number, while `right-triangle` and `isosceles-triangle` need two numbers and `triangle/sas` needs three?

3) Write `right-triangle` expressions for the images below. One argument for each should be `100`.



4) What do you think the numbers in `right-triangle` represent? _____

5) Write `isosceles-triangle` expressions for the images below. 1 argument for each should be `100`.



6) What do you think the numbers in `isosceles-triangle` represent?

7) Write 2 expressions that would build right-isosceles triangles. Use `right-triangle` for one expression and `isosceles-triangle` for the other expression.



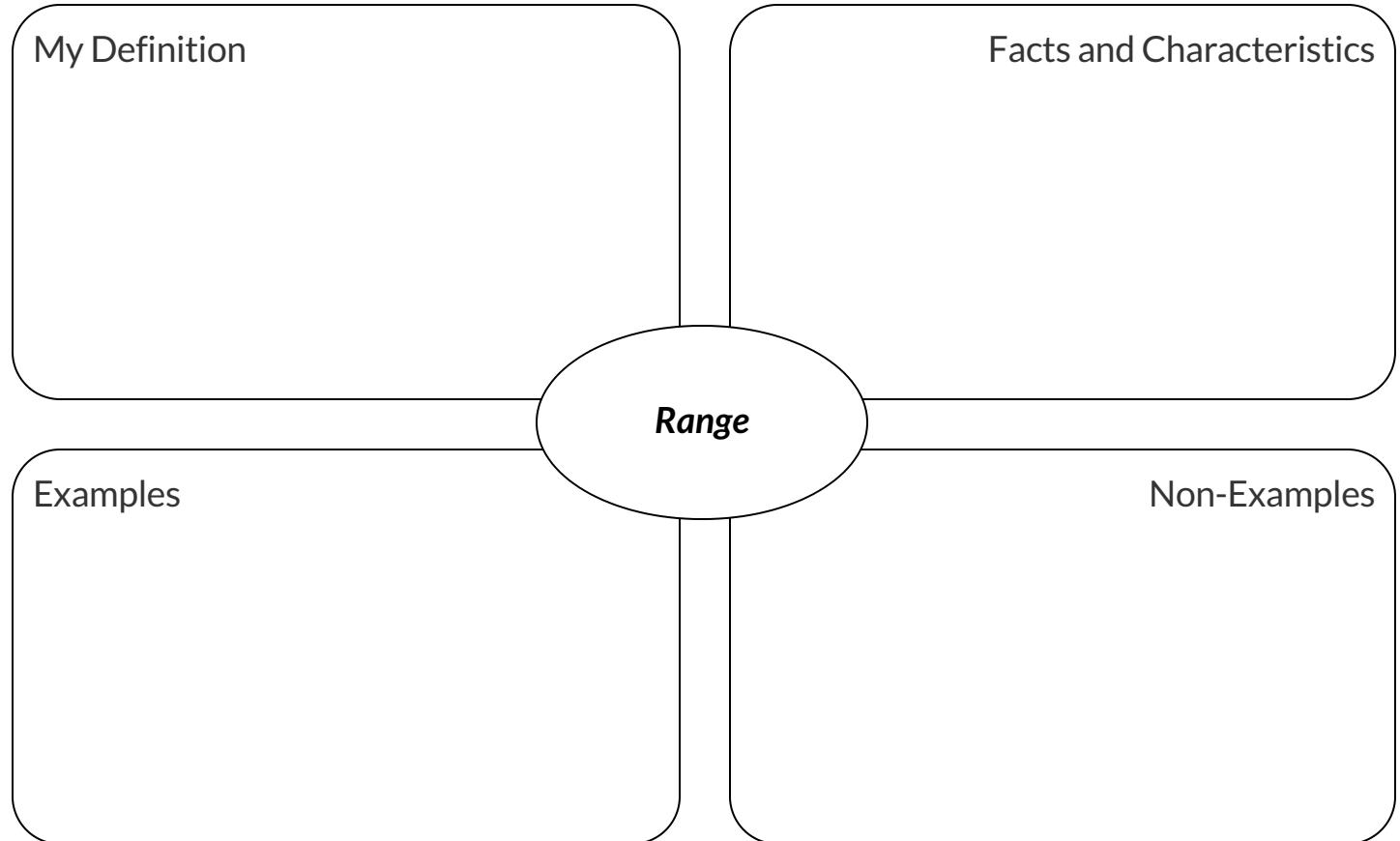
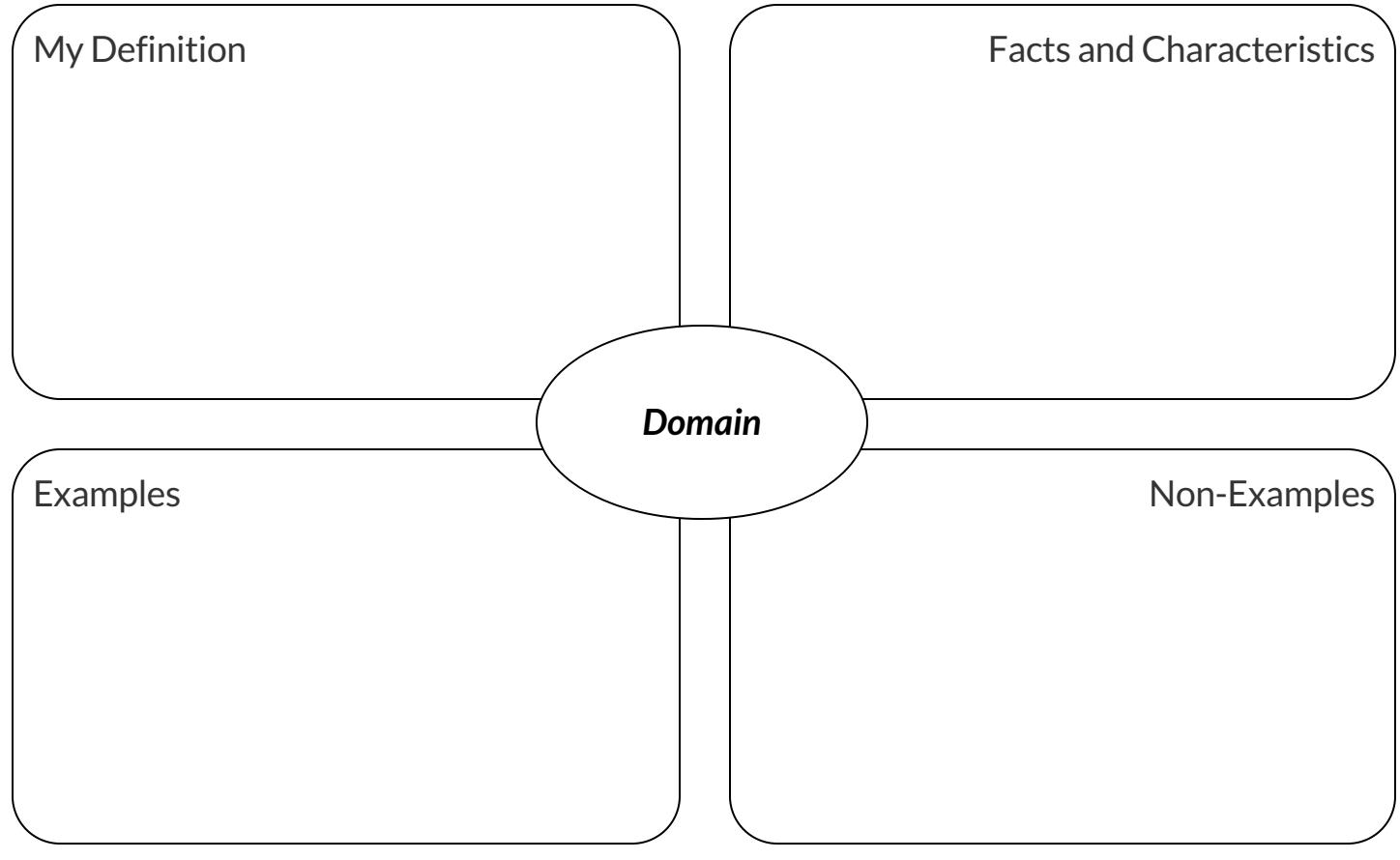
Radial Star

```
radial-star :: (  
    points :: Number,  
    inner-radius :: Number,  
    full-radius :: Number,  
    style :: String,  
    color :: String  
) -> Image
```

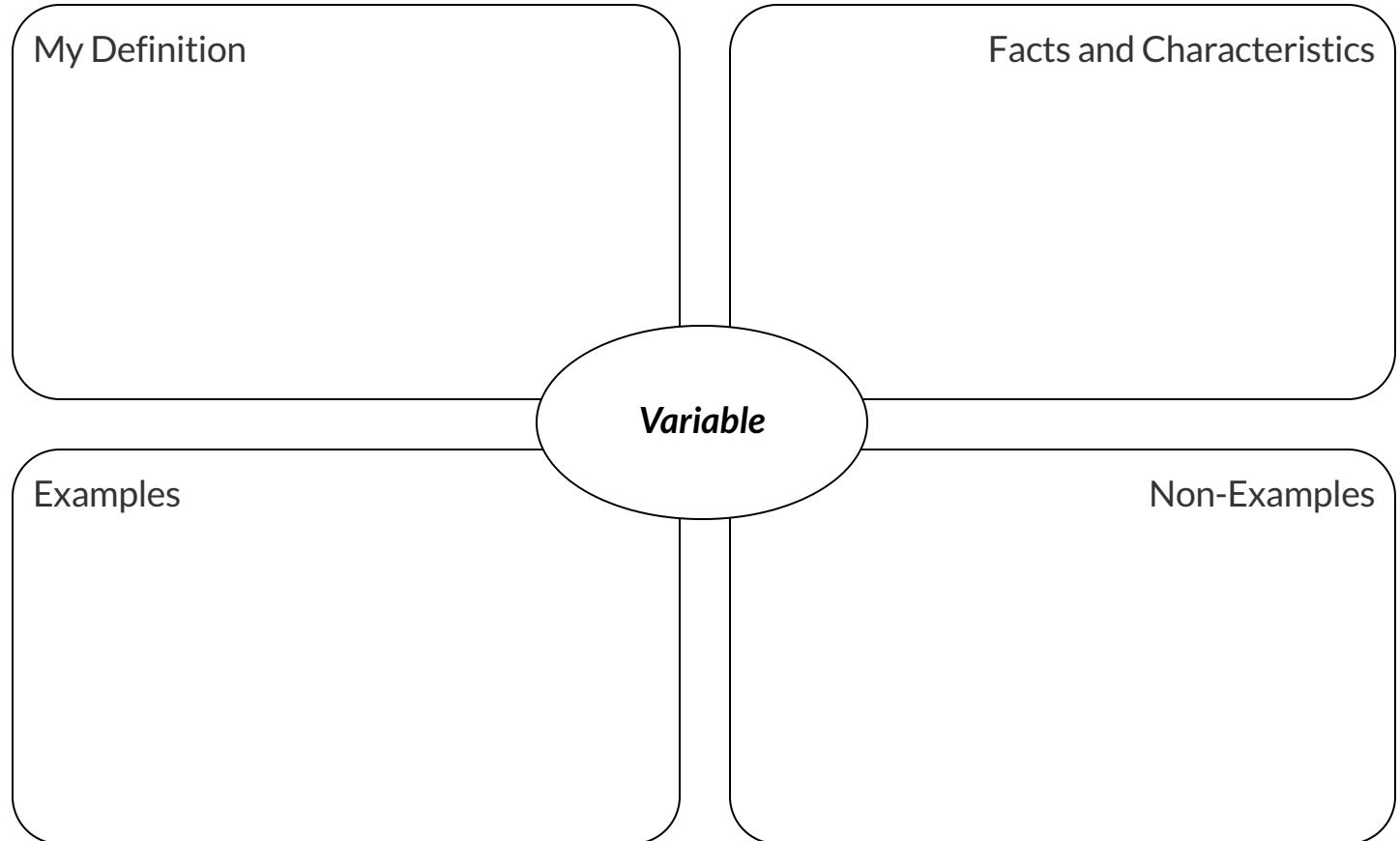
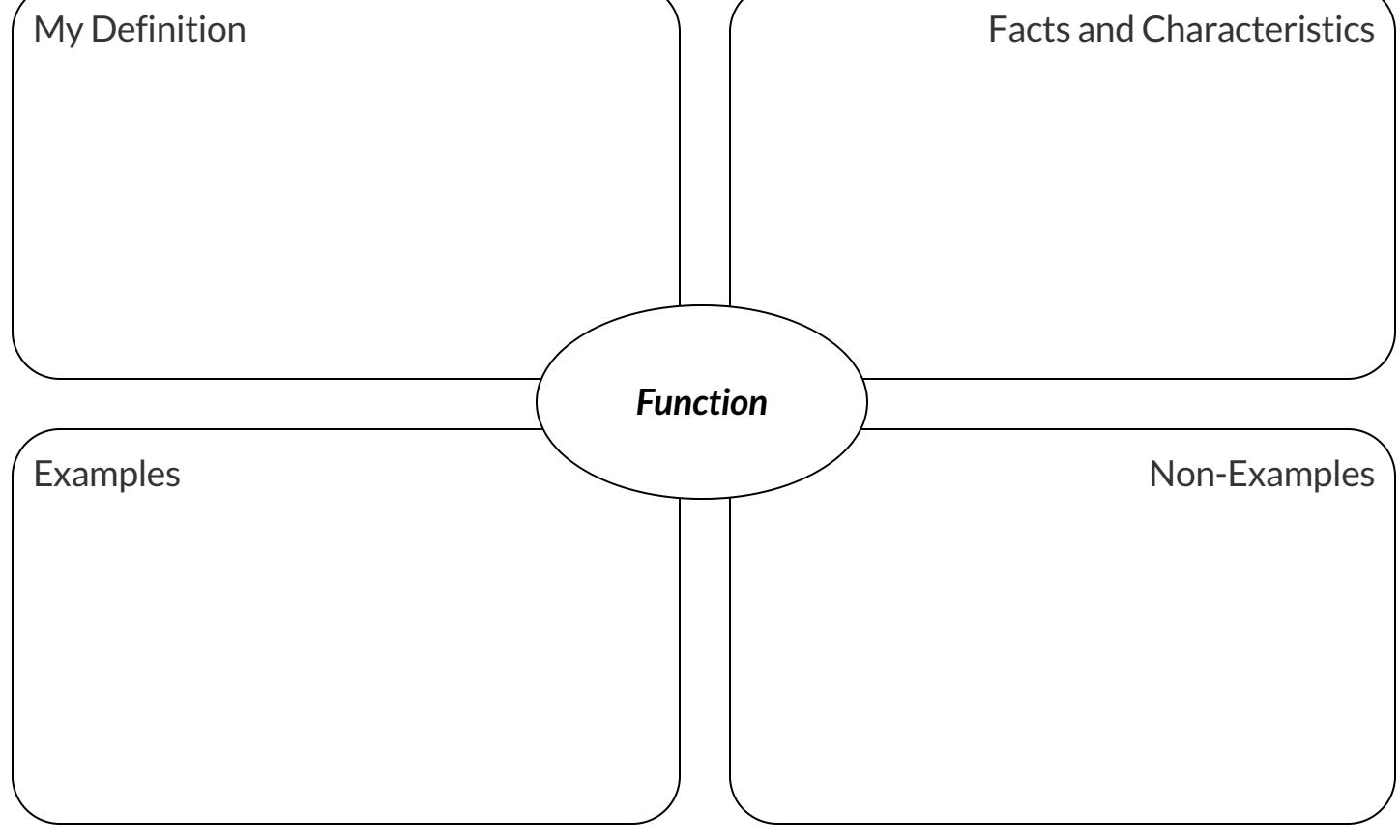
Using the detailed Contract above, match each image to the expression that describes it. Go to [WeScheme](#) to test your code.

Image		Expression
	1	A <code>(radial-star 5 50 200 "solid" "black")</code>
	2	B <code>(radial-star 7 100 200 "solid" "black")</code>
	3	C <code>(radial-star 7 100 200 "outline" "black")</code>
	4	D <code>(radial-star 10 150 200 "solid" "black")</code>
	5	E <code>(radial-star 10 20 200 "solid" "black")</code>
	6	F <code>(radial-star 100 20 200 "outline" "black")</code>
	7	G <code>(radial-star 100 100 200 "outline" "black")</code>

Frayer Model: Domain and Range



Frayer Model: Function and Variable



Contracts for Image-Producing Functions

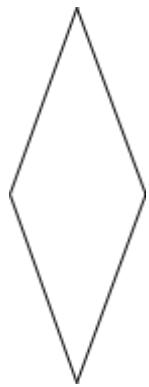
Contracts tell us how to use a function. e.g. `ellipse :: Number, Number, String, String -> Image` tells us that the name of the function is `ellipse`, and that it takes four inputs (two Numbers and two Strings). From the Contract, we know `(ellipse 100 50 "outline" "teal")` will evaluate to an `Image`.

Name	Domain	Range
<code>; ; triangle</code>	<code>:: Number, String, String</code>	<code>-> Image</code>
<code>(triangle 80 "solid" "green")</code>		
<code>; ; star</code>	<code>::</code>	<code>-></code>
<code>; ; circle</code>	<code>::</code>	<code>-></code>
<code>; ; square</code>	<code>::</code>	<code>-></code>
<code>; ; rectangle</code>	<code>::</code>	<code>-></code>
<code>; ; text</code>	<code>::</code>	<code>-></code>
<code>; ; ellipse</code>	<code>::</code>	<code>-></code>
<code>; ; regular-polygon</code>	<code>::</code>	<code>-></code>
<code>; ; rhombus</code>	<code>::</code>	<code>-></code>
<code>; ; right-triangle</code>	<code>::</code>	<code>-></code>
<code>; ; isosceles-triangle</code>	<code>::</code>	<code>-></code>
<code>; ; radial-star</code>	<code>::</code>	<code>-></code>
<code>; ; star-polygon</code>	<code>::</code>	<code>-></code>
<code>; ; triangle/sas</code>	<code>::</code>	<code>-></code>
<code>; ; triangle/asa</code>	<code>::</code>	<code>-></code>

Using Contracts (2)

Use the contracts to write expressions to generate images similar to those pictured. Go to [WeScheme](#) to test your code.

; rhombus :: Number, Number, String, String -> Image



Write an expression for a square (rotated)
using `rhombus` !

What variable changes with the first
Number?

What variable changes with the second
Number?

Triangle Contracts (SAS & ASA)

Type each expression (left) below into the [WeScheme](#) and match it to the image it creates (right).

Expression		Image
(triangle-sas 120 45 70 "solid" "black")	1	A
(triangle-sas 120 90 70 "solid" "black")	2	B
(triangle-sas 120 135 70 "solid" "black")	3	C
(triangle-sas 70 135 120 "solid" "black")	4	D

Think about how you would describe each of the arguments that `triangle-sas` takes in to someone who'd never used the function before and annotate the Contract below using descriptive variable names.

```
triangle-sas :: (  
    _____ :: Number,  
    _____ :: Number,  
    _____ :: Number,  
    _____ :: String,  
    _____ :: String  
) -> Image
```

If you have a printed workbook, add examples of each of the triangle functions we've explored to your contracts pages.

If you have time, experiment with the `triangle-asa` function.

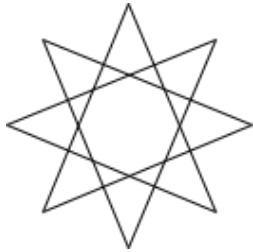
```
triangle-asa :: (  
    left-angle :: Number,  
    left-side :: Number,  
    bottom-angle :: Number,  
    style :: String  
    color :: String  
) -> Image
```

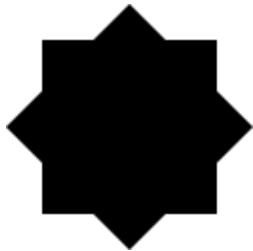
Star Polygon

```
star-polygon :: (  
    side-length :: Number,  
    points-on-polygon :: Number,  
    polygon-points-to-skip-between-star-points :: Number,  
    shading-style :: String,  
    color :: String  
) -> Image
```

Using the detailed Contract above, write expressions to create each image below.

Then make two more star polygons of your choosing. Sketch them and write expressions to generate them. Go to [WeScheme](#) to test your code.

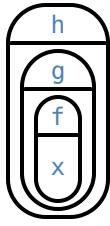
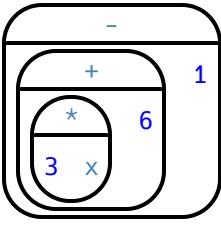
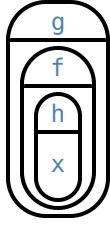
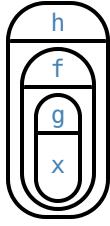
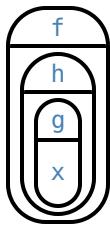




Diagramming Function Composition

f :: Number → Number Consumes a number, multiplies by 3 to produce the result	g :: Number → Number Consumes a number, adds six to produce the result	h :: Number → Number Consumes a number, subtracts one to produce the result
$f(x) = 3x$	$g(x) = x + 6$	$h(x) = x - 1$

For each function composition diagrammed below, translate it into the equivalent Circle of Evaluation for Order of Operations. Then write expressions for *both* versions of the Circles of Evaluation, and evaluate them for $x = 4$. The first one has been completed for you.

	Function Composition	Order of Operations	Translate & Evaluate	
1			Composition: $(h(g(f(x))))$	
			Operations: $(- (+ (* 3 x) 6) 1)$	
			Evaluate for $x = 4$ $h(g(f(4))) = 17$	
2			Composition:	
			Operations:	
			Evaluate for $x = 4$	
3			Composition:	
			Operations:	
			Evaluate for $x = 4$	
4			Composition:	
			Operations:	
			Evaluate for $x = 4$	

Function Composition – Green Star

1) Draw a Circle of Evaluation and write the Code for a **solid, green star, size 50**. Go to [WeScheme](#) to test your code.

Circle of Evaluation:

Code: _____

Using the star described above as the **original**, draw the Circles of Evaluation and write the Code for each exercise below. Test your code in the editor.

2) A solid, green star, that is triple the size of the original (using **scale**)

3) A solid, green star, that is half the size of the original (using **scale**)

4) A solid, green star of size 50 that has been rotated 45 degrees counter-clockwise

5) A solid, green star that is 3 times the size of the original **and** has been rotated 45 degrees

Function Composition – Your Name

You'll be investigating these functions with your partner:

```
; text :: String, Number, String -> Image  
; flip-horizontal :: Image -> Image  
; flip-vertical :: Image -> Image
```

```
; frame :: Image -> Image  
; above :: Image, Image -> Image  
; beside :: Image, Image -> Image
```

- 1) In the editor, write the code to make an image of your name in big letters in a color of your choosing using `text`. Then draw the Circle of Evaluation and write the Code that will create the image.

Circle of Evaluation for an "image of your name":

Code for an "image of your name": _____

Using the "image of your name" described above as the **original**, draw the Circles of Evaluation and write the Code for each exercise below. Test your ideas in the editor to make sure they work.

2) The framed "image of your name".

3) The "image of your name" flipped vertically.

4) The "image of your name" above "the image of your name" flipped vertically.

5) The "image of your name" flipped horizontally beside "the image of your name".

Function Composition – scale-xy

You'll be investigating these two functions with your partner:

`; scale/xy :: (Number , Number , Image) -> Image`
x-scale-factor y-scale-factor img-to-scale

`; overlay :: (Image , Image) -> Image`
top bottom

The Image:	Circle of Evaluation:	Code:
	<p>rhombus 40 90 "solid" "purple"</p>	<code>(rhombus 40 90 "solid" "purple")</code>

Starting with the image described above, write the Circles of Evaluation and Code for each exercise below. Be sure to test your code in the editor!

1) A purple rhombus that is stretched 4 times as wide.

2) A purple rhombus that is stretched 4 times as tall

3) The tall rhombus from #1 overlayed on the wide rhombus (#2).

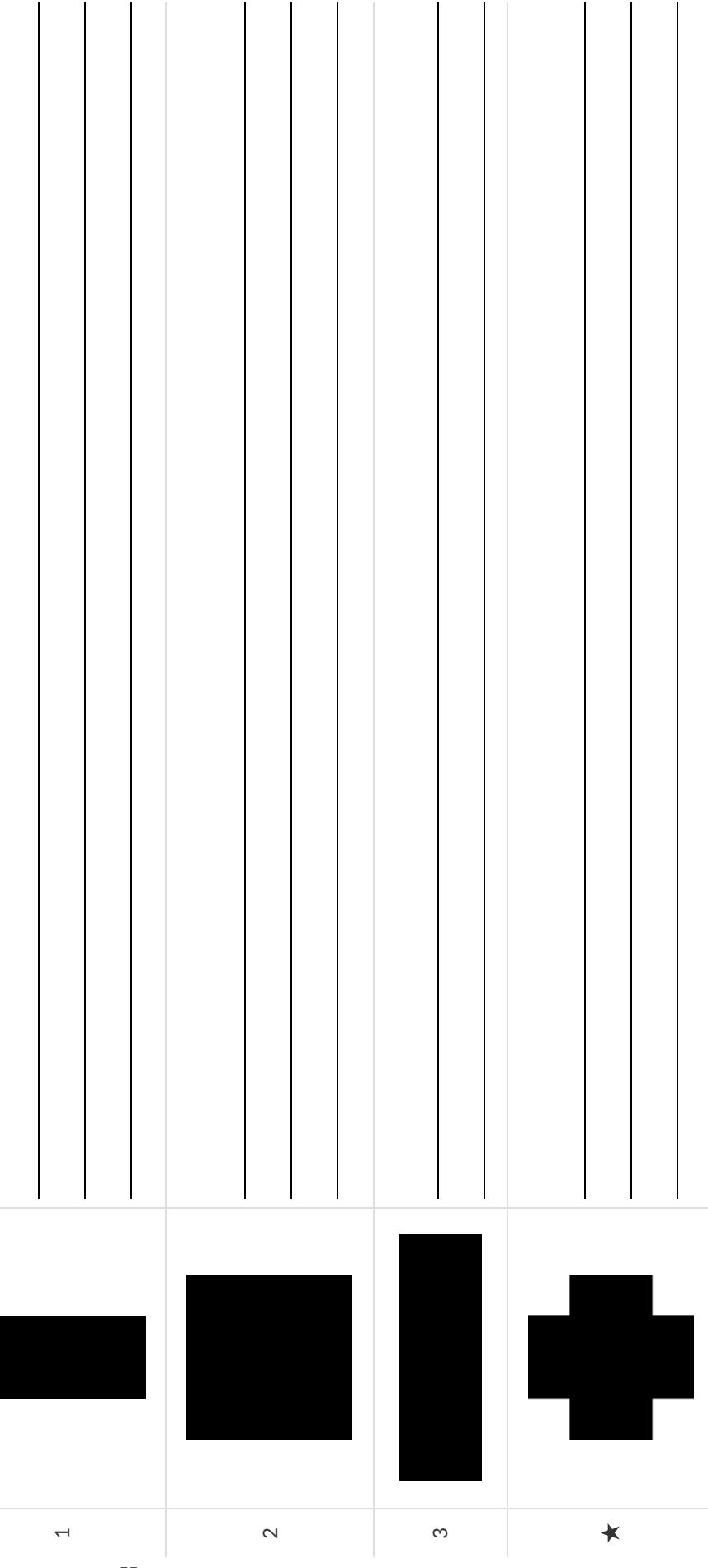
★ Overlay a red rhombus onto the last image you made in #3.

More than one way to Compose an Image!

What image will each of the four expressions below evaluate to? If you're not sure, go to [WeScheme](#) and type them into the Interactions Area and see if you can figure out how the code constructs its image.

```
(beside (rectangle 200 100 "solid" "black") (square 100 "solid" "black"))  
(scale/xy 1 2 (square 100 "solid" "black"))  
(scale 2 (rectangle 100 100 "solid" "black"))  
(above  
  (rectangle 100 50 "solid" "black")  
  (above  
    (rectangle 200 100 "solid" "black")  
    (rectangle 100 50 "solid" "black"))))
```

For each image below, identify 2 expressions that could be used to compose it. The bank of expressions at the top of the page includes one possible option for each image.



Function Composition: Matching

g :: Number → Number Consumes a number, multiplies by 6 to produce the result	h :: Number → Number Consumes a number, subtracts 6 to produce the result	j :: Number → Number Consumes a number, adds 6 to produce the result	k :: Number → Number Consumes a number, divides by 6 to produce the result
$g(n) = n \times 6$	$h(n) = n - 6$	$j(n) = n + 6$	$k(n) = n \div 6$

Draw a line from each expression on the left to the corresponding Circle of Evaluation on the right.

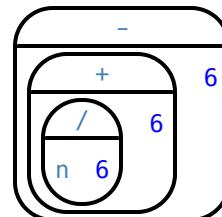
Function Notation

Circle of Evaluation

g($h(j(n))$)

1

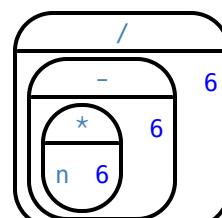
A



$h(j(k(n)))$

2

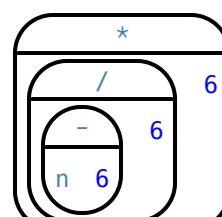
B



$g(k(h(n)))$

3

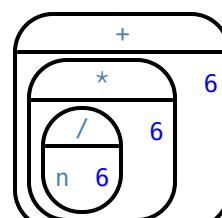
C



$k(h(g(n)))$

4

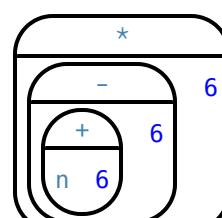
D



$j(g(k(n)))$

5

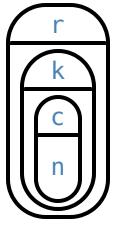
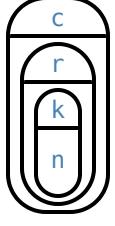
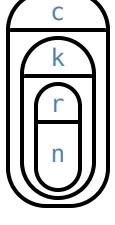
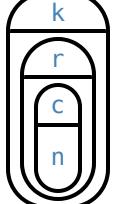
E



Diagramming Function Composition (2)

$m :: \text{Number} \rightarrow \text{Number}$ Consumes a number, divides by 2 to produce the result	$r :: \text{Number} \rightarrow \text{Number}$ Consumes a number, subtracts 5 to produce the result	$w :: \text{Number} \rightarrow \text{Number}$ Consumes a number, adds 4 to produce the result
$k(n) = n \div 2$	$r(n) = n - 5$	$c(n) = n + 4$

For each function composition diagrammed below, translate it into the equivalent Circle of Evaluation for Order of Operations. Then write expressions for *both* versions of the Circles of Evaluation, and evaluate them for $n = 7$.

	Function Composition	Order of Operations	Translate & Evaluate
1			<p>Composition:</p> <hr/> <p>Operations:</p> <hr/> <p>Evaluate for $n = 7$</p>
2			<p>Composition:</p> <hr/> <p>Operations:</p> <hr/> <p>Evaluate for $n = 7$</p>
3			<p>Composition:</p> <hr/> <p>Operations:</p> <hr/> <p>Evaluate for $n = 7$</p>
4			<p>Composition:</p> <hr/> <p>Operations:</p> <hr/> <p>Evaluate for $n = 7$</p>

Defining Values

In math, we use **values** like -98.1 , $\frac{2}{3}$ and 42 . In math, we also use **expressions** like 1×3 , $\sqrt{16}$, and $5 - 2$. These evaluate to results, and typing any of them in as code produces some answer.

Math also has **definitions**. These are different from values and expressions, because *they do not produce results*. Instead, they simply create names for values, so that those names can be re-used to make the Math simpler and more efficient.

Definitions always have both a name and an expression. The name goes on the left and the value-producing expression goes on the right, separated by an equals sign:

`x = 4`

`y = 9 + x`

The name is defined to be the result of evaluating the expression. Using the above examples, we get "x is defined to be 4, and y is defined to be 13. **Important: there is no "answer" to a definition**, and typing in a definition as code will produce no result.

Notice that *definitions can refer to previous definitions*. In the example above, the definition of `y` refers to `x`. But `x`, on the other hand, *cannot* refer to `y`. Once a value has been defined, it can be used in later expressions.

In WeScheme, these definitions are written a little differently, making it clear that we're talking about definitions:

Try typing these definitions into the Definitions Area on the left, clicking "Run", and then *using* them in the Interactions Area on the right.

`(define x 4)`

`(define y (+ 9 x))`

Just like in math, definitions in our programming language can only refer to previously-defined values.

Here are a few more value definitions. Feel free to type them in, and make sure you understand them.

`(define x (+ 5 1))`

`(define y (* x 7))`

`(define food "Pizza!")`

`(define dot (circle y "solid" "red"))`

Defining Values - Explore

Open the [Defining Values Starter File](#) and click "Run".

1) What do you Notice?

2) What do you Wonder?

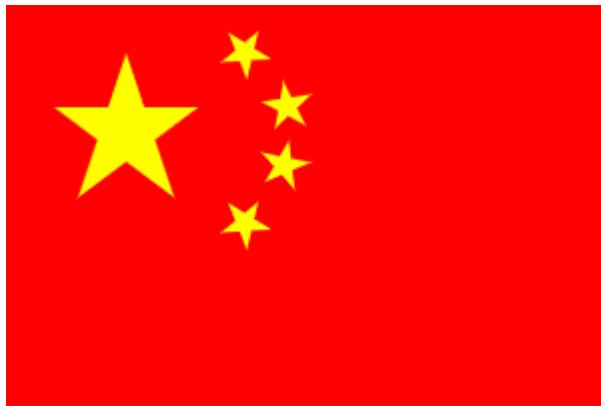
3) Look at the expressions listed below. What do you expect each of them to produce? Write your predictions below, and then test them out one at a time in the Interactions Area.

- `x` _____
- `(+ x 5)` _____
- `(- y 9)` _____
- `(* x y)` _____
- `z` _____
- `t` _____
- `gold-star` _____
- `my-name` _____
- `swamp` _____
- `(* 5 pi)` _____

4) What have you learned about defining values?

5) Define at least 2 more variables in the Definitions Area, click "Run" and test them out. Once you know they're working, record the code you used below.

Defining Values - Chinese Flag



1) What image do you see repeated in the flag? _____

2) In the code below, highlight or circle all instances of the expression that makes the repeated image.

```
(define China
  (put-image (
    rotate 40 (star 15 "solid" "yellow"))
  120 175
    (put-image (
      rotate 80 (star 15 "solid" "yellow")))
  140 150
    (put-image (
      rotate 60 (star 15 "solid" "yellow")))
  140 120
    (put-image (
      rotate 40 (star 15 "solid" "yellow")))
  120 90
    (put-image (
      scale 3 (star 15 "solid" "yellow")))
  60 140
    (rectangle 300 200 "solid" "red"))))))
```

3) Write the code to define a value for the repeated expression.

4) Open the [Chinese Flag Starter File](#) and click "Run".

- Type `china` into the Interactions Area and hit **Enter**.
- **Save a copy** of the file, and simplify the flag code using the value you defined.
- Click "Run", and confirm that you still get the same image as the original.
- Now change the color of all of the stars to black, in both files.
- Then change the size of the stars.

5) Why is it helpful to define values for repeated images?

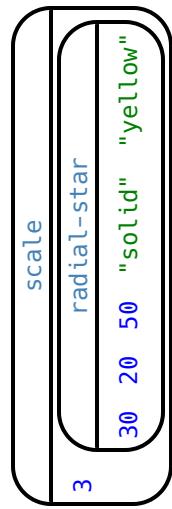
★ This file uses a function we haven't seen before! What is it? _____ Can you figure out its Contract?
Hint: Focus on the last instance of the function.

Why Define Values?

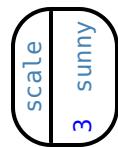
1) Complete the table using the first row as an example.

2) Write the code to define the value of `sunny`.

Original Circle of Evaluation & Code



Use the defined value `sunny` to simplify!

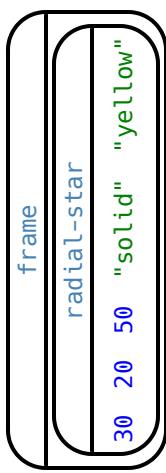


Code:

```
(scale 3 (radial-star 30 20 50 "solid" "yellow"))
```

→

Use the defined value `sunny` to simplify!



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Code:

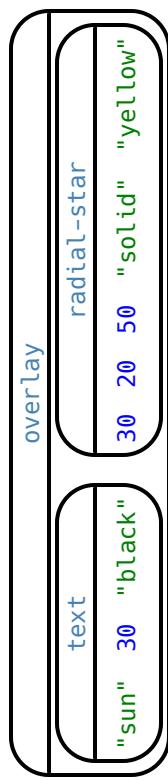
```
(frame (radial-star 30 20 50 "solid" "yellow"))
```

→

Code:

```
(scale 3 sunny)
```

→



Code:

```
(frame (sunny))
```

→

Code:

```
(scale 3 sunny)
```

→

Code:

```
(overlay (text "sun" 30 "black") (radial-star 30 20 50 "solid" "yellow"))
```

3) Test your code in the editor and make sure it produces what you would expect it to.

Which Value(s) Would it Make Sense to Define?

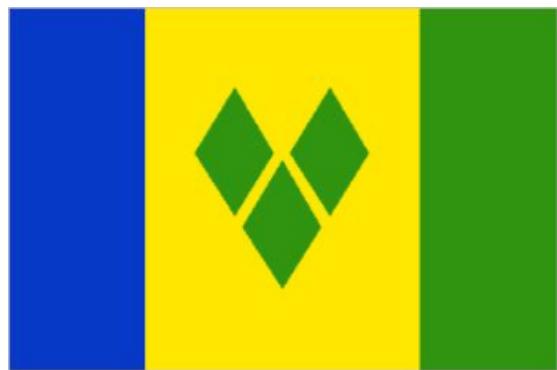
For each of the images below, identify which element(s) you would want to define before writing code to compose the image.

Hint: what gets repeated?

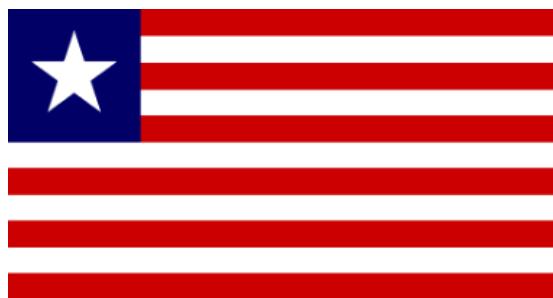
Philippines



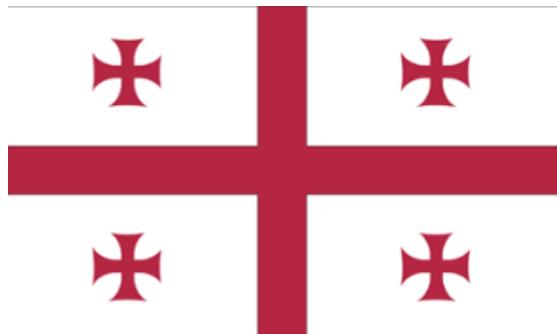
St. Vincent & the Grenadines



Liberia



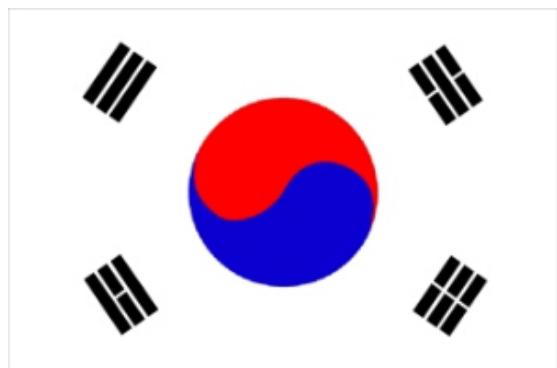
Republic of Georgia



Quebec



South Korea



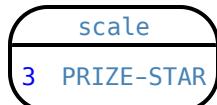
Writing Code using Defined Values

1) On the line below, write the Code to define `PRIZE-STAR` as a pink, outline star of size 65.

Using the `PRIZE-STAR` definition from above, draw the Circle of Evaluation and write the Code for each of the exercises. Be sure to test out your code in [WeScheme](#) before moving onto the next item. One Circle of Evaluation has been done for you.

2 The outline of a pink star that is three times the size of the original (using `scale`)

Circle of Evaluation:



3 The outline of a pink star that is half the size of the original (using `scale`)

Circle of Evaluation:

Code:

4 The outline of a pink star that is rotated 45 degrees
(*It should be the same size as the original.*)

Circle of Evaluation:

Code:

5 The outline of a pink star that is three times as big as the original and has been rotated 45 degrees

Circle of Evaluation:

Code:

Code:

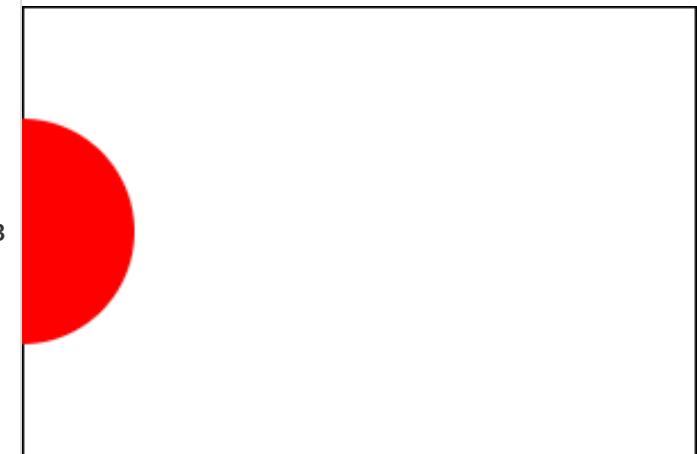
6) How does defining values help you as a programmer?

Estimating Coordinates

```
(define dot (circle 50 "solid" "red"))
(define background (rectangle 300 200 "outline" "black"))
```

Think of the background image as a sheet of graph paper with the origin (0,0) in the bottom left corner. The width of the rectangle is 300 and the height is 200. The numbers in `put-image` specify a point on that graph paper, where the center of the top image (in this case `dot`) should be placed.

Estimate: What coordinates for the `dot` created each of the following images?

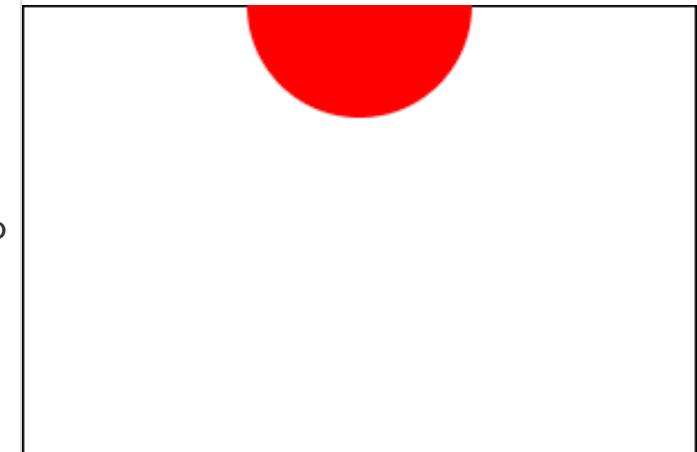


(`put-image dot _____ background`)

(`put-image dot _____ background`)



(`put-image dot _____ background`)



(`put-image dot _____ background`)

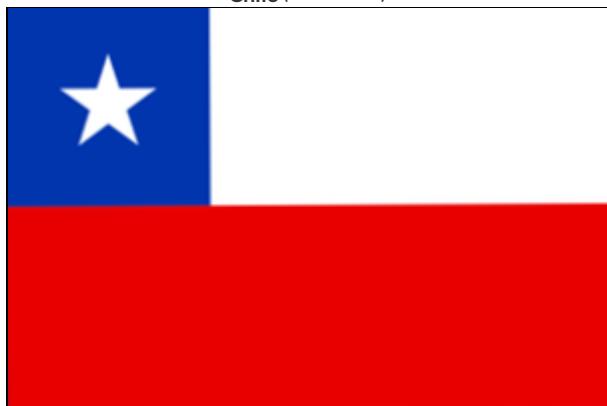
Decomposing Flags

Each of the flags below is shown with their width and height. Identify the shapes that make up each flag. Use the flag's dimensions to estimate the dimensions of the different shapes. Then estimate the x and y coordinates for the point at which the center of each shape should be located on the flag. Hint: The bottom left corner of each flag is at (0,0) and the top right corner is given by the flags dimensions.

Cameroon (450 x 300)



Chile (420 x 280)



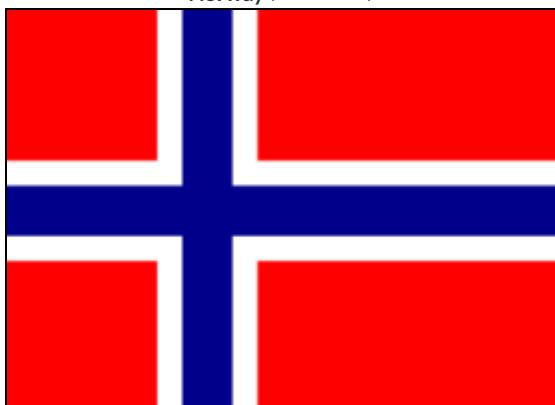
shape:	color:	width:	height:	x	y

shape:	color:	width:	height:	x	y

Panama (300 x 200)



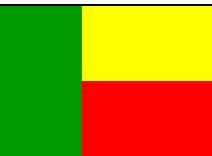
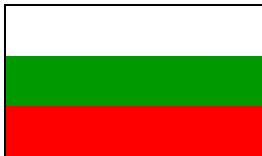
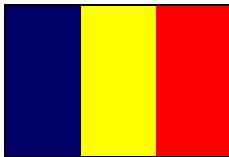
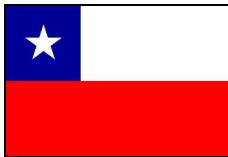
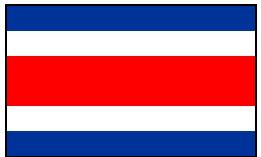
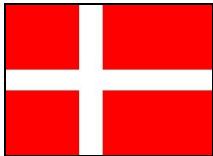
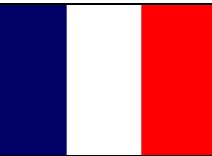
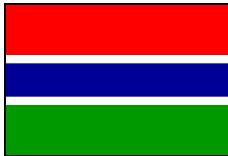
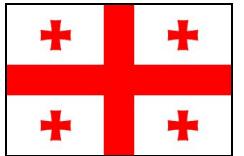
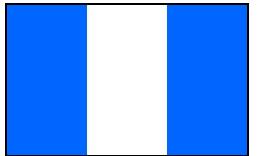
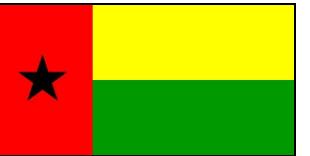
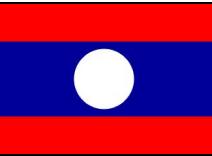
Norway (330 x 240)



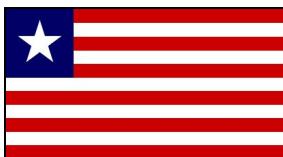
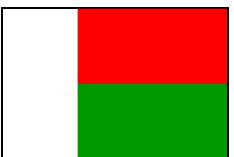
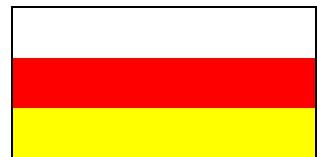
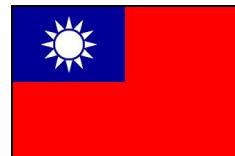
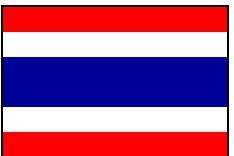
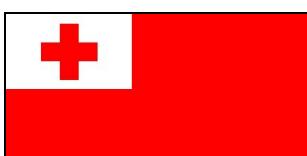
shape:	color:	width:	height:	x	y

shape:	color:	width:	height:	x	y

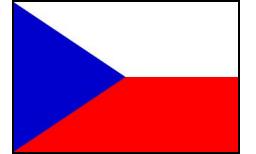
These flags can all be made using a combination of put-image, above, beside, and rotate.

 Armenia (1:2)	 Belgium (13:15)	 Bolivia (15:22)	 Benin (2:3)
 Bulgaria (3:5)	 Botswana (2:3)	 Burkina Faso (2:3)	 Cameroon (2:3)
 Chad (2:3)	 Chile (2:3)	 Costa Rica (3:5)	 Cote d'Ivoire (2:3)
 Denmark (28:37)	 Estonia (7:11)	 Finland (11:18)	 France (2:3)
 Gabon (3:4)	 The Gambia (2:3)	 Georgia (2:3)	 Germany (3:5)
 Ghana (2:3)	 Greece (2:3)	 Guatemala (5:8)	 Guinea-Bissau (1:2)
 Hungary (1:2)	 Ireland (1:2)	 Italy (2:3)	 Laos (2:3)

These flags can all be made using a combination of put-image, above, beside, and rotate.

			
Liberia (1:2)	Lithuania (1:2)	Madagascar (2:3)	Mali (2:3)
			
Mauritius (2:3)	Myanmar (2:3)	The Netherlands (2:3)	Niger (6:7)
			
Panama (2:3)	Romania (2:3)	Russia (2:3)	Senegal (2:3)
			
Sierra Leone (2:3)	South Ossetia (1:2)	Suriname (2:3)	Sweden (5:8)
			
Switzerland (1:1)	Taiwan (Republic of China) (2:3)	Thailand (2:3)	Togo (1:1.618)
			
Tonga (1:2)	United Arab Emirates (1:2)	Yemen (2:3)	

These flags can all be made using a combination of `put-image`, `above`, `beside`, and `rotate`.

			
Bahamas (1:2)	Cuba (1:2)	Czech Republic (2:3)	Djibouti (2:3)
			
Timor-Leste (1:2)	Guyana (3:5)	Jordan (1:2)	Palestine (1:2)
			
Saint Lucia (1:2)	South Sudan (1:2)	Sudan (1:2)	Tunisia (2:3)
			
Artsakh (1:2)	Azerbaijan (1:2)	Sao Tome & Principe (1:2)	

Notice and Wonder

As you investigate the [Blank Game Starter File](#) with your partner, record what you Notice, and then what you Wonder.
Remember, "Notices" are statements, not questions.

What do you Notice?	What do you Wonder?

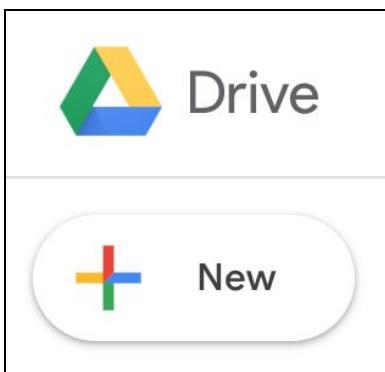
Quick Guide to Saving Images to Google Drive

Windows/MacOS:

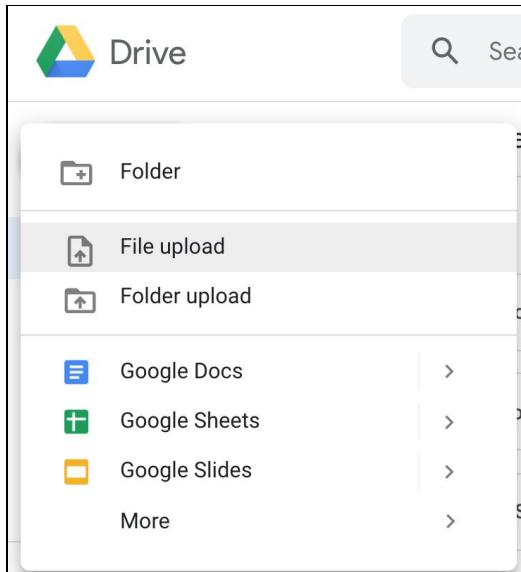
1. Find the image you'd like to save. If using Google Image Search or a similar search engine, click once on the image to expand it.
2. Right-click (or 2-finger click on trackpad) on the expanded image.
3. Select "Save Image As" (or "Save Picture As").



4. Name the file and select a location on your computer to save it to. (If saving several images, you can make a folder to make uploading faster.)
5. Open Google Drive (drive.google.com) and sign in if needed.
6. Click the "New" button near the top left.

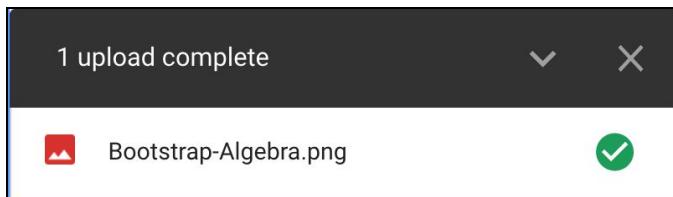


7. Select “File upload” (or “Folder upload” if you have a folder of images to upload).



8. Select the file (or folder) you want and click “Open”.

9. Wait for the upload to finish (a green checkmark will appear).

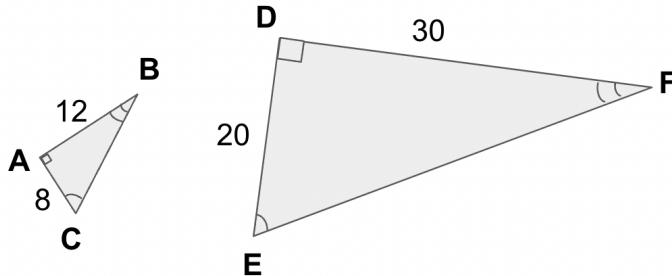


10. In WeScheme (www.wescheme.org), click the button and, if prompted, select the Google account you’re using. (If the “Images” button isn’t there, you’ll need to sign in to WeScheme.)

Select your image and you’ll see the code for your image (using the `bitmap/url` function) appear!

Scaling Practice

`; scale :: (Number, Image) -> Image`
scale-factor image-producing-expression



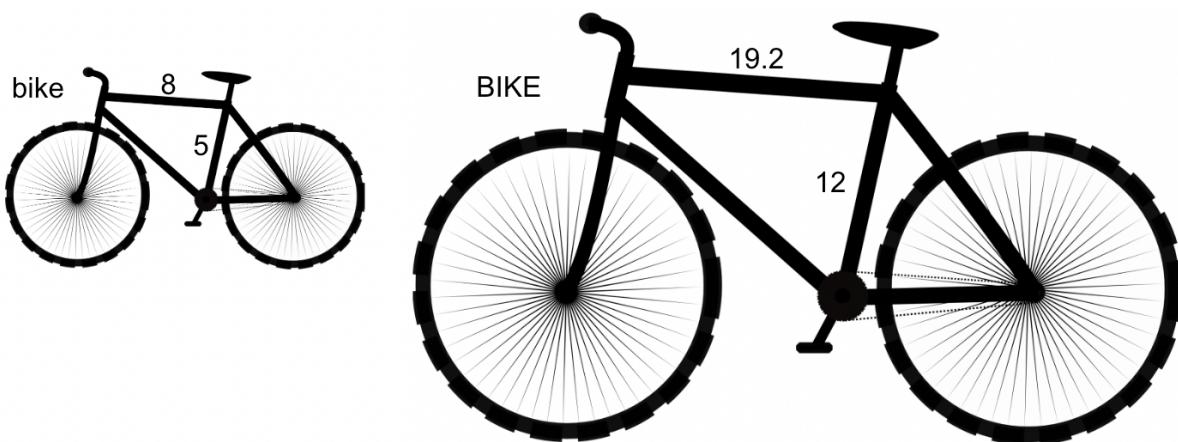
The class was given an assignment to generate triangle DEF by scaling triangle ABC .

- Jourdan wrote: (`scale 1.5 ABC`)
- Roux wrote: (`scale (/ 30 12) ABC`)
- Zuni wrote: (`scale (/ 8 20) ABC`)
- Cedric wrote: (`scale (/ 30 20) ABC`)
- Josie wrote: (`scale 2.5 ABC`)
- Celine wrote: (`scale (/ 20 8) ABC`)

1) Whose expressions will work? _____

2) How do you know? _____

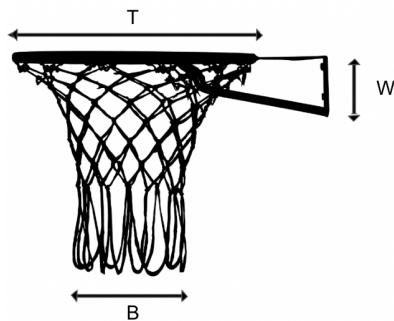
3) Which one would you use and why? _____



4) Write at least two expressions for generating the image titled BIKE by scaling `bike` .

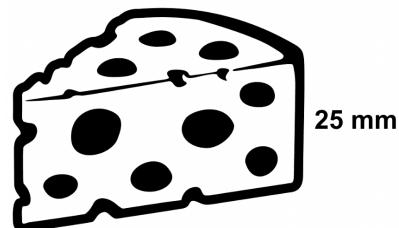
Scaling Practice (2)

Part 1: Complete the table below by filling in the missing fields for the original image and the three transformations.



Description	Original	Double-size	Triple-size	
expression	hoop	(scale 2 hoop)		(scale 0.5 hoop)
percent of original	100%		300%	50%
length of T		36	54	9
length of B	6			3
length of W		4		1

Part 2: Raffi wants to use this `cheese` image in his game. In thinking through what size he wants it to be, he comes up with the list of transformations described below. Help him to translate his ideas into code by matching each description to a `scale` expression.



Desired Resizing			Expression
New height of 75 mm	1	A	(scale 1.5 cheese)
60% as tall	2	B	(scale 3 cheese)
New height of 30 mm	3	C	(scale 2 cheese)
One and a half times as tall	4	D	(scale 1.2 cheese)
New height of 5 mm	5	E	(scale 0.2 cheese)
200% of the original size	6	F	(scale 0.6 cheese)
3/4 as tall	7	G	(scale 0.75 cheese)
New height of 12.5 mm	8	H	(scale 0.05 cheese)
5% as tall	9	I	(scale 0.5 cheese)

The Great gt domain debate!

Kermit: The domain of `gt` is `Number`, `String`, `String`.

Oscar: The domain of `gt` is `Number`.

Ernie: I'm not sure who's right!

In order to make a triangle, we need a size, a color and a fill style...

but all we had to tell our actor was (`gt 20`) ...and they returned (`triangle 20 "solid" "green"`).

Please help us!

1) What is the correct domain for `gt`?

2) What could you tell Ernie to help him understand how you know?

Let's Define Some New Functions!

1) Let's define a function `rs` to generate solid red squares of whatever size we give them!

If I say `(rs 5)`, what would our actor need to say?

Let's write a few more examples:

`(rs ____)` → _____

`(rs ____)` → _____

`(rs ____)` → _____

What changes in these examples? Name your variable(s): _____

Let's define our function using the variable:

`(define (rs _____))` _____

2) Let's define a function `bigc` to generate big solid circles of size 100 in whatever color we give them!

If I say `(bigc "orange")`, what would our actor need to say?

Let's write a few more examples:

`(bigc _____)` → _____

`(bigc _____)` → _____

`(bigc _____)` → _____

What changes in these examples? Name your variable(s): _____

Let's define our function using the variable:

`(define (bigc _____))` _____

3) Let's define a function `ps` to build a pink star of size 50, with the input determining whether it's solid or outline!

If I say `(ps "outline")`, what would our actor need to say?

Write examples for all other possible inputs:

`(ps _____)` → _____

`(ps _____)` → _____

What changes in these examples? Name your variable(s): _____

Let's define our function using the variable:

`(define (ps _____))` _____

4) Add these new function definitions to your [gt Starter File](#) and test them out!

Let's Define Some More New Functions!

1) Let's define a function `sun` to write SUNSHINE in whatever color and size we give it!

If I say `(sun 5 "blue")`, what would our actor need to say?

Let's write a few more examples:

`(sun _____) → _____`

`(sun _____) → _____`

`(sun _____) → _____`

What changes in these examples? Name your variable(s): _____
Let's define our function using the variable(s): _____

`(define (sun _____))`

2) Let's define a function `me` to generate your name in whatever size and color we give it!

If I say `(me 18 "gold")`, what would our actor need to say?

Let's write a few more examples:

`(me _____) → _____`

`(me _____) → _____`

`(me _____) → _____`

What changes in these examples? Name your variable(s): _____
Let's define our function using the variable(s): _____

`(define (me _____))`

3) Let's define a function `gr` to build a solid, green rectangle of whatever height and width we give it!

If I say `(gr 10 80)`, what would our actor need to say?

Let's write a few more examples:

`(gr _____) → (rectangle _____ "solid" "green")`

`(gr _____) → (rectangle _____ "solid" "green")`

`(gr _____) → (rectangle _____ "solid" "green")`

What changes in these examples? Name your variable(s): _____
Let's define our function using the variable(s): _____

`(define (gr _____))`

4) Add these new function definitions to your [gt Starter File](#) and test them out!

Describe and Define Your Own Functions!

1) Let's define a function _____ to generate...

If I say _____, what would our actor need to say? _____

Let's write a few more examples:

(_____) → (_____)

(_____) → (_____)

(_____) → (_____)

What changes in these examples? Name your variable(s): _____

Let's define our function using the variable.

fun _____ (_____) : _____(_____) **end**

2) Let's define a function _____ to generate...

If I say _____, what would our actor need to say? _____

Let's write a few more examples:

(_____) → (_____)

(_____) → (_____)

(_____) → (_____)

What changes in these examples? Name your variable(s): _____

Let's define our function using the variable.

fun _____ (_____) : _____(_____) **end**

3) Let's define a function _____ to generate...

If I say _____, what would our actor need to say? _____

Let's write a few more examples:

(_____) → (_____)

(_____) → (_____)

(_____) → (_____)

What changes in these examples? Name your variable(s): _____

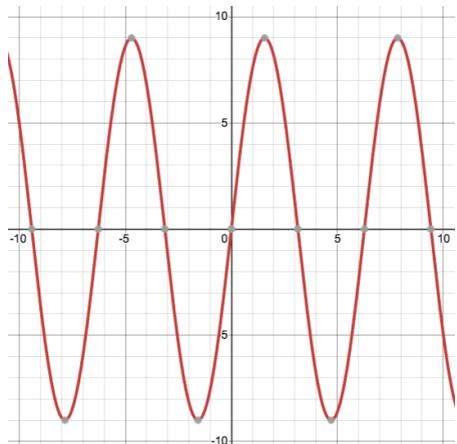
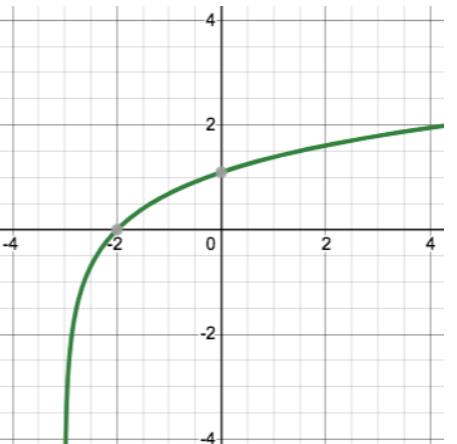
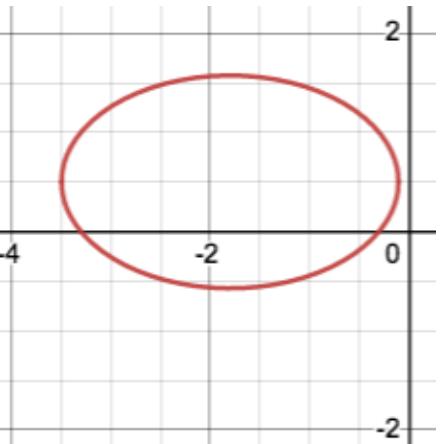
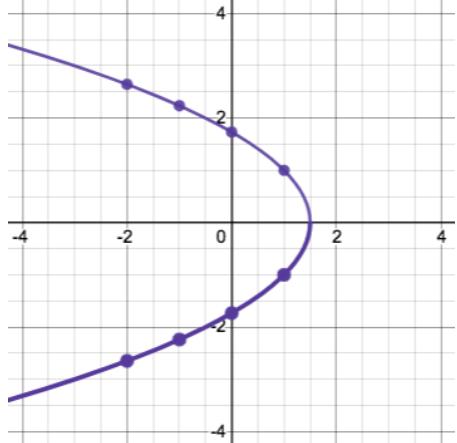
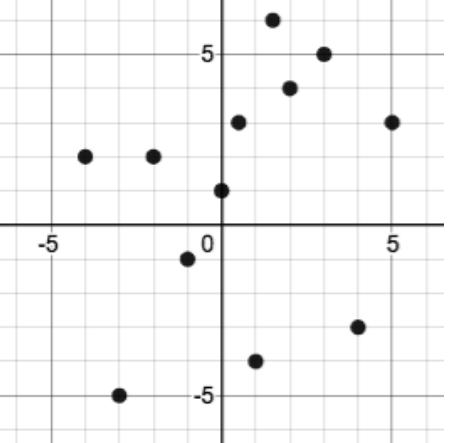
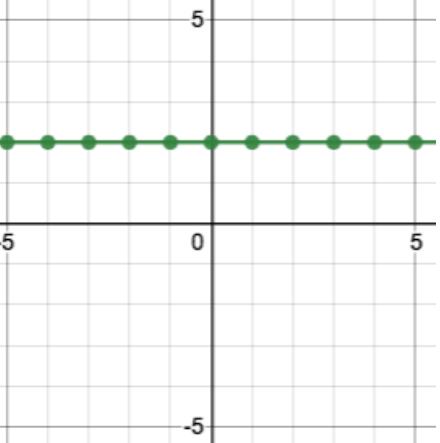
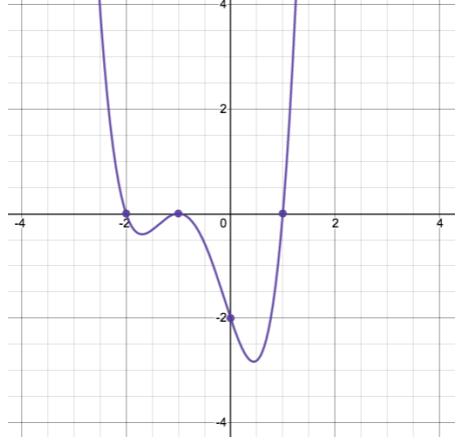
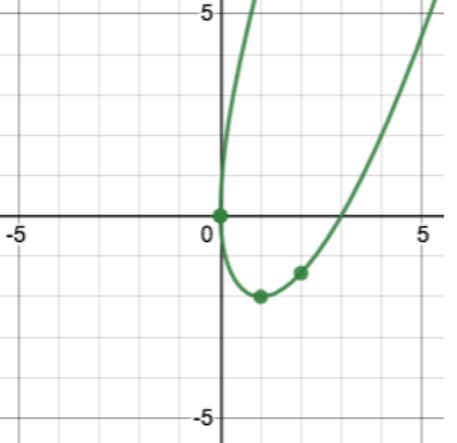
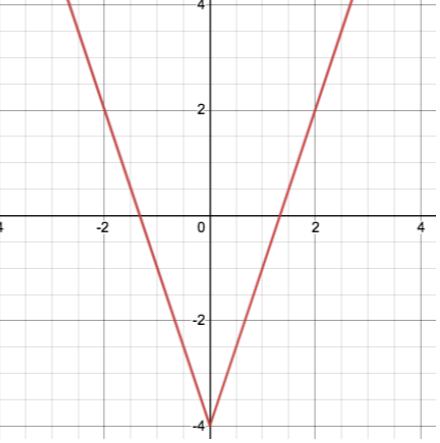
Let's define our function using the variable.

fun _____ (_____) : _____(_____) **end**

4) Add your new function definitions to your [gt Starter File](#) and test them out!

Identifying Functions from Graphs

Decide whether each graph below is a function. If it's not, prove it by drawing a vertical line that crosses the plot at more than one point.

		
Function or Not a Function? 	Function or Not a Function? 	Function or Not a Function? 
Function or Not a Function? 	Function or Not a Function? 	Function or Not a Function? 
Function or Not a Function?	Function or Not a Function?	Function or Not a Function?

Identifying Functions from Graphs (2)

Decide whether each graph below is a function. If it's not, prove it by drawing a vertical line that crosses the plot at more than one point.

Function or Not a Function? 	Function or Not a Function? 	Function or Not a Function?
Function or Not a Function? 	Function or Not a Function? 	Function or Not a Function?
Function or Not a Function?	Function or Not a Function?	Function or Not a Function?

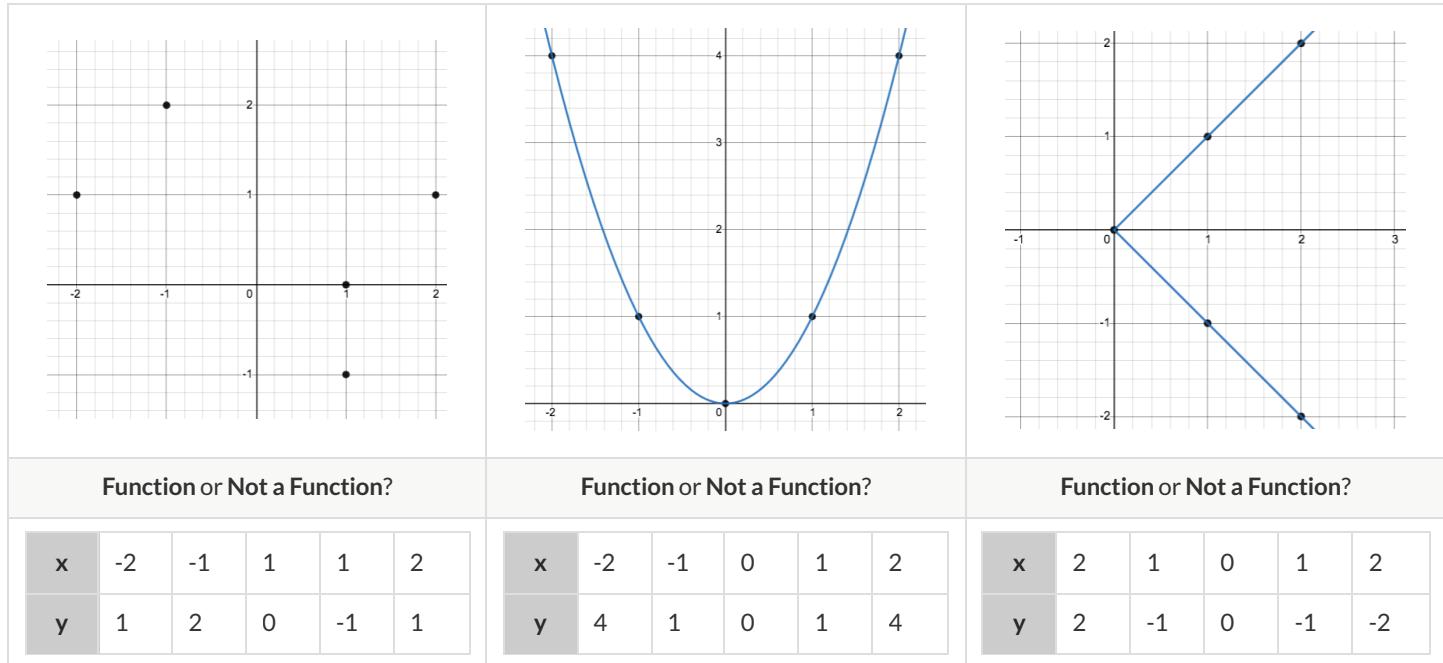
Notice and Wonder - Functions

Write down what you Notice and Wonder about the graphs you've just seen. At a later point you will *also* use this page to record what you Notice and Wonder about the tables you'll see. Remember: "Notices" should be statements, not questions!

What do you Notice?	What do you Wonder?

How Tables Fail the Vertical Line Test

1) Each of the graphs below is also represented by a table. Use the vertical line test to determine whether or not each graph represents a function.



2) For each graph that failed the vertical line test, label the offending points with their coordinates.

3) Find the same coordinates in the table below the graph and circle or highlight them.

4) What do the tables of the non-functions have in common? What could you look for in other tables to identify whether or not they could represent a function?

5) Use the process you just described to determine whether each table below could represent a function. Circle or highlight the points that would end up on the same vertical line.

x	y
0	-2
1	-2
2	-2
3	-2
4	-2

x	y
0	-2
1	1
2	4
3	7
3	10

x	y
0	3
1	4
-1	5
2	6
-2	7

x	y
1	0
0	1
1	2
2	3
3	4

Function or Not?

Function or Not?

Function or Not?

Function or Not?

Identifying Functions from Tables

Decide whether or not each table below could represent a function. If not, circle what you see that tells you it's not a function. *In a function, there is exactly one y-value (or output) for each x-value (or input). If a table has more than one y-value (or output) for the same x-value (or input), it can not represent a function.*

x	y
0	3
1	2
2	5
3	6
4	5

x	y
5	3
1	4
-3	5
3	6
2	7

input	output
0	2
5	2
2	2
6	2
3	2

x	y
1	0
1	1
1	2
1	3
1	4

Function or Not?

Function or Not?

Function or Not?

Function or Not?

tickets	\$
2	0
1	2
2	4
3	6
4	8

input	output
-4	-2
-3	-1
-2	0
-1	1
0	2

x	y
10	9
3	2
9	8
17	16
3	5

C	F
-40	-40
0	32
10	50
37	98.6
100	212

Function or Not?

Function or Not?

Function or Not?

Function or Not?

input	output
0	7
-1	2
4	3
8	6
-5	-8

\$	games
10	5
11	25
12	45
13	65
14	85

x	y
8	10
6	5
4	0
6	-5
8	-10

miles	minutes
0	0
1	2
2	4
3	6
4	8

Function or Not?

Function or Not?

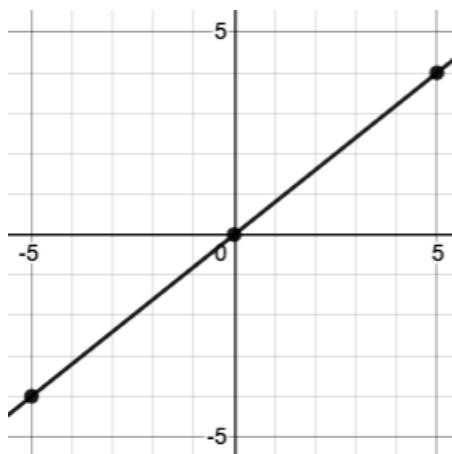
Function or Not?

Function or Not?

Identifying Functions from Tables & Graphs

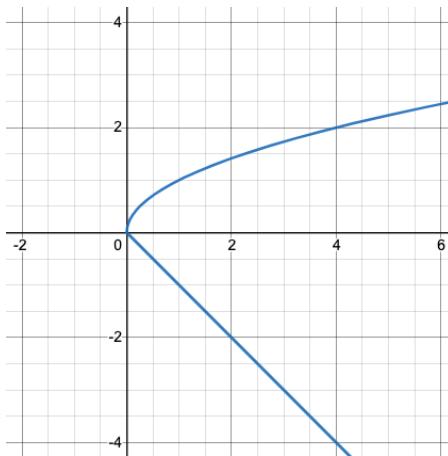
Decide whether or not each table or graph below could represent a function. If not, circle what tells you it's not a function. *In a function, there is exactly one y-value for each x-value. If a table or graph has more than one y-value for the same x-value, it can not represent a function.*

x	y
-2	5
0	2
2	4
4	7
6	8



x	y
0	7
1	2
1	3
2	6
3	-8

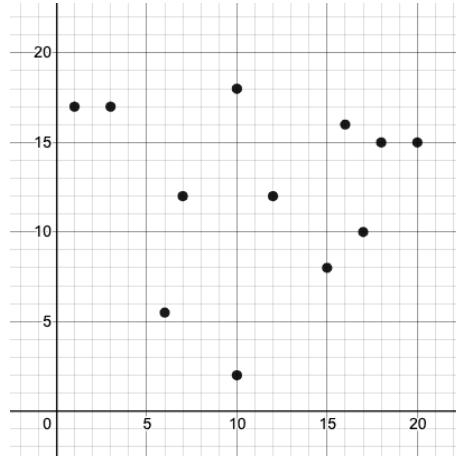
Function or Not a Function?



Function or Not a Function?

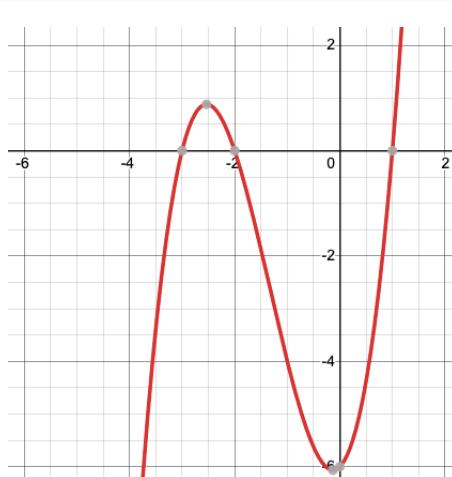
x	y
-1.5	-2
-1	-1
-0.5	0
0	1
0.5	2

Function or Not a Function?



Function or Not a Function?

x	y
-1	1.5
0	1.5
1	1.5
2	1.5
3	1.5



x	y
8	1
5	2
4	3
5	4
8	5

Function or Not a Function?

Function or Not a Function?

Function or Not a Function?

Matching Examples and Definitions (Math)

Match each of the function definitions on the left with the corresponding table on the right.

It may help to circle or highlight what's changing in the $f(x)$ column of the table!

Function Definitions

Example Tables

$$f(x) = x - 2$$

1

A

x	$f(x)$
1	2×1
2	2×2
3	2×3

$$f(x) = 2x$$

2

B

x	$f(x)$
15	$15 - 2$
25	$25 - 2$
35	$35 - 2$

$$f(x) = 2x + 1$$

3

C

x	$f(x)$
10	$2 + 10$
15	$2 + 15$
20	$2 + 20$

$$f(x) = 1 - 2x$$

4

D

x	$f(x)$
0	$1 - 2(0)$
1	$1 - 2(1)$
2	$1 - 2(2)$

$$f(x) = 2 + x$$

5

E

x	$f(x)$
10	$2(10) + 1$
20	$2(20) + 1$
30	$2(30) + 1$

Function Notation - Substitution

Complete the table below, by substituting the given value into the expression and evaluating.

Function Definition	Expression	Substitution	Evaluates to
$f(x) = x + 2$	$f(3)$	$3 + 2$	5
$g(x) = x - 1$	$g(6)$		
$h(x) = 3x$	$h(4)$		
$k(x) = 2x - 1$	$k(5)$		

Now that you understand how to evaluate an expression, let's get some more practice! The table below includes four different functions. Beneath each of them are a collection of different expressions to evaluate.

$m(x) = -2x + 3$	$n(x) = -x + 7$	$v(x) = 10x - 8$	$w(x) = x^2$
$m(3) = -2(3) + 3$	$n(5) =$	$v(7) =$	$w(-2) =$
-3			
$m(-4) =$	$n(-2) =$	$v(0) =$	$w(10) =$
$m(0) =$	$n(3.5) =$	$v(-10) =$	$w(0) =$
$m(0.5) =$	$n(0) =$	$v(2.5) =$	$w(1.5) =$

What do you Notice?

What do you Wonder?

Function Notation - Graphs

Find the values described by the expressions below each graph.

$f(-5) = \underline{\hspace{2cm}}$ $f(5) = \underline{\hspace{2cm}}$	$g(-2) = \underline{\hspace{2cm}}$ $g(0) = \underline{\hspace{2cm}}$	$h(0) = \underline{\hspace{2cm}}$ $h(1) = \underline{\hspace{2cm}}$
$j(-2) = \underline{\hspace{2cm}}$ $j(0) = \underline{\hspace{2cm}}$	$k(3) = \underline{\hspace{2cm}}$ $k(-2.5) = \underline{\hspace{2cm}}$	$m(0) = \underline{\hspace{2cm}}$ $m(1) = \underline{\hspace{2cm}}$
$n(2) = \underline{\hspace{2cm}}$ $n(-\infty) = \underline{\hspace{2cm}}$	$v(5) = \underline{\hspace{2cm}}$ $v(2) = \underline{\hspace{2cm}}$	$w(-2) = \underline{\hspace{2cm}}$ $w(0) = \underline{\hspace{2cm}}$

Function Notation - Tables

Find the values described by the expressions below each table.

Note: not all of the relationships here are actually functions!

x	$f(x)$
0	0
1	2
2	4
3	6
4	8

x	$g(x)$
5	3
1	4
-3	5
3	6
2	7

x	$h(x)$
0	2
5	2
2	2
6	2
3	2

x	$y(x)$
1	0
1	1
1	2
1	3
1	4

$$f(3) = \underline{\hspace{2cm}}$$

$$g(1) = \underline{\hspace{2cm}}$$

$$h(0) = \underline{\hspace{2cm}}$$

$$y(1) = \underline{\hspace{2cm}}$$

$$f(4) = \underline{\hspace{2cm}}$$

$$g(3) = \underline{\hspace{2cm}}$$

$$h(3) = \underline{\hspace{2cm}}$$

$$y(8) = \underline{\hspace{2cm}}$$

a	$b(a)$
-4	-2
-3	-1
-2	0
-1	1
0	2

c	$d(c)$
0	3
1	2
2	5
3	6
4	5

n	$m(n)$
0	0
-1	-1
-2	-2
-3	-3
-4	-4

q	$p(q)$
2	0
1	2
2	4
3	6
4	8

$$b(-1) = \underline{\hspace{2cm}}$$

$$d(2) = \underline{\hspace{2cm}}$$

$$m(0) = \underline{\hspace{2cm}}$$

$$p(1) = \underline{\hspace{2cm}}$$

$$b(0) = \underline{\hspace{2cm}}$$

$$d(4) = \underline{\hspace{2cm}}$$

$$m(-3) = \underline{\hspace{2cm}}$$

$$p(2) = \underline{\hspace{2cm}}$$

s	$r(s)$
0	7
-1	2
4	3
8	6
-5	-8

w	$v(w)$
10	5
11	25
12	45
13	65
14	85

y	$z(y)$
8	10
6	5
4	0
5	-5
8	-10

$time$	$l(time)$
10	9
3	2
9	8
17	16
5	5

$$r(-1) = \underline{\hspace{2cm}}$$

$$v(11) = \underline{\hspace{2cm}}$$

$$z(6) = \underline{\hspace{2cm}}$$

$$l(10) = \underline{\hspace{2cm}}$$

$$r(8) = \underline{\hspace{2cm}}$$

$$v(14) = \underline{\hspace{2cm}}$$

$$z(2) = \underline{\hspace{2cm}}$$

$$l(3) = \underline{\hspace{2cm}}$$

Function Notation Challenge

$$f(x) = 2x - 3$$

$$g(x) = 3x + 2$$

$$h(x) = x^2$$

$$k(x) = 2^x$$

Evaluate each expression below using the function definitions above.

$$f(4)$$

$$f(4) - 3$$

$$f(4 - 3)$$

$$g(4) + h(4)$$

$$3 - f(5)$$

$$h(3) - k(3)$$

$$f(-5)$$

$$g^{(1/3)}$$

$$5 \times g(4)$$

$$h(4) + f(6) - 5$$

$$h(2) - 5$$

$$h(2-5)$$

$$k(4-1)$$

$$k(4) - 1$$

Function Notation - Piecewise Graphs

Find the values described by the expressions below each graph.

$f(-2) = \underline{\hspace{2cm}}$	$g(1) = \underline{\hspace{2cm}}$	$h(1) = \underline{\hspace{2cm}}$
$f(0) = \underline{\hspace{2cm}}$ 	$g(4) = \underline{\hspace{2cm}}$ 	$h(2) = \underline{\hspace{2cm}}$
$j(0) = \underline{\hspace{2cm}}$	$k(1) = \underline{\hspace{2cm}}$	$m(2) = \underline{\hspace{2cm}}$
$j(4) = \underline{\hspace{2cm}}$	$k(3) = \underline{\hspace{2cm}}$	$m(4) = \underline{\hspace{2cm}}$
$n(2) = \underline{\hspace{2cm}}$	$v(3) = \underline{\hspace{2cm}}$	$w(1) = \underline{\hspace{2cm}}$
$n(5) = \underline{\hspace{2cm}}$	$v(4) = \underline{\hspace{2cm}}$	$w(4) = \underline{\hspace{2cm}}$

Defining Functions

Functions can be viewed in *multiple representations*. You already know one of them: **Contracts**, which specify the Name, Domain, and Range of a function. Contracts are a way of thinking of functions as a *mapping* between one set of data and another. For example, a mapping from Numbers to Strings:

`; f :: Number -> String`

Another way to view functions is with **Examples**. Examples are essentially input-output tables, showing what the function would do for a specific input:

In our programming language, we focus on the last two columns and write them as code:

```
(EXAMPLE (f 1) (+ 1 2))
(EXAMPLE (f 2) (+ 2 2))
(EXAMPLE (f 3) (+ 3 2))
(EXAMPLE (f 4) (+ 4 2))
```

Finally, we write a formal **function definition** ourselves. The pattern in the Examples becomes *abstract* (or "general"), replacing the inputs with **variables**. In the example below, the same definition is written in both math and code:

$$f(x) = x + 2$$

```
(define (f x) (+ x 2))
```

Look for connections between these three representations!

- The function name is always the same, whether looking at the Contract, Examples, or Definition.
- The number of inputs in the Examples is always the same as the number of types in the Domain, which is always the same as the number of variables in the Definition.
- The "what the function does" pattern in the Examples is almost the same in the Definition, but with specific inputs replaced by variables.

Matching Examples and Contracts

Match each set of examples (left) with the Contract that best describes it (right).

Examples	Contract
(EXAMPLE (f 5) (/ 5 2)) (EXAMPLE (f 9) (/ 9 2)) (EXAMPLE (f 24) (/ 24 2))	1 A ; f :: Number -> Number
(EXAMPLE (f 1) (rectangle 1 1 "outline" "red")) (EXAMPLE (f 6) (rectangle 6 6 "outline" "red"))	2 B ; f :: String -> Image
(EXAMPLE (f "pink" 5) (star 5 "solid" "pink")) (EXAMPLE (f "blue" 8) (star 8 "solid" "blue"))	3 C ; f :: Number -> Image
(EXAMPLE (f "Hi!") (text "Hi!" 50 "red")) (EXAMPLE (f "Ciao!") (text "Ciao!" 50 "red"))	4 D ; f :: Number, String -> Image
(EXAMPLE (f 5 "outline") (star 5 "outline" "yellow")) (EXAMPLE (f 5 "solid") (star 5 "solid" "yellow"))	5 E ; f :: String, Number -> Image

Matching Examples and Function Definitions

Find the variables in `gt` and label them with the word "size".

```
(EXAMPLE (gt 20) (triangle 20 "solid" "green"))
(EXAMPLE (gt 50) (triangle 50 "solid" "green"))
(define (gt size) (gt size "solid" "green"))
```

Highlight and label the variables in the example lists below. Then, using `gt` as a model, match the examples to their corresponding function definitions.

Examples		Definition
<pre>(EXAMPLE (f "solid") (circle 8 "solid" "red")) (EXAMPLE (f "outline") (circle 8 "outline" "red"))</pre>	1	A <pre>(define (f s) (star s "outline" "red"))</pre>
<pre>(EXAMPLE (f 2) (+ 2 2)) (EXAMPLE (f 4) (+ 4 4)) (EXAMPLE (f 5) (+ 5 5))</pre>	2	B <pre>(define (f num) (+ num num))</pre>
<pre>(EXAMPLE (f "red") (circle 7 "solid" "red")) (EXAMPLE (f "teal") (circle 7 "solid" "teal"))</pre>	3	C <pre>(define (f c) (star 9 "solid" c))</pre>
<pre>(EXAMPLE (f "red") (star 9 "solid" "red")) (EXAMPLE (f "grey") (star 9 "solid" "grey")) (EXAMPLE (f "pink") (star 9 "solid" "pink"))</pre>	4	D <pre>(define (f s) (circle 8 s "red"))</pre>
<pre>(EXAMPLE (f 3) (star 3 "outline" "red")) (EXAMPLE (f 8) (star 8 "outline" "red"))</pre>	5	E <pre>(define (f c) (circle 7 "solid" c))</pre>

Creating Contracts From Examples

Write the contracts used to create each of the following collections of examples. The first one has been done for you.

1) ; **big-triangle** :: Number, String -> Image

```
(EXAMPLE (big-triangle 100 "red"))
  (triangle 100 "solid" "red"))
(EXAMPLE (big-triangle 200 "orange")
  (triangle 200 "solid" "orange"))
```

2)

```
(EXAMPLE (purple-square 15)
  (rectangle 15 15 "outline" "purple"))
(EXAMPLE (purple-square 6)
  (rectangle 6 6 "outline" "purple"))
```

3)

```
(EXAMPLE (banner "Game Today!")
  (text "Game Today!" 50 "red"))
(EXAMPLE (banner "Go Team!")
  (text "Go Team!" 50 "red"))
(EXAMPLE (banner "Exit")
  (text "Exit" 50 "red"))
```

4)

```
(EXAMPLE (twinkle "outline" "red")
  (star 5 "outline" "red"))
(EXAMPLE (twinkle "solid" "pink")
  (star 5 "solid" "pink"))
(EXAMPLE (twinkle "outline" "grey")
  (star 5 "outline" "grey"))
```

5)

```
(EXAMPLE (half 5) (/ 5 2))
(EXAMPLE (half 8) (/ 8 2))
(EXAMPLE (half 900) (/ 900 2))
```

Contracts, Examples & Definitions - bc

gt

Directions: Define a function called gt, which makes solid green triangles of whatever size we want.

Every contract has three parts...

: gt : Number -> Image
function name Domain Range

Write some examples, then circle and label what changes...

(EXAMPLE (gt 10) (triangle 10 "solid" "green")
function name input(s) what the function produces

(EXAMPLE (gt 20) (triangle 20 "solid" "green")
function name input(s) what the function produces

Write the definition, giving variable names to all your input values...

(define (gt size)
function name variable(s)

(triangle size "solid" "green")
what the function does with those variable(s)

bc

Directions: Define a function called bc, which makes solid blue circles of whatever radius we want.

Every contract has three parts...

: : ->
function name Domain Range

Write some examples, then circle and label what changes...

(EXAMPLE ()
function name input(s) what the function produces

(EXAMPLE ()
function name input(s) what the function produces

Write the definition, giving variable names to all your input values...

(define ()
function name variable(s)

what the function does with those variable(s)

Contracts, Examples & Definitions - Stars

sticker

Directions: Define a function called `sticker`, which consumes a color and draws a 50px solid star of the given color.

Every contract has three parts...

`;-----:` _____ Domain _____ -> _____ Range _____

Write some examples, then circle and label what changes...

(EXAMPLE (_____ function name _____) _____ input(s) _____) _____ what the function produces _____)

(EXAMPLE (_____ function name _____) _____ input(s) _____) _____ what the function produces _____)

Write the definition, giving variable names to all your input values...

(define (_____ function name _____) _____ variable(s) _____)

_____ what the function does with those variable(s) _____)

gold-star

Directions: Define a function called `gold-star`, which takes in a radius and draws a solid gold star of that given size.

Every contract has three parts...

`;-----:` _____ Domain _____ -> _____ Range _____

Write some examples, then circle and label what changes...

(EXAMPLE (_____ function name _____) _____ input(s) _____) _____ what the function produces _____)

(EXAMPLE (_____ function name _____) _____ input(s) _____) _____ what the function produces _____)

Write the definition, giving variable names to all your input values...

(define (_____ function name _____) _____ variable(s) _____)

_____ what the function does with those variable(s) _____)

Contracts, Examples & Definitions - Name

name-color

Directions: Define a function called name-color, which makes an image of your name at size 50 in whatever color is given.

Every contract has three parts...

:
function name :
Domain → Range

Write some examples, then circle and label what changes...

(EXAMPLE ()
function name input(s)) what the function produces

(EXAMPLE ()
function name input(s)) what the function produces

Write the definition, giving variable names to all your input values...

(define ()
function name variable(s)
what the function does with those variable(s))

name-size

Directions: Define a function called name-size, which makes an image of your name in your favorite color (be sure to specify your name and favorite color!) in whatever size is given.

Every contract has three parts...

:
function name :
Domain → Range

Write some examples, then circle and label what changes...

(EXAMPLE ()
function name input(s)) what the function produces

(EXAMPLE ()
function name input(s)) what the function produces

Write the definition, giving variable names to all your input values...

(define ()
function name variable(s)
what the function does with those variable(s))

Do the Examples Have the Same Contracts?

For each pair of Examples below, decide whether the two examples have the same Contract. If they do, fill in the Contract in the space provided. If not, write a few words explaining how you know their contracts aren't the same.

1) _____

```
(EXAMPLE (mystery 30) (* 30 50))
(EXAMPLE (mystery 10)
  (text "Welcome!" 10 "darkgreen"))
```

2) _____

```
(EXAMPLE (mystery 30 40) (- 40 (* 2 30)))
(EXAMPLE (mystery 10 15) (- 15 (* 2 10))))
```

3) _____

```
(EXAMPLE (mystery "New York")
  (text "New York" 20 "red"))
(EXAMPLE (mystery 20)
  (text "New York" 20 "red"))
```

4) _____

```
(EXAMPLE (mystery "green" 32)
  (circle 32 "outline" "green"))
(EXAMPLE (mystery 18 "green")
  (circle 18 "outline" "green"))
```

5) _____

```
(EXAMPLE (mystery 6 9 10) (/ 6 (+ 9 10)))
(EXAMPLE (mystery 3 7) (/ 3 (+ 7 10))))
```

6) _____

```
(EXAMPLE (mystery "red" "blue")
  (text "blue" 25 "red"))
(EXAMPLE (mystery "purple" "Go Team!")
  (text "Go Team!" 25 "purple"))
```

Do the Examples Have the Same Contracts? (2)

For each pair of Examples below, decide whether the two examples have the same Contract. If they do, fill in the Contract in the space provided. If not, write a few words explaining how you know their contracts aren't the same.

1) _____

```
(EXAMPLE (mystery (triangle 70 "solid" "green"))
  (triangle 140 "solid" "green"))
(EXAMPLE (mystery (circle 100 "solid" "blue"))
  (circle 200 "solid" "blue"))
```

2) _____

```
(EXAMPLE (mystery "red")
  (triangle 140 "solid" "red"))
(EXAMPLE (mystery "blue" "circle")
  (circle 140 "solid" "blue"))
```

3) _____

```
(EXAMPLE (mystery "" 4 5) ( 4 5))
(EXAMPLE (mystery "sqrt" 25) (sqrt 25))
```

4) _____

```
(EXAMPLE (mystery "circle" 4) (* pi (sqr 4)))
(EXAMPLE (mystery "square" 5) (sqr 5))
```

5) _____

```
(EXAMPLE (mystery "dog") 3)
(EXAMPLE (mystery "cat") "kitten")
```

6) _____

```
(EXAMPLE (mystery "dog") 3)
(EXAMPLE (mystery "kitten") 6)
```

Matching Contracts and Examples

Match each Example on the left with its Contract on the right. NOTE: Multiple examples may match to the same Contract!

Contract	Examples
(EXAMPLE (match (circle 10 "solid" "green")) (rotate 37 (circle 10 "solid" "green"))))	1 A ; match :: Number, Image -> Image
(EXAMPLE (match (triangle 20 "solid" "blue") 3) (scale 3 (triangle 20 "solid" "blue"))))	2
(EXAMPLE (match (circle 20 "outline" "gold")) (rotate 37 (circle 20 "outline" "gold"))))	3 B ; match :: Image, Number -> Image
(EXAMPLE (match 30 "red") (+ 30 (string-length "red"))))	4
(EXAMPLE (match (circle 10 "solid" "orange") 22) (scale 22 (circle 10 "solid" "orange"))))	5
(EXAMPLE (match 10 "blue") (+ 10 (string-length "blue"))))	6 C ; match :: Image -> Image
(EXAMPLE (match 5 (star 20 "solid" "red")) (rotate (- 90 5) (star 20 "solid" "red"))))	7
(EXAMPLE (match (abs -4) "45") 4)	8 D ; match :: Number, String -> Number

Matching Contracts and Examples (2)

Match each Example on the left with its Contract on the right. NOTE: Multiple examples may match to the same Contract!

Contract	Examples
(EXAMPLE (match 1.5) "greater than 1")	1
(EXAMPLE (match 24) (star (* 24 2) "outline" "purple"))	2
(EXAMPLE (match (string-length "tabletop")) "8")	3
(EXAMPLE (match (star 20 "outline" "red") 3) (* 3 (image-height (star 20 "outline" "red"))))	4
(EXAMPLE (match (circle 10 "solid" "silver") 16) (* 16 (image-height (circle 10 "solid" "silver"))))	5
(EXAMPLE (match "triangle" "blue") (triangle 40 "outline" "blue"))	6
(EXAMPLE (match 30) (star (* 30 2) "outline" "purple"))	7
(EXAMPLE (match (string-length "coffee") (string-length "tea")) (+ 6 3))	8

A ; match :: Number -> String

B ; match :: Number -> Image

C ; match :: Number, Number -> Number

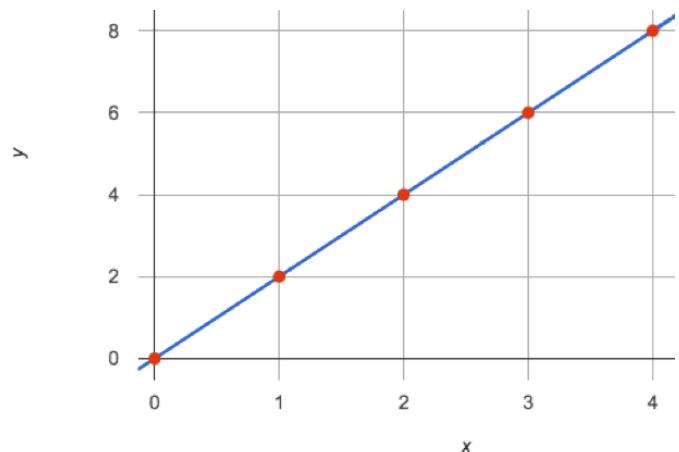
D ; match :: String, String -> Image

E ; match :: Images, Number -> Number

Notice and Wonder (Linearity)

Part 1:

x	y
0	0
1	2
2	4
3	6
4	8



What do you Notice?

What do you Wonder?

Part 2:

- What would be the next (x,y) pair for each of the tables?
- What would the y-value for each table be when x is 0?

x	y
0	
1	2
2	3
3	4
4	5
5	6

x	y
0	
1	20
2	17
3	14
4	11
5	8

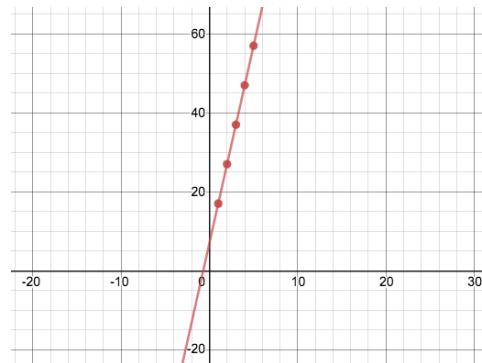
Matching Tables to Graphs

For each of the tables below, find the graph that matches.

Note: Scales on the graphs vary. The tables are shown sideways to save space.

x	1	2	3	4	5
y	4	5	6	7	8

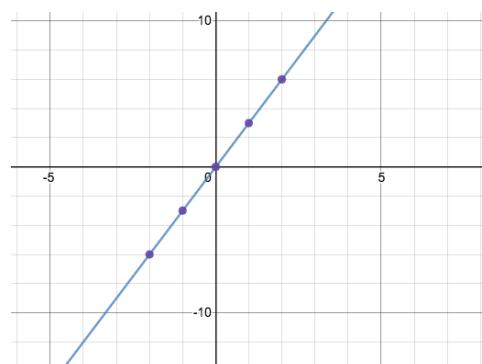
1



A

x	-5	-4	-3	-2	-1
y	5	4	3	2	1

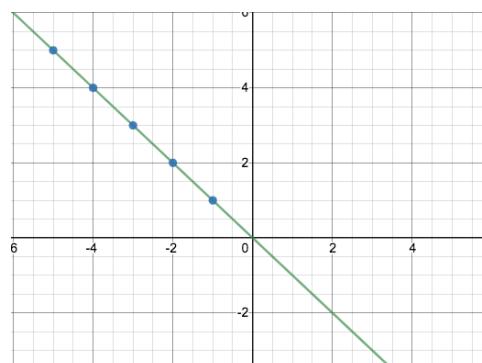
2



B

x	1	2	3	4	5
y	17	27	37	47	57

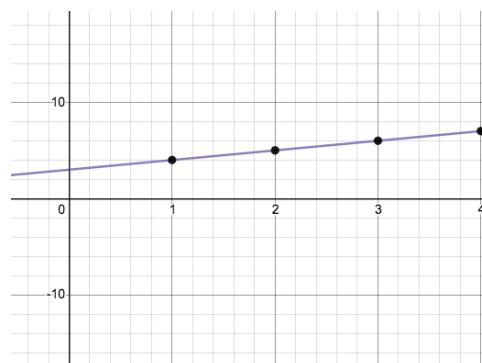
3



C

x	-2	-1	0	1	2
y	-6	-3	0	3	6

4

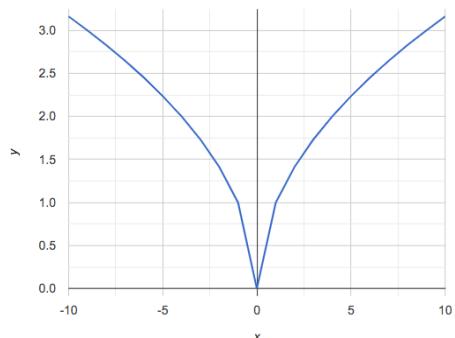
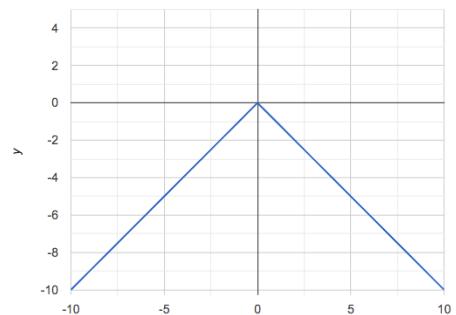
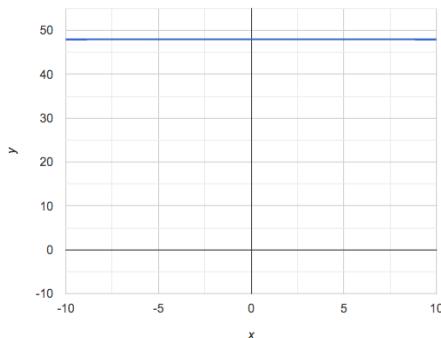


D

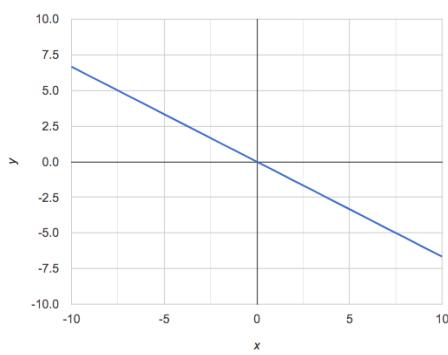
Are All Graphs Linear?

If all linear relationships can be shown as points on a graph, does that mean all graphs are linear?

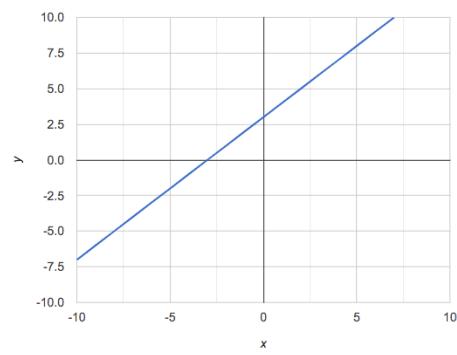
Beneath each graph circle **Linear** or **Not Linear**.



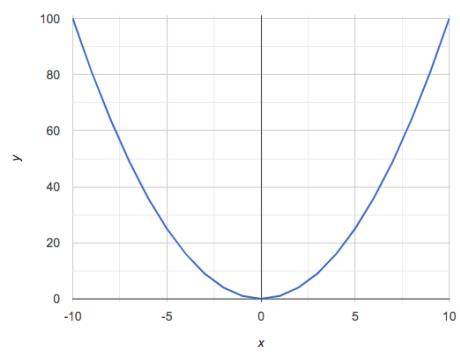
1) Linear or Not Linear?



2) Linear or Not Linear?



3) Linear or Not Linear?



4) Linear or Not Linear?

5) Linear or Not Linear?

6) Linear or Not Linear?

What do you Notice?

What do you Wonder?

Are All Tables Linear?

If all linear relationships can be shown as tables, does that mean all tables are linear? Look at the six tables shown below.

1) Extend as many of the tables as you can by adding the next (x,y) pair in the sequence.

2) If the table is linear, write down your prediction of what the y-value will be when x = 0.

3) If the table is not linear, write **not linear** instead of an answer for y.

A

x	-2	-1	0	1	2	
y	-2	-3	-4	-5	-6	

when $x=0$, y will equal _____

B

x	2	3	4	5	6	
y	-12	-14	-16	-18	-20	

when $x=0$, y will equal _____

C

x	1	2	3	4	5	
y	1	4	9	16	25	

when $x=0$, y will equal _____

D

x	5	6	7	8	9	
y	3	3	3	3	3	

when $x=0$, y will equal _____

E

x	1	2	3	4	5	
y	84	94	104	114	124	

when $x=0$, y will equal _____

F

x	-10	-9	-8	-7	-6	
y	$-\frac{1}{10}$	$-\frac{1}{9}$	$-\frac{1}{8}$	$-\frac{1}{7}$	$-\frac{1}{6}$	

when $x=0$, y will equal _____

What do you Notice?

What do you Wonder?

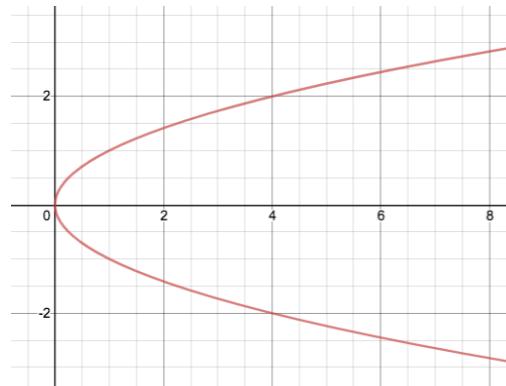
Linear, Non-linear, or Bust?

Circle whether each representation is of a linear function, a nonlinear function or is not a function at all!

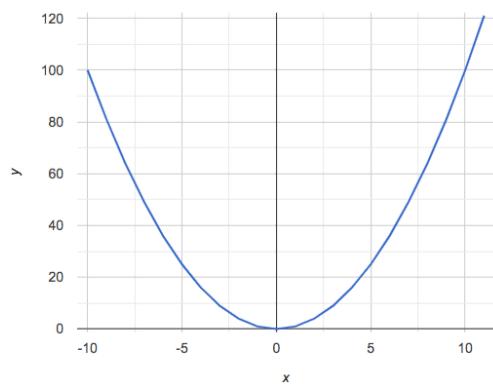
Remember: Functions will pass the Vertical Line Test!

x	y
1	5
2	10
3	15
4	20
5	25
6	30
7	35

- 1) Linear Nonlinear Not a Function



- 2) Linear Nonlinear Not a Function



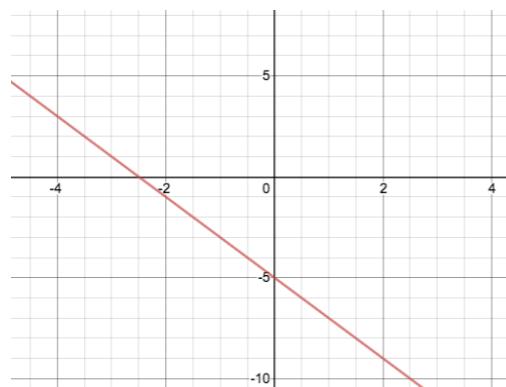
- 3) Linear Nonlinear Not a Function

x	y
1	1
2	4
3	9
4	16
5	25
6	36
7	49

- 4) Linear Nonlinear Not a Function

x	y
1	1
2	2
3	3
4	4
4	5
6	6
7	9

- 5) Linear Nonlinear Not a Function



- 6) Linear Nonlinear Not a Function

Slope & y-Intercept from Tables (Intro)

slope (rate): how much y changes as x -increases by 1

y-intercept: the y -value when $x = 0$

x	-1	0	1	2	3	4
y	-1	1	3	5	7	9

1) Compute the slope: _____

2) Compute the y-intercept: _____

3) What strategies did you use to compute the slope and y-intercept?

The slope and y-intercept in this table are harder to find, because the x-values don't go up by 1 and we can't see a value for $x = 0$. Try filling in the points that have been skipped to compute the slope and y-intercept.

x	2	5	8	11
y	3	9	15	21

4) Compute the slope: _____

5) Compute the y-intercept: _____

The slope and y-intercept in this table are even harder to find, because the x-values are out of order!

Calculate the slope and y-intercept from any two points! Be sure to show your work.

x	3	20	5	9	1
y	5	56	11	23	-1

6) Compute the slope: _____

7) Compute the y-intercept: _____

Slope & y-Intercept from Tables (Practice)

x	-1	0	1	2	3	4
y	-1	2	5	8	11	14

1) slope: _____ y-intercept: _____

x	-2	-1	0	1	2	3
y	15	10	5	0	-5	-10

2) slope: _____ y-intercept: _____

x	-3	-2	-1	0	1	2
y	-1	-0.5	0	0.5	1	1.5

3) slope: _____ y-intercept: _____

x	-1	0	1	2	3	4
y	-7	-3	1	5	9	13

4) slope: _____ y-intercept: _____

x	-5	-4	-3	-2	-1	0
y	1	2.5	4	5.5	7	8.5

5) slope: _____ y-intercept: _____

x	-3	-2	-1	0	1	2
y	0	12.5	25	37.5	50	62.5

6) slope: _____ y-intercept: _____

x	1	2	3	4	5	6
y	5	3	1	-1	-3	-5

7) slope: _____ y-intercept: _____

x	-4	-2	0	2	4	6
y	0	4	8	12	16	20

8) slope: _____ y-intercept: _____

Identifying Slope in Tables

Can you identify the **slope** for the functions represented in each of these tables?

Note: Some tables may have their rows out of order!

x	y
0	3
1	5
2	7
3	9

slope/rate: _____

x	y
-5	35
-4	28
-3	21
-2	14

slope/rate: _____

x	y
12	15
13	15.5
14	16
16	17

slope/rate: _____

x	y
1	39
4	36
3	37
2	38

slope/rate: _____

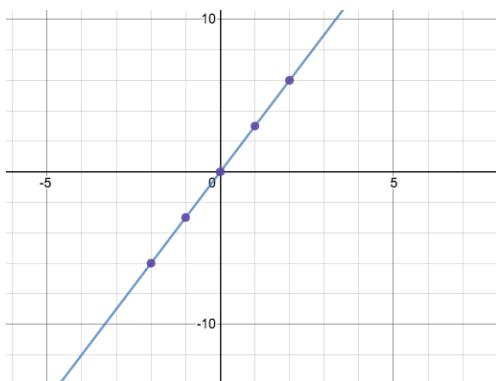
x	y
13	57
9	41
11	49
7	33

slope/rate: _____

Identifying Slope and y-intercept in Graphs

Can you identify the **slope** and **y-intercept** for each of these graphs?

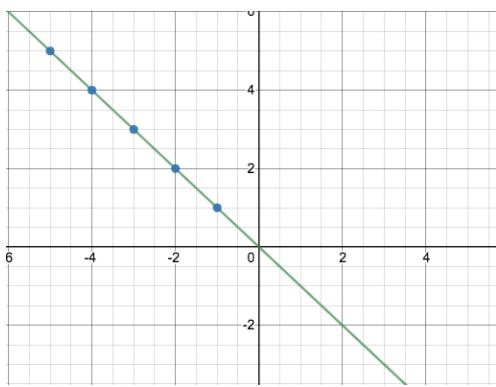
1



slope/rate: _____

y-intercept: _____

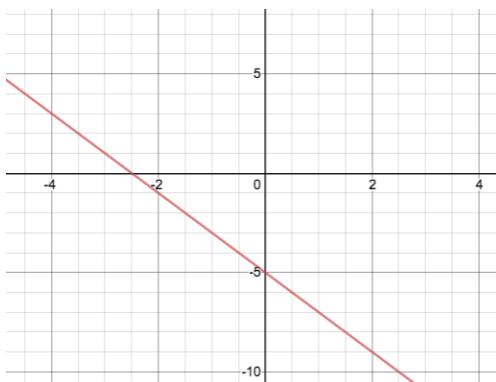
2



slope/rate: _____

y-intercept: _____

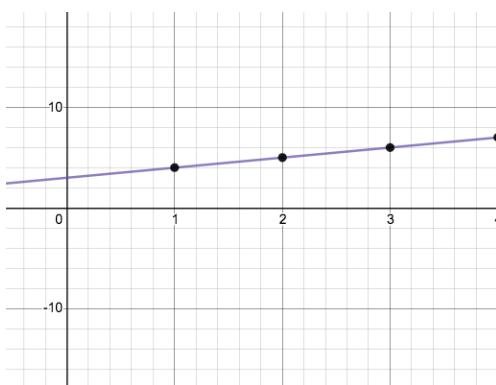
3



slope/rate: _____

y-intercept: _____

4



slope/rate: _____

y-intercept: _____

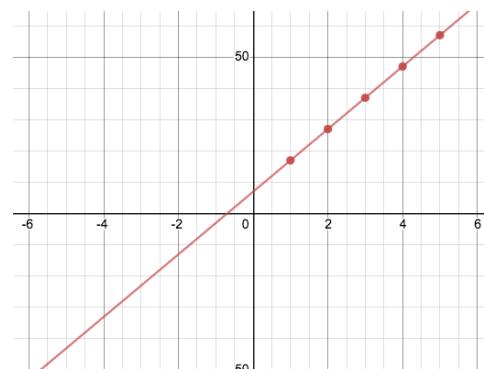
Matching Tables to Graphs (Challenge)

For each of the tables below, find the graph that matches. **Note:** The tables are shown sideways to save space.

- The scales on the graphs are not the same! Look at the axes to help you find the right match!

x	-3	-4	-1	-5	-2
y	3	4	1	5	2

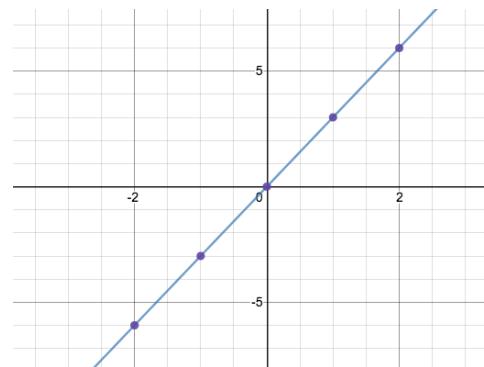
1



A

x	4	1	3	5	2
y	7	4	6	8	5

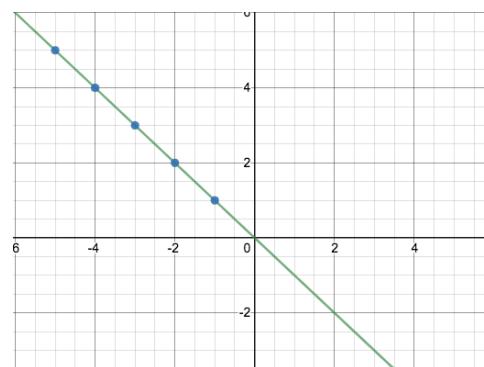
2



B

x	3	4	5	2	1
y	37	47	57	27	17

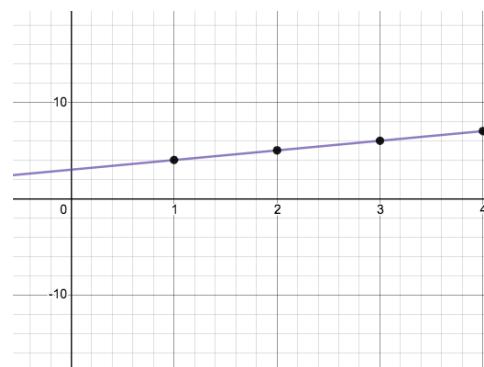
3



C

x	3	5	2	1	4
y	9	15	6	3	12

4



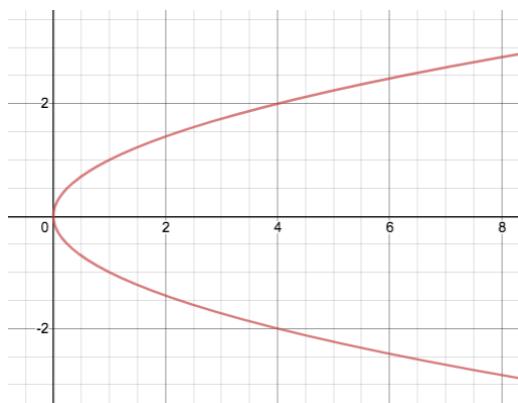
D

Graphs: Linear, Non-linear, or Bust?

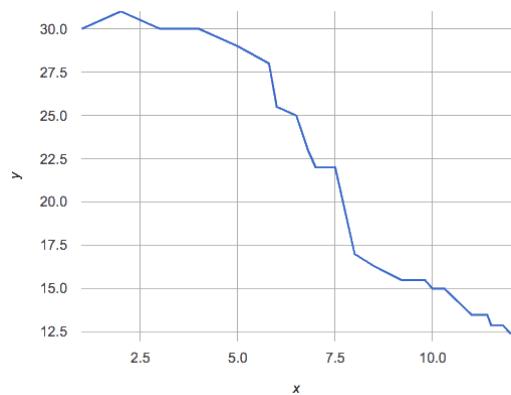
Decide whether each representation is of a linear function, a nonlinear function or is not a function at all!



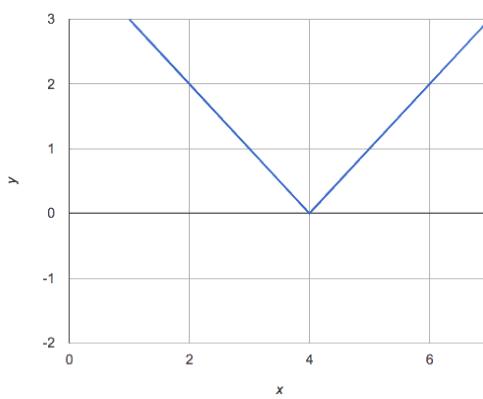
- 1) Linear Nonlinear Not a Function



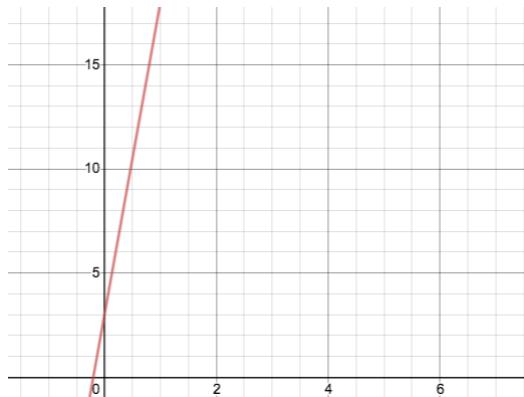
- 2) Linear Nonlinear Not a Function



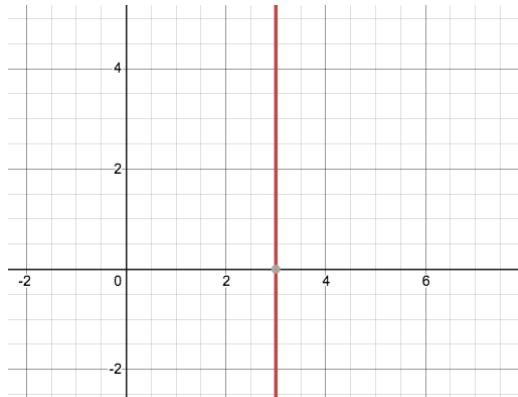
- 3) Linear Nonlinear Not a Function



- 4) Linear Function Nonlinear Function Not a Function



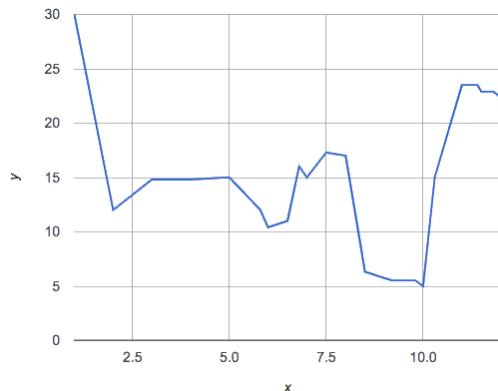
- 5) Linear Nonlinear Not a Function



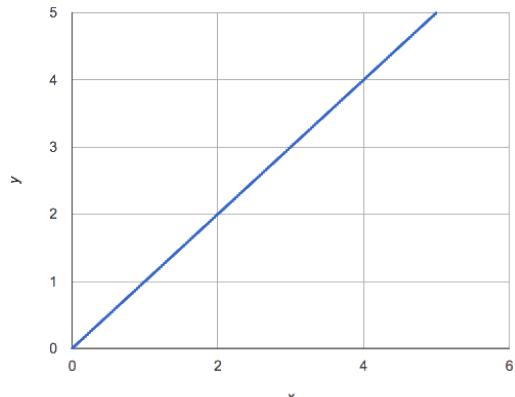
- 6) Linear Nonlinear Not a Function

Graphs: Linear, Non-linear, or Bust?(2)

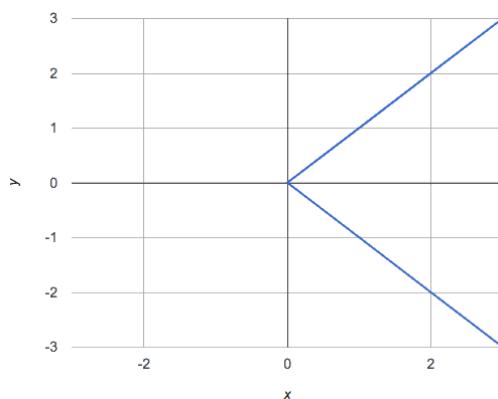
Decide whether each representation is of a linear function, a nonlinear function or is not a function at all!



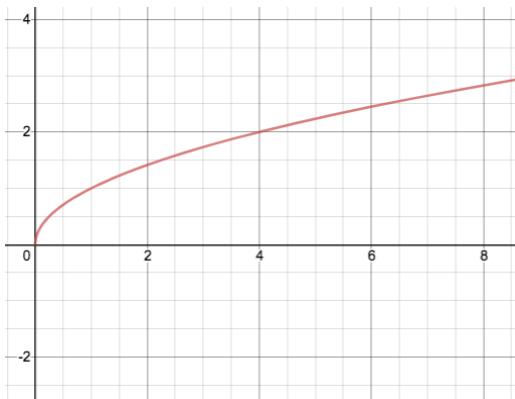
- 1) Linear Nonlinear Not a Function



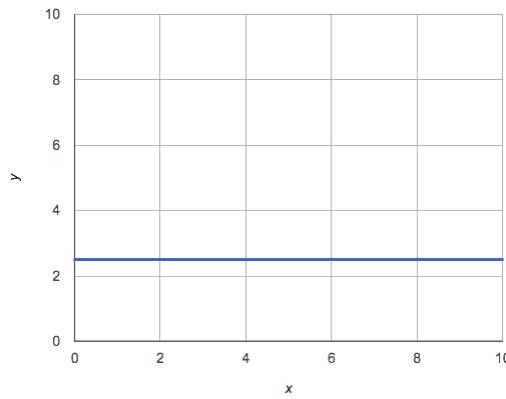
- 2) Linear Nonlinear Not a Function



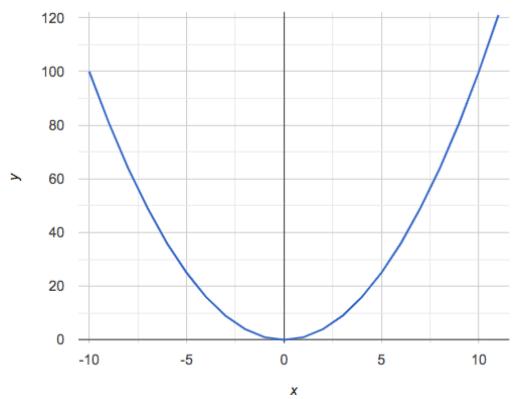
- 3) Linear Nonlinear Not a Function



- 4) Linear Nonlinear Not a Function



- 5) Linear Nonlinear Not a Function

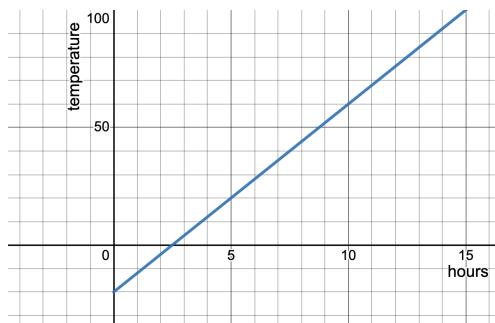


- 6) Linear Nonlinear Not a Function

What Story does the Graph tell?

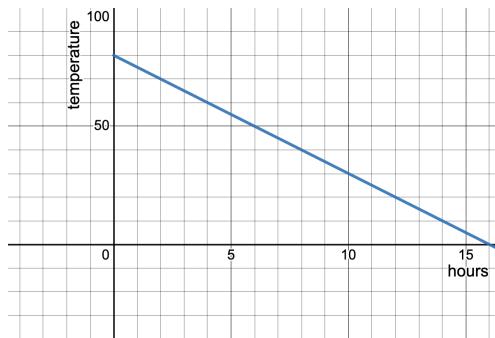
For each of the Graphs below, write the story that it tells. The first one has been done for you.

1

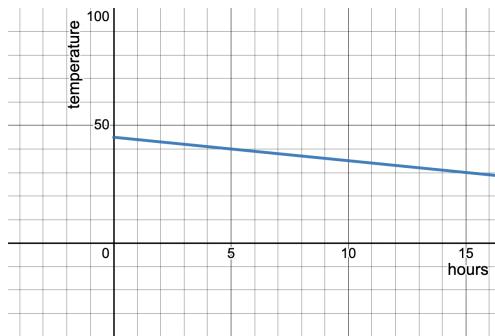


The temperature started at -20 degrees and increased by 8 degrees per hour.

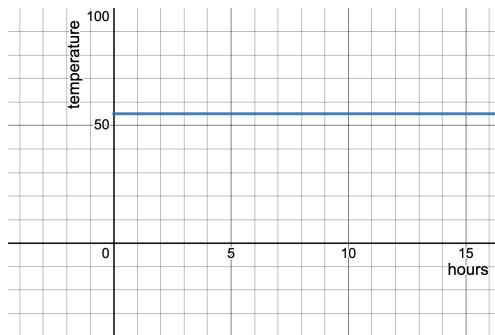
2



3



4



What Story does the Table tell?

For each of the Tables below, write the story that it tells.

1

maple syrup produced (gallons)	0	1	2	3	4
gallons of sap boiled	0	40	80	120	160

2

seconds on stove	0	10	20	30	40	50
water temp in deg F	50	59	68	77	86	95

3

tickets sold	0	10	20	30	40
profit in dollars	-560	-360	-160	40	240

4

bowls served	0	10	20	30	40
gallons of gumbo in the pot	19	18	17	16	15

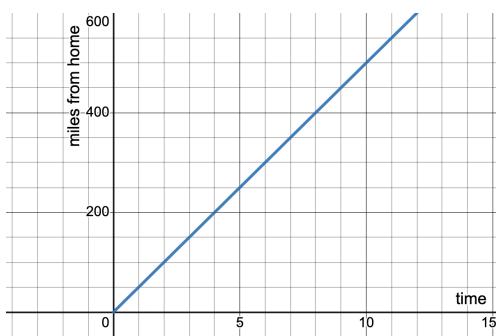
5

month	1	2	3	4	5	6	7	8	9	10	11	12
hours of daylight in Berlin	8.3	9.8	11.9	13.8	15.8	16.9	16.4	14.8	12.8	10.8	8.8	7.8

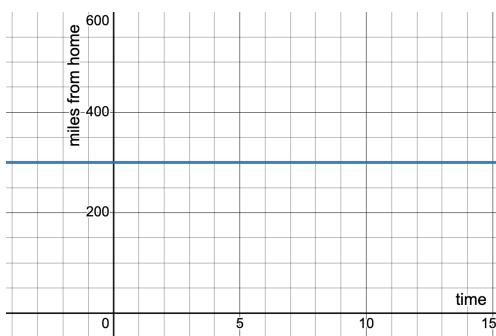
What Story does the Graph tell?(2)

For each of the Graphs below, write the story that it tells.

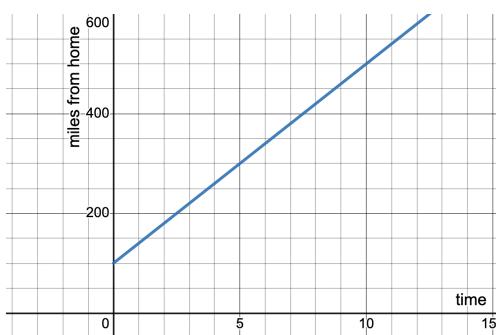
1



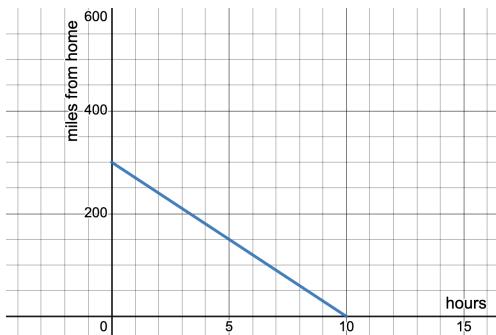
2



3



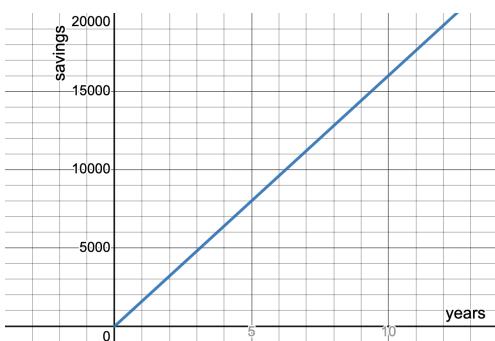
4



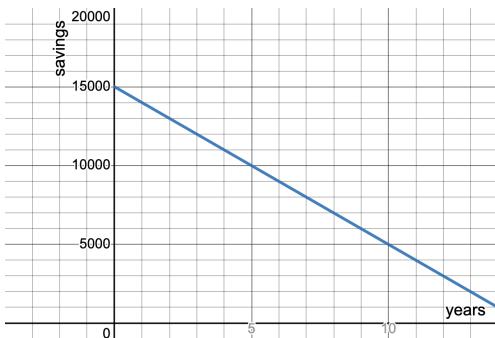
What Story does the Graph tell?(3)

For each of the Graphs below, write the story that it tells.

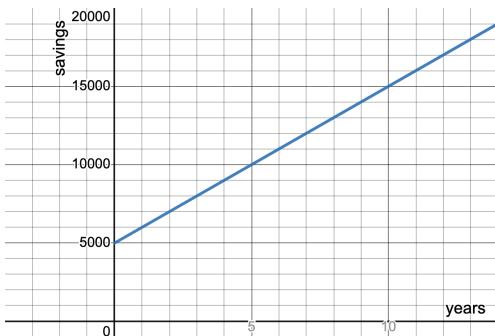
1



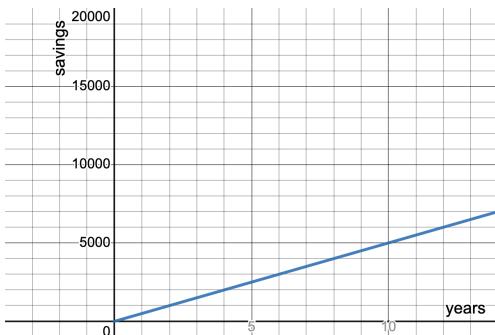
2



3



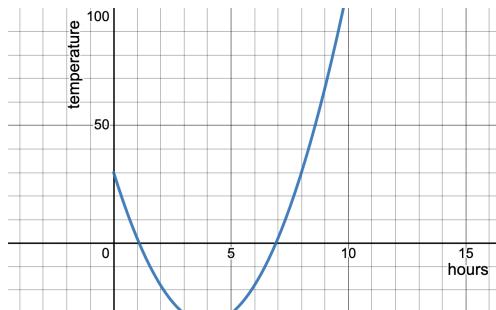
4



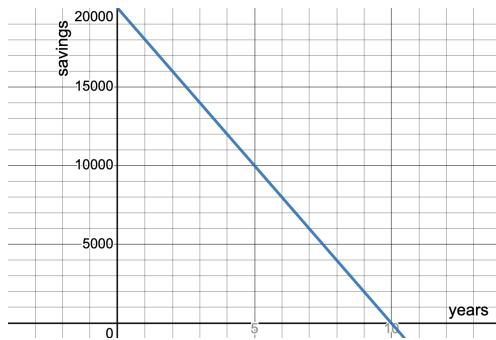
What Story does the Graph tell? (challenge)

For each of the Graphs below, write the story that it tells.

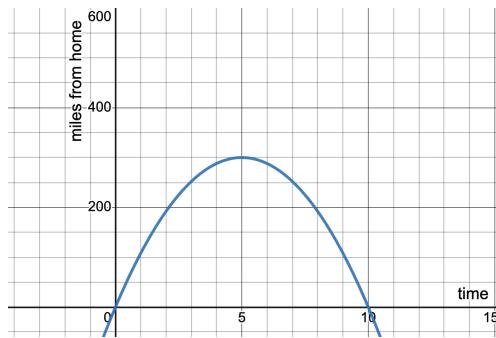
1



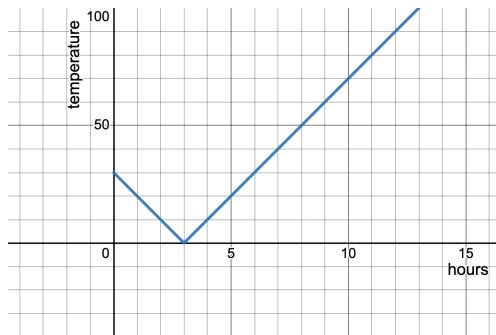
2



3



4



Solving Word Problems

Being able to see functions as Contracts, Examples or Definitions is like having three powerful tools. These representations can be used together to solve word problems!

- 1) When reading a word problem, the first step is to figure out the **Contract** for the function you want to build. Remember, a Contract must include the Name, Domain and Range for the function!
- 2) Then we write a **Purpose Statement**, which is a short note that tells us what the function *should do*. Professional programmers work hard to write good purpose statements, so that other people can understand the code they wrote! Programmers work on teams; the programs they write must outlast the moment that they are written.
- 3) Next, we write at least two **Examples**. These are lines of code that show what the function should do for a *specific* input. Once we see examples of at least two inputs, we can *find a pattern* and see which parts are changing and which parts aren't.
- 4) To finish the Examples, we circle the parts that are changing, and label them with a short **variable name** that explains what they do.
- 5) Finally, we **define the function** itself! This is pretty easy after you have some examples to work from: we copy everything that didn't change, and replace the changeable stuff with the variable name!

Matching Word Problems and Purpose Statements

Match each word problem below to its corresponding purpose statement.

Annie got a new dog, Xavier, that eats about 5 times as much as her little dog, Rex, who is 10 years old. She hasn't gotten used to buying enough dog food for the household yet. Write a function that generates an estimate for how many pounds of food Xavier will eat, given the amount of food that Rex usually consumes in the same amount of time.

1 A Consume the pounds of food Rex eats and add 5.

Adrienne's raccoon, Rex, eats 5 more pounds of food each week than her pet squirrel, Lili, who is 7 years older. Write a function to determine how much Lili eats in a week, given how much Rex eats.

2 B Consume the pounds of food Rex eats and subtract 5.

Alejandro's rabbit, Rex, poops about $1/5$ of what it eats. His rabbit hutch is 10 cubic feet. Write a function to figure out how much rabbit poop Alejandro will have to clean up depending on how much Rex has eaten.

3 C Consume the pounds of food Rex eats and multiply by 5.

Max's turtle, Rex, eats 5 pounds less per week than his turtle, Harry, who is 2 inches taller. Write a function to calculate how much food Harry eats, given the weight of Rex's food.

4 D Consume the pounds of food Rex eats and divide by 5.

Writing Examples from Purpose Statements

We've provided contracts and purpose statements to describe two different functions. Write examples for each of those functions.

Contract and Purpose Statement

Every contract has three parts...

; triple : Number -> Number
function name Domain Range

; Consumes a Number and triples it. what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (function name) input(s)) what the function produces

(EXAMPLE (function name) input(s)) what the function produces

Contract and Purpose Statement

Every contract has three parts...

; upside-down : Image -> Image
function name Domain Range

; Consumes an image, and turns it upside down by rotating it 180 degrees. what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (function name) input(s))
what the function produces

(EXAMPLE (function name) input(s)) what the function produces

Fixing Purpose Statements

Beneath each of the word problems below is a purpose statement (generated by ChatGPT!) that is either missing information or includes unnecessary information. Write an improved version of each purpose statement beneath the original, then explain what was wrong with the ChatGPT-generated Purpose Statement.

1) Word Problem: *The New York City ferry costs \$2.75 per ride. The Earth School requires two chaperones for any field trip. Write a function fare that takes in the number of students in the class and returns the total fare for the students and chaperones.*

ChatGPT's Purpose Statement: Take in the number of students and add 2.

Improved Purpose Statement: _____

Problem with ChatGPT's Purpose Statement: _____

2) Word Problem: *It is tradition for the Green Machines to go to Humpy Dumpty's for ice cream with their families after their soccer games. Write a function cones to take in the number of kids and calculate the total bill for the team, assuming that each kid brings two family members and cones cost \$1.25.*

ChatGPT's Purpose Statement: Take in the number of kids on the team and multiply it by 1.25.

Improved Purpose Statement: _____

Problem with ChatGPT's Purpose Statement: _____

3) Word Problem: *The cost of renting an ebike is \$3 plus an additional \$0.12 per minute. Write a function ebike that will calculate the cost of a ride, given the number of minutes ridden.*

ChatGPT's Purpose Statement: Take in the number of minutes and multiply it by 3.12.

Improved Purpose Statement: _____

Problem with ChatGPT's Purpose Statement: _____

4) Word Problem: *Suleika is a skilled house painter at only age 21. She has painted hundreds of rooms and can paint about 175 square feet an hour. Write a function paint that takes in the number of square feet of the job and calculates how many hours it will take her.*

ChatGPT's Purpose Statement: Take in the number of square feet of walls in a house and divide them by 175 then add 21 years.

Improved Purpose Statement: _____

Problem with ChatGPT's Purpose Statement: _____

Word Problem: rocket-height

Directions: A rocket blasts off, and is now traveling at a constant velocity of 7 meters per second. Use the Design Recipe to write a function `rocket-height`, which takes in a number of seconds and calculates the height.

Contract and Purpose Statement

Every contract has three parts...

: _____ function name : _____ Domain -> _____ Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (_____ function name _____ variable(s)) _____)

what the function does with those variable(s)

Writing Examples from Purpose Statements (2)

We've provided contracts and purpose statements to describe two different functions. Write examples for each of those functions.

Contract and Purpose Statement

Every contract has three parts...

; half-image : Image Domain -> Image Range

; Consumes an image, and produces that image scaled to half its size.
what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (function name input(s))
what the function produces)

(EXAMPLE (function name input(s))
what the function produces)

Contract and Purpose Statement

Every contract has three parts...

; product-squared : Number Number Domain -> Number Range

; Consumes two numbers and squares their product
what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (function name input(s))
what the function produces)

(EXAMPLE (function name input(s))
what the function produces)

Rocket Height Challenges

1) Can you make the rocket fly faster?

2) Can you make the rocket fly slower?

3) Can you make the rocket sink down instead of fly up?

4) Can you make the rocket accelerate over time, so that it moves faster the longer it flies?

5) Can you make the rocket blast off and then land again?

6) Can you make the rocket blast off, reach a maximum height of exactly 1000 meters, and then land?

7) Can you make the rocket blast off, reach a maximum height of exactly 1000 meters, and then land after exactly 100 seconds?

8) Can you make the rocket fly to the edge of the universe?

The Design Recipe (Restaurants)

Directions: Use the Design Recipe to write a function `split-tab` that takes in a cost and the number of people sharing the bill and splits the cost equally.

Contract and Purpose Statement

Every contract has three parts...

: _____ function name : _____ Domain -> _____ Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (_____ function name _____ variable(s)) _____)

what the function does with those variable(s)

Directions: Use the Design Recipe to write a function `tip-calculator` that takes in the cost of a meal and returns the 15% tip for that meal.

Contract and Purpose Statement

Every contract has three parts...

: _____ function name : _____ Domain -> _____ Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (_____ function name _____ variable(s)) _____)

what the function does with those variable(s)

The Design Recipe (Direct Variation)

Directions: Use the Design Recipe to write a function wage, that takes in a number of hours worked and returns the amount a worker will get paid if their rate is \$10.25/hr.

Contract and Purpose Statement

Every contract has three parts...

; _____ : _____ Domain -> _____ Range

; _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____ function name input(s)) _____ what the function produces)

(EXAMPLE (_____ function name input(s)) _____ what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (_____ function name variable(s)) _____)

what the function does with those variable(s)

Directions: On average, people burn about 11 calories/minute riding a bike. Use the Design Recipe to write a function calories-burned that takes in the number of minutes you bike and returns the number of calories burned..

Contract and Purpose Statement

Every contract has three parts...

; _____ : _____ Number Domain -> _____ Number Range

; _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____ function name input(s)) _____ what the function produces)

(EXAMPLE (_____ function name input(s)) _____ what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (_____ function name variable(s)) _____)

what the function does with those variable(s)

The Design Recipe (Slope/Intercept)

Directions: For his birthday, James' family decided to open a savings account for him. He started with \$50 and committed to adding \$10 a week from his afterschool job teaching basketball to kindergartners. Use the Design Recipe to write a function `savings` that takes in the number of weeks since his birthday and calculates how much money he has saved.

Contract and Purpose Statement

Every contract has three parts...

;
function name :
Domain -> Range

;
what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____)
function name input(s) what the function produces

(EXAMPLE (_____)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values...

(define (_____)
function name variable(s)
)
what the function does with those variable(s)

Directions: Use the Design Recipe to write a function `moving` that takes in the days and number of miles driven and returns the cost of renting a truck. The truck is \$45 per day and each driven mile is 15¢.

Contract and Purpose Statement

Every contract has three parts...

;
function name :
Domain -> Range

;
what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____)
function name input(s) what the function produces

(EXAMPLE (_____)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values...

(define (_____)
function name variable(s)
)
what the function does with those variable(s)

The Design Recipe (Negative Slope/Intercept)

Directions: An Olympic pool holds 660,000 gallons of water. A fire hose can spray about 250 gallons per minute. Use the Design Recipe to write a function `pool` that takes in the number of minutes that have passed and calculates how much water is still needed to fill it.

Contract and Purpose Statement

Every contract has three parts...

: _____ : _____ Domain -> _____ Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____ function name input(s)) _____ what the function produces)

(EXAMPLE (_____ function name input(s)) _____ what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (_____ function name variable(s)) _____)

what the function does with those variable(s)

Directions: The community arts fund awards a \$1500 grant each month to support a new mural. They started with \$50000 in their account. Use the Design Recipe to write a function `funds-available` that takes in the number of months and calculates how much money they have left.

Contract and Purpose Statement

Every contract has three parts...

: _____ : _____ Domain -> _____ Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____ function name input(s)) _____ what the function produces)

(EXAMPLE (_____ function name input(s)) _____ what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (_____ function name variable(s)) _____)

what the function does with those variable(s)

The Design Recipe (Geometry - Rectangles)

Directions: Use the Design Recipe to write a function `Lawn-area` that takes in the length and width of a rectangular lawn and returns its area.

Contract and Purpose Statement

Every contract has three parts...

: _____ function name : _____ Domain -> _____ Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (_____ function name _____ variable(s)) _____)

what the function does with those variable(s)

Directions: Use the Design Recipe to write a function `rect-perimeter` that takes in the length and width of a rectangle and returns the perimeter of that rectangle.

Contract and Purpose Statement

Every contract has three parts...

: _____ function name : _____ Domain -> _____ Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (_____ function name _____ variable(s)) _____)

what the function does with those variable(s)

The Design Recipe (Geometry - Rectangular Prisms)

Directions: Use the Design Recipe to write a function `rectprism-vol` that takes in the length, width, and height of a rectangular prism and returns the Volume of a rectangular prism.

Contract and Purpose Statement

Every contract has three parts...

;
function name :
; Domain -> Range
;

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (function name input(s)) what the function produces)

(EXAMPLE (function name input(s)) what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (function name variable(s))
what the function does with those variable(s))

Directions: Use the Design Recipe to write a function `rect-prism-sa` that takes in the width, length and height of a rectangular prism and calculates its surface area (the sum of the areas of each of its six faces)

Contract and Purpose Statement

Every contract has three parts...

;
function name :
; Domain -> Range
;

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (function name input(s))
what the function produces)

(EXAMPLE (function name input(s))
what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (function name variable(s))
what the function does with those variable(s))

The Design Recipe (Geometry - Circles)

Directions: Use the Design Recipe to write a function `circle-area-dec` that takes in a radius and uses the decimal approximation of pi (3.14) to return the area of the circle.

Contract and Purpose Statement

Every contract has three parts...

: _____ : _____ Domain -> _____ Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (_____ function name _____ variable(s)) _____)

what the function does with those variable(s)

Directions: Use the Design Recipe to write a function `circumference` that takes in a radius and uses the decimal approximation of pi (3.14) to return the circumference of the circle.

Contract and Purpose Statement

Every contract has three parts...

: _____ : _____ Domain -> _____ Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (_____ function name _____ variable(s)) _____)

what the function does with those variable(s)

The Design Recipe (Geometry - Cylinders)

Directions: Use the Design Recipe to write a function `circle-area` that takes in a radius and uses the fraction approximation of pi ($\frac{22}{7}$) to return the area of the circle.

Contract and Purpose Statement

Every contract has three parts...

;
function name :
Domain → Range

;
what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (function name input(s)) what the function produces)

(EXAMPLE (function name input(s)) what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (function name variable(s)))

what the function does with those variable(s)

Directions: Use the Design Recipe to write a function `cylinder` that takes in a cylinder's radius and height and calculates its volume, making use of the function `circle-area`.

Contract and Purpose Statement

Every contract has three parts...

;
function name :
Domain → Range

;
what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (function name input(s)) what the function produces)

(EXAMPLE (function name input(s)) what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (function name variable(s)))

what the function does with those variable(s)

The Design Recipe (Breaking Even)

Directions: The Swamp in the City Festival is ordering t-shirts. The production cost is \$75 to set up the silk screen and \$9 per shirt. Use the Design Recipe to write a function `min-shirt-price` that takes in the number of shirts to be ordered, n , and returns the minimum amount the festival should charge for the shirts in order to break even. (Assume that they will sell all of the shirts.)

Contract and Purpose Statement

Every contract has three parts...

:
function name :
Domain -> Range

:
what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (:
function name input(s)):
what the function produces)

(EXAMPLE (:
function name input(s)):
what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (:
function name variable(s)):
what the function does with those variable(s))

The Design Recipe (Marquee & Cubing)

Directions: Use the Design Recipe to write a function `marquee` that takes in a message and returns that message in large gold letters.

Contract and Purpose Statement

Every contract has three parts...

`; _____` function name `;` _____ Domain \rightarrow _____ Range

`;` _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____) function name _____ input(s) _____ what the function produces)

(EXAMPLE (_____) function name _____ input(s) _____ what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (_____) function name _____ variable(s) _____)

what the function does with those variable(s)

Directions: Use the Design Recipe to write a function `num-cube` that takes in a number and returns the cube of that number.

Contract and Purpose Statement

Every contract has three parts...

`; _____` function name `;` _____ Domain \rightarrow _____ Range

`;` _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____) function name _____ input(s) _____ what the function produces)

(EXAMPLE (_____) function name _____ input(s) _____ what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (_____) function name _____ variable(s) _____)

what the function does with those variable(s)

Danger and Target Movement

Directions: Use the Design Recipe to write a function `update-danger`, which takes in the danger's x-coordinate and produces the next x-coordinate, which is 50 pixels to the left.

Contract and Purpose Statement

Every contract has three parts...

: _____ : _____ -> _____
function name Domain Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____) _____ what the function produces
function name input(s) _____)

(EXAMPLE (_____) _____ what the function produces
function name input(s) _____)

Definition

Write the definition, giving variable names to all your input values...

(define (_____)
function name variable(s) _____)

what the function does with those variable(s)

Directions: Use the Design Recipe to write a function `update-target`, which takes in the target's x-coordinate and produces the next x-coordinate, which is 50 pixels to the right.

Contract and Purpose Statement

Every contract has three parts...

: _____ : _____ -> _____
function name Domain Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____) _____ what the function produces
function name input(s) _____)

(EXAMPLE (_____) _____ what the function produces
function name input(s) _____)

Definition

Write the definition, giving variable names to all your input values...

(define (_____)
function name variable(s) _____)

what the function does with those variable(s)

Surface Area of a Rectangular Prism - Explore

1) What do you picture in your mind when you hear *rectangular prism* ?

2) What do you picture in your mind when you hear *surface area* ?

3) Open the [Surface Area of a Rectangular Prism Starter File](#) and click "Run". Type `prism` into the Interactions Area (on the right) and hit "enter" to see an image of a rectangular prism. What do you notice about the image?

4) How many faces does this prism have? _____

Find PART 1 in the Definitions Area of the starter file (on the left). You will see a definition for `front` and `back` .

5) How did the author know to use width and height as the dimensions for `front` and `back` ?

6) Why are `front` and `back` defined to be the same thing?

7) Add definitions for the other faces of the prism, using these definitions as a model, and the image of the prism as a support.

Find PART 2 in the starter file. You'll see `faces = [list: front, back,]` ... so far the list only includes `front` and `back` .

8) Complete the faces list, then type ([print-imgs faces](#)) into the Interactions Area. What do you see?

We're going to print the faces following directions in PART 3 and build a paper model of a rectangular prism.

Before you print and build your prism, you can change the length, width, and height of your prism at the top of the starter file. Be sure that all 3 dimensions are different, and that they are all small enough to fit on a sheet of paper. If you change them, record your new dimensions here.

LENGTH: _____ WIDTH: _____ HEIGHT: _____

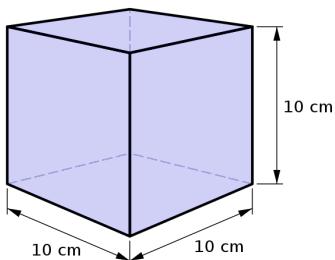
11) Calculate the surface area of your prism, by adding the area of each face. _____ Show your work below.

12) In PART 4 of the starter file, you wrote code to calculate the surface area. How many definitions did you use? _____

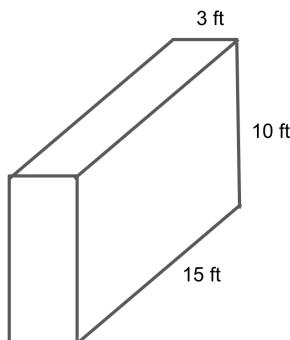
13) How does the surface area that the computer returns compare to the surface area you calculated by hand?

Surface Area of a Prism - Practice

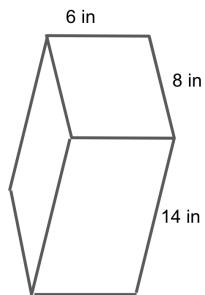
Find the Surface Area of each rectangular prism below. Show your work.



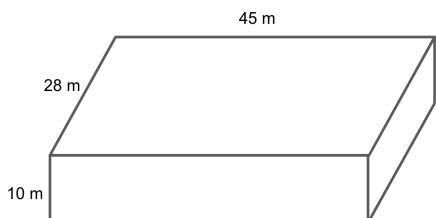
Surface Area: _____



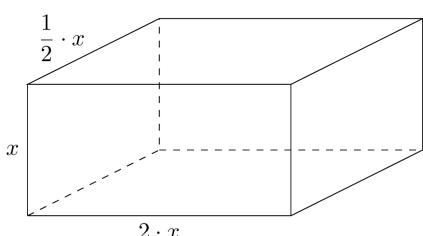
Surface Area: _____



Surface Area: _____



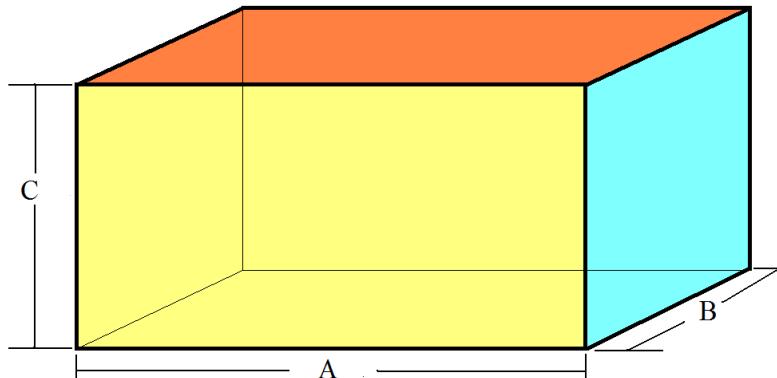
Surface Area: _____



Surface Area: _____

Surface Area of a Prism - More than One Way

Students in Mr. Grattan's class were asked to write code that would calculate the surface area of this rectangular prism. Help them convert their strategies into algebraic expressions and code, and double check that each strategy works.



1) Della says, "Just find the area of the top, bottom, left, right, front and back and add them all together!" **Will it work?** _____

• Algebraic Expression: _____

• Code: _____

2) Orion says, "Just find the area of the front, top and right faces, add them together, and double the sum." **Will it work?** _____

• Algebraic Expression: _____

• Code: _____

3) Jules says, "Double the area of the front, double the area of the top, double the area of the side. Then add them." **Will it work?** _____

• Algebraic Expression: _____

• Code: _____

4) Tate says, "Just multiply the length times the width times the height and double." **Will it work?** _____

• Algebraic Expression: _____

• Code: _____

5) Can you think of one other way to find the surface area of the prism?

• Description: _____

• Algebraic Expression: _____

• Code: _____

6) Whose strategy do you like best? _____

Why? _____

Problem Decomposition

- Sometimes a problem is too complicated to solve all at once. Maybe there are too many variables, or there is just so much information that we can't get a handle on it!
- We can use **Problem Decomposition** to break those problems down into simpler pieces, and then work with the pieces to solve the whole. There are two strategies we can use for decomposition:
 - **Top-Down** - Start with the "big picture", writing functions or equations that describe the connections between parts of the problem. Then, work on defining those parts.
 - **Bottom-Up** - Start with the smaller parts, writing functions or equations that describe the parts we understand. Then, connect those parts together to solve the whole problem.
- You may find that one strategy works better for some types of problems than another, so make sure you're comfortable using either one!

Word Problems: revenue, cost

Directions: Use the Design Recipe to write a function `revenue`, which takes in the number of glasses sold at \$1.75 apiece and calculates the total revenue.

Contract and Purpose Statement

Every contract has three parts...

: _____ function name : _____ Domain -> _____ Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (_____ function name _____ variable(s)) _____)

what the function does with those variable(s)

Directions: Use the Design Recipe to write a function `cost`, which takes in the number of glasses sold and calculates the total cost of materials if each glass costs \$.30 to make.

Contract and Purpose Statement

Every contract has three parts...

: _____ function name : _____ Domain -> _____ Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (_____ function name _____ variable(s)) _____)

what the function does with those variable(s)

Word Problem: profit

Directions: Use the Design Recipe to write a function `profit` that calculates total profit from glasses sold, which is computed by subtracting the total cost from the total revenue.

Contract and Purpose Statement

Every contract has three parts...

: _____ function name : _____ Domain -> _____ Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (_____ function name _____ variable(s)) _____)

what the function does with those variable(s)

Profit - More than one Way!

Four students defined the same `revenue` and `cost` functions, shown below:

```
(define (revenue g) (* 1.75 g))  
(define (cost g) (* 0.3 g))
```

However, they came up with **four different definitions** for `profit`:

Khalil:	(define (profit g) (- (* 1.75 g) (* 0.3 g)))
Samaria:	(define (profit g) (* (- 1.75 0.3) g))
Alenka:	(define (profit g) (* 1.45 g))
Fauzi:	(define (profit g) (- (revenue g) (cost g)))

1) Which of these four definitions do you think is "best", and why?

2) If lemons get more expensive, which definitions of `profit` need to be changed?

3) If Sally raises her prices, which definitions of `profit` need to be changed?

4) Which definition of `profit` is the most flexible? Why?

Top Down or Bottom Up

Jamal's trip requires him to drive 20 mi to the airport, fly 2,300 mi, and then take a bus 6 mi to his hotel. His average speed driving to the airport is 40 mph, the average speed of an airplane is 575 mph, and the average speed of his bus is 15 mph. Aside from time waiting for the plane or bus, how long is Jamal in transit?

Bear's Strategy:	Lion's Strategy:
$\text{Drive Time} = 20 \text{ miles} \times \frac{1 \text{ hour}}{40 \text{ miles}} = 0.5 \text{ hours}$	$\text{In Transit Time} = \text{Drive Time} + \text{Fly Time} + \text{Bus Time}$
$\text{Fly Time} = 2300 \text{ miles} \times \frac{1 \text{ hour}}{575 \text{ miles}} = 4 \text{ hours}$	$\text{Drive Time} = 20 \text{ miles} \times \frac{1 \text{ hour}}{40 \text{ miles}} = 0.5 \text{ hours}$
$\text{Bus Time} = 6 \text{ miles} \times \frac{1 \text{ hour}}{15 \text{ miles}} = 0.4 \text{ hours}$	$\text{Fly Time} = 2300 \text{ miles} \times \frac{1 \text{ hour}}{575 \text{ miles}} = 4 \text{ hours}$
$\text{In Transit Time} = \text{Drive Time} + \text{Fly Time} + \text{Bus Time}$	$\text{Bus Time} = 6 \text{ miles} \times \frac{1 \text{ hour}}{15 \text{ miles}} = 0.4 \text{ hours}$
$0.5 + 4 + 0.4 = 4.9 \text{ hours}$	$0.5 + 4 + 0.4 = 4.9 \text{ hours}$

1) Whose Strategy was Top Down? How do you know?

2) Whose Strategy was Bottom Up? How do you know?

3) Which way of thinking about the problem makes more sense to you?

What's happening with that Math?!

When calculating Jamal's drive time, we multiplied distance by speed. More specifically, we multiplied the starting value (20 miles) by $\frac{1 \text{ hour}}{40 \text{ miles}}$. Why? Why not reverse it, to use $\frac{40 \text{ miles}}{1 \text{ hour}}$, as stated in the problem?

Time is the desired outcome. Looking at the units, we can see that speed must have miles as its denominator to *cancel out* the miles in the starting value.

$$\frac{20 \text{ miles}}{1} \times \frac{1 \text{ hour}}{40 \text{ miles}} = \frac{20 \text{ miles} \times 1 \text{ hour}}{40 \text{ miles}} = \frac{20}{40} \text{ hour} = \frac{1}{2} \text{ hour}$$

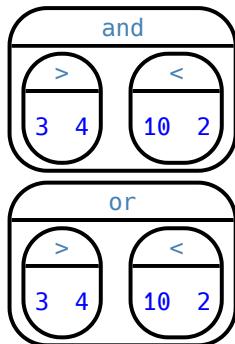
Sally's Bike

We know that it costs Sally 30cents to make a cup of lemonade and she's selling each cup for \$1.75. If the bike Sally wants costs \$198 and sales tax in her town is 7 percent, how many cups of lemonade will Sally have to sell in order to buy the bike?

Use the open space below to find the answer, being sure to show your work!

Inequalities

- Sometimes we want to *ask questions* about data. For example, is `x` greater than `y`? Is one string equal to another? These questions can't be answered with **Numbers**. Instead, they are answered with a new data type called a **Boolean**.
- Video games use Booleans for many things: asking when a player's health is equal to zero, whether two characters are close enough to bump into one another, or if a character's coordinates put it off the edge of the screen.
- A Boolean value is either `true` or `false`. Unlike Numbers, Strings, and Images, Booleans have only two possible values.
- You already know some functions that produce Booleans, such as `<` and `>!` Our programming language has them, too: `(< 3 4)`, `(> 10 2)`, and `(= -10 19)`.
- We also have ways of writing **Compound Inequalities**, so we can ask more complicated questions using the `and` and `or` functions.
 - `(and (> 3 4) (< 10 2))` translates to "three is greater than four *and* ten is less than two". This will evaluate to `false`, since the `and` function requires that both sub-expressions be `true`.
 - `(or (> 3 4) (< 10 2))`, which translates to "three is greater than four *or* ten is less than two". This will evaluate to `true`, since the `or` function only requires that one sub-expression be `true`.
- The Circles of Evaluation work the same way with Booleans that they do with Numbers, Strings and Images:



Boolean Functions

Make a prediction about what each function in the [Boolean Starter File](#) does.

Now, experiment with the functions. Fill in the blanks below so that each of the five functions returns **true**.

- 1) (`odd?`)
- 2) (`even?`)
- 3) (`less-than-one?`)
- 4) (`continent?`)
- 5) (`primary-color?`)

Fill in the blanks below so that each of the five functions returns **false**.

- 6) (`odd?`)
- 7) (`even?`)
- 8) (`less-than-one?`)
- 9) (`continent?`)
- 10) (`primary-color?`)

Simple Inequalities

Each inequality expression in the first column contains a number.

Decide whether or not that number is a solution to the expression and place it in the appropriate column.

Then identify 4 *solution* values and 4 *non-solution* values for x .

- **Solutions** will make the expression **true**.
- **Non-Solutions** will make the expression **false**.

You can see graphs of the solution sets of these inequalities and test out each of your lists in the [Simple Inequalities Starter File](#).

The comments in the starter file will help you learn how it works!

★ Challenge yourself to use negatives, positives, fractions, decimals, etc. for your x values.

	Expression	4 solutions that evaluate to true	4 non-solutions that evaluate to false
a	($> x 2$)		
b	($\leq x -2$)		
c	($< x 3.5$)		
d	($\geq x -1$)		
e	($> x -4$)		
f	($\neq x 2$)		

1) For which inequalities was the number from the expression part of the solution?

2) For which inequalities was the number from the expression not part of the solution?

3) For which inequalities were the solutions on the left end of the number line?

4) For which inequalities were the solutions on the right end of the number line?

Word Problem: hot?

Directions: Use the Design Recipe to write a function `hot?`, which takes in a temperature in Fahrenheit and determines if it is above 80 degrees

Contract and Purpose Statement

Every contract has three parts...

: _____ function name : _____ Domain -> _____ Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

Definition

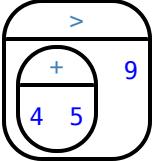
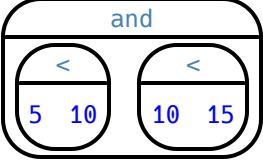
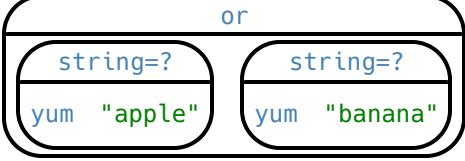
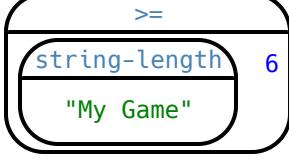
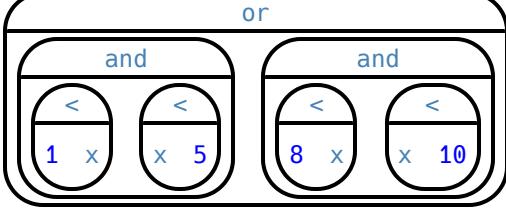
Write the definition, giving variable names to all your input values...

(`define` (_____ function name _____ variable(s)) _____)

what the function does with those variable(s)

Converting Circles of Evaluation to Code

For each Circle of Evaluation on the left-hand side, write the Code for the Circle on the right-hand side.

	Circle of Evaluation	Code
1		
2		
3		
4		
5		

Compound Inequalities – Practice

Create the Circles of Evaluation, then convert the expressions into Code in the space provided.

1) 2 is less than 5, and 0 is equal to 6

What will this evaluate to? _____

2) 6 is greater than 8, or -4 is less than 1

What will this evaluate to? _____

3) The String “purple” is the same as the String “blue”, and 3 plus 5 equals 8

What will this evaluate to? _____

4) Write the contracts for and & or in your Contracts page.

Compound Inequality Warmup

1) What are 4 solutions for $x > 5$?

2) What are 4 non-solutions for $x > 5$?

3) What are 4 solutions for $x \leq 15$?

4) What are 4 non-solutions for $x \leq 15$?

5) What 4 numbers are in the solution set of $x > 5$ **and** $x \leq 15$, making both of these inequalities true?

6) How would that be different from the solution set of $x > 5$ **or** $x \leq 15$, making at least one of these inequalities true?

Exploring Compound Inequality

This page is designed to accompany [Compound Inequalities Starter File](#). When you click "Run" you will see 4 graphs. The first two are simple inequalities and the second two are compound inequalities.

1) What does and–intersection do?

2) Why is the circle on 5 red and the circle on 15 green?

3) Do you think every graph made with and–intersection will have different color dots at the ends? Why or why not?

4) What does or–union do?

5) Why did the graph of this or–union result in the whole numberline being shaded blue?

6) Not all graphs of or–union will look like this. Can you think of a pair of inequalities whose union won't shade the whole graph?

Change the function definition on *line 8* to $x < 5$ and the definition on *line 9* to $x \geq 15$ and, before you click "Run", take a moment to think about what the new graphs of and–intersection and or–union will look like. Then click "Run" and take a look.

7) What does the new and–intersection graph look like?

8) What does the new or–union graph look like?

9) Why is the dot for 5 red and the dot for 15 green?

10) Which of the 8 numbers from the list are part of the solution set? How do you know?

11) Is 3 part of the solution set? Explain. _____

12) Is 10 part of the solution set? Explain. _____

Compound Inequalities: Solutions & Non-Solutions

For each Compound Inequality listed below, identify 4 *solutions* and 4 *non-solutions*.

If there are **no solutions** or the solution set includes **all real numbers**, write that instead of making a list.

- Solutions for **intersections**, which use **and** will make both of the expressions true.
- Solutions for **unions**, which use **or** will make at least one of the expressions true.

Pay special attention to the numbers in the sample expression! Challenge yourself to use negatives, positives, fractions, decimals, etc. for your x values.

The first two have been done for you - Answers will vary!

	Expression	4 solutions that evaluate to true	4 non-solutions that evaluate to false
a	$x > 5 \text{ and } x < 15$	6, 9.5, 12, 14.9	-2, 5, 15, 16.1
b	$x > 5 \text{ or } x < 15$	All real numbers	No non-solutions
c	$x \leq -2 \text{ and } x > 7$		
d	$x \leq -2 \text{ or } x > 7$		
e	$x < 3.5 \text{ and } x > -4$		
f	$x < 3.5 \text{ or } x > -4$		
g	$x \geq -1 \text{ and } x > -5$		
h	$x \geq -1 \text{ or } x > -5$		
i	$x < -4 \text{ and } x > 2$		

1) Could there ever be a union with *no solutions*? Explain your thinking.

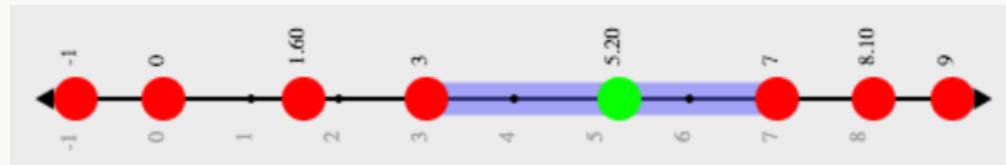
2) Could there ever be an intersection whose solution is *all real numbers*? Explain your thinking.

Compound Inequality Functions

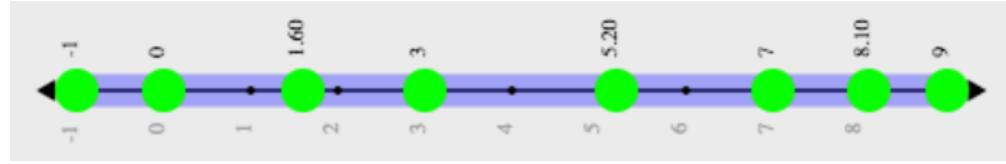
Each of the plots below was generated using the code `inequality(comp-ineq, [list: -1, 0, 1.6, 3, 5.2, 7, 8.1, 9])`. Using the numbers 3 and 7, write the code to define `comp-ineq` for each plot. Note: The example is defined using 0 and 8.1 rather than 3 and 7.



code: _____



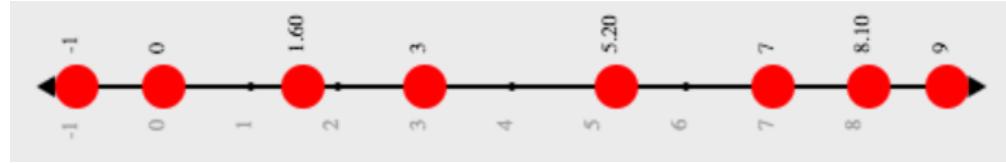
code: _____



code: _____



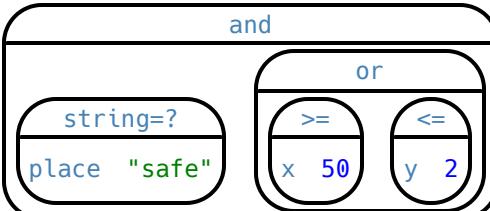
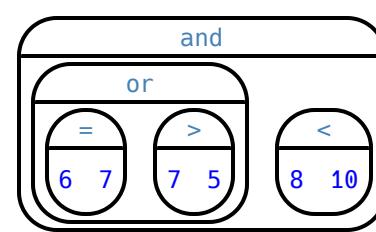
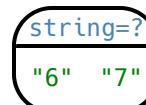
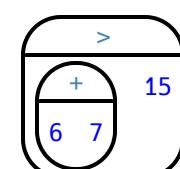
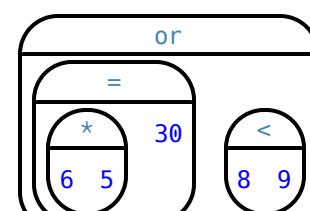
code: _____



code: _____

Converting Circles of Evaluation with Booleans to Code 2

For each Circle of Evaluation on the left-hand side, write the code for the Circle on the right-hand side.

	Circle of Evaluation	Code
1	 <p>and</p> <p>string=? place "safe"</p> <p>or</p> <p>>= x 50 <= y 2</p>	
2	 <p>and</p> <p>or</p> <p>= 6 7 > 7 5 < 8 10</p>	
3	 <p>string=? "6" "7"</p>	
4	 <p>> + 6 7 15</p>	
5	 <p>or</p> <p>= * 6 5 30 < 8 9</p>	

Sam the Butterfly

Open the [Sam the Butterfly Starter File](#) starter file and click "Run". (*Hi, Sam!*)

Move Sam around the screen using the arrow keys.

1) What do you Notice about the program?

2) What do you Wonder?

3) What do you see when Sam is at (0,0)? Why is that?

4) What changes as the butterfly moves left and right?

5) Sam is in a 640×480 yard. Sam's mom wants Sam to stay in sight. **How far to the left and right can Sam go and still remain visible?**

Use the new inequality functions to answer the following questions *with code*:

6) Sam hasn't gone off the left edge of the screen as long as... _____

7) Sam hasn't gone off the right edge of the screen as long as... _____

8) Use the space below to draw Circles of Evaluation for these two expressions:

Left and Right

Directions: Use the Design Recipe to write a function `safe-left?`, which takes in an x-coordinate and checks to see if it is greater than -50.

Contract and Purpose Statement

Every contract has three parts...

: function name : Domain -> Range

: what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (function name) input(s)) what the function produces)

(EXAMPLE (function name) input(s)) what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (function name variable(s))
what the function does with those variable(s))

Directions: Use the Design Recipe to write a function `safe-right?`, which takes in an x-coordinate and checks to see if it is less than 690.

Contract and Purpose Statement

Every contract has three parts...

: function name : Domain -> Range

: what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (function name) input(s)) what the function produces)

(EXAMPLE (function name) input(s)) what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (function name variable(s))
what the function does with those variable(s))

Word Problem: onscreen?

Directions: Use the Design Recipe to write a function `onscreen?`, which takes in an x-coordinate and checks to see if Sam is safe on the left while also being safe on the right.

Contract and Purpose Statement

Every contract has three parts...

: _____ function name : _____ Domain -> _____ Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

(EXAMPLE (_____ function name _____ input(s)) _____ what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (_____ function name _____ variable(s)) _____)

what the function does with those variable(s)

Onscreen - More than One Way

Nokosee's Thinking

```
(define (safe-bottom? y) (>= y -30))  
(define (safe-top? y) (<= y 510))  
(define (onscreen? y) (and (safe-bottom? y) (safe-top? y)))
```

Sabra's Thinking

```
(define (safe-bottom? y) (> y -40))  
(define (safe-top? y) (< y 520))  
(define (onscreen? y) (and (> y -40) (< y 520)))
```

1) Nokosee and Sabra have different strategies for keeping Sam on the screen. How does Nokosee's strategy work?

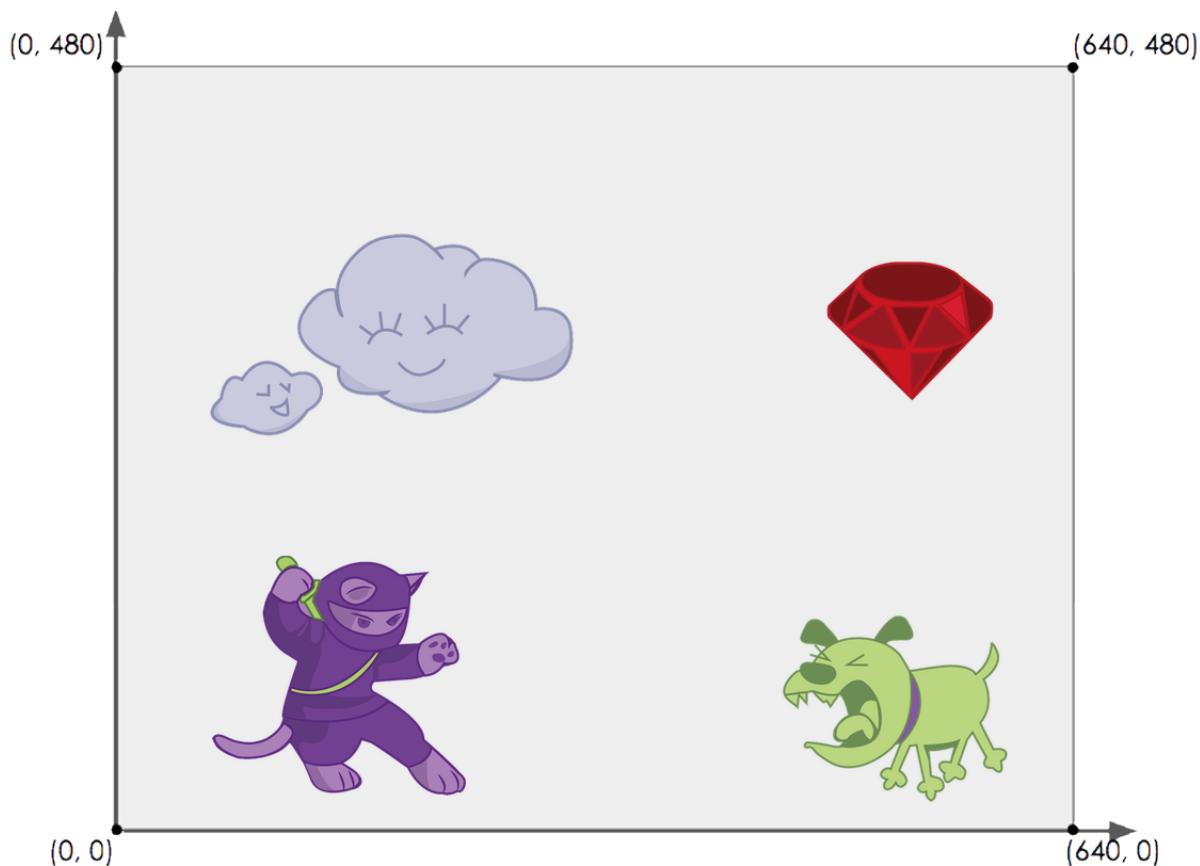
2) How does Sabra's strategy work?

3) What's an advantage of Nokosee's strategy?

4) What's an advantage of Sabra's strategy?

5) Which strategy do you prefer? Why?

Keeping NinjaCat in the Game



After each jump, Ninjacat lands at the bottom of the screen. It wouldn't be much fun if Ninjacat kept falling past the bottom of the screen and couldn't be seen anymore!

1) What changes as Ninjacat moves up and down? _____

Use the new inequality functions to answer the following questions with code:

2) Ninjacat is still visible on the bottom of the screen as long as... _____

3) Ninjacat hasn't gone off the top edge of the screen as long as... _____

4) Use the space below to draw Circles of Evaluation for these two expressions:

Piecewise Functions

- Sometimes we want to build functions that act differently for different inputs. For example, suppose a business charges \$10/pizza, but only \$5 for orders of six or more. How could we write a function that computes the total price based on the number of pizzas?
- In math, **Piecewise Functions** are functions that can behave one way for part of their Domain, and another way for a different part. In our pizza example, our function would act like $\text{cost}(\text{pizzas}) = 10 * \text{pizzas}$ for anywhere from 1-5 pizzas. But after 5, it acts like $\text{cost}(\text{pizzas}) = 5 * \text{pizzas}$.
- Piecewise functions are divided into "pieces". Each piece is divided into two parts:
 1. How the function should behave
 2. The domain where it behaves that way
- Our programming language can be used to write piecewise functions, too! Just as in math, each piece has two parts:

```
(define (cost pizzas)
  (cond
    [(>= pizzas 6) (* 5 pizzas)]))
```

Piecewise functions are powerful, and let us solve more complex problems. We can use piecewise functions in a video game to add or subtract from a character's x-coordinate, moving it left or right depending on which key was pressed.

Red Shape - Explore

1) Open the [Red Shape Starter File](#), and read through the code you find there. This code contains new programming that you haven't seen yet! Take a moment to list everything you Notice, and then everything you Wonder...

Notice	Wonder

2) What happens if you click "Run" and type `(red-shape "ellipse")` ?

3) Add **another example** for "triangle".

4) Add another line of code to the definition, to define what the function should do with the input "triangle".

5) Come up with some new shapes, and add them to the code. Make sure you include examples or you will get an error message!

6) In your own words, describe how *piecewise functions* work in this programming environment.

Word Problem: red-shape

Directions: A friend loves red shapes so we've decided to write a program that makes it easy to generate them. Write a function called red-shape which takes in the name of a shape and makes a 20-pixel, solid, red image of the shape.

Contract and Purpose Statement

Every contract has three parts...

; red-shape : String -> Image
function name Domain Range

; Given a shape name, produce a solid, red, 20-pixel image of the shape.
what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (red-shape
function name) "circle") (circle 20 "solid" "red")
input(s) what the function produces

(EXAMPLE (red-shape
function name) "triangle") (triangle 20 "solid" "red")
input(s) what the function produces

(EXAMPLE (red-shape
function name) "rectangle") (rectangle 20 20 "solid" "red")
input(s) what the function produces

(EXAMPLE (red-shape
function name) "star") (star 20 "solid" "red")
input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values...

(define (function name) variable(s))

(cond _____

[_____] _____

[_____] _____

[_____] _____

[_____] _____

[_____]))

Decide & Defend - Piecewise Onto Functions

Joy and Marianna have written two different sets of code to accomplish the same goal of helping a caterer direct people with dietary restrictions to a menu item that works for them. Look at the code below.

Joy's Code:

```
(define (entree diet)
  (cond
    [(string=? diet "gluten-free") salmon]
    [(string=? diet "kosher") salmon]
    [(string=? diet "lactose-int") salmon]
    [(string=? diet "nut allergy") lasagna]
    [(string=? diet "vegan") stir-fry]
    [(string=? diet "vegetarian") stir-fry]
    [else (text "unknown diet" 20 "red")]))
```

Marianna's Code:

```
(define (entree diet)
  (cond
    [(or (or (string=? diet "gluten-free") (string=? diet "kosher")) (string=? diet "lactose-int"))
       salmon]
    [(or (string=? diet "vegan") (string=? diet "vegetarian")) stir-fry]
    [else (text "unknown diet" 20 "red")]))
```

Whose method do you like better? Why?

Word Problem: Mood Generator

NOTE: This file uses emojis. Even though emojis look like images, they are actually characters in a string! They can be accessed from your keyboard, just like any other character.

Directions: They say a picture is worth a thousand words. Write a function mood that translates moods into emojis so that we can "see" what someone is feeling.

Contract and Purpose Statement

Every contract has three parts...

; mood : String -> String

; Consumes a mood and produces the emoji for that mood.
what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (mood "happy") "😊") what the function produces

(EXAMPLE (mood "sad") "😢") what the function produces

(EXAMPLE (mood "angry") "😡") what the function produces

(EXAMPLE (mood "sick") "🤒") what the function produces

Definition

Write the definition, giving variable names to all your input values...

(define (function name) variable(s))

(cond

[]

[]

[]

[]

[]))

Alice's Restaurant - Explore

Alice's code has some new elements we haven't seen before, so let's experiment a bit to figure out how it works! Open the [Alice's Restaurant Starter File](#), click "Run", and try using the `cost` function in the Interactions window.

1) What does `(cost "hamburger")` evaluate to? _____

2) What does `(cost "pie")` evaluate to? _____

3) What if you ask for `(cost "fries")`? _____

4) Explain what the function is doing in your own words.

5) What is the function's name? _____ Domain? _____ Range? _____

6) What is the name of its variable? _____

7) Alice says onion rings have gone up to \$3.75. Change the `cost` function to reflect this.

8) Try adding menu items of your own. What's your favorite? _____

9) For an unknown food item, the function produces the String `"That's not on the menu!"` Is this a problem? Why or why not?

10) Suppose Alice wants to calculate the price of a hamburger, *including a 5% sales tax*. Draw a Circle of Evaluation for the expression below.

Word Problem: Alice's Restaurant

Directions: Alice's Restaurant has hired you as a programmer. They offer the following menu items: hamburger (\$6.00), onion rings (\$3.50), fried tofu (\$5.25) and pie (\$2.25). Write a function called Alice's Restaurant which takes in the name of a menu item and outputs the price of that item.

Contract and Purpose Statement

Every contract has three parts...

: function name : Domain -> Range

: what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (function name input(s)) what the function produces)

(EXAMPLE (function name input(s)) what the function produces)

(EXAMPLE (function name input(s)) what the function produces)

(EXAMPLE (function name input(s)) what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (function name variable(s))

(cond _____

[_____]

[_____]

[_____]

[_____]

[_____]))

Word Problem: update-player

Directions: The player moves by 20 pixels each time the up or down key is pressed. Write a function called `update-player`, which takes in the player's y-coordinate and the name of the key pressed ("up" or "down"), and returns the new y-coordinate.

Contract and Purpose Statement

Every contract has three parts...

: _____ function name : _____ Domain -> _____ Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (update-player 300 "up") _____) what the function produces
function name input(s)

(EXAMPLE (_____) _____) what the function produces
function name input(s)

(EXAMPLE (_____) _____) what the function produces
function name input(s)

(EXAMPLE (_____) _____) what the function produces
function name input(s)

Definition

Write the definition, giving variable names to all your input values...

(define (_____)
function name variable(s)

(cond _____

[_____]

[_____]

[_____]))

Challenges for update-player

For each of the challenges below, see if you can come up with two EXAMPLEs of how it should work!

1) **Warping** - Program one key to "warp" the player to a set location, such as the center of the screen.

(EXAMPLE (update-player _____ _____)
_____)

(EXAMPLE (update-player _____ _____)
_____)

2) **Boundaries** - Change `update-player` such that `PLAYER` cannot move off the top or bottom of the screen.

(EXAMPLE (update-player _____ _____)
_____)

(EXAMPLE (update-player _____ _____)
_____)

3) **Wrapping** - Add code to `update-player` such that when `PLAYER` moves to the top of the screen, it reappears at the bottom, and vice versa.

(EXAMPLE (update-player _____ _____)
_____)

(EXAMPLE (update-player _____ _____)
_____)

4) **Hiding** - Add a key that will make `PLAYER` seem to disappear, and reappear when the same key is pressed again.

(EXAMPLE (update-player _____ _____)
_____)

(EXAMPLE (update-player _____ _____)
_____)

Challenge: Character Movement in Two Dimensions

If your game is working and...

- both the Danger and Target return to the screen
 - your Player moves up and down with the arrow keys
- ... then you have all the tools you need to begin this challenge!

```
; update-danger :: Number -> Number  
; consumes danger's x-coordinate and produces the next one
```

The update-danger function only moves our DANGER left or right... because it doesn't do anything with the y-coordinate!

Suppose we wanted to write a new function, update-danger-2 that moves the DANGER diagonally...

1) What will have to change about the Domain? _____

2) What, if anything, will have to change about the Range if we want to get our character moving diagonally? _____

Since an (x, y) coordinate has two Numbers, one idea might be to write the Contract this way:

```
; update-danger-2 :: Number Number -> Number Number  
; consumes danger's x- and y-coordinate, and produces the next x- and next y-coordinate
```

...But that Contract breaks an important rule about functions:

Given an input, all functions must produce one output!

We need some way to package two Numbers together into a single value.

Fortunately, our programming language has another data type, called a Posn, which utilizes two Numbers to describe a single "position"!

We can make a Posn to represent the position (100, 200) with the following code: (`make-posn 100 200`)

3) What expression will make a Posn representing the origin? _____

4) Write the Contract for the `make-posn` function on the line below.

Challenge: Character Movement in Two Dimensions (2)

Directions: On the lines below, write the new Contract and Purpose for update-danger-2, so that it produces a Posn instead of a Number. Then complete the Design Recipe.

Contract and Purpose Statement

Every contract has three parts...

: function name : Domain -> Range

: what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (function name input(s)) what the function produces)

(EXAMPLE (function name input(s)) what the function produces)

Definition

Write the definition, giving variable names to all your input values...

(define (function name variable(s))

what the function does with those variable(s))

Adding Your New Function to Your Game File

1) Find update-danger in your game file.

2) Directly beneath it, add update-danger-2 (including Contract, Purpose, Examples, and Definition) to your game file.

3) Scroll down to the very end of your game file and find the following PROVIDED CODE.

```
(define g (make-game TITLE TITLE-COLOR
    BACKGROUND
    DANGER update-danger
    TARGET update-target
    PLAYER update-player
    mystery update-mystery
    distances-color line-length distance
    collide? onscreen?))
(play g)
```

4) Change update-danger to update-danger-2 in the list and click "Run" so that your program will use your new function with two-dimensional movement, instead of the old function with one-dimensional movement.

Note: If, at any point, you would like to use the old function, all you have to do is change this list so that it says update-danger instead of update-danger-2!

Challenge: update-player-2

If your game is working and...

- both the Danger and Target return to the screen
- your Player moves up and down with the arrow keys
- you've completed [Challenge: Character Movement in Two Dimensions](#)
- your Danger is moving diagonally

you have all the tools you need to work through this Design Recipe and get your player moving in all four directions!

Directions: Write a new function update-player-2 that takes in the player's x-coordinate, y-coordinate, and an arrow key (described by a String) and moves the player to a new Posn. Your goal is to get all 4 arrow keys working as you would expect them to by moving the player 50 pixels in the corresponding direction!

Contract and Purpose Statement

Every contract has three parts...

; _____ : _____
; _____ Domain _____ -> Range
; _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____)
; _____ function name _____ input(s) _____ what the function produces
(EXAMPLE (_____)
; _____ function name _____ input(s) _____ what the function produces
(EXAMPLE (_____)
; _____ function name _____ input(s) _____ what the function produces
(EXAMPLE (_____)
; _____ function name _____ input(s) _____ what the function produces

Definition

Write the definition, giving variable names to all your input values...

(define (_____)
; _____ function name _____ variable(s)

(cond _____

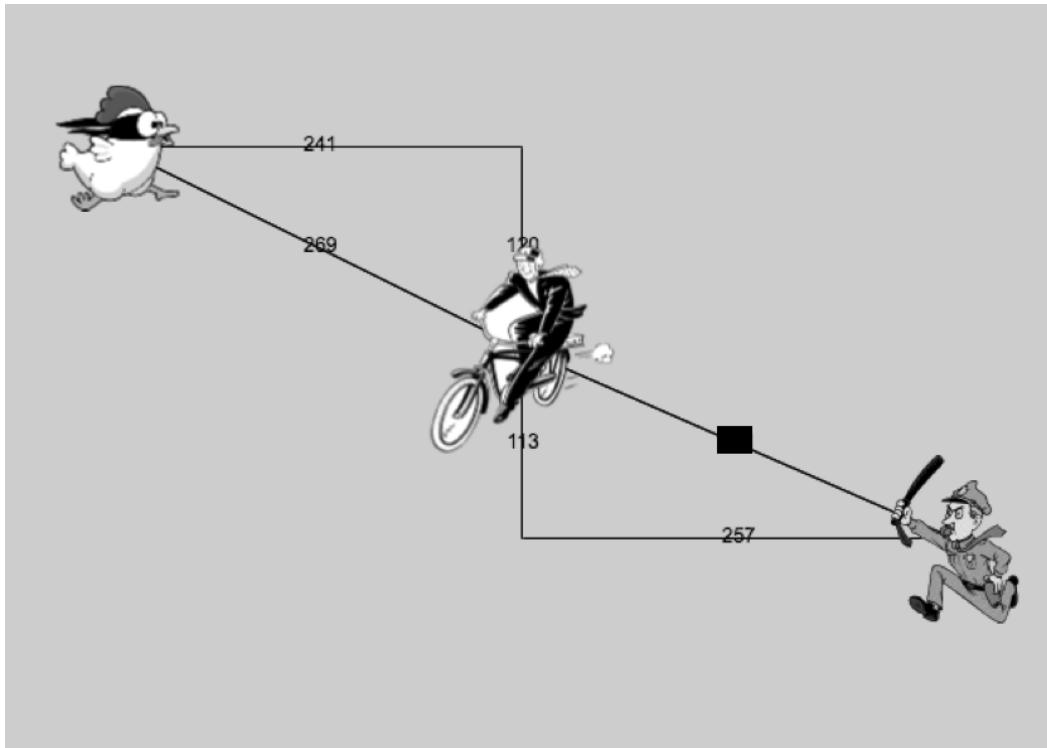
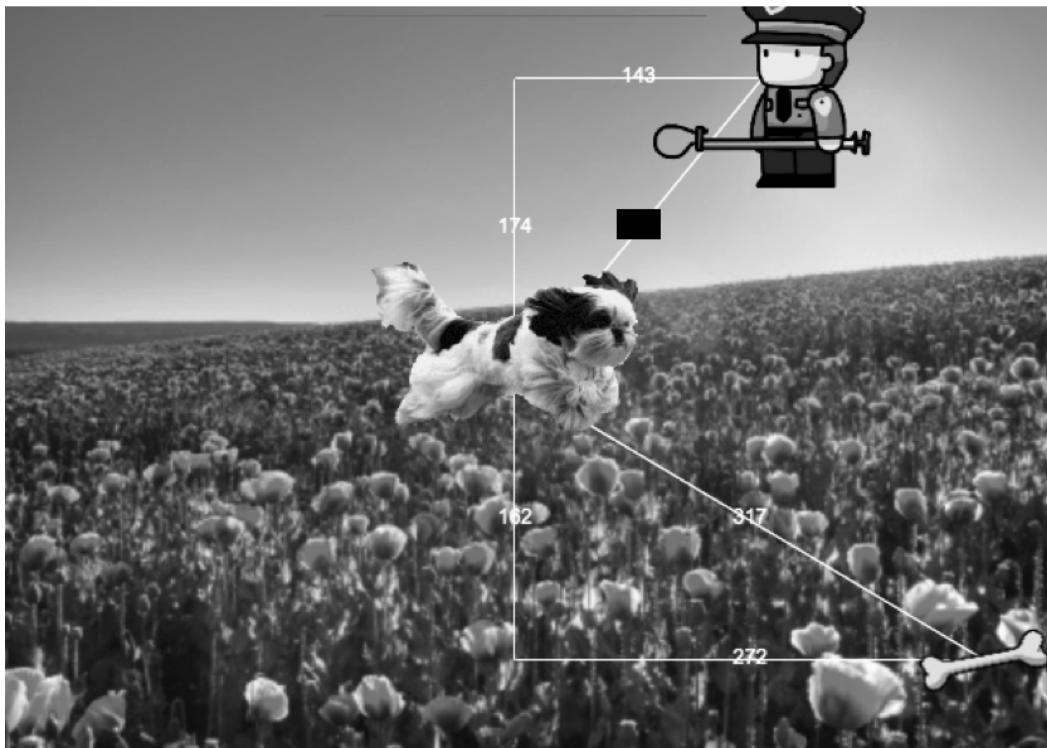
[_____]
[_____]
[_____]
[_____]
[_____]

Follow the directions on [Challenge: Character Movement in Two Dimensions \(2\)](#) for adding your new function to your game file, this time changing update-player to update-player-2.

Once you've mastered 2-dimensional movement, you might want to add secret functionality for some of your favorite letters on the keyboard...

Writing Code to Calculate Missing Lengths

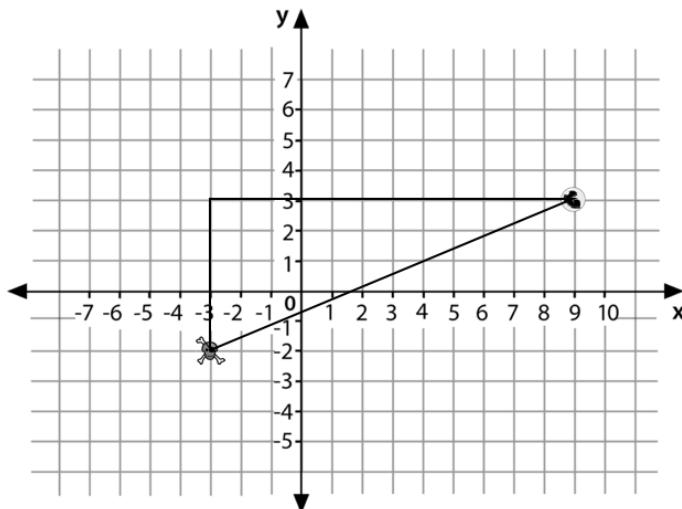
In each of the game screenshots below, one of the distance labels has been hidden. Write the code to generate the missing distance on the line below each image. Hint: Remember the Pythagorean Theorem!



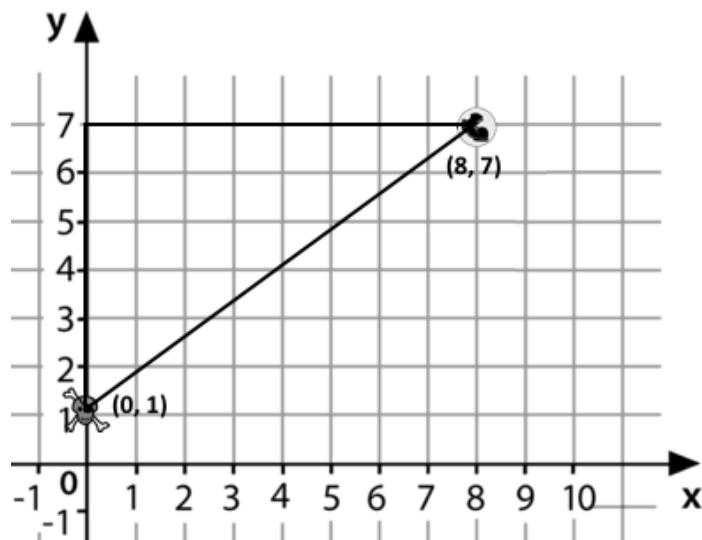
Distance on the Coordinate Plane

Distance between the pyret and the boot:

```
(sqrt (+ (sqr (line-length 9 -3)) (sqr (line-length 3 -2)))))
```



Explain how the code works.



Now write the code to find the distance between this boot and pyret.

Circles of Evaluation: Distance between (0, 2) and (4, 5)

The distance between x_1 and x_2 is computed by (`line-length x2 x1`) . The distance between y_2 and y_1 is computed by (`line-length y2 y1`) . Below is the equation to compute the hypotenuse of a right triangle with legs the lengths of those distances:

$$\sqrt{\text{line-length}(x_2, x_1)^2 + \text{line-length}(y_2, y_1)^2}$$

Suppose your player is at (0, 2) and a character is at (4, 5). What is the distance between them?

1. Identify the values of x_1, y_1, x_2 , and y_2

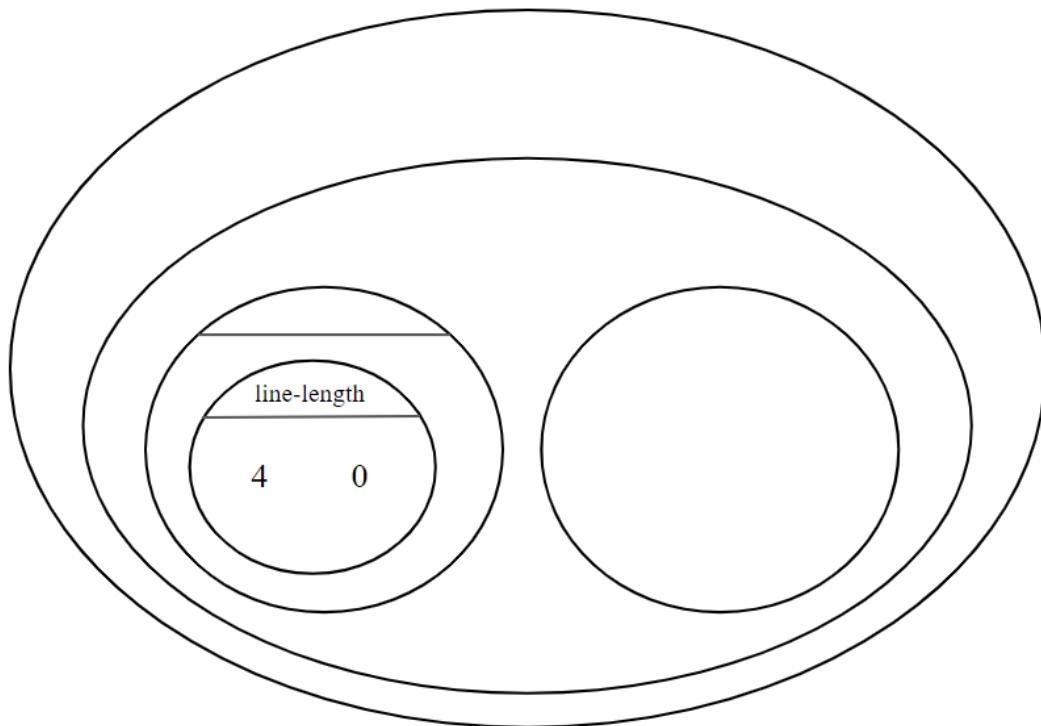
x_1 (x -value of 1st point)	y_1 (y -value of 1st point)	x_2 (x -value of 2nd point)	y_2 (y -value of 2nd point)

The equation to compute the distance between these points is:

$$\sqrt{\text{line-length}(4, 0)^2 + \text{line-length}(5, 2)^2}$$

2. Translate the expression above, for (0,2) and (4,5) into a Circle of Evaluation below.

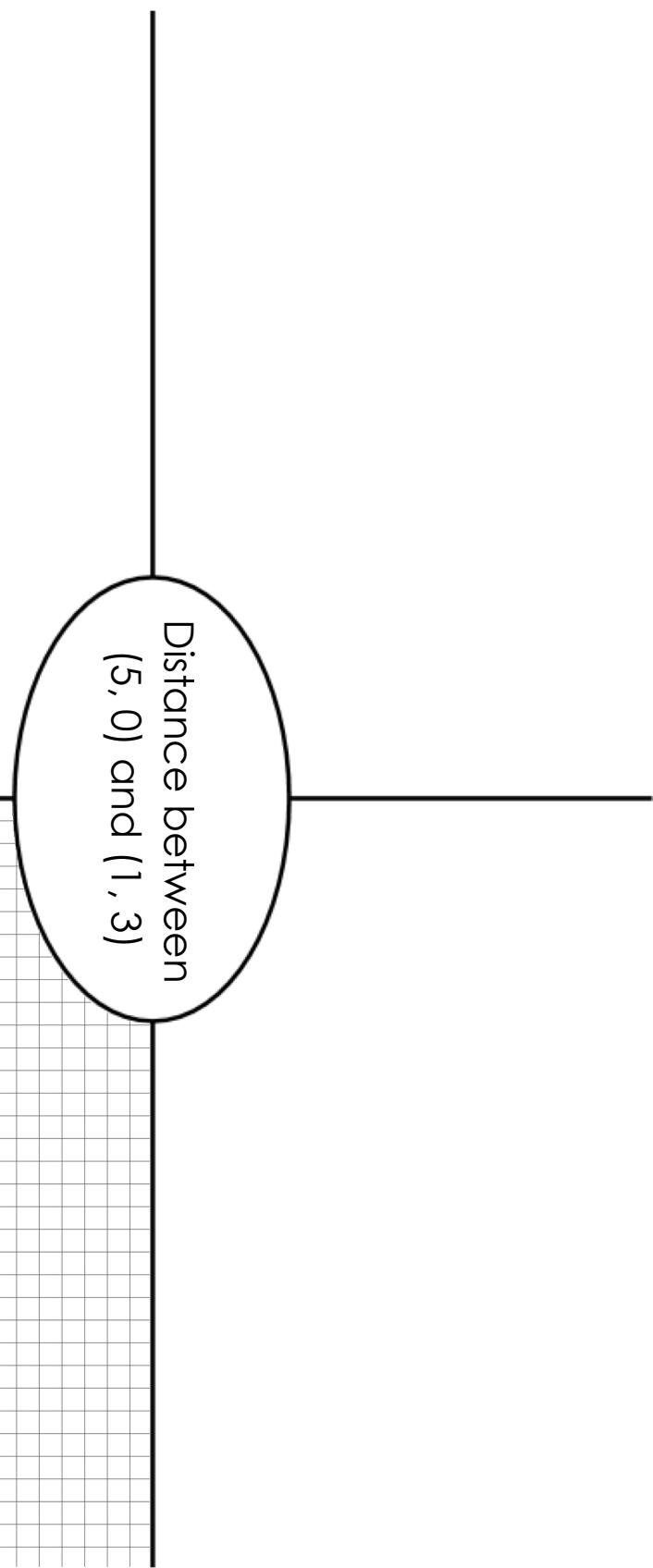
Hint: In our programming language `sqr` is used for x^2 and `sqrt` is used for \sqrt{x}



3. Convert the Circle of Evaluation to Code below.

Circle of Evaluation

Code

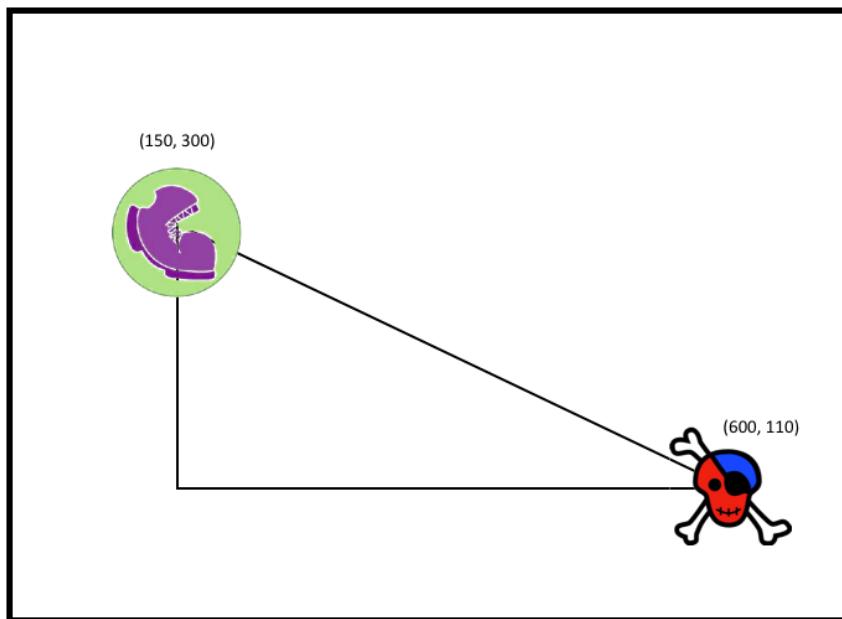


Computed distance
between (5, 0) and (1, 3)

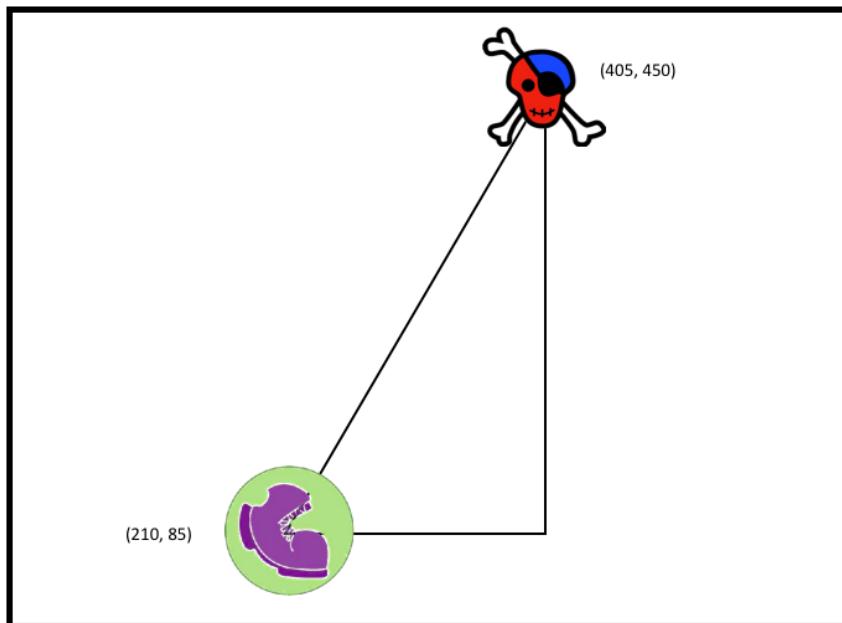
Graph

Distance From Game Coordinates

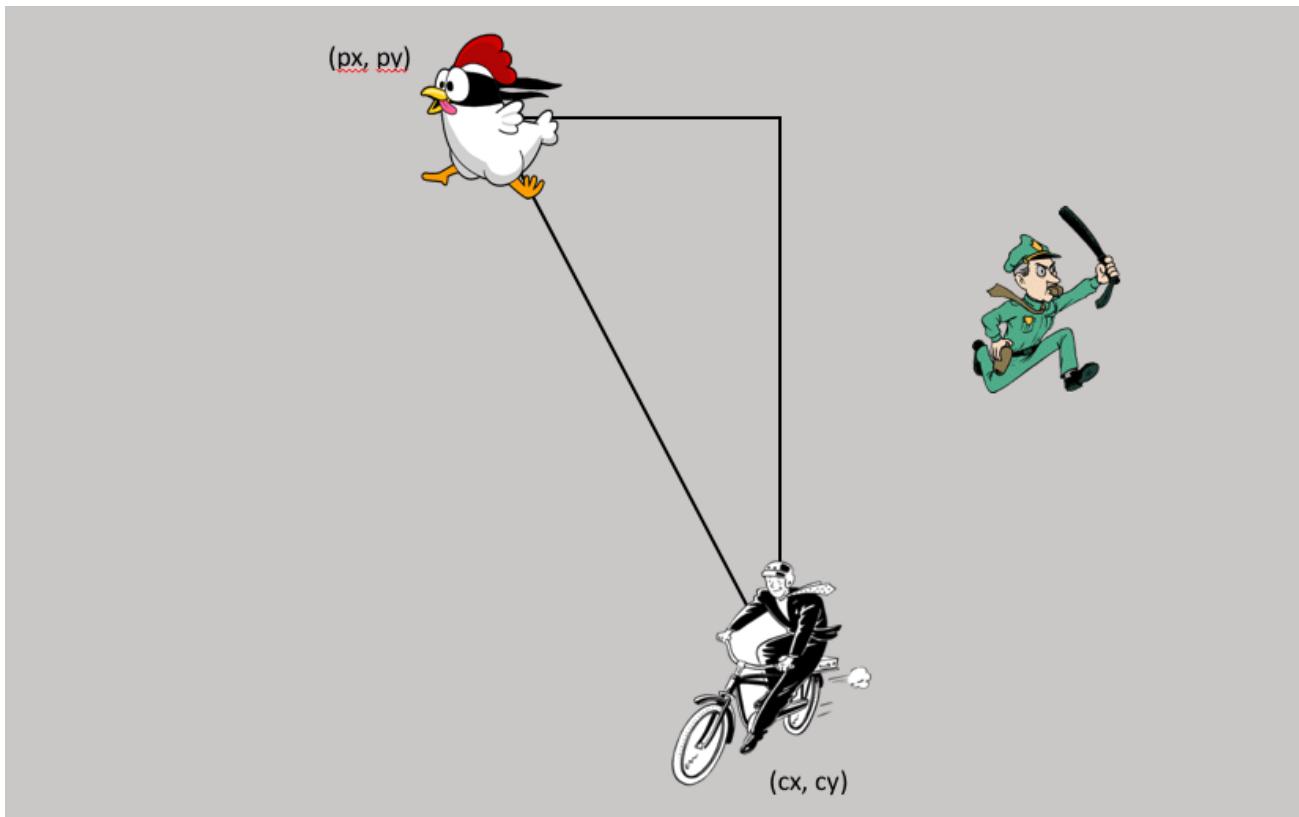
For each of the game screenshots, write the code to calculate the distance between the indicated characters. *The first one has been done for you.*



```
(sqrt (+ (sqr (line-length 600 150)) (sqr (line-length 110 300)))))
```



Distance (px, py) to (cx, cy)



Directions: Use the Design Recipe to write a function `distance`, which takes in FOUR inputs: `px` and `py` (the x- and y-coordinate of the Player) and `cx` and `cy` (the x- and y-coordinates of another character), and produces the distance between them in pixels.

Contract and Purpose Statement

Every contract has three parts...

: _____ ; _____ -> _____
function name Domain Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____) _____)
function name input(s) what the function produces

(EXAMPLE (_____) _____)
function name input(s) what the function produces

Definition

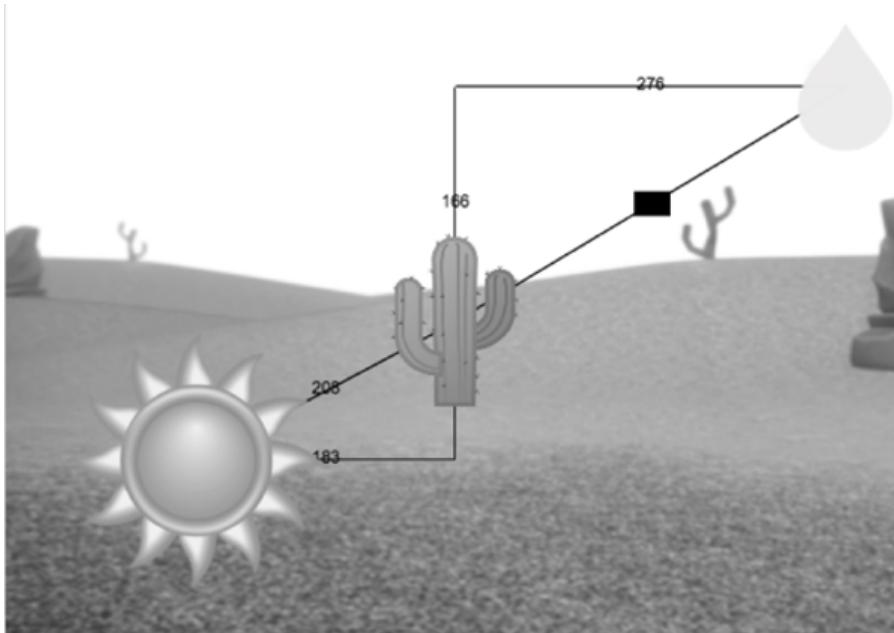
Write the definition, giving variable names to all your input values...

(define (_____)
function name variable(s)

)
what the function does with those variable(s)

Comparing Code: Finding Missing Distances

For each of the game screenshots below, the math and the code for computing the covered distance is shown. Notice what is similar and what is different about how the top and bottom distances are calculated. Think about why those similarities and differences exist and record your thinking.



$$\sqrt{166^2 + 276^2}$$

```
(sqrt (+ (sqr 166) (sqr 276)))
```



$$\sqrt{276^2 - 194^2}$$

```
(sqrt (- (sqr 276) (sqr 194)))
```

Line Length Explore

Find the `line-length` function in your game files and take a minute to look at the code.

1) What do you Notice?

2) What do you Wonder?

Click Run, and practice using `line-length` in the *Interactions Area* with different values for `a` and `b`.

3) What does the `line-length` function *do*?

4) Why does it use conditionals?

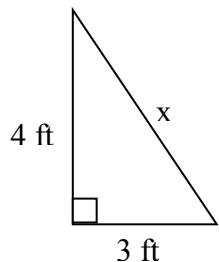
5) Why is the distance between two points always positive?

Name: _____ Date: _____ Pythagorean Theorem Practice

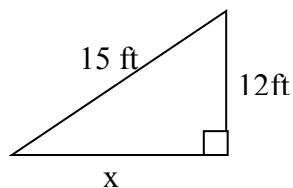
Label the hypotenuse of the triangle c . In each triangle find the length of the side marked x to the nearest unit (foot, cm, etc.). Show your work.

$$a^2 + b^2 = c^2$$

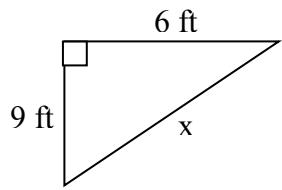
1.



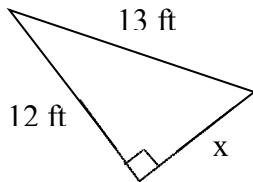
2.



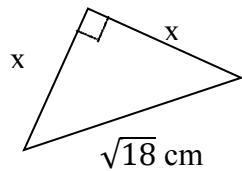
3.



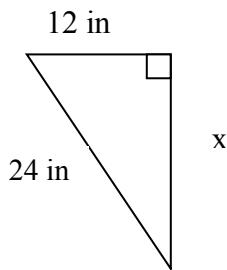
4.

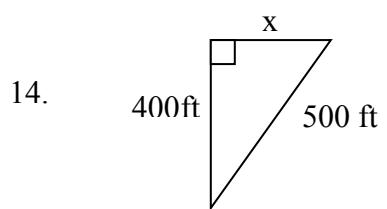
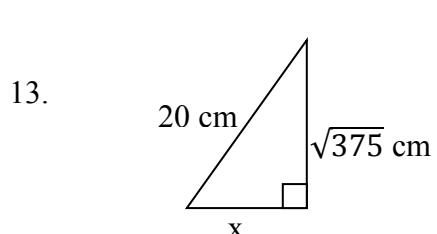
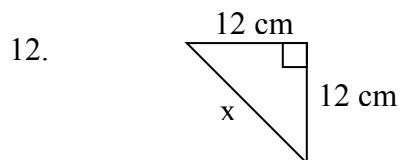
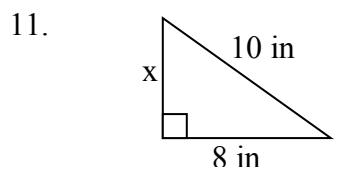
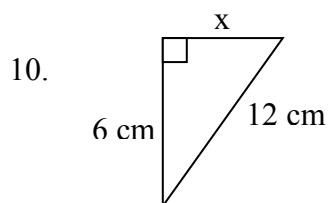
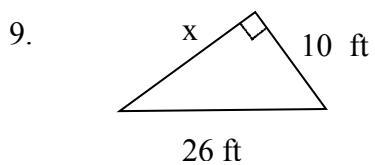
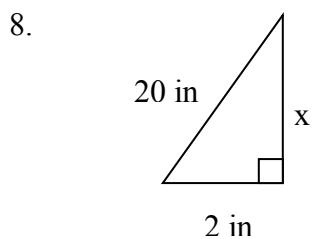
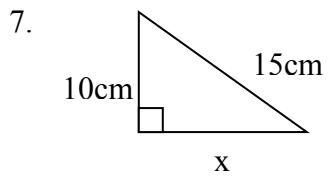


5.



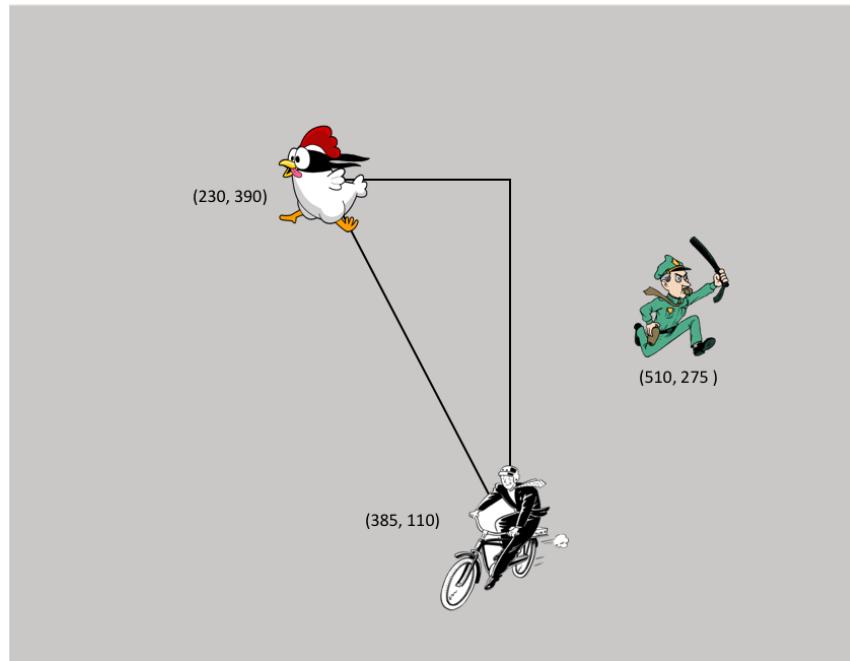
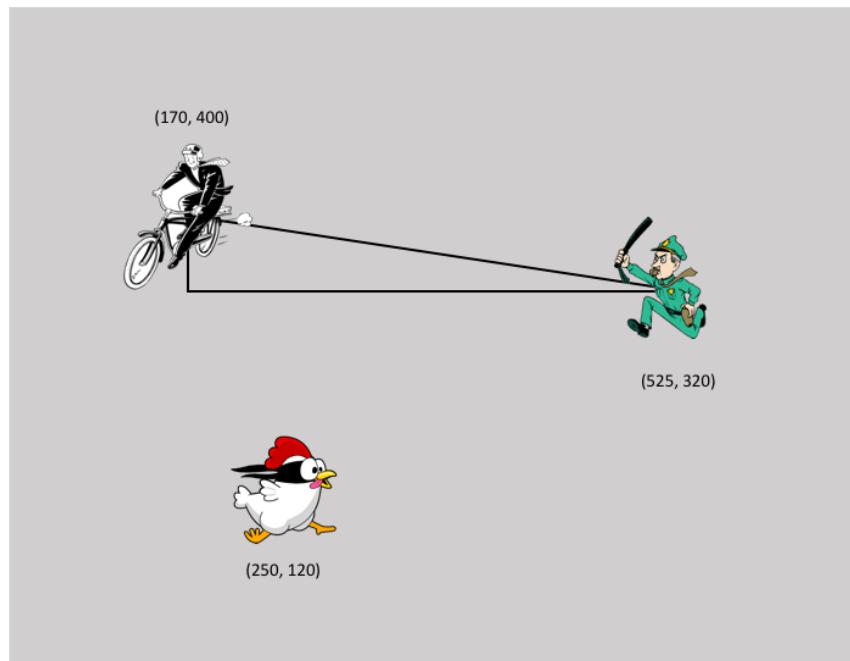
6.





Distance From Game Coordinates 2

For each of the game screenshots below, write the code to calculate the distance between the indicated characters. Refer to *Distance from Game Coordinates for an Example*.



Word Problem: line-length

Directions: Write a function called `line-length`, which takes in two numbers and returns the **positive difference** between them. It should always subtract the smaller number from the bigger one. If they are equal, it should return zero.

Contract and Purpose Statement

Every contract has three parts...

: _____ : _____ -> _____
function name Domain Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (line-length
function name) 10 5) (- 10 5) what the function produces

(EXAMPLE (line-length
function name) 2 8) (- 8 2) what the function produces

Definition

Write the definition, giving variable names to all your input values...

(define (_____
function name) _____
variable(s))

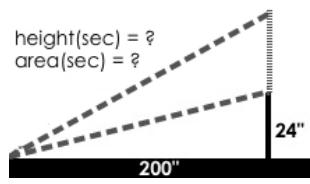
(cond _____

[_____ _____]

[_____ _____]))

Top Down/Bottom Up

A retractable flag pole starts out 24 inches tall, and grows taller at a rate of 0.6in/sec. An elastic is anchored 200 inches from the base and attached to the top of the pole, forming a right triangle. Using a top-down or bottom-up strategy, define functions that compute the *height* of the pole and the *area* of the triangle after a given number of seconds.



;

function name : Domain -> Range

;

what does the function do?

(EXAMPLE (function name input(s)) what the function produces

(EXAMPLE (function name input(s)) what the function produces

(define (function name variable(s))

what the function does with those variable(s))

;

function name : Domain -> Range

;

what does the function do?

(EXAMPLE (function name input(s)) what the function produces

(EXAMPLE (function name input(s)) what the function produces

(define (function name variable(s))

what the function does with those variable(s))

Word Problem: collision?

Directions: Use the Design Recipe to write a function `collision?`, which takes in FOUR inputs: `px` and `py` (the x- and y-coordinate of the Player) and and `cy` (the x- and y-coordinates of another character), and checks if they are close enough to collide.

Contract and Purpose Statement

Every contract has three parts...

: _____ : _____
function name Domain -> Range

: _____ what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (_____) _____)
function name input(s) what the function produces

(EXAMPLE (_____) _____)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values...

(define (_____)
function name variable(s)

)
what the function does with those variable(s)

Contracts for Algebra (Wescheme)

Contracts tell us how to use a function, by telling us three important things:

1. The **Name**
2. The **Domain** of the function - what kinds of inputs do we need to give the function, and how many?
3. The **Range** of the function - what kind of output will the function give us back?

For example: The contract `triangle :: (Number, String, String) -> Image` tells us that the name of the function is `triangle`, it needs three inputs (a Number and two Strings), and it produces an Image.

With these three pieces of information, we know that typing `(triangle 20 "solid" "green")` will evaluate to an Image.

Name	Domain	Range
<code>; *</code>	<code>:: (Number , Number)</code>	<code>-> Number</code>
<code>(* 1 2)</code>		
<code>; +</code>	<code>:: (Number , Number)</code>	<code>-> Number</code>
<code>(+ 1 2)</code>		
<code>; -</code>	<code>:: (Number , Number)</code>	<code>-> Number</code>
<code>(- 1 2)</code>		
<code>; /</code>	<code>:: (Number , Number)</code>	<code>-> Number</code>
<code>(/ 1 2)</code>		
<code>; <</code>	<code>:: (Number , Number)</code>	<code>-> Boolean</code>
<code>(< 3 4) ; produces true</code>		
<code>; <=</code>	<code>:: (Number , Number)</code>	<code>-> Boolean</code>
<code>(<= 3 3) ; produces true, because 3 is equal to 3</code>		
<code>; =</code>	<code>:: (Number , Number)</code>	<code>-> Boolean</code>
<code>(= 3 4) ; produces false</code>		
<code>; ></code>	<code>:: (Number , Number)</code>	<code>-> Boolean</code>
<code>(> "a" "b") ; produces false</code>		
<code>; >=</code>	<code>:: (Number , Number)</code>	<code>-> Boolean</code>
<code>(>= 3 4) ; produces false, because 3 is neither greater-than nor equal-to 4</code>		
<code>; above</code>	<code>:: (Image , Image)</code>	<code>-> Image</code>
<code>(above (circle 10 "solid" "black") (square 50 "solid" "red"))</code>		
<code>; and</code>	<code>:: (Boolean condition1 , Boolean condition2)</code>	<code>-> Boolean</code>
<code>(and (> 0 1) (= 4 4)) ; produces false because both conditions must be true</code>		

Name	Domain	Range
; <code>beside</code>	:: (<u>Image</u> _{left} , <u>Image</u> _{right})	-> <u>Image</u>
	(<i>beside (circle 10 "solid" "black") (square 50 "solid" "red")</i>)	
; <code>circle</code>	:: (<u>Number</u> _{radius} , <u>String</u> _{fill-style} , <u>String</u> _{color})	-> <u>Image</u>
	(<i>circle 50 "solid" "purple"</i>)	
; <code>ellipse</code>	:: (<u>Number</u> _{width} , <u>Number</u> _{height} , <u>String</u> _{fill-style} , <u>String</u> _{color})	-> <u>Image</u>
	(<i>ellipse 100 50 "outline" "orange"</i>)	
; <code>expt</code>	:: (<u>Number</u> _{base} , <u>Number</u> _{power})	-> <u>Number</u>
	(<i>expt 3 4</i> ; three to the fourth power)	
; <code>flip-horizontal</code>	:: (<u>Image</u>)	-> <u>Image</u>
	(<i>flip-horizontal (text "Lion" 50 "maroon")</i>)	
; <code>flip-vertical</code>	:: (<u>Image</u>)	-> <u>Image</u>
	(<i>flip-vertical (text "Orion" 65 "teal")</i>)	
; <code>image-url</code>	:: (<u>String</u> _{url})	-> <u>Image</u>
	(<i>image-url "https://bootstrapworld.org/images/icon.png"</i>)	
; <code>isosceles-triangle</code>	:: (<u>Number</u> _{size} , <u>Number</u> _{vertex-angle} , <u>String</u> _{fill-style} , <u>String</u> _{color})	-> <u>Image</u>
	(<i>isosceles-triangle 50 20 "solid" "grey"</i>)	
; <code>or</code>	:: (<u>Boolean</u> _{condition1} , <u>Boolean</u> _{condition2})	-> <u>Boolean</u>
	(<i>or (> 1 0) (= 4 4))</i> ; produces true if one or more conditions are be true	
; <code>overlay</code>	:: (<u>Image</u> _{top} , <u>Image</u> _{bottom})	-> <u>Image</u>
	(<i>overlay (circle 10 "solid" "black") (square 50 "solid" "red")</i>)	
; <code>put-image</code>	:: (<u>Image</u> _{front} , <u>Number</u> _{x-coordinate} , <u>Number</u> _{y-coordinate} , <u>Image</u> _{behind})	-> <u>Image</u>
	(<i>put-image (circle 10 "solid" "black") 10 10 (square 50 "solid" "red")</i>)	
; <code>radial-star</code>	:: (<u>Num</u> _{points} , <u>Num</u> _{inner} , <u>Num</u> _{outer} , <u>Str</u> _{fill-style} , <u>Str</u> _{color})	-> <u>Image</u>
	(<i>radial-star 6 20 50 "solid" "red"</i>)	
; <code>rectangle</code>	:: (<u>Number</u> _{width} , <u>Number</u> _{height} , <u>String</u> _{fill-style} , <u>String</u> _{color})	-> <u>Image</u>
	(<i>rectangle 100 50 "outline" "green"</i>)	
; <code>regular-polygon</code>	:: (<u>Number</u> _{vertices} , <u>Number</u> _{size} , <u>String</u> _{fill-style} , <u>String</u> _{color})	-> <u>Image</u>
	(<i>regular-polygon 25 5 "solid" "purple"</i>)	
; <code>rhombus</code>	:: (<u>Number</u> _{angle} , <u>Number</u> _{size} , <u>String</u> _{fill-style} , <u>String</u> _{color})	-> <u>Image</u>
	(<i>rhombus 60 90 "outline" "pink"</i>)	

Name	Domain	Range
<code>; right-triangle</code>	<code>:: (Number leg1 , Number leg2 , String fill-style , String color) -> Image</code> <code>(right-triangle 50 60 "outline" "blue")</code>	
<code>; rotate</code>	<code>:: (Number degrees , Image img) -> Image</code> <code>(rotate 45 (star 50 "solid" "darkblue"))</code>	
<code>; scale</code>	<code>:: (Number factor , Image img) -> Image</code> <code>(scale 1/2 (star 50 "solid" "lightblue"))</code>	
<code>; sqr</code>	<code>:: (Number) -> Number</code> <code>(sqr 4)</code>	
<code>; sqrt</code>	<code>:: (Number) -> Number</code> <code>(sqrt 4)</code>	
<code>; square</code>	<code>:: (Number size , String fill-style , String color) -> Image</code> <code>(square 50 "solid" "red")</code>	
<code>; star</code>	<code>:: (Number radius , String fill-style , String color) -> Image</code> <code>(star 50 "solid" "red")</code>	
<code>; star-polygon</code>	<code>:: (Number size , Number point-count , Number step-count , String fill-style , String color) -> Image</code> <code>(star-polygon 100 10 3 "outline" "red")</code>	
<code>; string-contains?</code>	<code>:: (String haystack , String needle) -> Boolean</code> <code>(string-contains? "hotdog" "dog")</code>	
<code>; string-length</code>	<code>:: (String) -> Number</code> <code>(string-length "rainbow")</code>	
<code>; text</code>	<code>:: (String message , Number size , String color) -> Image</code> <code>(text "Zari" 85 "orange")</code>	
<code>; triangle</code>	<code>:: (Number size , String fill-style , String color) -> Image</code> <code>(triangle 50 "solid" "fuchsia")</code>	
<code>; triangle/asa</code>	<code>:: (Number top-left-angle , Number left-side , Number bottom-angle , String fill-style , String color) -> Image</code> <code>(triangle/asa 90 200 10 "solid" "purple")</code>	
<code>; triangle/sas</code>	<code>:: (Number top-side , Number top-R-angle , Number bottom-R-side , String fill-style , String color) -> Image</code> <code>(triangle/sas 50 20 70 "outline" "darkgreen")</code>	



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