Practice Questions

Q.1 (a). A random variable X is distributed randomly between the values 0 and 1, such that the PDF is $\mathbf{f(x) = kx^2(1 - x^3)}$, where k is a constant. Find the value of k. Then using the value of k, find the mean and variance

(b). A variable X is distributed randomly between the values 0 and 4, and its pdf is given by $\mathbf{f(x) = kx^3(4 - x)^2}$. Find the value of k and, thus, the mean and standard deviation of the distribution.

Q2. The random process $\mathbf{X}(t) = 2e^{-At} sin(\omega t + B)u(t)$ where $u(t)$ is the unit step function and random variables A and B are independent. A is distributed uniformly in (0,2), and B is distributed uniformly in $(-\pi, \pi)$. Verify whether the process is wide-sense stationary.

Q3 Let X be a discrete random variable with the following PMF

$$P_X(k) = \begin{cases} \frac{1}{4} & \text{for } k = -2, \\ \frac{1}{8} & \text{for } k = -1, \\ \frac{1}{8} & \text{for } k = 0, \\ \frac{1}{4} & \text{for } k = 1, \\ \frac{1}{4} & \text{for } k = 2, \\ 0 & \text{otherwise} \end{cases}$$

We define a new random variable as Y as $Y = (X + 1)^2$

a) Find the range of Y.
b) Find the PMF of Y.

Q4. Calculate the means for X and Y variables and the coefficient of correlation between them from the following two regression equations:

$$4x - 5y + 33 = 0$$
$$20x - 9y - 107 = 0$$

Q5. Let $X_1, X_2, ...$ be identically distributed random variables with mean μ, and variance σ^2. Let N be a random variable taking values in the non-negative integers and independent of the $X_{i's}$. Let $S = X_1 + X_2 + ... + X_N$.

a) Show that E[S] = μE[N]

b) The variance of S can be written as var[S] =σ^2 E [N] + σ^2 var[N]. Suppose now that the random variable N obeys the Poisson distribution with the parameter lambda. Then, compute this variance var[S].

Q6: Use steepest descent method for 2 iterations on
$f(x_1, x_2, x_3) = (x_1 - 4)^4 + (x_2 - 3)^2 + 4(x_3 + 5)^4$ with initial point $x^{(0)} = [4, 2, -1]^T$

Q7: Derive the gradient descent training rule assuming that the target function representation is:

$$O_d = w_0 + w_1 x_1 + ... + w_n x_n$$

Define explicitly the cost/error function E, assuming that a set of training examples D is provided, where each training example d ∈ D is associated with the target output t_d.

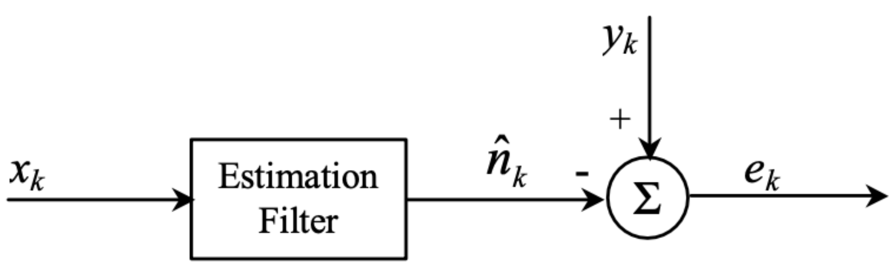
Q8: Let $\mathbf{c}$ and $\mathbf{w}$ be complex vectors of dimension ($M \times 1$). For $g = \mathbf{c}^H \mathbf{w}$ find $\nabla_{\mathbf{w}}(g)$? (weights are complex)

Q9: Consider a WSS random process $X(t)$ with

$$R_X(\tau) = \begin{cases} 1 & -1 \leq \tau \leq 1 \\ 0 & otherwise \end{cases}$$

Find the PSD of $X(t)$ and $E[X(t)^2]$

Q10: A signal estimation problem is illustrated in the diagram below, where the observed input sequence is x_k and the desired (ideal) signal is y_k, such that



η_k is a noise sequence of power = 0.1, and the estimation filter is of order 2 (i.e. it has two coefficients).

Calculate: a). The 2 x 2 autocorrelation matrix $\mathbf{R_{XX}}$

b). The 1 x 2 cross-correlation matrix $\mathbf{R_{yX}}$

c). The optimum Wiener filter coefficients for this case.