भारतीय प्रौद्योगिकी संस्थान जम्मू
INDIAN INSTITUTE OF TECHNOLOGY JAMMU
विद्याधनं सर्वधन प्रधानम्

Mid-Semester Examination

M.Tech 2023

Attempt all questions. Max. Marks = 40

Q1: A plane crashed, and it is presumed that it likely went down in any of the three possible regions. Let $1-\alpha_i$ denote the probability the plane will be found upon a search of the ith region when the plane is, in fact, in that region, i =1,2,3. (The constant α_i are called overlook probabilities because they represent the probability of overlooking the plane; they are generally attributed to the geographical and environmental conditions of the region). What is the conditional probability that the plane is in the i^{th} region, given that a search of region 1 is unsuccessful? (5)

Q2: Data from the National Oceanic and Atmospheric Administration indicate that the yearly precipitation in Los Angeles is a normal random variable with a mean of 12.08 inches and a standard deviation of 3.1 inches.
(a) Find the probability that the total precipitation during the next 2 years will exceed 25 inches.
(b) Find the probability that next year's precipitation will exceed that of the following year by more than 3 inches.
Assume that the precipitation totals for the next 2 years are independent. (7)

Q3: Let $\mathbf{y}=[7, 6]^T$; and $\mathbf{v}=[4, 2]^T$. Find the orthogonal projection of $\mathbf{y}$ on to $\mathbf{v}$. Then write $\mathbf{y}$ as a sum of two orthogonal vectors, one in span{$\mathbf{v}$} and one orthogonal to $\mathbf{v}$. (7)

Q4: Given $\mathbf{v}_1$ and $\mathbf{v}_2$ in a vector space V, let H = Span{$\mathbf{v}_1$, $\mathbf{v}_2$}. Show that H is a subspace of V. (7)

Q5: With the help of data points given in the table below, fit a second-order polynomial to the following data using the least square method (7)

I	1	2	3	4	5	6
x	0	0.5	1.0	1.5	2.0	2.5
y	0	0.25	1.0	2.25	4.0	6.25

Q6: Let X be a continuous random variable with PDF given as

$$\mathbf{f_X(x)} = \frac{\mathbf{1}}{\sqrt{\mathbf{2}\pi}} \exp^{-\frac{\mathbf{x^2}}{\mathbf{2}}}, \qquad \text{for all } x \in \mathcal{R}$$

And let $\mathbf{Y}= \mathbf{X^2}$. Find $\mathbf{f_Y(y)}$. (7)