Quiz 1 - Statistics | CHRIS SUNNY THALIYATH
11th JUNE 2023

Q1

Question: [use row reduction to solve]

$$2x_1 + 4x_2 + 6x_3 = 8$$

$$x_1 + 2x_2 + 4x_3 = 8$$

$$3x_1 + 6x_2 + 9x_3 = 12$$

Answer

$$[2x_1 + 4x_2 + 6x_3] = 8$$

$$\downarrow$$

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} = 8$$

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = 8$$

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} = 12$$

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 6 & 4 & 9 \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \\ 12 \end{bmatrix}$$

We know if

$$AB = C$$

$$A B B^{-1} = C B^{-1}$$

$$A = C B^{-1}$$

where

B^{-1} - inverse of B

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \\ 12 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 6 & 4 & 9 \end{bmatrix}^{-1}$$

Now we have to find the Inverse

$$A A^{-1} = I$$

this is identity matrix

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 6 & 4 & 9 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q2] ~~i~~

$v_1 \ldots\ldots v_r$: Eigen vectors

$\lambda_1 \ldots\ldots \lambda_r$: Eigen values

$n \times n$ matrix A_{nn}

Show that $\{v_1 \ldots v_r\}$ is linearly independent

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linear Independent :

Q3] A & B are n×n matrices

$$\det(AB) = \det(A)\det(B)$$

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Answer:

$\det(AB) =$

lets take example

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$\det(A) = 1\times4 - 3\times2$$

$$= 4 - 6 = -2$$

$$\det(B) = 5\times8 - 6\times7$$

$$= 40 - 42 = -2$$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} (1\times 5 + 2\times 7) & (1\times 6 + 2\times 8) \\ (3\times 5 + 4\times 7) & (3\times 6 + 4\times 8) \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 22 \\ 43 & 40 \end{bmatrix}$$

$$\det(AB) = \det \begin{bmatrix} 19 & 22 \\ 43 & 40 \end{bmatrix}$$

$$= (19\times 40 - 22\times 43)$$

in this example

$$\det(AB) \neq \det(A)\det(B)$$

thus not true

The only situation where this is true is where A & B are Identity matrix

4] Cramers rule to Solve

$$3x_1 - 2x_2 = 6$$

$$-5x_1 + 4x_2 = 8$$

Answer:

$$\begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}^{-I}$$

let $A = \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}$

$$A A^{-I} = I$$

$$\begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$