

⑤

$$y = ax^2 + bx + c$$

y	x
0	0
0.25	0.5
1	1
2.25	1.5
4	2
6.25	2.5

OLS

$$\Rightarrow \sum (y_a - y_p)^2$$

$$= (0 - (0 - 0 + c))^2 +$$

$$(0.25 - (0.05a + 0.5b + c))^2$$

$$+ (1 - (a + b + c))^2 +$$

$$+ (2.25 - (2.5a + 1.5b + c))^2 +$$

$$= \sum_{i=1}^6 [y_a - (ax^2 + bx + c)]^2$$

$$= \sum_{i=1}^6 (y_a - ax^2 - bx - c)^2$$

$$= \sum_{i=1}^6 y_a^2$$



Q6: Let  $X$  be a continuous random variable with PDF given as

$$f_X(x) = \frac{1}{\sqrt{2\pi}} \exp^{-\frac{x^2}{2}}, \quad \text{for all } x \in \mathcal{R}$$

And let  $Y = X^2$ . Find  $f_Y(y)$ .



P=1

$$P(A_1|B) = \frac{P(B|A_1) \times P(A_1)}{P(B)}$$

$$= \frac{(1-\alpha_1) \times 1/3}{\frac{(1-\alpha_1)}{3} + \frac{2}{3}}$$

$$P(A_1|B) = \frac{1-\alpha_1}{3-\alpha_1}$$

P=2

$$P(A_2|B) = \frac{P(B|A_2) \times P(A_2)}{P(B)}$$

$$= \frac{1 \times 1/3}{\frac{(1-\alpha_1)}{3} + \frac{2}{3}}$$

$$P(A_2|B) = \frac{1}{3-\alpha_1} = P(A_3|B)$$



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$P(B)$  = probability that a search of region is unsuccessful

$P(A_i)$   $\Rightarrow$  a prior probability that the plane is in that region

Now we need  $P(A_i|B)$

$$P(A_i|B) = \frac{P(B|A_i) + P(A_i)}{P(B)}$$

$$\underline{i=1} \Rightarrow P(B) = P(A_1) P(B|A_1) + P(A_2) P(B|A_2) + P(A_3) \times P(B|A_3)$$

$P(B)/2 \times \frac{1}{3} (P(A) = 1/3)$  All region equal probab<sup>l</sup> for plane is there

$$P(B) = \frac{1}{3} (1 - \alpha) + \frac{1}{3} (1) + \frac{1}{3} (1)$$

$P(B|A_2)$  is 1 because of your search plane in first region but plane is in ②

$$\text{Hence } P(B|A_2) = 1$$



$P=1$

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$$\mu_{y_2} - \mu_{y_1} \geq 3$$

~~$$\mu_{y_2} - \mu_{y_1} \geq 3$$~~

$$\mu_{diff} = \mu_{y_2} - \mu_{y_1} = 0$$

$$\sigma_{diff}^2 = \sigma_{y_1}^2 + \sigma_{y_2}^2 =$$

$$= 2(3.1)^2$$

$$\sigma_{diff} = \sqrt{2 \times 3.1} = 4.38$$

$$z = \frac{y_2 - y_1}{\sigma} = \frac{3}{4.38}$$

$$P(z > 0.6849) = 1 - 0.7517$$

$$p = 0.2483$$



$$y = \begin{bmatrix} 7 \\ 6 \end{bmatrix} \quad v = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

Orthogonal projection of  $y$  onto  $v$

$$\text{proj}_v y = \left( \frac{y \cdot v}{v \cdot v} \right) v.$$

$$y \cdot v = \begin{bmatrix} 7 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 2 \end{bmatrix} = 7 \times 4 + 6 \times 2$$

$$= 40$$

$$v \cdot v = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 2 \end{bmatrix} = 20$$

$$\text{proj}_v y = \left( \frac{40}{20} \right) v = 2v = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$$

write  $y$  as sum of orthogonal vectors

One in  $\text{Span}\{v\}$  and one is  
Orthogonal to  $v$

$$y = \text{Span}\{v\} + \text{proj}_v y.$$

$$y - \text{proj}_v y = \begin{bmatrix} 7 \\ 6 \end{bmatrix} - \begin{bmatrix} 8 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$y = \begin{bmatrix} 8 \\ 4 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$y = \text{span of } \begin{bmatrix} 8 \\ 4 \end{bmatrix} +$$

vector is orthogonal to  $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$$\begin{bmatrix} -1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 4 \end{bmatrix} = -8 + 8 = 0$$

↑ ~~other~~ orthogonal to  $y$ .



④ Subspace properties (H is Subspace of V)

→ The zero vector of V is in H

→ Closed under addition (vector)

→ Closed under scalar multiplication

① Zero vector

$$0 \cdot v_1 + 0 \cdot v_2 = 0$$

②  $a_1 = c_1 v_1 + c_2 v_2$

$$a_2 = d_1 v_1 + d_2 v_2$$

$$a_1 + a_2 = c_1 v_1 + c_2 v_2 + d_1 v_1 + d_2 v_2$$

$$= (c_1 + d_1) v_1 + (c_2 + d_2) v_2$$

③  $a = c_1 v_1 + c_2 v_2$

$$k \cdot a = k(c_1 v_1 + c_2 v_2)$$

$$= k c_1 v_1 + k c_2 v_2$$



$$\sum x = 75$$

$$\sum x^2 = 13.75$$

$$\sum x^3 = 28.125$$

$$\sum x^4 = 61.1875$$

$$6c + 7.5b + 13.75a = 13.75 \rightarrow$$

$$7.5c + 13.75b + 28.125a = 28.125 \rightarrow$$

$$13.75c + 28.125b + 61.1875a = 78.0625 \rightarrow$$

$$a = -13.75/112 = -11.83$$

$$b = 39.75/112 = 35.49$$

$$a = \frac{-738}{56} = -13.196$$

some mistake there

DO calculation correctly



$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$y = x^2$$

$$x = \pm \sqrt{y}$$

Jacobian transformation

$$\left| \frac{dx}{dy} \right| = \frac{1}{2\sqrt{y}}$$

$$f_y(y) = f_x(\sqrt{y}) \left| \frac{dx}{dy} \right| +$$

$$f_x(-\sqrt{y}) \left| \frac{dx}{dy} \right|$$

$$= \frac{1}{\sqrt{2\pi}} x e^{-\frac{y}{2}} \frac{1}{2\sqrt{y}} + \frac{1}{\sqrt{2\pi}} x e^{-\frac{y}{2}} \frac{1}{2\sqrt{y}}$$

$$= \frac{1}{\sqrt{2\pi}} x e^{-\frac{y}{2}} \times \frac{1}{\sqrt{y}}$$

$$y = x^2$$



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$$\text{Hence } P(B|A_2) = 1$$