Practice Examples

Q1:

1. The weight update expression of steepest descent algorithm is given by $w(n+1) = w(n) + \mu[p - Rw(n)]$. This algorithm is used to estimate the coefficient of a second order moving average filter where the cross-correlation vector $p = \begin{bmatrix} 0.8 \\ 0.3 \end{bmatrix}$, autocorrelation matrix of the observed sequence is $R = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$, the learning rate $\mu = 1.0$. With these set of observed values, determine the following:
   (i) Optimal weight as per Wiener-Hopf expression
   (ii) Weight obtained after three iterations of the steepest descent algorithm with the initial estimate of the weight as $w(n) = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix}$.

Q2 : Prove that for a real auto correlation matrix R all the eigen values are must be real and eigen vector corresponding to distinct eigenvalues of R are mutually orthogonal.

Q3:

Suppose the signal $x(n)$ is *wide-sense stationary.* We develop a first-order linear predictor of the form

$$\hat{x}(n+1) = w(0)x(n) + w(1)x(n-1) = \mathbf{w}^H\mathbf{x}$$

where $\mathbf{w} = [w(0)\ w(1)]^T$ and $\mathbf{x} = [x(n)\ x(n-1)]^T$.

(1pts) (a) Show that the autocorrelation sequence of a wide-sense stationary random process is a conjugate symmetric function of the lag k, i.e., $r_x(k) = r_x^*(-k)$.

(4pts) (b) Derive the optimum $\mathbf{w}$ by minimizing the mean-squared error

$$J(\mathbf{w}) = E\{|\mathbf{w}^H\mathbf{x} - x(n+1)|^2\}.$$

Q4: Prove that the error vector while optimizing the weights when projected onto the space span by the input data vector is orthogonal.

Q5: https://www.cs.cmu.edu/~epxing/Class/10701/exams/10f-601-midterm.pdf (Question 2, 3 and similar questions).