Q1 :- $2x_1 + 4x_2 + 6x_3 = 8$

$x_1 + 2x_2 + 4x_3 = 8$

$3x_1 + 6x_2 + 9x_3 = 12$

Can be written as

$$\begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 4 \\ 3 & 6 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \\ 12 \end{bmatrix}$$

$$AX = D$$

$$\left[\begin{array}{ccc:c} 2 & 4 & 6 & 8 \\ 1 & 2 & 4 & 8 \\ 3 & 6 & 9 & 12 \end{array}\right]$$

$R_2 \longrightarrow 2R_2 - R_1$

$$= \left[\begin{array}{ccc:c} 2 & 4 & 6 & 8 \\ 0 & 0 & 2 & 8 \\ 3 & 6 & 9 & 12 \end{array}\right]$$

$R_3 = R_3/3 - \frac{R_1}{2}$

$$= \left[\begin{array}{ccc:c} 2 & 4 & 6 & 8 \\ 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

i.e $2x_3 = 8 \longrightarrow \underline{\underline{x_3 = 4}}$

$2x_1 + 4x_2 + 6x_3 = 8$

OR $\quad x_1 + 2x_2 + 3x_3 = 4$

Put value of $x_3 = 4$

$$x_1 + 2x_2 = -8$$

Q2 Proof:- Suppose $\{v_1, \ldots v_k\}$ is linearly dependant.

Let 'p' be the least index such that v_{p+1} is a linear combination of the preceding (linearly independant vectors.

Then there exists scalars $c_1, \ldots c_p$ such that

$$c_1 v_1 + c_2 v_2 + \ldots \ldots c_p v_p = v_{p+1}$$

multiplying b/s by A and with $Av_k = \lambda_k v_k$ for each 'k' we get.

$$c_1 Av_1 + \ldots \ldots + c_p Av_p = Av_{p+1}$$

$$c_1 \lambda_1 v_1 + \ldots \ldots + c_p \lambda_p v_p = \lambda_{p+1} v_{p+1} \quad \text{— } A$$

multiplying b/s by λ_{p+1} and subtracting from A we get.

$$c_1(\lambda_1 - \lambda_{p+1})v_1 + \ldots \ldots + c_p(\lambda_p - \lambda_{p+1})v_p = 0 \quad \text{— } B$$

As $\{v_1 \ldots \ldots v_p\}$ are linearly independant, weights ... are $(\lambda_i - \lambda_{p+1})$ are in $\eta(B)$ are all zero. Also none

zero, as eigen values are distinct. Thus $C_i = 0 \; \forall \, i = 1 \ldots p$.

But $v_{p+1} = 0$, which is impossible, hence

$\{v_1, \ldots, v_r\}$ cannot be linearly dependant.

Q3 Solution:-

We know if A is not invertible then AB is neither, then in that case

$$\det(AB) = \det(A) \cdot \det(B)$$

if A is invertible, then A and identity matrix I_n are row equivalent by invertible matrix theorum. Then there exists elementary matricies $E_1 \ldots E_p$ such that

$$A = E_p E_{p-1} \cdots E_1 \cdot I_n = E_p E_{p-1} \cdots E_1$$

Then by the theorum of row operations

$$\det(AB) = \det(E_p \cdots E_1 B) = \det(E_p)\det(E_{p-1} \cdots E_1 B) =$$
$$= \det(E_p) \cdots \det(E_1)\det(B) = \cdots$$
$$= \det(E_p \cdots E_1)\det(B)$$
$$= \det(A)\det(B).$$

Q4 Solution: Using Cramer's rule.

$$Ax = b$$

$$A = \begin{bmatrix} 3 & -2 \\ -5 & 4 \end{bmatrix} ; \quad A_1(b) = \begin{bmatrix} 6 & -2 \\ 8 & 4 \end{bmatrix} ; \quad A_2(b) = \begin{bmatrix} 3 & 6 \\ -5 & 8 \end{bmatrix}$$

As $\det(A) = 2$, then system has a unique solution. By Cramer's rule

$$x_1 = \frac{\det(A_1(b))}{\det(A)} = \frac{24+16}{2} = 20$$

$$x_2 = \frac{\det(A_2(b))}{\det(A)} = \frac{24+30}{2} = 27.$$

Thus $\boxed{x_1 = 20 \; ; \; x_2 = 27}$