#### **Today**

#### Uninformed Search Methods:

Depth-First Search

Breadth-First Search

**Uniform-Cost Search** 

#### Informed Search:

A\* or "A star".

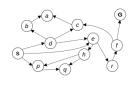
Sirius? Brightest star in sky.

No! search! Main idea: Heuristics.

Admissible.

Graph Search: Consistent.

### State Space Graphs



Tiny search graph for a tiny search problem

# State space graph: A mathematical representation of a search problem

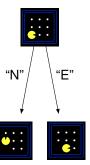
- Nodes are (abstracted) world configurations
- Arcs represent successors (action results)
- The goal test is a set of goal nodes (maybe only one)

In a state space graph, each state occurs only once!

We can rarely build this full graph in memory (it's too big), but it's a useful idea.

#### Search Trees

#### This is now / start



Possible futures.

#### A search tree:

A "what if" tree of plans and their outcomes

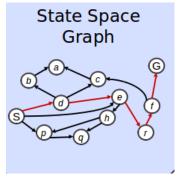
The start state is the root node

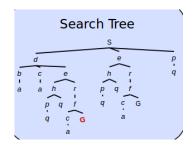
Children correspond to successors

Nodes show states, but correspond to PLANS that achieve those states

For most problems, we can never actually build the whole tree

# State Space Graphs vs. Search Trees

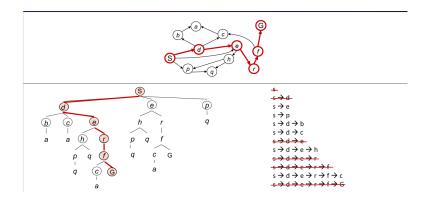




Each NODE in search tree is an entire PATH in state space graph.

We construct both on demand – and we construct as little as possible.

## Tree Search: example.



# Depth-First Search



## Depth-First Search

Strategy: expand a deepest node first.

Implementation: Fringe is a LIFO stack

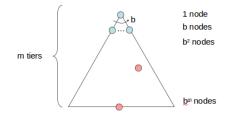
### Tree Search: example.

Strategy: expand a deepest node first Implementation: Fringe is a LIFO stack

# Search Algorithm Properties



## Search Algorithm Properties



Complete: Guaranteed to find a

solution if one exists?

Optimal: Guaranteed to find the

least cost path?

Time complexity?

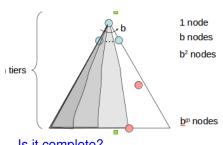
Space complexity?

Sketch of search tree: b is the branching factor m is the maximum depth solutions at various depths

Number of nodes in entire tree?

$$1+b+b^2+\cdots=O(b^m)$$

#### Depth-First Search (DFS) Properties



#### Which nodes expanded?

Some left prefix of the tree.

Could process the whole tree!

If m is finite, takes time  $O(b^m)$ 

How much space does the fringe take?

Only has siblings on path to root, so O(bm)

Is it complete?

*m* could be infinite, so only if we prevent cycles (more later)

Is it optimal?

No, it finds the "leftmost" solution, regardless of depth or cost

#### **Breadth-First Search**



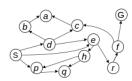
#### **Breadth-First Search**

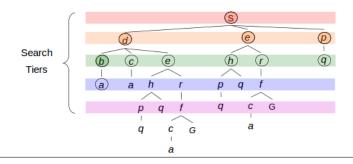
Strategy: expand a shallowest

node first

Implementation: Fringe is a FIFO

queue

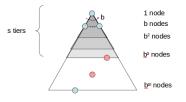




## Breadth-First Search (BFS) Properties

What nodes does BFS expand? Processes all nodes above shallowest solution

Let depth of shallowest solution be sSearch takes time  $O(b^s)$ 



How much space does the fringe take? Has roughly the last tier, so  $O(b^s)$ 

Is it complete?

s must be finite if a solution exists, so yes!

#### Is it optimal?

Only if costs are all 1 (more on costs later).

### Quiz: DFS vs BFS





#### DFS vs BFS

```
When will BFS outperform DFS?
When will DFS outperform BFS?
[Demo
:
dfs
/
bfs
maze water (L2D6)]
Space versus Time or Quality of Solution.
```

# Video of Demo Maze Water DFS/BFS (part 1)



# Video of Demo Maze Water DFS/BFS (part 2)

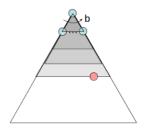


Next Up.

Iterative Deepening.

Uniform Cost Search.

### **Iterative Deepening**



Idea: get DFS's space advantage with BFS's time / shallow-solution advantages

Run a DFS with depth limit 1.
If no solution...
Run a DFS with depth limit 2.

If no solution ...
Run a DFS with depth limit 3. ....

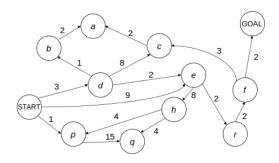
#### Isn't that wastefully redundant?

Generally most work happens in the lowest level searched, so not so bad!

#### Cost-Sensitive Search

BFS finds the shortest path in terms of number of actions.

It does not find the least-cost path. We will now cover a similar algorithm which does find the least-cost path.

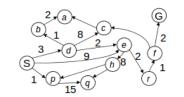


How?

# Uniform Cost Search (Djikstra's algorithm.)

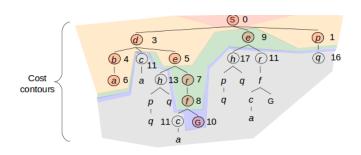


#### **Uniform Cost Search**

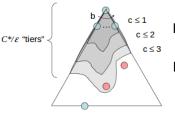


Strategy: expand a cheapest node first:

Fringe is a priority queue (priority: cumulative cost)



# Uniform Cost Search (UCS) Properties



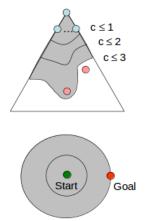
What nodes does UCS expand? All nodes cheaper than solution! Solution cost C\* and arc cost  $> \varepsilon$ : depth  $O(C^*/\varepsilon)$ . Time:  $O(b^{C^*/\varepsilon})$ .

How much space does the fringe take? Last Tier Space:  $O(b^{C^*/\varepsilon})$ .

Is it complete? Finite solution cost/positive arc weights? Then yes.

Is it optimal? Yes.

#### **Uniform Cost Issues**



Remember: UCS explores increasing cost contours.

The good:

UCS is complete and optimal!

The bad:

Goes in every "direction".

The ugly? Huh?

No information about goal location.

We'll fix that soon!

[Demo: empty grid UCS (L2D5)]

[Demo: maze with deep/shallow water DFS/BFS/UCS (L2D7)]

# Video of Demo Empty UCS



Video of Demo Maze with Deep/Shallow Water — DFS, BFS, or UCS? (part 1)



Video of Demo Maze with Deep/Shallow Water — DFS, BFS, or UCS? (part 2)



Video of Demo Maze with Deep/Shallow Water — DFS, BFS, or UCS? (part 3)



#### The One Queue

All these search algorithms are the same except for fringe strategies.

Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)

Practically, for DFS and BFS, you can avoid the log(n) overhead from an actual priority queue, by using stacks and queues

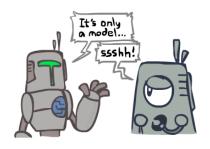
Can even code one implementation that takes a variable queuing object.



#### Search and Models

Search operates over models (state spaces) of the world The agent doesn't actually try all the plans out in the real world! Planning is all "in simulation"

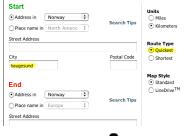
Your search is only as good as your models. . .



# Search Gone Wrong?



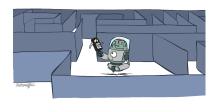
Model or State Space gone wrong!







### CS 188: Artificial Intelligence



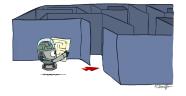
Informed Search

#### Informed Search



Informed Search Heuristics Greedy Search A\* Search Graph Search

# Recap: Search



## Recap: Search

#### Search problem:

States (configurations of the world)

Actions and costs

Successor function (world dynamics)

Start state and goal test

#### Search tree:

Nodes: represent plans for reaching states Plans have costs (sum of action costs)

#### Search algorithm:

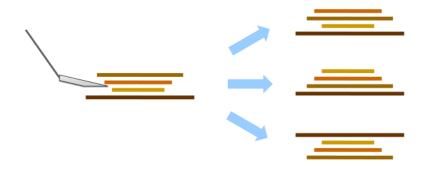
Systematically builds a search tree Orders the fringe (unexplored nodes)

Complete: finds a plan.

Optimal: finds least-cost plans

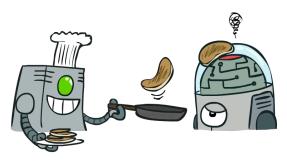


# Example: Pancake Problem



Cost: Number of pancakes flipped

## Example: Pancake Problem



#### BOUNDS FOR SORTING BY PREFIX REVERSAL

William H. GATES

Microsoft, Albuquerque, New Mexico

Christos H. PAPADIMITRIOU\*†

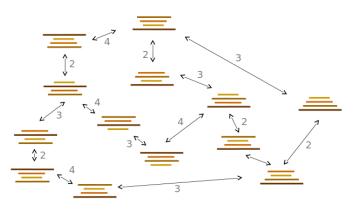
Department of Electrical Engineering, University of California, Berkeley, CA 94720, U.S.A.

Received 18 January 1978 Revised 28 August 1978

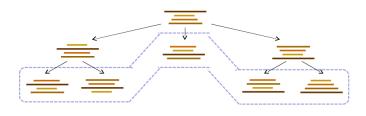
For a permutation  $\sigma$  of the integers from 1 to n, let  $f(\sigma)$  be the smallest number of prefix reversals that will transform  $\sigma$  to the identity permutation, and let f(n) be the largest such  $f(\sigma)$  for all  $\sigma$  in (the symmetric group)  $S_n$ . We show that  $f(n) \approx (5n+5)/3$ , and that  $f(n) \approx 17nt/16$  for n a multiple of 16. If, furthermore, each integer is required to participate in an even number of reversed prefixes, the corresponding function g(n) is shown to obey  $3nt/2 - 1 \leqslant g(n) \leqslant 2n + 3$ .

# Example: Pancake Problem

State space graph with costs as weights.



#### General Tree Search



function TREE-SEARCH(problem, strategy) returns a solution, or failure initialize the search tree using the initial state of problem loop do

if there are no candidates for expansion then return failure choose a leaf node for expansion according to strategy if the node contains a goal state then return the corresponding solution else expand the node and add the resulting nodes to the search tree

end

Action: flip top two

Cost: 2

Action: flip all four

Cost: 4

Path to reach goal: Flip four, flip three

Total cost: 7

### The One Queue



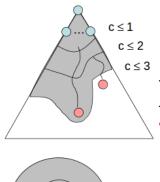
# Our search algorithms are the same except for fringe strategies

- All fringes are priority queues: states with priorities.
- DFS, BFS a bit faster using simple stack/queues.
- Can code one implementation with variable queuing object.

## **Uninformed Search**



#### **Uniform Cost Search**



Strategy: expand lowest path cost.

The good: Complete and optimal!

The bad: Explores options in every

"direction"

No information about goal location

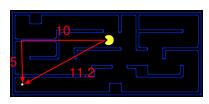


## **Informed Search**

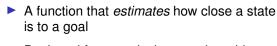


## Search Heuristics





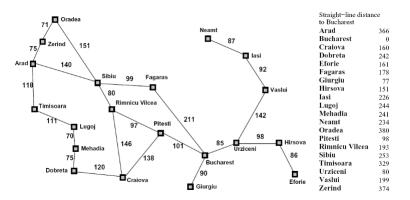
#### A heuristic is:



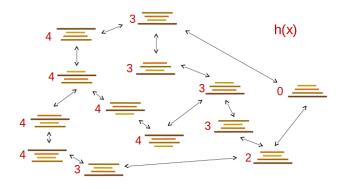


- Designed for a particular search problem
- Examples: Euclidean distance for pathing. Manhattan distance.

## Example: Heuristic Function



## Example: Heuristic Function



Heuristic: the number of the largest pancake that is still out of place