भारतीय प्रौद्योगिकी संस्थान जम्मू
INDIAN INSTITUTE OF TECHNOLOGY JAMMU
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End-Semester Examination

Ex. M.Tech 2023

Attempt all questions. Max. Marks = 40

Q1. The random process $\mathbf{X}(t) = 2e^{-At} sin(\omega t + B)u(t)$ where $u(t)$ is the unit step function and random variables A and B are independent. A is distributed uniformly in (0,2), and B is distributed uniformly in $(-\pi, \pi)$. Verify whether the process is wide-sense stationary. (5)

Q2. George has 4 umbrellas, some at home, and some in the office. He keeps moving between home and office. He takes an umbrella with him only if it rains. If it does not rain, he leaves the umbrella behind (at home or in the office). It may happen that all umbrellas are in one place and George in the other, and it starts raining, and he must leave, so he gets wet.
a) If the probability of rain is p, what is the probability that he gets wet?
b) Estimates show that p = 0.6 in London. How many umbrellas should George have so that, if he follows the strategy above, the probability of him getting wet is less than 0.1?

Q3. Prove that n × n matrix A is diagonalizable if and only if it has n linearly independent eigenvectors. In this case, A is similar to a matrix D, whose diagonal elements are the eigenvalues of A. (5)

Q4 Let X be a discrete random variable with the following PMF

$$P_X(k) = \begin{cases} \frac{1}{4} & \text{for } k = -2, \\ \frac{1}{8} & \text{for } k = -1, \\ \frac{1}{8} & \text{for } k = 0, \\ \frac{1}{4} & \text{for } k = 1, \\ \frac{1}{4} & \text{for } k = 2, \\ 0 & \text{otherwise} \end{cases}$$

We define a new random variable as Y as $Y = (X+1)^2$

a) Find the range of Y.
b) Find the PMF of Y. (5)

Q5. A box contains 3 coins; Two regular and one fake coin with Heads on both sides $(P(H) = 1)$
a) Pick a coin at random and toss it. What is the probability that it will land with a head? (5)
b) Pick a coin randomly, toss it, and get a Head. What is the probability that it is a two-headed coin?

Q6. Calculate the means for X and Y variables and the coefficient of correlation between them from the following two regression equations:

$$4x - 5y + 33 = 0$$
$$20x - 9y - 107 = 0$$

(5)

Q7. Let $X_1, X_2, ...$ be identically distributed random variables with mean μ, and variance σ^2. Let N be a random variable taking values in the non-negative integers and independent of the $X_{i's}$. Let $S = X_1 + X_2 + ... + X_N$.

a) Show that E[S] = μ E[N]

b) The variance of S can be written as var[S] = σ^2 E [N] + σ^2 var[N]. Suppose now that the random variable N obeys the Poisson distribution with the parameter lambda. Then, compute this variance var[S]. (5)

Q8. In a school, female students are selected randomly and weighted. Suppose the weights of selected students are normally distributed with unknown mean μ and standard deviation σ. Upon randomly choosing a sample of 10 female students, the following weights are recorded:

115, 122, 130, 127, 149, 160, 152, 138, 149, 180

With the declarations mentioned above, identify the likelihood function and the maximum likelihood estimator of μ, the mean weight of all female children in the school. Using the given sample, find a maximum likelihood estimate of μ as well. (5)