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MID SEM - STATS & PROB

Q1) $y = [7, 6]^T$

$v = [4, 2]^T$

Orthogonal projection of y onto v

which means

$\text{Proj}(y) = \left(\frac{y \cdot v}{v \cdot v}\right) v$

$y \cdot v = \begin{bmatrix} 7 \\ 6 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix} = 7 \times 4 + 6 \times 2$

$= 40$

$v \cdot v = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix} = 20$

$\text{Proj}_v\, y = \left(\frac{40}{20}\right) v = 2v = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$

Write y as sum of orthogonal vectors

One in span $\{v\}$

One in orthogonal to v

$y = \text{Span}\{v\} + \text{Proj}_v\, y$

$y = \text{Proj}_v\, y = \begin{bmatrix} 7 \\ 6 \end{bmatrix} - \begin{bmatrix} 8 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$y = \begin{bmatrix} 8 \\ 4 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$y =$ Span of $v \begin{bmatrix} 8 \\ 4 \end{bmatrix}$ + vector is Orthogonal to $v \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$\begin{bmatrix} -1 & 2 \end{bmatrix} \begin{bmatrix} 8 \\ 4 \end{bmatrix} \Rightarrow -8 + 8 = 0$ ie orthogonal to v

Q2) Box contains 3 coins

1 Regular coin
1 coin with head on both side ie $P(H) = 1$

i) Pick a coin at random and toss it, what is the probability it will end with a head?

$Pr(\text{heads}) = Pr(\text{heads} \cap \text{two headed coin}) + Pr(\text{heads} \cap \text{normal coin})$

$Pr(\text{heads} \cap \text{two headed coin}) = P(\text{heads} \mid \text{two headed coin})$

$P(\text{two headed coin}) = 1 \times \frac{1}{3}$

$P(\text{heads} \cap \text{normal coin}) = P(\text{heads} \mid \text{normal coin}) \times P(\text{normal coin})$

There fore $Pr(\text{heads}) = \frac{1}{3} + \frac{1}{3}$

$= 2/3$ //

Final Answer

b) $Pr(\text{two headed coin} \mid \text{heads})$

$$= \frac{Pr(\text{heads} \cap \text{two headed coin})}{Pr(\text{heads})}$$

We know this from Part (A)

$$= \frac{(1/3)}{(2/3)} = \boxed{1/2}$$

Final Answer

Q3)

x	0	0.5	1.0	1.5	2.0	2.5
y	0	0.25	1.0	2.25	4.0	6.25

Mean of x $= \{0 + 0.5 + 1.0 + 1.5 + 2.0 + 2.5\}/6$

$= 7.5/6 = 1.25$

Mean of y $= \{0 + 0.25 + 1.0 + 2.25 + 4.0 + 6.25\}/6$

$= 13.75/6 = 2.291$

$$\boxed{y = a + bx}$$ ← Equation of a straight line

$$\sum y = an + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

x	y	x^2	xy
0	0	0	0
0.5	0.25	0.25	0.125
1.0	1.0	1	1
1.5	2.25	2.25	3.375
2.0	4.0	4.0	8.0
2.5	6.25	6.25	15.625
$\Sigma x = 7.5$	$\Sigma y = 13.75$	$\Sigma x^2 = 13.75$	$\Sigma xy = 28.125$

Substituting the above values in the following equations

$$\Sigma y = an + b\Sigma x$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

$$13.75 = a \times 6 + b \times 7.5$$

$$28.125 = a \times 7.5 + b \times 13.75$$

$$\frac{13.75 - b \times 7.5}{6} = a$$

Substituting a in $28.125 = \left(\frac{13.75 - b \times 7.5}{6}\right) \times 7.5 + b \times 13.75$

$$\Rightarrow \quad 28.125 = (13.75 - b \times 7.5)\, 1.25 + b \times 13.75$$

$$= 17.18 - b\, 11.25 + 13.75\, b$$

$$11.57 = 2.5\, b$$

$$\Rightarrow \boxed{b = \frac{11.57}{2.5} = 4.628}$$

Substituting b in equation for a

$$a = \frac{13.75 - b \times 7.5}{6} = \frac{13.75 - 34.71}{6}$$

$$= -3.493$$

Therefore $y = a + bx$

This is what I got from calculator

$$\boxed{y = -3.493 + 4.628x}$$

From Python calculation

$$\boxed{y = -3.081 + 5.112x}$$

Question 4

$$f(x,y) = \begin{cases} \frac{12}{5}x(2-x-y), & 0<x<1, 0<y<1 \\ 0, & \text{otherwise} \end{cases}$$

Compute conditional density of x given that $Y=y$ where $0<y<1$

$f_{x|y}(x|y)$ for $0<y<1$

Marginal density of Y

$$f_Y(y) = \begin{cases} \frac{12}{5}\left[\frac{2-y}{2} - \frac{1}{3}\right] & \text{for } 0<y<1 \\ 0 & \text{otherwise} \end{cases}$$

$$f_{x|y}(x|y) = \frac{\frac{12}{5}x(2-x-y)}{\frac{2}{5}(4-3y)} = \frac{6x(2-x-y)}{4-3y}$$

Q.5)

$$X = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix}$$

Change the matrix to a row - echelon form then we can get the RANK of the matrix

If the RANK is 3, This would imply that the three vectors are linearly independent

Reduced Row echelon of the above matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, [0, 1, 2]$$

Rank of the Matrix is equal to the number of rows / Columns of the largest Square Submatrix of X that has non zero determinant

There fore the Rank of the row - echelon form of X is 3

Thus the above matrix is linearly independent