A HILBERT space $\mathcal{H}$ is a

Complete complex vector space,

$\psi, \varphi \in \mathcal{H}$ and $a, b \in \mathbb{C}$ =>$a\psi + b\varphi \in \mathcal{H}$

With a positive-definite scalar product, this scalar product induces a norm.

Subset $\mathcal{H}_{sub} \subset \mathcal{H}$ which is a vector space and inherits the scalar product and the norm from $\mathcal{H}$. This is called sub HILBERT space or subspace of $\mathcal{H}$.

We can define Quantum Fourier Transform (QFT) on $\mathcal{H}^{\otimes n}$ as

$$QFT \coloneqq \frac{1}{\sqrt{2^n}} \sum_{x,y=0}^{2^n-1} \exp\left(2\pi i \frac{xy}{2^n}\right) |x\rangle\langle y|$$

where $|x\rangle$ and $\langle y|$ denote vector of the computational basis of $\mathcal{H}^{\otimes n}$

Let $n \in \mathbb{N}$ and

$$x = \sum_{j=0}^{n-1} x_j 2^j$$

where $x_j \in \{0,1\}$ for $j \in \{0, \cdots, n-1\}$

Then the quantum Fourier transform (QFT) on any vector $|x\rangle$ of the computational basis (Z) of $\mathcal{H}^n$ is

$$QFT\ |x\rangle = \frac{1}{\sqrt{2^n}} \otimes_{j=0}^{n-1} \left[|0\rangle + e^{2\pi i 0.x_j \ldots x_0} |x\rangle\right]$$