Project 0, MAT 362

Due Sunday September 10th, midnight.

Modify the Matlab demo codes to explore the following four algorithms, and then write a short report containing your results. Upload one or several Matlab .m files along with a PDF copy of your report. This is a solo project - you may collaborate but write your own code and report. *Consult the Project Guidelines*!!!

For the following questions, let the functions $f, g : \mathbb{R} \to \mathbb{R}$ be defined by:

$$f(x) = x^2 \quad \text{and} \quad g(x) = x^4.$$

(1) Approximate $\int_0^1 f(x)\,dx$ and $\int_0^1 g(x)\,dx$. Use the midpoint rule with $n = 10, 20, 40, 80$, and 160 divisions. Compare approximations to actual answers, and observe the magnitude of the error as a function of n. See demo program mp.m.

(2) Consider the IVP $y' = f(y)$ and $y' = g(y)$ over the intervals $[0, b]$, with $y(0) = 1$, for appropriate values of $b < 1$. Approximate via Euler's method with $n = 10, 20, 40, 80$, and 160 divisions. Compare approximations to actual answers, and observe the magnitude of the error as a function of n. See demo programs edrive.m and myeuler.m.

(3) Approximate the zeros of $g - f$. Use Newton's method with initial guesses $x_0 = 2$ and $x_0 = \frac{1}{4}$. Compare approximations to actual answers. Which roots are converged to at the faster rate? See demo program iterative.m.

(4) Approximate first and second derivatives of f and g over the interval $[0, 1]$ by calculating the matrix products $D_1\hat{f}$, $D_1\hat{g}$, $D_2\hat{f}$ and $D_2\hat{g}$. Use first and second sparse difference matrices for interior points with $n = 10, 20, 40, 80$, and 160 divisions. Compare approximate derivatives with actual derivatives, and observe the magnitude of the error as a function of n. See demo diffOpDemo.m.