LEBANESE AMERICAN UNIVERSITY

DEPARTMENT OF COMPUTER SCIENCE AND MATHEMATICS

MTH 304 - Differential Equations

Online Exam 1, Summer 2021

Duration: 60 minutes

You should submit your work before **09:35**

INSTRUCTIONS:

1. *This quiz consists of 4 questions.To receive full credits, you have to justify your answers.*
2. *Calculators are not allowed; you don't have to find the numerical values of your answers.*
3. *You need to submit your work as a scanned document (as a pdf file) by email to*

   laumath.online@gmail.com

4. *You should submit only one email containing one pdf file with a filename of the form FullName.pdf*
5. *Your name, student ID, and signature should appear on each page.*
6. *Submissions sent to my LAU email address WILL NOT be considered.*
7. *Avoid sending large photos, otherwise this will delay your submission/receipt of your email.*
8. *Make sure to submit your work before the deadline otherwise you loose grades.*
   Emails received after the submission deadline will loose credits according to the below rules:
   - *If I receive your quiz 1-5 minutes late you will loose* 15% *of your grade.*
   - *If I receive your quiz 6-10 minutes late you will loose* 30% *of your grade.*
   - *If I receive your quiz 11-15 minutes late you will loose* 60% *of your grade.*
   - *If I receive your quiz more than 15 minutes late, your quiz will not be considered and your grade will be Zero.*
9. ***For proctoring purposes, you should enable your camera and unmute your microphone. You are not allowed to talk or chat with the proctor during the quiz. The session will be recorded.***

1. **[15 Points]** Solve the following differential equations:

   (a) $y' + y\cos t = \cos t$

   (b) $ye^{-x}\frac{dy}{dx} = e^x$

   (c) $2y(t^2+4)\dfrac{dy}{dt} = y^2 + 1$

2. **[15 Points]** A new virus is spreading in a lake. Fish are dying at a rate proportional to the current size of the fish population with a proportionality constant $\alpha = 0.1$. On the other hand fish are reproducing and on average there are 500 new fish born per month.

   (a) Assume that initially the lake contained 10,000 fish, write a differential equation describing the fish population in the lake at any time t measured in months.

   (b) Plot the direction field that corresponds to the differential equation obtained in part (a) and describe (without solving the initial value problem) the fate of the fish population.

   (c) Solve the initial value problem and find the number of fish in the pond after one year.

3. **[8 Points]** Plot the phase line, direction field, and solution curves for the differential equation $\dfrac{dy}{dt} = y^2(2-y)$.

4. **[12 Points]** Let the initial value problem $\dfrac{dy}{dt} = y^{1/7}$ with $y(0) = 0$.

   (a) Verify that $y_1(t) = 0$ solves the given IVP.

   (b) Can you find a second solution $y_2(t)$ to the given IVP?

   (c) Does the result of part (2) contradict the existence and uniqueness theorem for initial value problems? Justify your answer.