

LEBANESE AMERICAN UNIVERSITY
DEPARTMENT OF COMPUTER SCIENCE AND MATHEMATICS
MTH 304 - Differential Equations

Online Final Exam, Summer 2021

Duration: 45 minutes

You should submit your work before **12:35 pm**

INSTRUCTIONS:

1. *This quiz consists of 3 questions. To receive full credits, you have to justify your answers.*
2. *Calculators are not allowed; you don't have to find the numerical values of your answers.*
3. *You need to submit your work as a scanned document (as a pdf file) by email to*

[*laumath.online@gmail.com*](mailto:laumath.online@gmail.com)
4. *You should submit only one email containing one pdf file with a filename of the form **FullName.pdf***
5. *Your name, student ID, and signature should appear on each page.*
6. *Submissions sent to my LAU email address WILL NOT be considered.*
7. *Avoid sending large photos, otherwise this will delay your submission/receipt of your email.*
8. *Make sure to submit your work before the deadline otherwise you lose grades.*
Emails received after the submission deadline will lose credits according to the below rules:
 - *If I receive your quiz 1-5 minutes late you will lose 15% of your grade.*
 - *If I receive your quiz 6-10 minutes late you will lose 30% of your grade.*
 - *If I receive your quiz 11-15 minutes late you will lose 60% of your grade.*
 - *If I receive your quiz more than 15 minutes late, your quiz will not be considered and your grade will be Zero.*
9. ***For proctoring purposes, you should enable your camera and unmute your microphone. You are not allowed to talk or chat with the proctor during the quiz. The session will be recorded.***

1. **[10 Points]** Solve the following differential equation

$$y'' + 2y' + 2y = \delta(t - 1),$$

subject to the initial conditions $y(0) = 1$ and $y'(0) = 0$.

2. **[15 Points]**

(a) Rewrite the second-order differential equation $y'' + 5y' + 6y = 0$ as an equivalent system of two linear first order differential equations of the form $X' = AX$ where $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ is the unknown solution and $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ is a constant matrix.

(b) Solve the linear system $X' = AX$ where $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and $A = \begin{pmatrix} 0 & 1 \\ -6 & -5 \end{pmatrix}$. Visualize the solution curves and describe their behavior as $t \rightarrow \infty$.

3. **[10 Points]** Use Laplace transforms to solve the system

$$\begin{cases} y_1'(t) + y_2(t) = 0 \\ y_1(t) + y_2'(t) = 2 \sin t \end{cases}$$

subject to the initial conditions $y_1(0) = 1$ and $y_2(0) = 0$.