Qualitative Analysis of Systems.

Given the system
$$\begin{cases} \dfrac{dx}{dt} = f(x,y) \\ \dfrac{dy}{dt} = g(x,y) \end{cases}$$
(autonomous)

We look at $\dfrac{dy}{dx} = \dfrac{g(x,y)}{f(x,y)}$

this determines the rate of change of y with respect to x at any point in the xy-plane.

The xy-plane is called the phase-plane for the system

A plot of tangent vectors to the curve y v.s x at any point of the phase plane produces a vector field. These tangent vectors ~~are~~ have slopes $= \dfrac{dy}{dx}$.

A plot of the curves y. v.s x in the phase plane forms the phase portrait of the system.

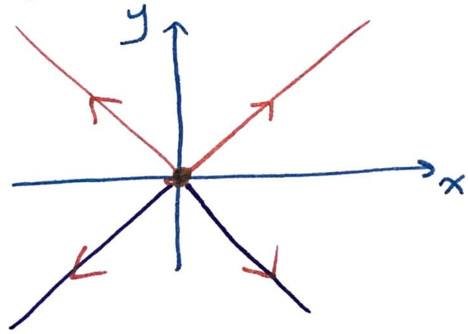
Phase portraits of linear systems.

① Linear systems with 2 real eigenvalues: (0, 0) is the only equilibrium solution.

λ is a real eigenvalues with $\vec{v} = \langle x, y \rangle$ is a corresponding eigenvalue.

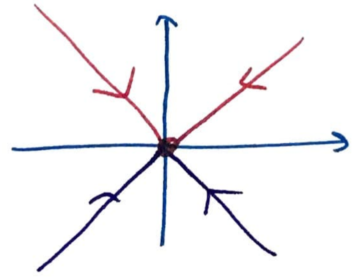
Each eigenvector introduces two "~~the~~" "straight-line" solutions in the phase plane - the straight lines are through the eigenvector.

(a) Two eigenvalues: $\lambda_1 > 0$, $\lambda_2 > 0$



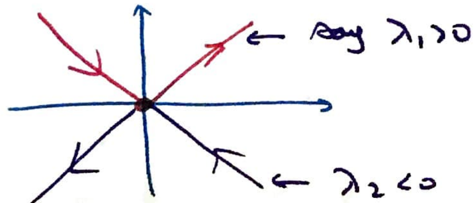
$\boxed{(0,0) = \text{source}}$

(b) two eigenvalues: $\lambda_1 < 0$, $\lambda_2 < 0$



$\boxed{(0,0) \text{ is a sink}}$

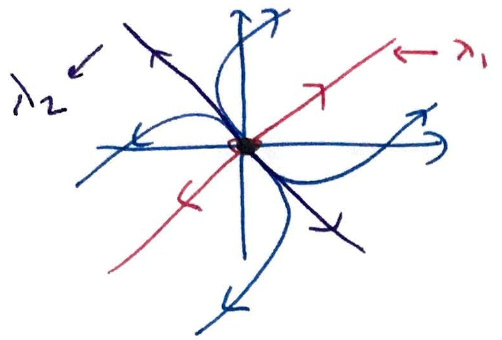
(c) two eigenvalues having opposite signs:



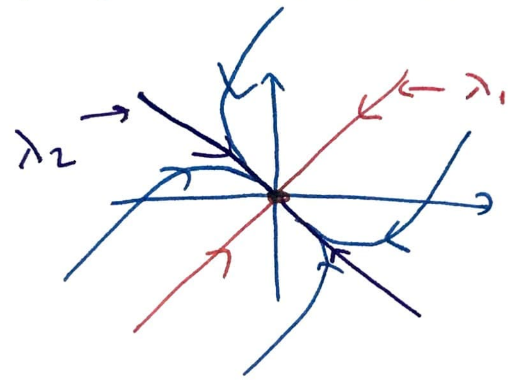
$\boxed{(0,0) = \text{saddle}}$

What about the other solutions?

(a) $\lambda_1 > 0$, $\lambda_2 > 0$; $\lambda_1 > \lambda_2$



(b) $\lambda_1 < 0$, $\lambda_2 < 0$; $|\lambda_1| > |\lambda_2|$



(c) λ_1, λ_2 opposite signs, say $\lambda_1 > 0$, $\lambda_2 < 0$