LEBANESE AMERICAN UNIVERSITY
DEPARTMENT OF COMPUTER SCIENCE AND MATHEMATICS
MTH 304 - Differential Equations

Online Exam 2, Summer 2021

Duration: 60 minutes
You should submit your work before **09:35**

**INSTRUCTIONS**:

1. *This quiz consists of 4 questions. To receive full credits, you have to justify your answers.*

2. *Calculators are not allowed; you don't have to find the numerical values of your answers.*

3. *You need to submit your work as a scanned document (as a pdf file) by email to*

   laumath.online@gmail.com

4. *You should submit only one email containing one pdf file with a filename of the form FullName.pdf*

5. *Your name, student ID, and signature should appear on each page.*

6. *Submissions sent to my LAU email address WILL NOT be considered.*

7. *Avoid sending large photos, otherwise this will delay your submission/receipt of your email.*

8. *Make sure to submit your work before the deadline otherwise you loose grades.*
   Emails received after the submission deadline will loose credits according to the below rules:

   - *If I receive your quiz 1-5 minutes late you will loose 15% of your grade.*
   - *If I receive your quiz 6-10 minutes late you will loose 30% of your grade.*
   - *If I receive your quiz 11-15 minutes late you will loose 60% of your grade.*
   - *If I receive your quiz more than 15 minutes late, your quiz will not be considered and your grade will be Zero.*

9. **_For proctoring purposes, you should enable your camera and unmute your microphone. You are not allowed to talk or chat with the proctor during the quiz. The session will be recorded._**

1. **[13 Points]**

    Use the method of undetermined coefficients to solve the differential equation

    $$y'' + 9y = 2\cos(3t) + 18.$$

2. **[15 Points]**

    Let the differential equation $\left(\dfrac{3x}{y^2} + \dfrac{1}{y}\right) + \left(\dfrac{x^2}{y^3} + \dfrac{x}{y^2}\right)\dfrac{dy}{dx} = 0$

    (a) Verify that the given differential equation is not exact.

    (b) Can you determine an integrating factor of the form $\mu(x, y) = xy^\alpha$ that transforms the given differential equation into an exact one? Here α is a constant to be determined.

    (c) Use the result of part (b) to solve the given differential equation.

3. **[12 Points]**

    (a) Solve the differential equation $t^2 y'' - 3ty' + 4y = 0$.

    (b) Use the method of variation of parameters to solve the nonhomogeneous differential equation $t^2 y'' - 3ty' + 4y = t^2 \ln t$, for $t > 0$. Take $y_1(t) = t^2$ and $y_2(t) = t^2 \ln t$

4. **[10 Points]**

    (a) Verify that $y_1(t) = e^t$ is a solution to the differential equation

    $$ty'' - (1 + 2t)y' + (t + 1)y = 0, \ t > 0.$$

    (b) Use the method of reduction of order to solve the differential equation

    $$ty'' - (1 + 2t)y' + (t + 1)y = t^2$$