TORQUE CALCULATIONS

Mass of parts - components included

- Body m_b = 1.386 kg (40% PLA infill)
- Coxa m_c = 0.04 kg (50% PLA infill)
- Femur m_f = 0.16 kg (50% PLA infill)
- Tibia m_t = 0.08 kg (50% PLA infill)

Weight

$$W = m \times g$$

$W_{body} = 1.386 \times 9.8 = 13.5828$ N

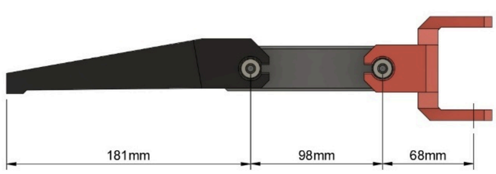
$W_{coxa} = 0.04 \times 9.8 = 0.392$ N

$W_{femur} = 0.16 \times 9.8 = 1.568$ N

$W_{tibia} = 0.08 \times 9.8 = 0.784$ N

Free Swing Torque

$$\tau = W \times L$$



$L_{coxa} = 0.068$ m $\quad l_{com,coxa} = 0.034$ m

$L_{femur} = 0.098$ m $\quad l_{com,femur} = 0.049$ m

$L_{tibia} = 0.181$m $\quad l_{com,tibia} = 0.078$ m

Femur Joint

$\tau_{femur} = W_{femur} \times l_{com,femur} + W_{tibia} \times (L_{femur} + l_{com,tibia})$

$\tau_{femur} = 1.568 \times 0.034 + 0.784 \times (0.098 + 0.078)2$

$\boxed{\tau_{femur} = 0.1913 \text{ Nm}}$

Tibia Joint

$\tau_{tibia} = W_{tibia} + l_{com,tibia}$

$\tau_{tibia} = 0.784 \times 0.078$

$\boxed{\tau_{tibia} = 0.06115 \text{ Nm}}$

Coxa

$$\tau = \tau_{inertia} + \tau_{gravity}$$

$$\tau_{inertia} = I \times \alpha$$

$$A = 4 \times \Delta\theta/t^2$$

Assume $\Delta\theta = 60° = 1.0472$ rad, t = 2 sec

$A = 4 \times 1.0472 / 2^2$

A = 1.0472 rad/sec

Assuming legs as uniform rods,

Coxa

$$I = 0.33 \times m \times L^2$$

$I_{coxa} = 0.33 \times 0.04 \times 0.068^2$

$I_{coxa} = 0.0001233$ kg m^2

Femur and Tibia

$$I_c = 0.0833 \times m \times L^2 \text{ (about centre)}$$

$$I_{l,coxa} = I_c + md^2$$

$I_{femur} = 0.0833 \times 0.16 \times 0.098^2$

$I_{femur} = 0.000128\ kg\ m^2$

$I_{femur,coxa} = 0.000128 + 0.016 \times (0.068 + 0.049)^2$

$I_{femur,coxa} = 0.000347\ kg\ m^2$

$I_{tibia} = 0.0833 \times 0.08 \times 0.181^2$

$I_{tibia} = 0.000218\ kg\ m^2$

$I_{tibia,coxa} = 0.000218 + 0.08 \times (0.068 + 0.098 + 0.078)^2$

$I_{tibia,coxa} = 0.00498\ kg\ m^2$

$I_{tot} = i_{coxa} + i_{femur,coxa} + i_{tibia,coxa}$

$I_{tot} = 0.0001233 + 0.000347 + 0.00498$

$I_{tot} = 0.0054503\ kg\ m^2$

$\tau_{inertia} = 0.0054503 \times 1.0472$

$\tau_{inertia} = 0.0057\ Nm$

$\tau_{gravity} = W_{coxa} \times l_{com,coxa} + W_{femur} \times (L_{coxa} + l_{com,femur}) + W_{tibia} \times (L_{coxa} + L_{femur} + l_{com,tibia})$

$\tau_{gravity} = 0.392 \times 0.034 + 1.568 \times (0.068 + 0.049) + 0.784 \times (0.068 + 0.098 + 0.078)$

$\tau_{gravity} = 0.38808\ Nm$

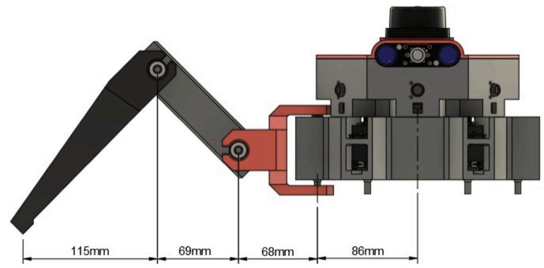
$\tau_{coxa} = \tau_{inertia} + \tau_{gravity}$

$\tau_{coxa} = 0.0057 + 0.38808$

$\boxed{\tau_{coxa} = 0.39378\ Nm}$

Stance Torque

Stance support and lift is provided by Femur and Tibia joints.



For tripod gait

Three legs on ground at a time

Total suspended mass

$m_{tot} = m_{body} + 3 \times (m_{coxa} + m_{femur} + m_{tibia})$

$m_{tot} = 1.386 + 3 \times (0.05 + 0.16 + 0.08)$

$m_{tot} = 2.226\ kg$

$W_{tot} = 2.226 \times 9.8$

$W_{tot =} 21.8148\ N$

The weight is balanced by three legs

$W_{each\ leg} = W_{tot}/3$

$W_{each\ leg} = 21.8148/3$

$W_{each\ leg} = 7.2716\ N$

Femur

$\tau_{femur} = W_{coxa} \times l_{com,coxa} + W_{each\ leg}\ x\ (L_{coxa} + l_{com,}bd)$

$\tau_{femur} = 0.392 \times 0.034 + 7.2716 \times (0.068 + 0.086)$

$\boxed{\tau_{femur} = 1.13315\ N}$

Tibia

$\tau_{tibia} = W_{femur} \times l_{com,femur} + W_{coxa} \times (L_{femur} + l_{com,coxa}) + W_{each\ leg} \times (L_{femur} + L_{coxa} + l_{com,}bd)$

$\tau_{tibia} = 1.568 \times 0.0345 + 0.392 \times (0.069 + 0.034) + 7.2716 \times (0.069 + 0.068 + 0.086)$

$\boxed{\tau_{tibia} = 1.716 \text{ Nm}}$

For Ripple gait

Five legs on ground at a time

Total suspended mass

$m_{tot} = m_{body} + (m_{coxa} + m_{femur} + m_{tibia})$

$m_{tot} = 1.386 + (0.05 + 0.16 + 0.08)$

$m_{tot} = 1.666$ kg

$W_{tot} = 1.666 \times 9.8$

$W_{tot} = 16.3268$ N

The weight is balanced by five legs

$W_{each\ leg} = W_{tot}/5$

$W_{each\ leg} = 16.3268/5$

$W_{each\ leg} = 3.26536$ N

Femur

$\tau_{femur} = W_{coxa} \times l_{com,coxa} + W_{each\ leg}\ x\ (L_{coxa} + l_{com,}bd)$

$\tau_{femur} = 0.392 \times 0.034 + 3.26536 \times (0.068 + 0.086)$

$\boxed{\tau_{femur} = 0.5162 \text{ N}}$

Tibia

$\tau_{tibia} = W_{femur} \times l_{com,femur} + W_{coxa} \times (L_{femur} + l_{com,coxa}) + W_{each\ leg} \times (L_{femur} + L_{coxa} + l_{com,}bd)$

$\tau_{tibia} = 1.568 \times 0.0345 + 0.392 \times (0.069 + 0.034) + 3.26536 \times (0.069 + 0.068 + 0.086)$

$\boxed{\tau_{tibia} = 0.8226 \text{ Nm}}$

Walking

During the walk phase the coxa joint pushes the bot forward.

Torque required for the coxa joint

$$\tau = \tau_{acceleration} + \tau_{inertia}$$

Ripple gait

Assume the bot as a cube

$I = 0.1666 \times ml^2 \times md^2$

$I = 0.1666 \times 1.666 \times 0.086 + 1.666 \times 0.086^2$

$I = 0.0362$ kg m^2

$\tau_{inertia} = I \times \alpha$

$\tau_{inertia} = 0.0362 \times 1.0472$

$\tau_{inertia} = 0.0379$ Nm

$\tau_{acceleration} = m \times a \times r$

Assume linear acceleration a = 1m/s

$\tau_{acceleration} = 1.666 \times 1 \times 0.086$

$\tau_{acceleration} = 0.143276$ Nm

$\tau = 0.143276 + 0.0362$

$$\boxed{\tau = 0.1794 \text{ Nm}}$$

With safety factor 3

$\tau = 0.1794 \times 3$

$$\boxed{\tau = 0.5382 \text{ Nm}}$$

Tripod gait

$I = 0.1666 \times ml^2 + md^2$

$I = 0.1666 \times 2.226 \times 0.086 + 2.226 \times 0.086^2$

$I = 0.04835 \text{ kg m}^2$

$\tau_{\text{inertia}} = I \times \alpha$

$\tau_{\text{inertia}} = 0.04835 \times 1.0472$

$\tau_{\text{inertia}} = 0.0506 \text{ Nm}$

$\tau_{\text{acceleration}} = m \times a \times r$

Assume linear acceleration, a = 1m/s

$\tau_{\text{acceleration}} = 2.226 \times 1 \times 0.086$

$\tau_{\text{acceleration}} = 0.191436 \text{ Nm}$

$\tau = 0.191436 + 0.0506$

$\tau = 0.2420 \text{ Nm}$

With safety factor 3

$\tau = 0.2420 \times 3$

$$\boxed{\tau = 0.7260 \text{ Nm}}$$

The **tripod gait** is most suitable for terrains that are relatively flat and when achieving **maximum speed** is the primary requirement. On the other hand, the **ripple gait** is better adapted for **rough or uneven terrains**, as it requires less torque and provides **greater stability**, though at the expense of speed. This trade-off allows the hexapod to balance performance between efficiency and reliability depending on the operating environment.

Inverse Kinematics

1. Coordinate frames & symbols

Global frames - G (Ground): world/inertial frame. - **B** (Body): origin at body center. Axes: $+X_B$ forward, $+Y_B$ left, $+Z_B$ up. (Right‑handed.)

Per‑leg frame - C (Coxa yaw joint frame): origin at the **coxa yaw axis** of the leg. This joint rotates about $+Z_C$ (same as $+Z_B$).

Known constants (mm) - Body‑to‑coxa radial offset: R_b = 86 - distance from body center to the coxa yaw axis in the XY plane. - Link lengths (from your drawing): - L_1 = 68 (coxa link: horizontal offset from yaw axis to femur pitch axis). - L_2 = 98 (femur link: distance femur‑to‑knee). - L_3 = 181 (tibia link: knee‑to‑foot tip).

Per‑leg mounting - Each leg is mounted around the body at a known **mounting angle** ψ_i measured in the

XY plane from $+X_B$ toward $+Y_B$. (E.g., front-right ψ might be -45° , front-left $+45^\circ$, mid-right -90° ,
etc.) - Optional body orientation (roll-pitch-yaw): (ϕ, θ, ψ) . If the body is level, treat them as zero. **Joint variables** - q_1 : coxa yaw (about $+Z$). - q_2 : femur pitch (sagittal plane). - q_3 : tibia pitch (knee). **Foot target** - Desired foot position in **B**: $\mathbf{p}_B = [x_B, y_B, z_B]^T$ (units: mm).

Sign conventions: positive z up; positive pitches lift the leg (counter-clockwise when viewed from the leg's left side). Adjust to match your servo zeros if needed (see §7).

2. Body → Coxa transform

Coxa yaw axis position in B

$$\mathbf{o}_{CB} = \begin{bmatrix} R_b \cos \psi_i \\ R_b \sin \psi_i \\ 0 \end{bmatrix}$$

If the body has nonzero attitude, first express the target foot point in **B** by removing

body rotation (use the inverse of the body rotation matrix $\mathbf{R}_{GB}(\phi, \theta, \psi)$):

$\mathbf{p}_B = \mathbf{R}^T_{GB}\ \mathbf{P}_G$ (skip if commands are already in B).

Vector from coxa yaw to the foot (in B)

$$\mathbf{v} = \mathbf{p}_B - \mathbf{o}_{CB} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

3. Coxa yaw angle q_1

Projection to the XY plane gives yaw:

$$q_1 = \operatorname{atan2}(v_y, v_x) \in (-\pi, \pi].$$

Horizontal reach from yaw axis

$$\rho = \sqrt{v_x^2 + v_y^2}$$

Effective horizontal distance after the coxa link

$\rho' = \rho - L_1$ (must be $\rho' \geq 0$ to be reachable).

4. Sagittal-plane triangle (femur–tibia)

Work in the plane defined by coxa yaw: horizontal distance ρ' and vertical v_z .

Hip-to-foot straight-line distance

$$D = \sqrt{\rho'^2 + v_z^2}$$

Law of cosines for the knee

$$\cos\beta = \frac{L_2^2 + L_3^2 D^2}{2L_2L_3}$$

$$\beta = \arccos(\text{clip}(\cos\beta, -1, 1))$$

The **knee (tibia) joint angle** is typically taken as

$q_3 = \pi - \beta$ (flexion positive; adjust sign to taste).

Hip elevation angle to the target

$$\gamma = \text{atan2}(v_z, \rho')$$

Law of cosines for the femur interior angle

$$\cos\alpha = \frac{L_2^2 + D^2 - L_3^2}{2L_2D}$$

$$\alpha = \arccos(\text{clip}(\cos\alpha, -1, 1))$$

Femur joint angle (hip pitch):

$$q_2 = \gamma + \alpha.$$

Alternative common convention: $q_2 = \gamma - \alpha$ and $q_3 = -\beta$. Choose one set and keep it consistent with your servo zeros (see §7).

5. Final IK – compact formula set

Given $\mathbf{p}_B = [x_B, y_B, z_B]^T$, constants R_b, L_1, L_2, L_3 , and leg mount ψ_i :

1. $\mathbf{o}_{CB} = [R_b \cos\psi_i, R_b \sin\psi_i, 0]^T$
2. $\mathbf{v} = \mathbf{p}_B - \mathbf{o}_{CB}$
3. $q_1 = \text{atan2}(v_y, v_x)$
4. $\rho = \sqrt{v_x^2 + v_y^2}$
5. $D = \sqrt{\rho'^2 + v_z^2}$
6. $\beta = \arccos\left(\frac{L_2^2 + L_3^2 - D^2}{2L_2L_3}\right)$
7. $\gamma = \text{atan2}(v_z, \rho')$
8. $\alpha = \arccos\left(\frac{L_2^2 + D^2 - L_3^2}{2L_2D}\right)$
9. **Joint outputs** (one common convention):
10. $q_1 = \text{atan2}(v_y, v_x)$
11. $q_2 = \gamma + \alpha$
12. $q_3 = \pi - \beta$

6. Reachability & workspace checks

- **Horizontal constraint:** $\rho' = \rho - L_1 \geq 0$.
- **Triangle inequality:** $|L_2 - L_3| \leq D \leq L_2 + L_3$.
- **Joint limits:** clamp q_1, q_2, q_3 to your mechanical/servo limits.
- **Numerical safety:** clip cosine arguments to $[-1, 1]$ before arccos .

Max nominal reach (foot from coxa yaw, level leg): $L_1 + L_2 + L_3 = 68 + 98 + 181 = 347$ mm in the horizontal direction (ignoring joint limits).

7. Aligning with servo zeros & directions

Servos rarely have mathematical zeros. Let **mechanical zeros** (in encoder units or radians) be ($q_{1,0}$, $q_{2,0}$, $q_{3,0}$) and sign multipliers (s_1,s_2,s_3)\in\{-1,+1\}. Send commands as

$\tilde{q}_1 = q_{1,0} + s_1\, q_1$

$\tilde{q}_2 = q_{2,0} + s_2\, q_2$

$\tilde{q}_3 = q_{3,0} + s_3\, q_3$.

Calibrate by placing the robot in a known pose (e.g., "home": straight, level) and solving for that yields your encoder readings.

8. Including body attitude (roll–pitch–yaw)

When commanding the robot's feet to specific locations on the ground while the body tilts, rotate the foot positions into the body's frame before calculating the joint movements:

$$\mathbf{p}_B = \mathbf{R}^T(\phi, \theta, \psi)\,(\mathbf{p}_G - \mathbf{o}_{BB,G})$$

where $\mathbf{o}_{BB,G}$ is the body-center position in the ground frame and $\mathbf{R}_{GB}$ is the standard ZYX RPY rotation matrix. Then apply §5.

9. Minimal reference implementation (pseudocode)

```
function legIK(pB, legMountPsi):
    Rb = 86
    L1, L2, L3 = 68, 98, 181

    oCB = [Rb*cos(legMountPsi), Rb*sin(legMountPsi), 0]
    v = pB - oCB

    q1 = atan2(v.y, v.x)

    rho  = sqrt(v.x^2 + v.y^2)
    rhoP = rho - L1
    D    = sqrt(rhoP^2 + v.z^2)

    beta   = acos( clip((L2^2 + L3^2 - D^2)/(2*L2*L3), -1, 1) )
    gamma  = atan2(v.z, rhoP)
    alpha  = acos( clip((L2^2 + D^2 - L3^2)/(2*L2*D), -1, 1) )

    q2 = gamma + alpha
    q3 = PI - beta

    return q1, q2, q3
```

10. Quick sanity checks

- **Flat stance**: set z_B negative (below body), q_2 should be slightly negative (down), q_3 near flexed.
- **Straight-line reach**: with $v_z = 0$ and increasing ρ , q_2 decreases toward 0 and q_3 approaches π (straight leg) as $D \to L_2 + L_3$.
- **Under-body targets**: as ρ decreases below L_1 , the leg becomes unreachable $\Rightarrow$ plan stances so

$$\rho \geq L_1 .$$