Week 8 Notes
Astro 2 (Discussion Section 102)

Department of Physics: University of California, Santa Barbara

Updated May 19, 2011

Review

Midterm 2 Wrap-Up

I'll go over question from Midterm 2.

Threshold Temperature

The **threshold temperature** of a particle is the temperature at which pair production of that particle and its anti-particle are no longer possible. That is, it is the lowest temperature where the following reaction is possible when considering only photons from the CMB:

$$2\gamma \longrightarrow \mathrm{A}^{+} + \mathrm{A}^{-}$$

where γ represents a gamma ray from the CMB, and A^{+} and A^{-} are the particle and anti-particle (call each whichever you want). In this reaction, energy must be conserved. Thus, if we consider the two particles to be at rest when they are created, the energy on the right side is twice the rest energy of the particle (anti-particles have the same mass as their partners). Thus, we see that the photons that create the particles much each have an energy equal to the rest energy of the particles to be created (assuming they are equal in energy).

Recall that the energy of a photon is given by

$$E = h\nu = \frac{hc}{\lambda}$$

where ν is the frequency of the photon, λ is its wavelength, and h is Planck's constant. If you are unfamiliar with this, see chapter 5 for a refresher. Since h and c are fundamental constants, we may determine the wavelength of these photons required for the reaction. Since the CMB has a blackbody distribution, we may find what temperature corresponds to this peak wavelength:

$$\lambda_{\text{peak}} = \frac{0.0028\ \text{K m}}{T}$$

The temperature that satisfies these requirements is the lowest possible one, and is thus the threshold temperature. Higher temperatures can still yield the same particles, they would just be created with some kinetic energy in addition to its rest energy.

Example Find the threshold for protons and anti-protons. Also find out when (what redshift) protons stopped being formed by pair production.

We first need to find the rest energy of the protons they have a mass of $m_p = 1.67 \times 10^{-27}$ kg. For now, let's just call this quantity m_p to keep numbers out of the problem until the very end. The rest energy is given by Einstein's mass-energy equivalence equation:

$$E_0 = m_p c^2$$

Now in order for a pair of photons to create a proton-anti-proton pair, each must have this same energy. We can use this to find the wavelength that these photons must have. Equating the photon energy to the rest energy, we have

$$E_0 = \frac{hc}{\lambda} \Rightarrow m_p c^2 = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{h}{m_p c}$$

Now we find the energy where this wavelength is the peak wavelength using Wien's law

$$\frac{h}{m_p c} = \frac{0.0028 \text{ K m}}{T_{\text{thresh}}}$$
$$T_{\text{thresh}} = \frac{m_p c}{h}(0.0028 \text{ K m})$$

Now we plug in numbers. The units here will differ from your homework, so be careful

$$T_{\text{thresh}} = \frac{(1.67 \times 10^{-27} \text{ kg})(3.0 \times 10^{8} \text{ m/s})}{6.62 \times 10^{-34} \text{ J s}}(0.0028 \text{ K m}) = 2.12 \times 10^{12} \text{ K}$$

We can find out when this temperature was reached by recalling that $T(z) = T_0(1+z)$. Setting $T(z)$ equal to T_{thresh} and solving for z will tell us when protons stopped being formed by pair production:

$$(1+z)T_0 = T_{\text{thresh}}$$
$$1+z = \frac{T_{\text{thresh}}}{T_0}$$
$$z = \frac{2.12 \times 10^{12} \text{ K}}{2.725 \text{ K}} - 1$$
$$z = 7.78 \times 10^{11}$$