Week 7 Notes
Astro 2 (Discussion Section 102)

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Review

Questions on HW 5

I'll field questions on Homework 5.

Midterm Review

Overview Expect an exam similar in form to Midterm 1, but focusing on Chapter 26 (that's all we've done since the last exam). Obviously previous material is assumed, but the emphasis will be on new material.

Conceptual Foundations Have a good idea of how to explain Olbers' Paradox, Newton's static universe, and how the big bang theory resolved the problem through having the universe be finite in time.

Expansion of Space Remember all of the scalings for redshift. That is, how lookback time affects quantities like length, area, volume, density, temperature, etc. These make for good computational problems, like those in your homework, so be very sure of your ability to apply these.

Density and Density Parameters

By Einstein's famous equation, $E = mc^2$, we can relate the energy from photons to an effective mass, or even a density. Combining the Stefan-Boltzmann law with $E = mc^2$ yields

$$\rho_{\mathrm{rad}} = \frac{4\sigma T^4}{c^3}$$

where σ is the Stefan-Boltzmann constant, T is the temperature of the radiation, in Kelvin, and c, as usual, is the speed of light. One could compute this for the universe using the cosmic microwave background radiation temperature of $T = 2.725$, since most of the universe's electromagnetic radiation is in the CMB. The result, as quoted in the text, is $\rho_{\mathrm{rad}} = 4.6 \times 10^{-31}\ \mathrm{kg/m^3}$. One can also compute an approximate matter density of the universe by observing the mass content of the universe. This is done by observing gravitational effects over large spaces to deduce the matter content and then dividing by the volume. We have an approximate measurement of $\rho_{\mathrm{m}} = 2.4 \times 10^{-27}\ \mathrm{kg/m^3}$. This has an uncertainty of approximately 15%.

Density Parameters The density of the universe actually defines what geometry it takes on. If the total density is above some critical density, it is closed. This density is

$$\rho_{\mathrm{c}} = \frac{3H_0^2}{8\pi G} = 1.0 \times 10^{-26}\ \mathrm{kg}$$

This is really the only relevant density, we often express the mass and radiation densities in terms of density parameters, which is the ratio of each density to the critical density:

$$\Omega_{\mathrm{m}} = \frac{\rho_{\mathrm{m}}}{\rho_{\mathrm{c}}} = 0.24$$

$$\Omega_{\mathrm{rad}} = \frac{\rho_{\mathrm{rad}}}{\rho_{\mathrm{c}}} = .000046$$

If we let ρ_0 be the total combined average density, that is the density due to matter, radiation, and any other forms of energy, we may define the **density parameter**:

$$\Omega_0 \equiv \frac{\rho_0}{\rho_\mathrm{c}}$$

We see then that if $\Omega_0 > 1$, we live in a closed universe. If $\Omega_0 = 1$, we live in a flat universe, and if $\Omega_0 < 1$, we live in an open universe. Scientists have studied the propagation of photons that have traveled vast distances (and thus their deviations from being parallel would be quite pronounced) and concluded that the universe is flat or very close to being flat. However, this is a problem, since our current value for the density is too small, just including matter and radiation:

$$\Omega_0 = \Omega_\mathrm{m} + \Omega_\mathrm{rad} = 0.24 < 1$$

We would expect to see an open universe. The "solution" to this problem, though not well understood at all, is the elusive **dark energy**. This gives rise to the **dark energy density parameter**:

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_\mathrm{c}}$$

Since ρ_rad is so small, we may approximate $\rho_0 = \rho_\mathrm{m} + \rho_\Lambda$, which then gives us

$$\Omega_0 = \Omega_\mathrm{m} + \Omega_\Lambda$$

We see then, by the assumption that the universe flat, or very close to flat, that $\Omega_\Lambda \approx 0.76$ in order to force $\Omega_0 = 1$.

Evolution of the Density Parameters The density parameters must necessarily change in time. For instance, the universe used to be dominated by radiation but this is no longer the case. Clearly Ω_rad was once greater than Ω_m. Since these Ω_m is dependent on the volume of interest, it must necessarily scale as $(z+1)^3$. Back in time, volumes were smaller, so it must be directly proportional, that is,

$$\Omega_\mathrm{m} = \Omega_{\mathrm{m},0}(z+1)^3$$

Similarly, accounting for the change in volume *and* wavelength, the radiation density parameter scales as

$$\Omega_\mathrm{rad} = \Omega_{\mathrm{rad},0}(z+1)^4$$

Example: Radiation to Mass Dominance When (at what redshift) did mass overtake radiation as the dominant source of energy in the universe?

The moment when mass overtook radiation was when the two density parameters were equal. Thus, we equate the previous two expressions:

$$\Omega_{\mathrm{m},0}(z+1)^3 = \Omega_{\mathrm{rad},0}(z+1)^4$$

which gives $z = \Omega_{\mathrm{m},0}/\Omega_{\mathrm{rad},0} - 1$.

Friedmann's Equation Revisited

Recall the cryptic Friedmann Equation from class:

$$H^2 = H_0^2\left[\Omega_{\mathrm{rad},0}(1+z)^4 + \Omega_{\mathrm{m},0}(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda\right]$$

Here the Ω's are density parameters (as measured now) and Ω_k is related to the overall geometry of the universe, being positive for a closed universe, zero for a flat universe, and negative for an open universe. This equation effectively governs the evolution of the universe, provided we can get good measurements on the current parameters. Note also how this equation reflects the time dependence on the different factors. Though now the universe is driven by dark energy, at sufficiently high z, mass catches up, and then later radiation also catches up.

Methods and Tools Be able to discuss some of the tools astronomers use to probe the structure and history of the universe (cosmography).

Horizon Problem Be able to talk about what the horizon problem is and how inflation theory solves it.