Week 4 Notes
Astro 1 (Discussion Section 105)

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Administrative Tasks

Stargazing Sign-In Pass sign-up sheet for stargazing night (March 3, 2011) around class, explaining that there is a 50 person limit, and that if that limit is breached, a second night will take place on March 8, 2011.

Website Announce website, which is `www.physics.ucsb.edu/~wmwolf`. Relevant files are under the "teaching" tab.

Test Results Hand out test forms. Curve was `RawScore/28 * 1.1 * 100%`. The average test score was around 17.5/30.

Homework Many mistakes are being made due to neglecting the use of units. While I will only take points off for not having units on your final answer, you get a good check on your answer by evaluating your units as you go. This is called **dimensional analysis**. Also, the answers in the back of the book are strictly a *reference*, not a goal to work towards. Many (and I mean many) students had completely incoherent work that led to bad answers. A result given in meters would have an arrow that pointed to the correct answer expressed in minutes. This is totally unacceptable and will gain you no points. It insults your graders' intelligence and will only hurt your grade. Work towards your own answer and check its validity with the provided one. You are better off doing honest work and evaluating your answer to see if it even makes physical sense rather than doing nonsense work and vomiting out the right answer.

Questions

Concept Review

Newton's Form of Kepler's Third Law With his laws of motion in place, Newton modified Kepler's third law to include more valid information. It's restatement is given as

$$P^2 = \left[\frac{4\pi^2}{G(m_1 + m_2)}\right] a^3$$

where P is the orbital period, m_1 and m_2 are the masses of the two objects, G is the gravitational constant, and a is the semi-major axis connecting the two *centers of mass.*

Average Density The density of an object measures how closely packed the matter is in a given object. For instance, the matter of a bag of popcorn is packed in less densely than a bag of coins. Thus, the bag of coins is heavier, even if it has the same volume. The density of an object is typically represented by the Greek letter, ρ ("rho"), and the **average density** is calculated thusly:

$$\rho_{\text{avg}} = \frac{M}{V}$$

where M is the mass of the object and V is the volume. For a sphere then,

$$V = \frac{4}{3}\pi R^3$$

Given the mass of the sphere, we can calculate its average density pretty easily.

Escape Velocity The escape velocity of an object is the velocity required to completely break free of the object's gravitational pull. This can be derived fairly easily by noting that the potential and kinetic energies are defined to be

$$E_p = -G\frac{Mm}{r}$$

$$E_k = \frac{1}{2}mv^2$$

Total energy is conserved, so $E_k + E_p = E_{\text{tot}} =$ a constant. Escaping gravitational pull completely essentially means that you are at an infinite distance from the object. We see that as our distance goes to infinity, potential energy goes to zero. The minimum amount of energy then (after escaping) is zero. So, let us set E_{tot} to zero and find what velocity is required when we start at the surface of the object $r = R$.

$$\begin{aligned}
0 &= E_{\text{tot}} \\
&= E_k + E_p \\
&= \frac{1}{2}mv_{\text{esc}}^2 - G\frac{Mm}{R} \\
G\frac{Mm}{R} &= \frac{1}{2}mv_{\text{esc}}^2 \\
\frac{2GM}{R} &= v_{\text{esc}}^2 \\
\sqrt{\frac{2GM}{R}} &= v_{\text{esc}}
\end{aligned}$$

So as it turns out, the minimum velocity needed to escape gravitational attraction is independent of mass, which is pretty cool! That being said, the acceleration required to get to that mass is directly proportional to mass, as shown by Newton's second law.

Example: Escape velocity of Earth What is the escape velocity of earth? Earth's radius is 6,378 km and its mass is 5.74×10^{24} kg.

We simply apply the formula we derived for escape velocity:

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.67 \times 10^{-11}\,\text{Nm}^2/\text{kg}^2)(5.74 \times 10^{24}\,\text{kg})}{6378\,\text{km}}} = 11,000\,\text{m/s} = 11\,\text{km/s}$$

Thermal Energy of Particles Temperature is an indicator of random motions of particles. This energy is completely random, and is due only to the kinetic energy of the particles. In equation form, it is given by

$$E_k = \frac{3}{2}kT$$

where k is Boltzmann's constant, $k = 1.38 \times 10^{-23}$ J/K and T is the temperature of the system in Kelvin.

Example: Speed of oxygen molecules at room temperature given that the average oxygen molecule has a mass of 5.32×10^{-26} kg.

We simply apply our two equation for kinetic energy of such a system:

$$E_k = \frac{3}{2}kT = \frac{1}{2}mv^2$$

Rearranging, we have then

$$v^2 = \frac{3kT}{m}$$

and then

$$v = \sqrt{\frac{3kT}{m}}$$

Suppose room temperature is around 20° C $= 293$ K, then from plugging everything in, we get $v = 478$ m/s.