Week 10 Notes
Astro 1 (Discussion Sections 101 & 102)

Department of Physics: University of California, Santa Barbara

Updated March 7, 2011

Administrative Tasks

Stargazing Event At present, only those on the roster that was distributed via e-mail are registered to attend either stargazing event (weather permitting). Unless you have been specifically told otherwise, failing to attend will result in a loss of 2% off of your final grade.

Today's Material I can either cover material from the next homework or work a few problems to review for the final exam. Whatever you all prefer to do is what we'll do.

Questions on Last Week's Homework

Concept Review

Main-Sequence Lifetimes

Stars remain on the main sequence only as long as they can successfully convert Hydrogen into Helium. Suppose that a star converts a fraction f of its total mass to energy over its lifetime. Then the total energy the star gives off during its lifetime due to fusion is

$$E = fMc^2$$

If we assume a constant luminosity of the star over its main-sequence lifetime, we see that

$$L = \frac{E}{t}$$

where t is the lifetime of the star. From this we get

$$E = Lt$$

and using our previous formula for energy released, we have

$$fMc^2 = Lt$$

Solving for t yields

$$t = \frac{fMc^2}{L}$$

We don't necessarily know what f is for a star, so we can just write a proportionality, that is

$$t \propto \frac{M}{L}$$

This means that t is equal to M/L multiplied by some unknown constant. We may also use the fact that the luminosity of a star is proportional to its mass raised to the 3.5 power, that is

$$L \propto M^{3.5}$$

Putting this all together, we obtain

$$t \propto \frac{M}{M^{3.5}} = \frac{1}{M^{2.5}} = \frac{1}{\sqrt{M}M^2}$$

This formula in and of itself is not extremely useful unless we have a reference star to work with. Then we may simply compute ratios, and the unknown constant, whatever it was, cancels out. Traditionally we use that the lifetime of the sun is $t_\odot = 1.2 \times 10^{10}$ years. In this case, the relevant equation becomes

$$\frac{t}{t_\odot} = \frac{\sqrt{M_\odot} M_\odot^2}{\sqrt{M} M^2} = \sqrt{\frac{M_\odot}{M}} \left(\frac{M_\odot}{M}\right)^2$$

This equation can be inverted to find what the mass of a given star would need to be in order for it to stay on the main sequence for a certain time. You are asked to do this on your homework, so I'll leave it to you.

Example: Lifetime of a Small Star Suppose we want to know how long a star of .1 solar masses will stay on the main sequence. We may then apply our equation with $M = .1 M_\odot$:

$$\frac{t}{t_\odot} = \sqrt{\frac{M_\odot}{.1 M_\odot}} \left(\frac{M_\odot}{.1 M_\odot}\right)^2$$

$$\frac{t}{t_\odot} = \sqrt{10}(10)^2$$

$$t \approx 316\, t_\odot = 3.8 \times 10^{12} \text{ years}$$

So the star will remain on the main sequence for about 4 trillion years. We have not seen any of these stars die out yet to form a planetary nebula, can you tell why?

Time Dilation

Einstein's Special Theory of Relativity has some funky consequences in our perception of the universe is that of time dilation. The flow of time is *not* the same for all observers. The faster an observer is moving with respect to another observer, the slower they will observe time to pass by. This result is mathematically represented as

$$T = \frac{T_0}{\sqrt{1 - (v/c)^2}}$$

Here T is the time interval measured by the moving observer, T_0 is the "proper time", that is, the time measured by the non moving observer. Then v is the speed of the moving observer relative to what is being observed, and c is the speed of light.

A More Concrete Example: Space Toast Suppose Bob is floating around in space and he is trying to make toast. He presses the button on the toaster. At the same time, Sally blasts off in a space ship and travels at a constant speed v near the toaster and observes how long it takes. Sally will measure the toast taking a time T to finish, whereas Bob will measure T_0, the proper time, because Bob is in the same inertial frame as the toaster (he is not moving relative to the phenomenon being observed).

Example: Making retirement count Suppose you retire and decide to go on a spaceship cruise through the universe at 95% of the speed of light for five years. When you return back to Earth, how much time will have passed?

Essentially this problem boils down to deciding what's the proper time and what's the dilated time. The proper time always measures the time of an event as it would be measured by an observer that is moving at the same speed as the object being measured. Thus, the proper time here is five years, since you measure the five years while you are traveling in your frame. You could even think of yourself as being still, while the *Earth* is zipping by, and we wish to know what time the *Earth* observes. Plugging this proper time and speed into the time dilation equation yields

$$T = \frac{5 \text{ years}}{\sqrt{1 - (.95\, c/c)^2}} = \frac{5}{\sqrt{1 - .9025}} \text{ years} = 16 \text{ years}$$

So when you return from your cruise, you will have aged five years, but earth will have passed through 16 years. As you can imagine, this strange nature of time can cause some wacky situations to take place when traveling at great speeds.

Length Contraction Of similar interest (though it's not on your homework this week) is the issue of length contraction. When an object is moving very fast, it appears to shrink in size, according to the equation

$$L = L_0\sqrt{1-(v/c)^2}$$

Here L is the length as measured by the stationary observer of the moving object, L_0 is the proper length of the object, measured at rest, and v and c are the same as before.

Black Holes and the Schwarzchild Radius

As I've mentioned before in class *any* object can be compressed into a black hole if it were made small enough. The "size" of any black hole is usually denoted by its Schwarzchild radius, or event horizon. It is defined thusly:

$$R_{\text{Sch}} = \frac{2GM}{c^2}$$

Interestingly the mass of the black hole is the only thing that defines the Schwarzchild radius.

Interesting facts about the Schwarzchild Radius If you were to approach a black hole and enter within the Schwarzchild radius, there is *nothing* you could do to escape it. I'm not saying it would be difficult to escape; I'm saying that there is no physically possible way to escape the pull of the black hole. Your body would be stretched by tidal forces to a hilariously thin state (this process is actually called "spaghettification", believe it or not) until it would eventually be torn apart atom by atom.

To be of any appreciable size, a black hole must be extremely massive. If one put the mass of the earth into the formula for the Schwarzchild radius, the resulting "black hole size" if Earth were compressed down to a point would be about the size of a peanut.

Mega Problem

Suppose an Astronomer takes some measurements on a certain star. She measures its parallax angle as $p = .12$ arcseconds and its angular diameter to be $\alpha = .02$ arcseconds. When she analyzes the star's spectra, she detects a line closely resembling the Hydrogen-alpha line at $\lambda = 655.89$ nm when in the lab this line would be measured to be $\lambda_0 = 656.28$ nm. Furthermore, the blackbody spectrum shows a peak wavelength at $\lambda_{\text{max}} = 300$ nm. From this information, determine:

(a) The radius, surface area, and volume of the star.

(b) The radial velocity of the star as measured from Earth.

(c) The surface temperature of the star.

(d) The luminosity (power) of the star.

(e) The energy flux from the star at Earth.

(f) Judging from the color and size of the star, what general type it is.

Solution

(a) We can use the small angle formula to get the diameter of the star, but first, we need the distance to the star, which can be obtained from the parallax angle formula:

$$d = \frac{1}{p} = \frac{1}{.12} = 8.3\,\mathrm{pc}$$

Using the small angle formula, we get the angular diameter:

$$D = \frac{d\alpha}{206,265} = \frac{(8.3\,\mathrm{pc})(.02)}{206,265} = 8 \times 10^{-7}\,\mathrm{pc} = 2.5 \times 10^{7}\,\mathrm{km}$$

The radius is simply half of the diameter, so

$$\boxed{r = 1.25 \times 10^{7}\ \mathrm{km} = 1.25 \times 10^{10}\ \mathrm{m}}$$

We may use the formulas for the surface area and volume of a sphere to round out this part of the question:

$$A = 4\pi r^2 = 2.0 \times 10^{15}\,\mathrm{km}^2 = 2.0 \times 10^{21}\,\mathrm{m}^2$$
$$V = \frac{4}{3}\pi r^3 = 8.2 \times 10^{21}\mathrm{km}^3 = 8.2 \times 10^{30}\,\mathrm{m}^3$$

So

$$\boxed{A = 2.0 \times 10^{21}\,\mathrm{m}^2 \qquad V = 8.2 \times 10^{30}\,\mathrm{m}^3}$$

(b) Radial velocity is measured by use of the doppler effect. We have a measured and an actual wavelength, so we may easily calculate the radial velocity.

$$\frac{v_r}{c} = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{655.89\,\mathrm{nm} - 656.28\,\mathrm{nm}}{656.28\,\mathrm{nm}} = -5.9 \times 10^{-4}$$

So the radial velocity is

$$\boxed{v_r = -5.9 \times 10^{-4}c = 1.77 \times 10^{5}\,\mathrm{m/s}\ \text{towards Earth}}$$

we then see the light from this star blue-shifted, which is consistent with our given result since the observed wavelength is shorter (bluer) than the expected one.

(c) The surface temperature of the star is related to its peak wavelength through Wien's displacement law, but the peak wavelength we see has been blue-shifted just as the Hydrogen-alpha line was. So before we can apply Wien's law, we must use the Doppler effect to get the true peak wavelength:

$$\frac{\lambda_{\mathrm{max}} - \lambda_{\mathrm{max},0}}{\lambda_{\mathrm{max},0}} = \frac{v}{c}$$

$$300\,\mathrm{nm} - \lambda_{\mathrm{max},0} = \left(-5.9 \times 10^{-4}\right)\lambda_{\mathrm{max},0}$$
$$300\,\mathrm{nm} = \left(1 - 5.9 \times 10^{-4}\right)\lambda_{\mathrm{max},0}$$
$$\lambda_{\mathrm{max},0} = 300.18\,\mathrm{nm}$$

Now we may use Wien's law to find the surface temperature of the star:

$$\lambda_{\mathrm{max},0} = \frac{.0029\,\mathrm{m\,K}}{T}$$

$$T = \frac{.0029\,\mathrm{m\,K}}{300.18\,\mathrm{nm}} = \frac{.0029\,\mathrm{m\,K}}{3.0018 \times 10^{-7}\,\mathrm{m}}$$

So the surface temperature is about

$$\boxed{T = 9660\,\mathrm{K}}$$

(d) With the surface temperature, we can find the flux passing through the surface of the star. This, in turn can be related to the luminosity, or power of the star. By combining the Stefan Boltzmann law with the definition of energy flux,

$$F = \sigma T^4 = \frac{L}{4\pi r^2}$$

Rearranging, we have

$$L = (4\pi r^2)\sigma T^4 = \left(2.0 \times 10^{21}\,\mathrm{m}^2\right)\left(5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}\right)(9600\,\mathrm{K})^4$$

So the luminosity is

$$\boxed{L = 9.6 \times 10^{29}\,\mathrm{W}}$$

(e) To find the flux at Earth, we relate the power of the star to the surface area through which the power must pass through at the distance of the earth. That is, we divide the power of the star by the area of the sphere with the radius equal to the distance from the star to the Earth:

$$F_{\mathrm{Earth}} = \frac{L}{4\pi d^2} = \frac{9.6 \times 10^{29}\,\mathrm{W}}{4\pi(8.3\,\mathrm{pc})^2} = \frac{9.6 \times 10^{29}\,\mathrm{W}}{4\pi\left(2.56 \times 10^{17}\,\mathrm{m}\right)^2}$$

So the flux at Earth is approximately

$$\boxed{F_{\mathrm{Earth}} = 1.2 \times 10^{-6}\,\mathrm{W/m^2}}$$

(f) The star is MUCH larger than the sun (compare radii), much more luminous than the sun, and, judging by its peak wavelength in the ultraviolet, would likely appear blue to our eyes. Thus, it seems likely that this is a blue supergiant star.

Other Questions