Week 3 Notes
Astro 2 (Discussion Section 101)

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Administrative Tasks

Add Codes All who have shown up to one of the first two weeks of section and signed in have been e-mailed add codes. If you don't have one in your e-mail and think you deserve one, let me know.

Survey I want to make sure this section is helpful to you. In Astro 1, material was being covered at a pace where I basically had to talk about problem solving skills that were pertinent to the homework each week. However, Astro 2 is a bit more slowly-paced, so I have a bit more freedom. I'll have you students fill out an annonomous survey to let me know what you'd like out of the class. I'll do this at the end of class.

Review

Number Density

We have use the concept of average density in the past. The average density of an object with mass M and volume V is

$$\rho \equiv \frac{M}{V}$$

However, we know that objects are typically composed of a large number of very small particles. If the object is made of only one kind of particle, we can introduce a **number density**. This number density gives us the number of particles per unit volume in the object, and thus has units of dimension $(\text{length})^{-3}$. If each particle has mass m, then the total mass is $Nm = M$ for some integer N. Thus we have

$$\rho = \frac{Nm}{V} = nm$$

where we have introduced the number density, n:

$$n \equiv \frac{N}{V}$$

Example We know that water has a mass density of approximately 1 gram per cubic centimeter. Find the number density of water in $(\text{m})^{-}3$ given that the mass of a hydrogen atom is 1.67×10^{-27} kg and the mass of one oxygen molecule is 2.66×10^{-26} kg.

Solution We can calculate the number density by using the equation $n = \rho/m$, where m is the mass of an individual water molecule. Since water is $\mathrm{H_2O}$, the mass is

$$m = 2m_{\mathrm{H}} + m_{\mathrm{O}} = 2\left(1.67 \times 10^{-27}\,\mathrm{kg}\right) + 2.66 \times 10^{-26}\,\mathrm{kg} = 3 \times 10^{-26}\,\mathrm{kg}$$

So the number density is

$$n = \frac{1\,\mathrm{g/cm^3}}{3 \times 10^{-26}\,\mathrm{kg}} \times \frac{1\,\mathrm{kg}}{1000\,\mathrm{g}} \times \left(\frac{100\,\mathrm{cm}}{1\,\mathrm{m}}\right)^3 = 3.3 \times 10^{28}\,\mathrm{m^{-3}}$$

Blast From the Past: Kepler's Third Law

Recall Newton's form of Kepler's Third Law:

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

where all units are understood to be in SI units.

Example: Suppose two galaxies are in orbit around each other, where galaxy A has mass $m_A = 1.4 \times 10^{42}$ kg and galaxy B has mass $m_B = 1.8 \times 10^{42}$ kg. If they are at most 900 kpc away from each other, how long will one orbit take?

Solution This is an application of Kepler's Third law on galactic scales. We can take Kepler's third law and solve for the period:

$$P = \sqrt{\frac{4\pi^2}{G(m_A + m_B)} a^3}$$

Here, $a = 450\,\mathrm{Mpc}$, or half the largest separation distance. Plugging in our values, we get

$$P = \sqrt{\frac{4\pi^2}{(6.67 \times 10^{-11})(3.2 \times 10^{42}\,\mathrm{kg})}(450 \times 3 \times 10^{19})^3\,\mathrm{m}} = 6.75 \times 10^{17}\,\mathrm{s} = 2.15\,\text{billion years}$$

Redshift Clarification

Last week we talked about the redshift of an object, z. While the definition

$$z = \frac{\Delta\lambda}{\lambda_0}$$

is always true, the formula

$$z = \frac{v}{c}$$

is *not* always true. It is valid for small z (typically $z = .1$ or so). For larger z, a relativistic treatment is needed. We can see this since $z > 1$ would imply that $v > c$. Thus, the following equations give a more complete treatment of the topic:

$$\frac{v}{c} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1}$$

$$z = \sqrt{\frac{c+v}{c-v}} - 1$$

Example (U.2.41) The galaxy RD1 has a redshift of $z = 5.34$.

(a) Determine its recessional velocity v in km/s and as a fraction of the speed of light.

(b) What recessional velocity would you have calculated if you had erroneously used the low-speed formula relating z and v? Would using this formula have been a small or large error?

(c) According to the Hubble law, what is the distance from Earth to RD!? Use $H_0 = 73$ km/s/Mpc for the Hubble constant, and give your answer in both megaparsecs and light-years.

Solution

(a) Since $z > .1$, we use the relativistic equations relating z, v, and c:

$$v = \frac{(z+1)^2 - 1}{(z+1)^2 + 1} c = \frac{6.34^2 - 1}{6.34^2 + 1} c = .95c$$

(b) Using the low-speed formula gives

$$v = zc = 5.34c$$

The percent error would be

$$\frac{5.34c - .95c}{.95c} \times 100\% = 462\%$$

This is obviously a huge error.

(c) Simply use Hubble's law:

$$d = \frac{v}{H_0} = \frac{.95(300,000\,\mathrm{km/s})}{73\,\mathrm{km/s/Mpc}} = 3.9 \times 10^3\,\mathrm{Mpc} = 1.27 \times 10^{10}\,\text{light years}$$