Week 10 Notes
Astro 1 (Discussion Section 105)

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Questions on Last Week's Homework

Mega Problem

Suppose an Astronomer takes some measurements on a certain star. She measures its parallax angle as $p = .12$ arcseconds and its angular diameter to be $\alpha = .02$ arcseconds. When she analyzes the star's spectra, she detects a line closely resembling the Hydrogen-alpha line at $\lambda = 655.89$ nm when in the lab this line would be measured to be $\lambda_0 = 656.28$ nm. Furthermore, the blackbody spectrum shows a peak wavelength at $\lambda_{\max} = 300$ nm. From this information, determine:

(a) The radius, surface area, and volume of the star.

(b) The radial velocity of the star as measured from Earth.

(c) The surface temperature of the star.

(d) The luminosity (power) of the star.

(e) The energy flux from the star at Earth.

(f) Judging from the color and size of the star, what general type it is.

Solution

(a) We can use the small angle formula to get the diameter of the star, but first, we need the distance to the star, which can be obtained from the parallax angle formula:

$$d = \frac{1}{p} = \frac{1}{.12} = 8.3\,\mathrm{pc}$$

Using the small angle formula, we get the angular diameter:

$$D = \frac{d\alpha}{206,265} = \frac{(8.3\,\mathrm{pc})(.02)}{206,265} = 8 \times 10^{-7}\,\mathrm{pc} = 2.5 \times 10^{7}\,\mathrm{km}$$

The radius is simply half of the diameter, so

$$\boxed{r = 1.25 \times 10^{7}\ \mathrm{km} = 1.25 \times 10^{10}\ \mathrm{m}}$$

We may use the formulas for the surface area and volume of a sphere to round out this part of the question:

$$A = 4\pi r^2 = 2.0 \times 10^{15}\,\mathrm{km}^2 = 2.0 \times 10^{21}\,\mathrm{m}^2$$

$$V = \frac{4}{3}\pi r^3 = 8.2 \times 10^{21}\mathrm{km}^3 = 8.2 \times 10^{30}\,\mathrm{m}^3$$

So

$$\boxed{A = 2.0 \times 10^{21}\,\mathrm{m}^2 \qquad V = 8.2 \times 10^{30}\,\mathrm{m}^3}$$

(b) Radial velocity is measured by use of the doppler effect. We have a measured and an actual wavelength, so we may easily calculate the radial velocity.

$$\frac{v_r}{c} = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{655.89\,\text{nm} - 656.28\,\text{nm}}{656.28\,\text{nm}} = -5.9 \times 10^{-4}$$

So the radial velocity is

$$\boxed{v_r = -5.9 \times 10^{-4} c = 1.77 \times 10^5\,\text{m/s towards Earth}}$$

we then see the light from this star blue-shifted, which is consistent with our given result since the observed wavelength is shorter (bluer) than the expected one.

(c) The surface temperature of the star is related to its peak wavelength through Wien's displacement law, but the peak wavelength we see has been blue-shifted just as the Hydrogen-alpha line was. So before we can apply Wien's law, we must use the Doppler effect to get the true peak wavelength:

$$\frac{\lambda_{\text{max}} - \lambda_{\text{max},0}}{\lambda_{\text{max},0}} = \frac{v}{c}$$

$$300\,\text{nm} - \lambda_{\text{max},0} = \left(-5.9 \times 10^{-4}\right) \lambda_{\text{max},0}$$

$$300\,\text{nm} = \left(1 - 5.9 \times 10^{-4}\right) \lambda_{\text{max},0}$$

$$\lambda_{\text{max},0} = 300.18\,\text{nm}$$

Now we may use Wien's law to find the surface temperature of the star:

$$\lambda_{\text{max},0} = \frac{.0029\,\text{m K}}{T}$$

$$T = \frac{.0029\,\text{m K}}{300.18\,\text{nm}} = \frac{.0029\,\text{m K}}{3.0018 \times 10^{-7}\,\text{m}}$$

So the surface temperature is about

$$\boxed{T = 9660\,\text{K}}$$

(d) With the surface temperature, we can find the flux passing through the surface of the star. This, in turn can be related to the luminosity, or power of the star. By combining the Stefan Boltzmann law with the definition of energy flux,

$$F = \sigma T^4 = \frac{L}{4\pi r^2}$$

Rearranging, we have

$$L = (4\pi r^2)\sigma T^4 = \left(2.0 \times 10^{21}\,\text{m}^2\right)\left(5.67 \times 10^{-8}\,\text{W m}^{-2}\,\text{K}^{-4}\right)(9600\,\text{K})^4$$

So the luminosity is

$$\boxed{L = 9.6 \times 10^{29}\,\text{W}}$$

(e) To find the flux at Earth, we relate the power of the star to the surface area through which the power must pass through at the distance of the earth. That is, we divide the power of the star by the area of the sphere with the radius equal to the distance from the star to the Earth:

$$F_{\text{Earth}} = \frac{L}{4\pi d^2} = \frac{9.6 \times 10^{29}\,\text{W}}{4\pi(8.3\,\text{pc})^2} = \frac{9.6 \times 10^{29}\,\text{W}}{4\pi\left(2.56 \times 10^{17}\,\text{m}\right)^2}$$

So the flux at Earth is approximately

$$\boxed{F_{\text{Earth}} = 1.2 \times 10^{-6}\,\text{W/m}^2}$$

(f) The star is MUCH larger than the sun (compare radii), much more luminous than the sun, and, judging by its peak wavelength in the ultraviolet, would likely appear blue to our eyes. Thus, it seems likely that this is a blue supergiant star.