Week 9 Notes
Astro 1 (Discussion Section 105)

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Administrative Tasks

Stargazing Event I'm still waiting on the addition of Sagar's students to the spreadsheet. It seems like we'll have the full 100 students attending, so sign-ups are likely closed, though you can still try if you wish. Most that have signed up this far have been put on the list, and if you expressed a preference, you have been given it.

Final Review Pass out index cards. Have students indicate one or more topics they are at all uncomfortable with and would like reviewed next week before the final.

Questions on Last Week's Homework

Concept Review

Main-Sequence Lifetimes

Stars remain on the main sequence only as long as they can successfully convert Hydrogen into Helium. Suppose that a star converts a fraction f of its total mass to energy over its lifetime. Then the total energy the star gives off during its lifetime due to fusion if

$$E = fMc^2$$

If we assume a constant luminosity of the star over its main-sequence lifetime, we see that

$$L = \frac{E}{t}$$

where t is the lifetime of the star. From this we get

$$E = Lt$$

and using our previous formula for energy released, we have

$$fMc^2 = Lt$$

Solving for t yields

$$t = \frac{fMc^2}{L}$$

We don't necessarily know what f is for a star, so we can just write a proportionality, that is

$$t \propto \frac{M}{L}$$

This means that t is equal to M/L multiplied by some unknown constant. We may also use the fact that the luminosity of a star is proportional to its mass raised to the 3.5 power, that is

$$L \propto M^{3.5}$$

Putting this all together, we obtain

$$t \propto \frac{M}{M^{3.5}} = \frac{1}{M^{2.5}} = \frac{1}{\sqrt{M}M^2}$$

This formula in and of itself is not extremely useful unless we have a reference star to work with. Then we may simply compute ratios, and the unknown constant, whatever it was, cancels out. Traditionally we use that the lifetime of the sun is $t_{\odot} = 1.2 \times 10^{10}$ years. In this case, the relevant equation becomes

$$\frac{t}{t_{\odot}} = \frac{\sqrt{M_{\odot}}M_{\odot}^2}{\sqrt{M}M^2} = \sqrt{\frac{M_{\odot}}{M}}\left(\frac{M_{\odot}}{M}\right)^2$$

This equation can be inverted to find what the mass of a given star would need to be in order for it to stay on the main sequence for a certain time. You are asked to do this on your homework, so I'll leave it to you.

Example: Lifetime of a Small Star Suppose we want to know how long a star of .1 solar masses will stay on the main sequence. We may then apply our equation with $M = .1M_{\odot}$:

$$\frac{t}{t_{\odot}} = \sqrt{\frac{M_{\odot}}{.1M_{\odot}}}\left(\frac{M_{\odot}}{.1M_{\odot}}\right)^2$$

$$\frac{t}{t_{\odot}} = \sqrt{10}(10)^2$$

$$t \approx 316\, t_{\odot} = 3.8 \times 10^{12}\,\text{years}$$

So the star will remain on the main sequence for about 4 trillion years. We have not seen any of these stars die out yet to form a planetary nebula, can you tell why?

Time Dilation

Einstein's Special Theory of Relativity has some funky consequences in our perception of the universe is that of time dilation. The flow of time is *not* the same for all observers. The faster an observer is moving with respect to another observer, the slower they will observe time to pass by. This result is mathematically represented as

$$T = \frac{T_0}{\sqrt{1-(v/c)^2}}$$

Here T is the time interval measured by the moving observer, T_0 is the "proper time", that is, the time measured by the non moving observer. Then v is the speed of the moving observer relative to what is being observed, and c is the speed of light.

A More Concrete Example: Space Toast Suppose Bob is floating around in space and he is trying to make toast. He presses the button on the toaster. At the same time, Sally blasts off in a space ship and travels at a constant speed v near the toaster and observes how long it takes. Sally will measure the toast taking a time T to finish, whereas Bob will measure T_0, the proper time, because Bob is in the same inertial frame as the toaster (he is not moving relative to the phenomenon being observed).

Example: Making retirement count Suppose you retire and decide to go on a spaceship cruise through the universe at 95% of the speed of light for five years. When you return back to Earth, how much time will have passed?

Essentially this problem boils down to deciding what's the proper time and what's the dilated time. The proper time always measures the time of an event as it would be measured by an observer that is moving at the same speed as the object being measured. Thus, the proper time here is five years, since you measure the five years while you are traveling in your frame. You could even think of yourself as being still, while the *Earth* is zipping by, and we wish to know what time the *Earth* observes. Plugging this proper time and speed into the time dilation equation yields

$$T = \frac{5\,\text{years}}{\sqrt{1-(.95\,c/c)^2}} = \frac{5}{\sqrt{1-.9025}}\,\text{years} = 16\,\text{years}$$

So when you return from your cruise, you will have aged five years, but earth will have passed through 16 years. As you can imagine, this strange nature of time can cause some wacky situations to take place when traveling at great speeds. Of similar interest (though it's not on your homework this week) is the issue of length contraction. When an object is moving very fast, it appears to shrink in size, according to the equation

$$L = L_0\sqrt{1-(v/c)^2}$$

Here L is the length as measured by the stationary observer of the moving object, L_0 is the proper length of the object, measured at rest, and v and c are the same as before.

Black Holes and the Schwarzchild Radius

As I've mentioned before in class *any* object can be compressed into a black hole if it were made small enough. The "size" of any black hole is usually denoted by its Schwarzchild radius, or event horizon. It is defined thusly:

$$R_{\mathrm{Sch}} = \frac{2GM}{c^2}$$

Interestingly the mass of the black hole is the only thing that defines the Schwarzchild radius.

Interesting facts about the Schwarzchild Radius If you were to approach a black hole and enter within the Schwarzchild radius, there is *nothing* you could do to escape it. I'm not saying it would be difficult to escape; I'm saying that there is no physically possible way to escape the pull of the black hole. Your body would be stretched by tidal forces to a hilariously thin state (this process is actually called "spaghettification", believe it or not) until it would eventually be torn apart atom by atom.

To be of any appreciable size, a black hole must be extremely massive. If one put the mass of the earth into the formula for the Schwarzchild radius, the resulting "black hole size" if Earth were compressed down to a point would be about the size of a peanut.

Other Questions