Lecture 12b

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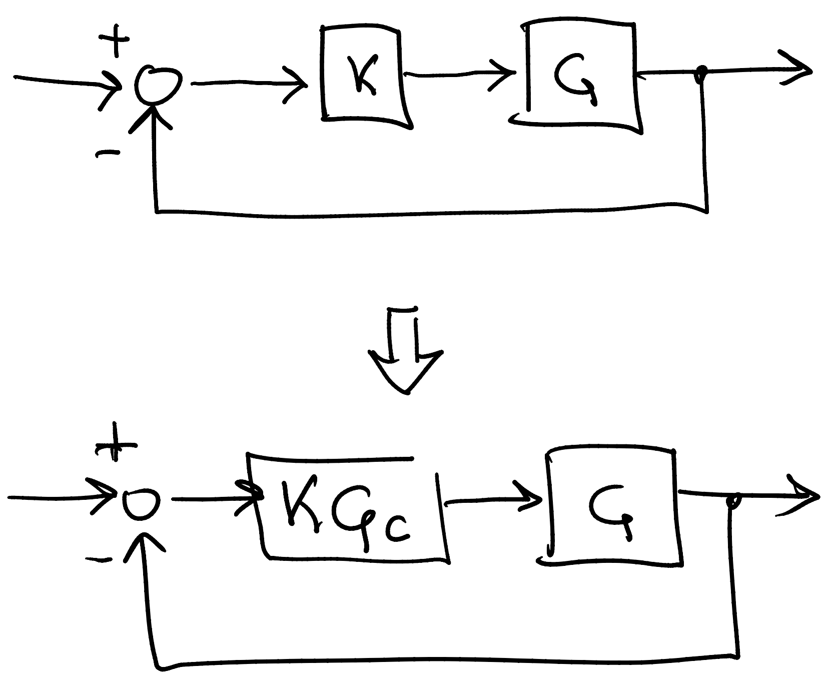
Lecture 12b: Lag, Lead and Lag-Lead Compensation

... in which we see how to improve the performance of a system by changing the root locus.

I. The Idea



stable system (G may be $G_{control} \cdot G_{plant}$)

improved performance via a "cascade compensator"

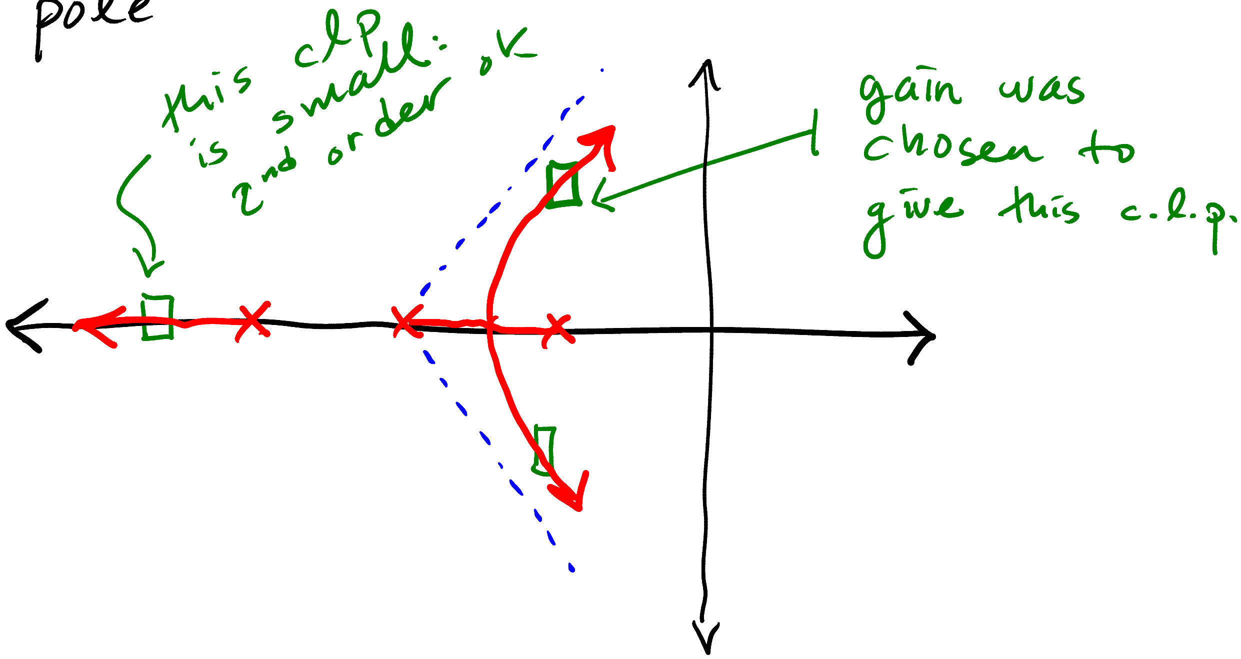
We will look at the following compensators:

PI	$\frac{K(s+z_c)}{s}$	active; eliminates steady state error.
Lag	$\frac{K(s+z_c)}{(s+p_c)}$ $-z_c < -p_c$ and $p_c \approx 0$.	passive; improves s.s.e. without eliminating it.
PD	$s + z_c$	active; noisy; improves settling time
Lead	$\frac{K(s+z_c)}{(s+p_c)}$ $-p_c < -z_c <$ system poles + zeros	passive; less noisy; improves settling time

II. Eliminating Steady State Error with a PI Compensator

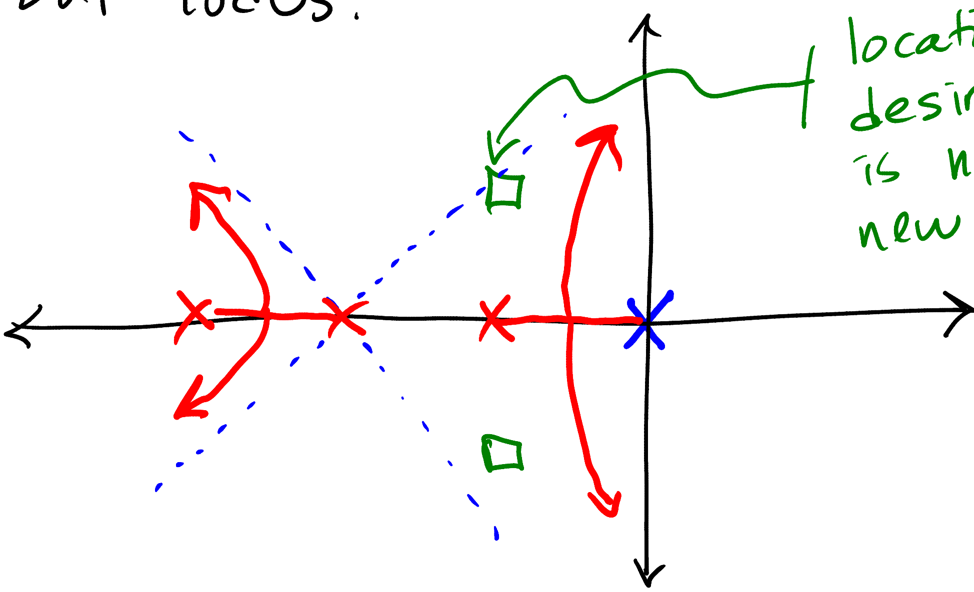
Say $G(s) = \dfrac{1}{(s+1)(s+2)(s+3)}$

and that K has been found to give a desired closed loop pole



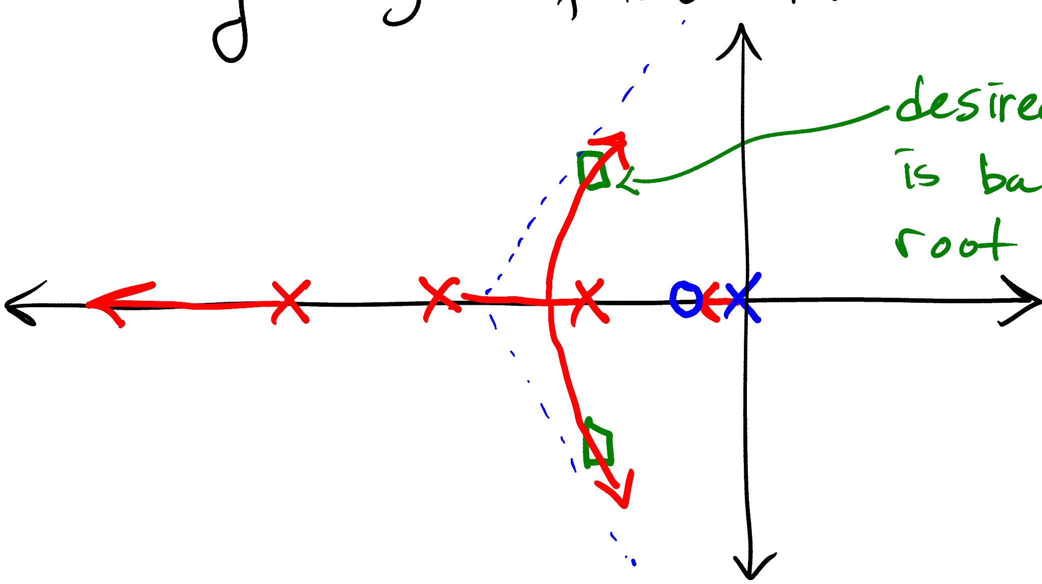
$$\sigma_0 = \frac{-1-2-3-0}{3} = -2$$

With a $\frac{K}{s}$ compensator we can track a step, but the desired clp is no longer on the root locus:



location of desired clp is not on new root locus

Adding a zero fixes this: $\frac{K(s+z_c)}{s}$



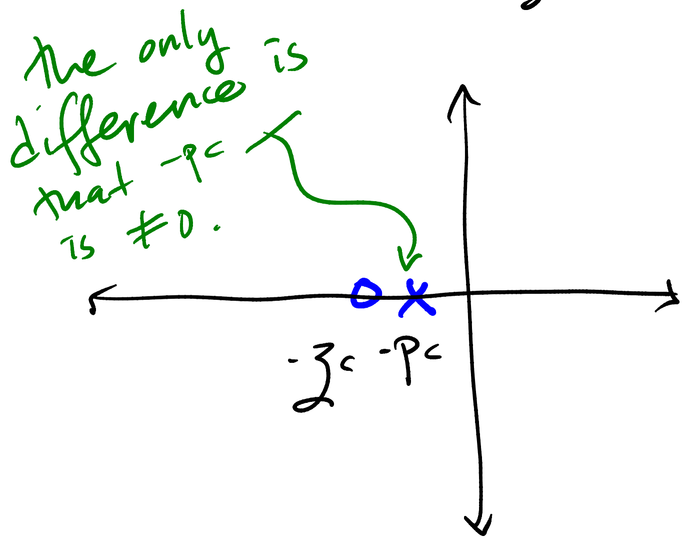
desired clp is back on the root locus

Thus, adding a PI compensator preserves the transient response while eliminating steady state error.

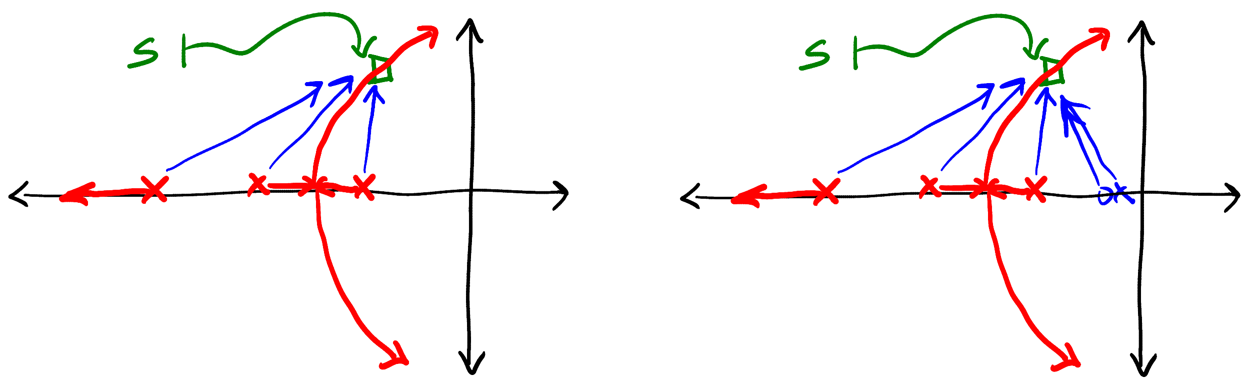
III. Lowering Steady State Error with a Lag Compensator

A PI controller is an <u>active compensator</u>. A similar compensator is the <u>Lag Compensator</u>, which is passive (more on this later):

$$\frac{K(s+z_c)}{s+p_c}$$



Notice that in the uncompensated system, a point on the R.L. has a particular angle:



$KG(s) = -1 \Rightarrow$

$-\angle(s+1) - \angle(s+2) - \angle(s+3) = 180°$

In the compensated system, the new pole and zero <u>must</u> almost cancel:

$$-\angle(s+1) - \angle(s+2) - \angle(s+3) - \underbrace{\angle(s+p_c) + \angle(s+z_c)}_{\approx 0}$$

$$\approx 180°$$

This is true for the PI and the Lag compensator.

Even though the lag compensator does not have a pole at zero, it still improves the steady state error.

Recall that

$$E(s) = \lim_{s \to 0} s\,(R(s) - Y(s))$$

$$= \lim_{s \to 0} s \left[\frac{1}{s} - \frac{KG}{1+KG} \cdot \frac{1}{s} \right]$$

$$= \lim_{s \to 0} \left(1 - \frac{KG}{1+KG} \right)$$

$$= \lim_{s \to 0} \left(\frac{1}{1+KG(s)} \right)$$

So the bigger $\lim_{s \to 0} KG(s)$ is, the smaller the error is

Uncompensated system:

$$K_u = \lim_{s\to 0} K\,G(s) = \lim_{s\to 0} \frac{\prod(s+z_i)}{\prod(s+p_i)} = \frac{\prod z_i}{\prod p_i}$$

Compensated system

$$K_c = \lim_{s\to 0} K \frac{(s+z_c)}{(s+p_c)} G(s) = \frac{z_c \prod z_i}{p_c \prod p_i}$$

$$= \frac{z_c}{p_c} K_u$$

Since $-z_c < -p_c$ we have

$$\frac{z_c}{p_c} > 1$$

$$\Rightarrow \frac{z_c}{p_c} K_u = \boxed{K_c > K_u}$$

So a lag compensator does practically the same thing as a PI controller.

Also, note that z_c and p_c need to be almost equal, so it seems like the ratio z_c/p_c can't be big. But if you take them near zero:

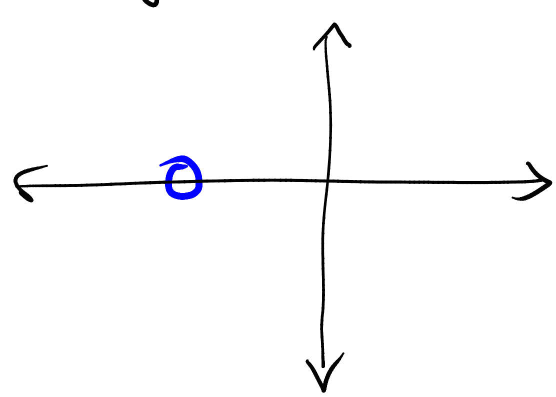
$$\frac{z_c}{p_c} = \frac{0.1}{0.001} = 100$$

it works.

IV. Decreasing T_s with a PD Compensator

A PD controller is just an added zero:

$K(s+z_c)$:



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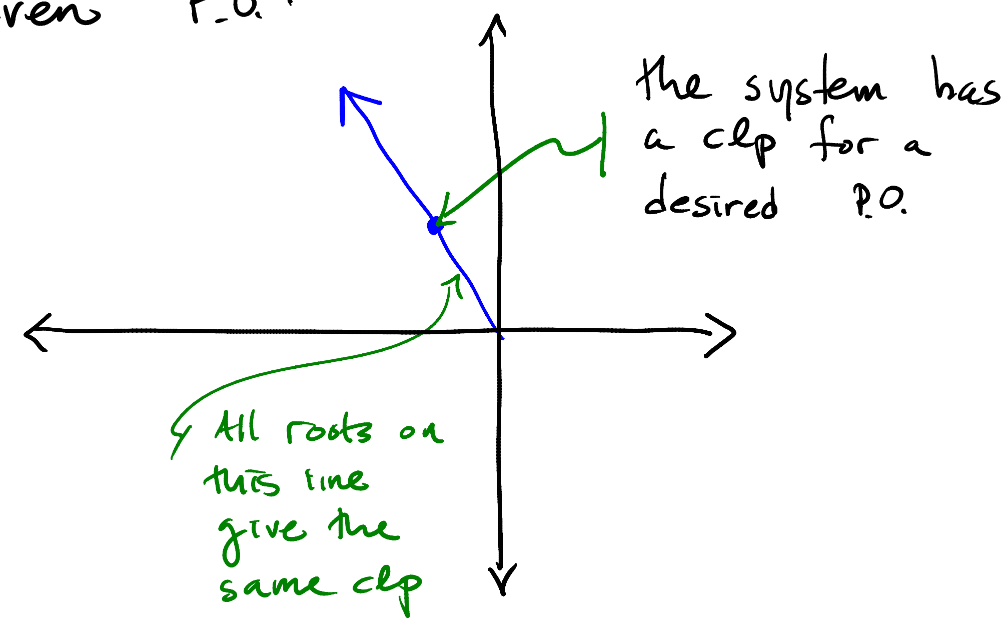
The goal with a PD controller is to preserve the transient response characteristics and to decrease T_s.

Say the uncompensated system has a given P.O.:



To improve T_s, we need to move the clp along the constant P.O. line away from the origin.

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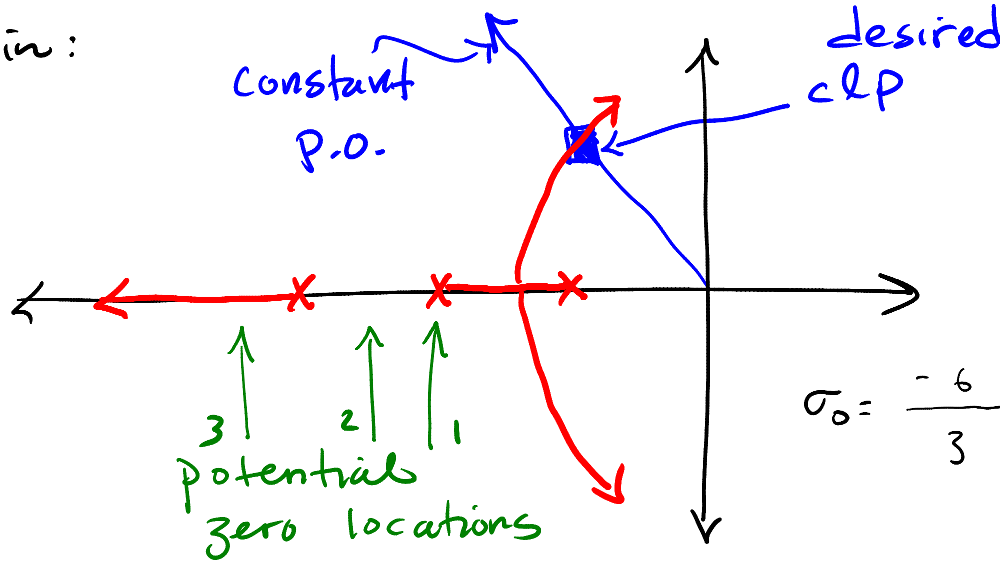
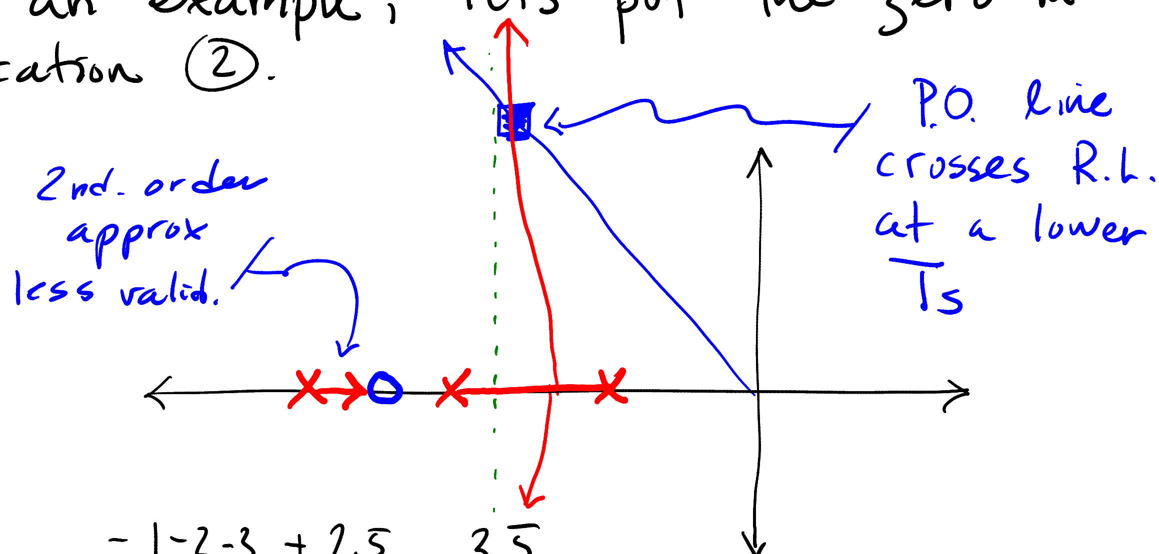
Lets look at $G(s) = \dfrac{1}{(s+1)(s+2)(s+3)}$

again:



$\sigma_0 = \dfrac{-6}{3} = -\dfrac{1}{2}$

As an example, let's put the zero in location ②.



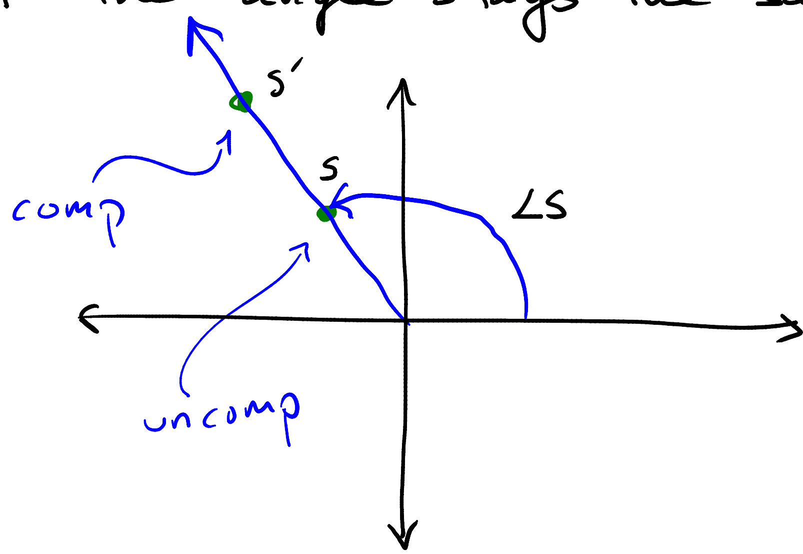
$\sigma_0 = \dfrac{-1-2-3+2.5}{2} = \dfrac{3.5}{2} = 1.75$

One again, The PD is an active device and, furthermore, PD is suceptable to noises.

A similar compensator that is passive is the lead:

$$\frac{K(s+z_c)}{s+p_c} \quad \text{with} \quad -p_c < -z_c$$

<u>How it works:</u> We add p_c and z_c so that the angle stays the same.



Example

Say $G(s) = \dfrac{1}{s(s+4)(s+6)}$ and

we put $-z_c = -5$. Then we want

$$\angle(s'+z_c) - \angle(s'+p_c) - \angle(s'+0) - \angle(s'+2) - \angle(s'+6)$$
$$= \pi$$

Solving for $\angle(s'+p_c)$ gives the angle.
Then use the diagram below an
do the geometry.