Lecture 10b

Thursday, November 10, 2005
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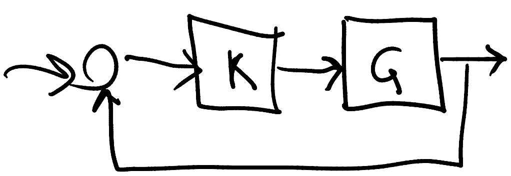
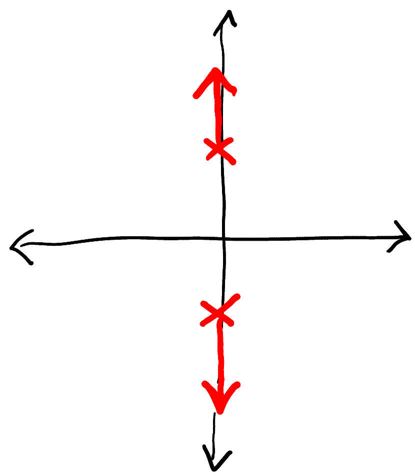
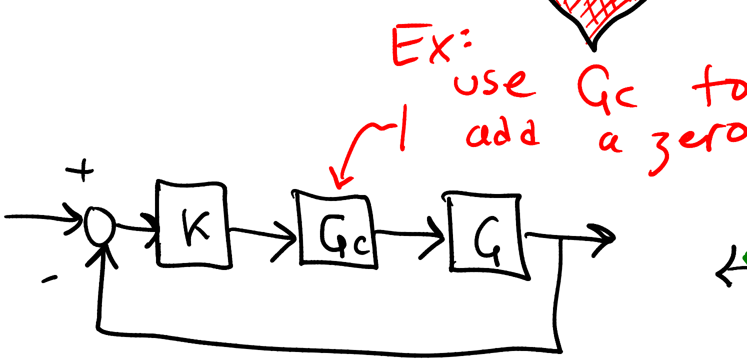
Lecture 10b: Controller Design Using the Root Locus

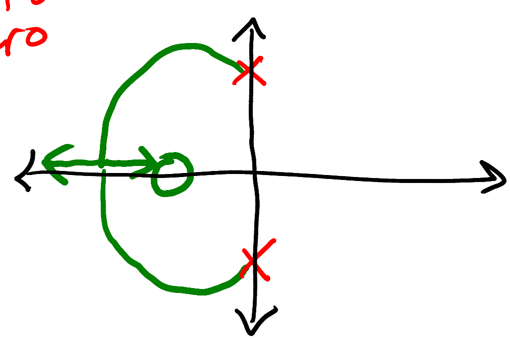
... in which we see how to build controllers by placing new poles and zeros.

I. The Idea







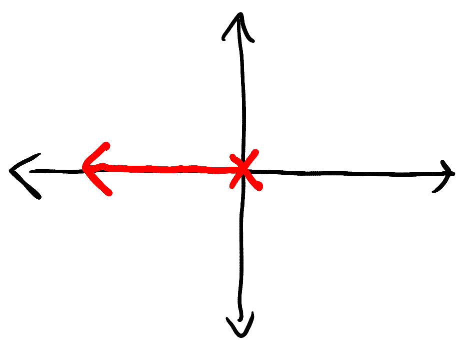


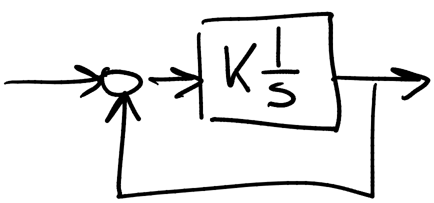
Example: PI control of the rocket

The PI controller is essentially a new pole at zero and a new zero.

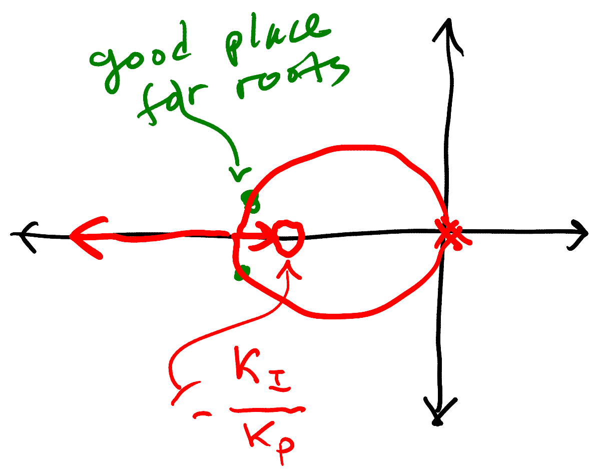
$$\frac{K_I}{s} + K_p = \frac{K_p\left(s + \frac{K_I}{K_p}\right)}{s}$$

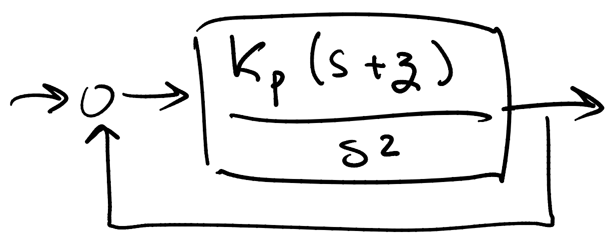
old R.L.





new R.L.





The new system can have overshoot and can be more responsive.

Example: The PID controller

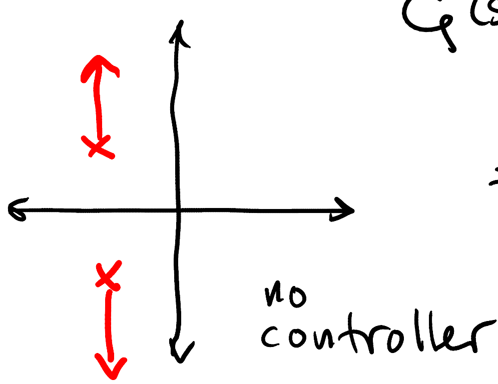
$$K_P + K_D s + \frac{K_I}{s} = \frac{K_D s^2 + K_P s + K_I}{s}$$

$$= \frac{K_D (s + z_1)(s + z_2)}{s}$$

So, picking z_1 and z_2 determines the shape of the RL. and K_D becomes the single gain.

For example, suppose

$$G_p(s) = \frac{1}{s^2 + 2s + 2}$$



$\Rightarrow$ poles $-1 \pm j$

Now say you want the system to have approximately:

$$T_s = 4s = \frac{1}{\zeta\omega}$$

$$P.O. = 0.1 = e^{-\zeta\pi/\beta}$$

Then we want

$$\zeta\omega = 4$$

$$-\zeta\pi/\beta = \ln 0.1 = -2.3$$

$$\Rightarrow \zeta = 0.6$$

$$\omega = 6.7$$

If we could place the poles, then we would want

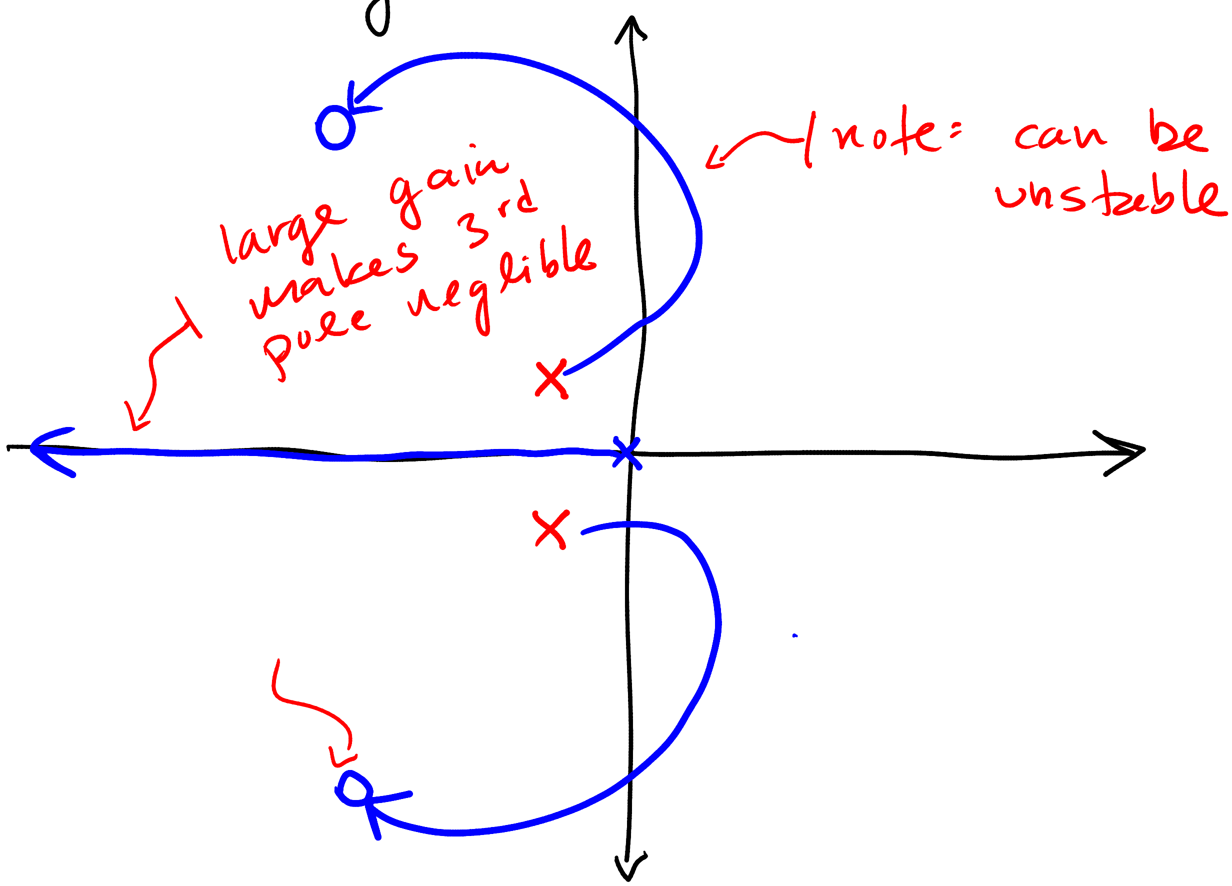
$$s^2 + 2\zeta\omega + \omega^2 = 0 \iff s^2 + 8s + (6.7)^2$$

$$\Rightarrow s = -4 \pm 5.4j$$

We cant place the poles anywhere, but we can place the zeros. So our PID controller becomes

$$\frac{K_D\left(s^2+8s+(6.7)^2\right)}{s}$$

Which gives the R.L.



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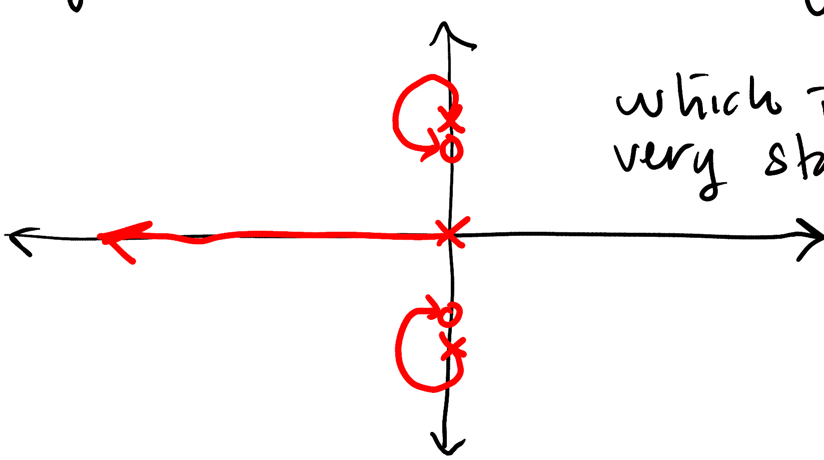
Example: What not to do:

Say $G(s) = \dfrac{1}{s^2+1}$.

Then one idea would be to have

$$KG_c(s) = \frac{K(s^2+1)}{s}$$

Then $KG_cG = \dfrac{K}{s}$, which is nice and stable. However, because $G(s)$ is the physical world, which we do not know very well, we actually get a RL:



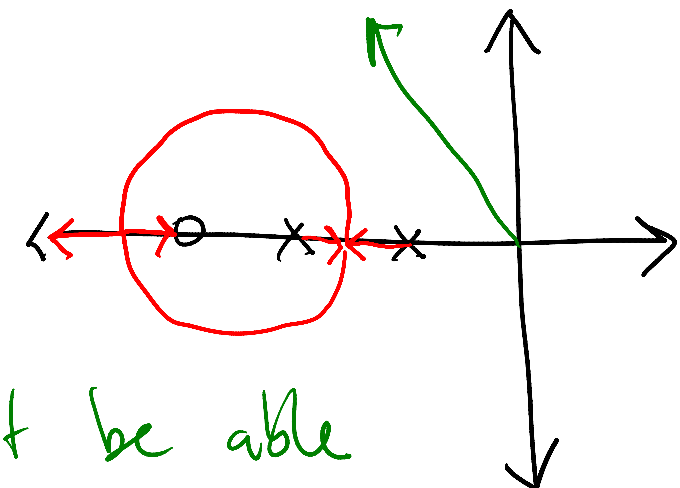
which is not very stable or even unstable.

Example: P, I, D and PID

Let's look at what each term in a PID does separately, and then compare to the system obtained from all of them.

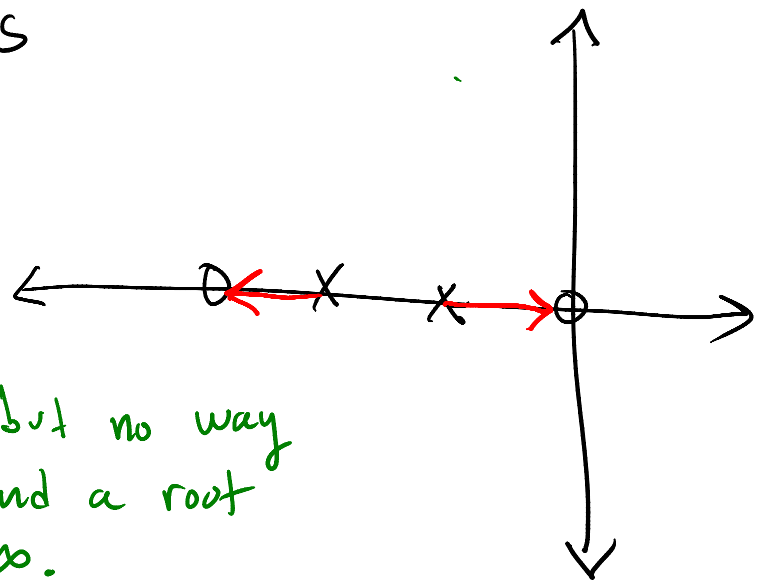
Say $G(s) = \dfrac{s+3}{(s+1)(s+2)}$

$KG_c(s) = K$



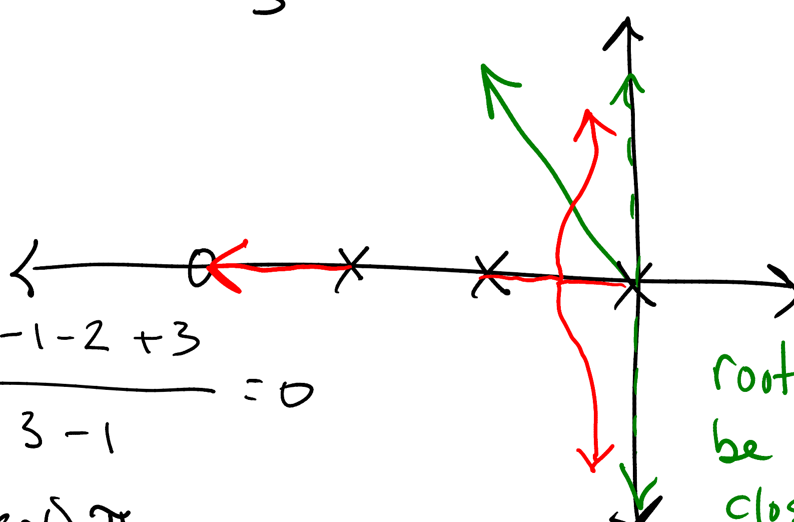
May not be able to get desired P.O.

$KG_c(s) = Ks$



Same, but no way to send a root to $-\infty$.

$KG_c(s) = \frac{K}{s}$



$$\sigma_a = \frac{0-1-2+3}{3-1} = 0$$

$$\theta_a = \frac{(2k+1)\pi}{2}$$

roots may be too close to the im. axis

Ex:

$$G_c(s) = K\left(10 + s + \frac{26}{s}\right)$$

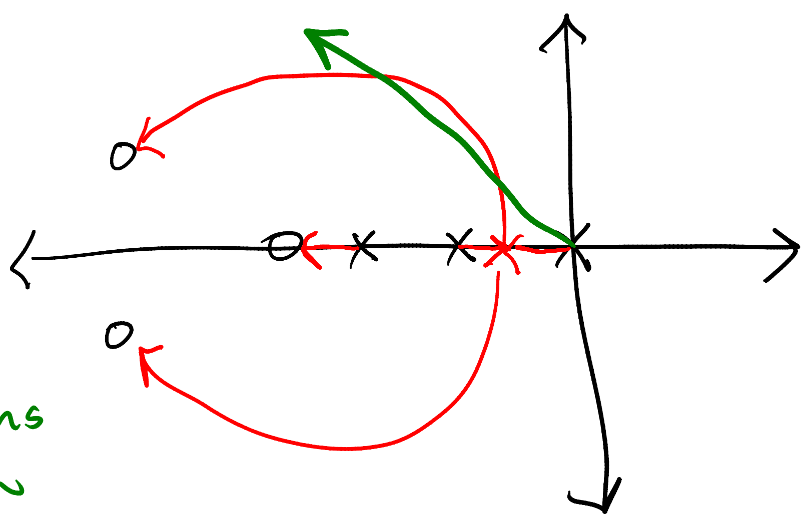
$$= K\,\frac{s^2 + 10s + 26}{s}$$

$$G_c G = K\,\frac{(s^2+10s+26)(s+3)}{s(s+1)(s+2)}$$

zeros: -3

$$-5 \pm \tfrac{1}{2}\sqrt{100-104}$$

$$= -5 \pm \tfrac{1}{2}\sqrt{-4} = -5 \pm j$$



PID wins again