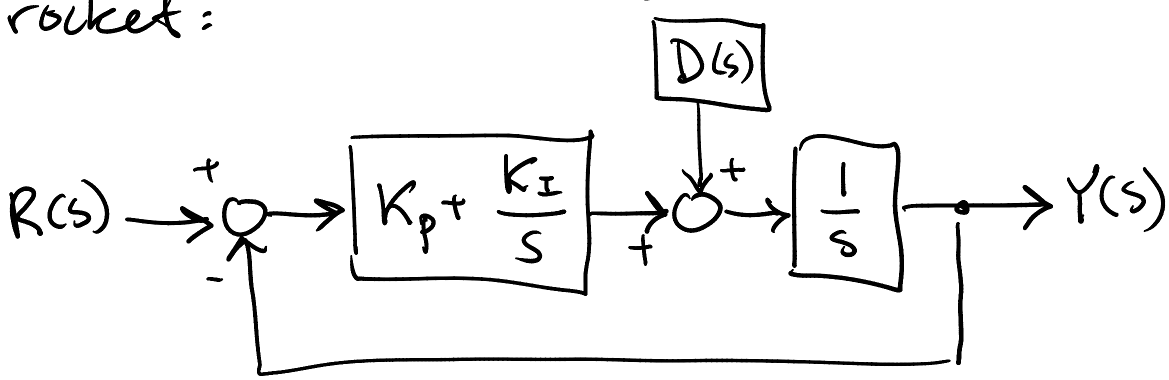
Lecture 7b: Disturbances and Noise

... in which we look at the effects of disturbances and noise in a specific example.

I. The effect of a disturbance

Recall our velocity controlled rocket:



Suppose the rocket engine suddenly drops in efficiency by 1 unit. Then $d(t) = -1$ and $D(s) = -1/s$.

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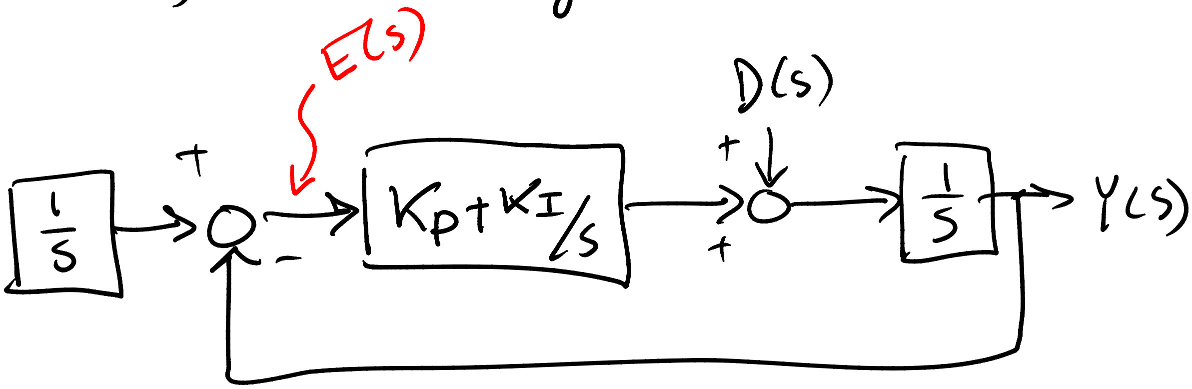
If $R(s) = \frac{1}{s}$ (the desired velocity is 1), then we get



We want $Y(s)$

$$Y(s) = \frac{1}{s}\left(D + \left(K_p + \frac{K_I}{s}\right) E \right)$$

$$= \frac{1}{s}\left[D + \left(K_p + \frac{K_I}{s}\right)\left(\frac{1}{s} - Y\right)\right].$$

$$= \frac{D}{s} + \frac{1}{s^2}\left(K_p + \frac{K_I}{s}\right) - \frac{1}{s}\left(K_p + \frac{K_I}{s}\right) Y$$

$$\Rightarrow s^2 Y(s) = Ds + K_p + \frac{K_I}{s} - s\left(K_p + \frac{K_I}{s}\right) Y$$

$$Y(s) = \frac{Ds + K_p + K_I/s}{s^2 + K_p s + K_I}.$$

Using the final value theorem, we get:

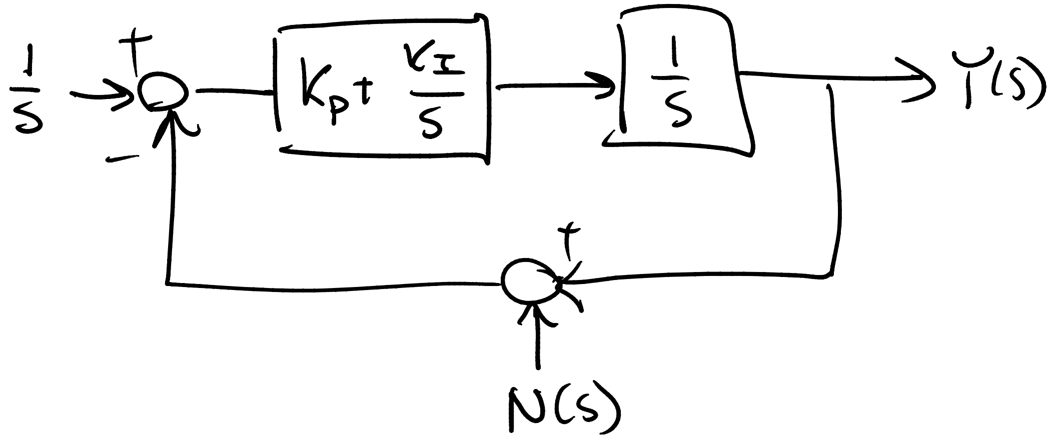
$$\lim_{s \to 0} sY(s) = \frac{K_I}{K_I} = 1.$$

Which means that, at $t \to \infty$, the system completely rejects the disturbance.

However, the time response is different.

II. Noise

We can also determine the transfer function $Y(s)/N(s)$ assuming that $R(s) = 1/s$. We get:



$$Y(s) = \frac{1}{s}\left(K_p + \frac{K_I}{s}\right)E$$

$$= \frac{1}{s}\left(K_p + \frac{K_I}{s}\right)\left(\frac{1}{s} - Y - N\right)$$

$$Y\left(s + K_p + \frac{K}{s}\right) = \left(K_p + \frac{K_I}{s}\right)\left(\frac{1}{s} - N\right)$$

$$Y = \frac{\left(K_p + K_I/s\right)\left(\frac{1}{s} - N\right)}{s + K_p + K/s}$$

Suppose that $N(s) = \frac{n}{s}$.

Using the Final Value thm, we see that

$$y(\infty) = \lim_{s \to 0} s\,Y(s)$$

$$= \lim_{s \to 0} s \frac{\left(K_p + K_I/s\right)\left(\frac{1}{s} - \frac{n}{s}\right)}{s + K_p + K_I/s}$$

$$= \lim_{s \to 0} \cancel{s} \frac{\frac{K_p s + K_I}{\cancel{s}} \cdot \frac{1-n}{s}}{s + K_p + K_I/s}$$

$$= \lim_{s \to 0} \frac{(K_p s + K_I)(1-n)}{s^2 + K_p s + K_I} = \boxed{1-n}$$

The magnitude of the effect is equal to the magnitude of the noise.

Now suppose the noise is an impulse: $N(s) = n$. Then

$$y(\infty) = \lim_{s \to 0} s\,Y(s)$$

$$= \lim_{s \to 0} s \frac{\left(K_p + K_I/s\right)\left(\frac{1}{s} - n\right)}{s + K_p + K_I/s}$$

$$= \lim_{s \to 0} \cancel{s} \frac{\frac{K_p s + K_I}{\cancel{s}} \cdot \frac{1 - ns}{s}}{s + K_p + K_I/s}$$

$$= \lim_{s \to 0} \frac{K_p s - K_p n s^2 + K_I - K_I n s}{s^2 + K_p s + K_I}$$

$$= \boxed{1}$$

So short blips in the sensor are recovered from, but we still don't know how long it takes.

(To Simulink)