Lecture 11b

Monday, November 06, 2006
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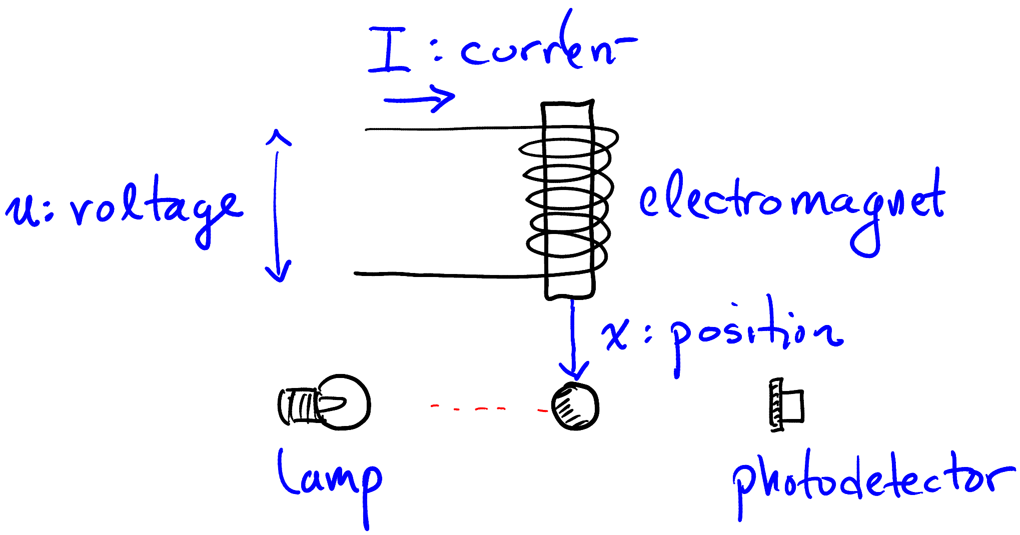
Lecture 11b: Levitation!

... in which we try to control the position of a levitating steel ball with an electromagnet.

I. The Model



$$\dot{x} = v$$

$$\dot{v} = -\frac{c}{M}\left(\frac{I}{x}\right)^2 + g$$

$$\dot{I} = \frac{1}{L}\left(-RI + 2c\frac{Iv}{x^2} + u\right)$$

c : Magnet Const
g : Gravity
L : Inductance
R : Resistance
M : Mass

We wish to control the ball so that it stays at

$$x^* = \frac{1}{2}\,\mathrm{cm} = \frac{1}{200}\,\mathrm{m}.$$

This requires a constant voltage, which we can determine.

$$\dot{v} = 0 = -\frac{c}{M}\left(\frac{I}{x^*}\right)^2 + g$$

$$\Rightarrow \quad I = x^* \sqrt{\frac{Mg}{c}}$$

$$\dot{I} = 0 = \frac{1}{L}\left(-RI + 2c\,\overbrace{\cancel{\frac{Iv}{x^2}}}^{0 \text{ since } \dot{x}=v=0} + u\right)$$

$$\Rightarrow \quad u^* = RI^* = R\,x^* \sqrt{\frac{Mg}{c}} = \frac{R}{200}\sqrt{\frac{Mg}{c}}.$$

However, there are uncertainties, so we need feedback. But what controller?

II. Error Coordinates

Define $x_e = x - x^* = x - 1/200$

$v_e = v - v^* = v$

$I_e = I - I^* = I - \frac{1}{200}\sqrt{\frac{Mg}{c}}$

$u_e = u^* - u$

↳ this is for convenience. with $u - u^*$ I get a negative Transfer function

Then the "error dynamics" are:

$$\dot{x}_e = v_e$$

$$\dot{v}_e = -\frac{c}{M}\left(\frac{I_e + I^*}{x_e + x^*}\right)^2 + g$$

$$\dot{I}_e = \frac{1}{L}\left[-R(I_e + I^*) + 2c\,\frac{(I_e + I^*)\,v}{(x_e + x^*)^2} + u^* - u_e\right]$$

Linearizing gives

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 400g & 0 & -400\sqrt{\frac{Cg}{M}} \\ 0 & \frac{1}{L}400\sqrt{CgM} & -R/L \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ -\frac{400}{R}\sqrt{\frac{Cg}{M}} \\ 0 \end{pmatrix}$$

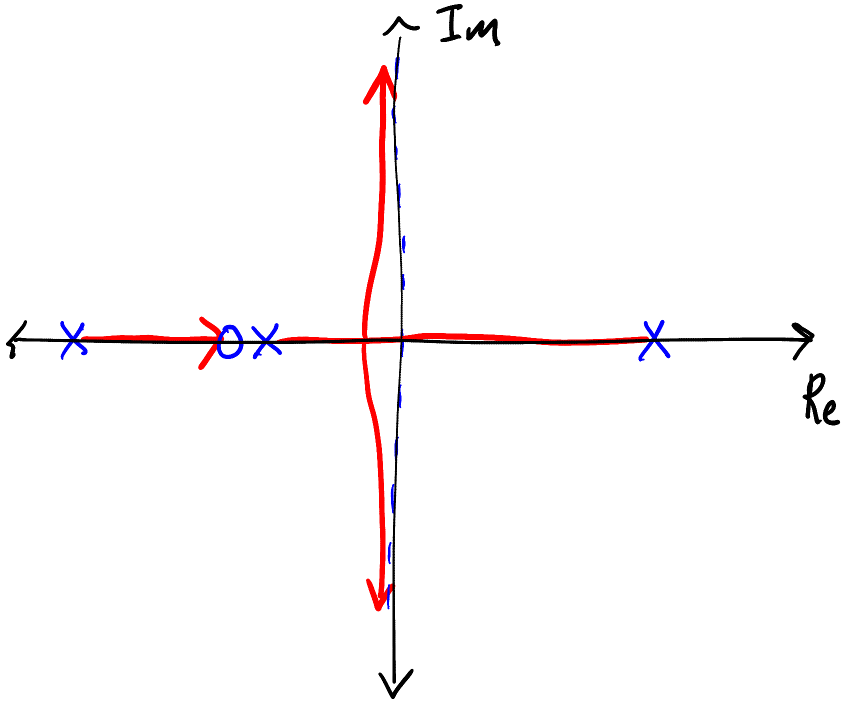
$$C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

Putting $G(s) = C(sI-A)^{-1}B$ and substituting $R \to 30$, $L \to 1$, $C \to 10^{-4}$, $g \to 10$, $M \to 12\times10^{-3}$ gives

$$G(s) = \frac{3.8(s+30)}{s^3+30s^2-3840s-120000}$$

There is one zero at -30.

The poles are -65.5, 64.1 and -28.6.



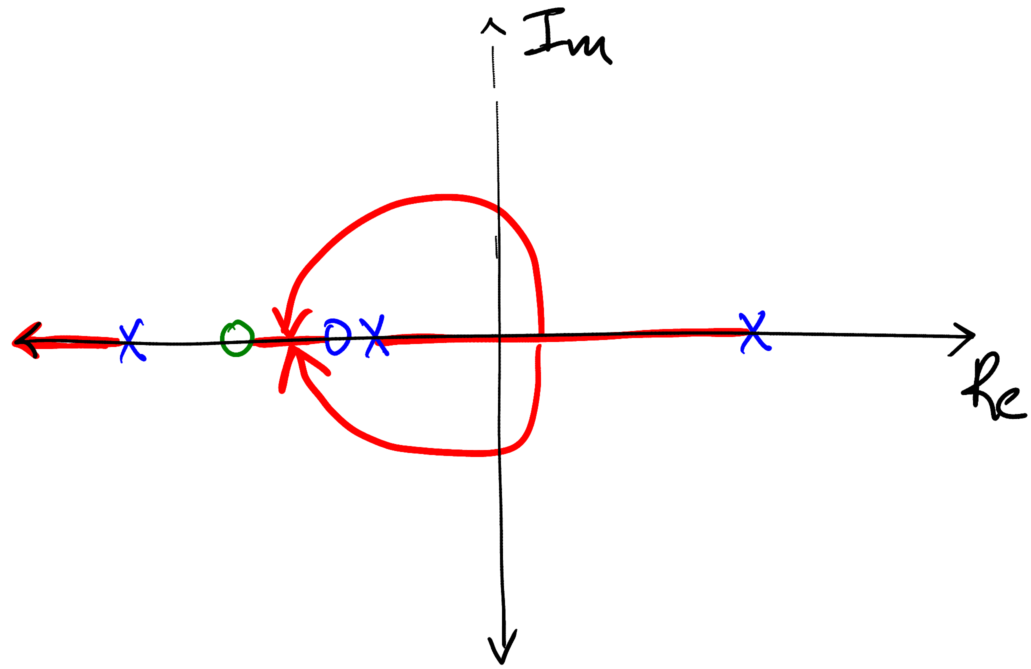
Two infinite zeros:

$$\sigma = \frac{(-65.5 + 64.1 - 28.6) - (-30)}{2} \approx 0$$

$$\theta = \pm 90^\circ$$

To control this, we need to get the RL into the LHP.

How about a zero:



So the proposed controller is

$$G_{PC} = K(s+45).$$

Note, this is good for this application: We want to regulate, so we need to track an impulse → which a type 0 can do!