Lecture 6b

Tuesday, October 18, 2005
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Lecture 6b: Physical Variables and Diagonal Form

... in which we explore other ways to convert block diagrams into state space.

I. Physical Variable Form

A good choice of states is to associate with each measurable output or block in the system a state variable.

There is no automatic method, as with the methods in Lec. 6a, but the basic idea is:

1. Draw $\overset{\dot{x}_i}{\circ} \xrightarrow{1/s} \overset{x_i}{\circ}$ for each var
2. Connect the above components according to the block diag.

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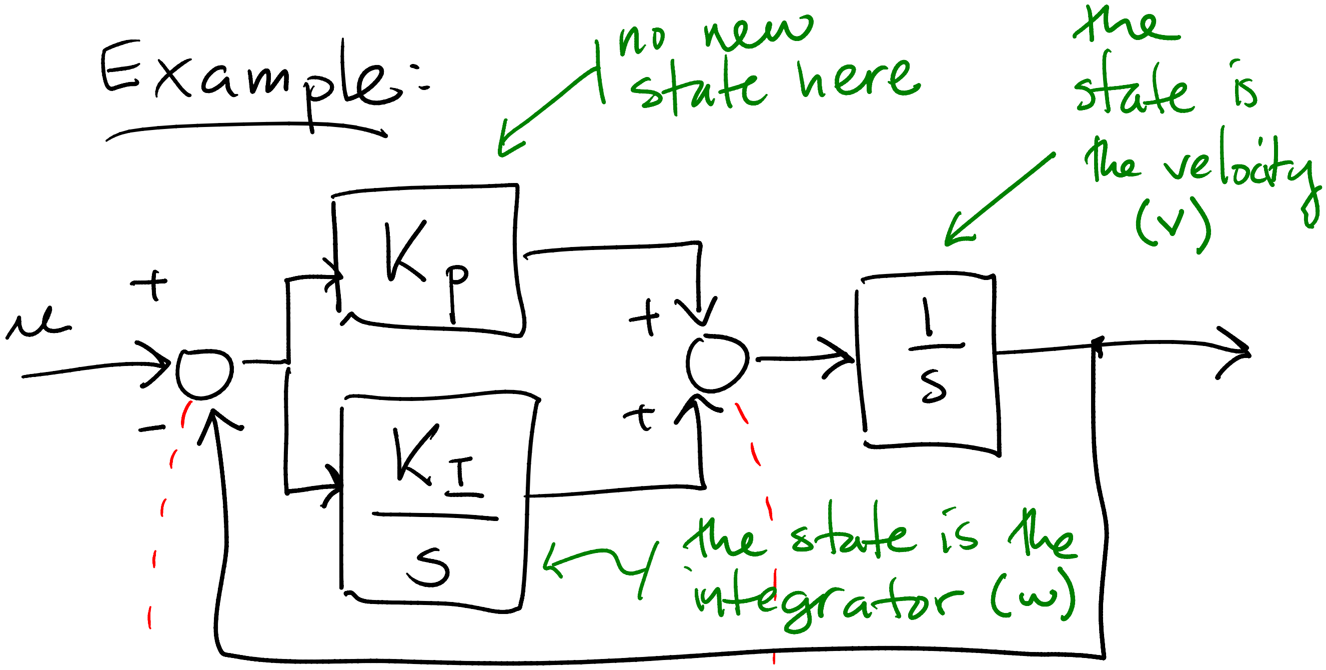
Example:



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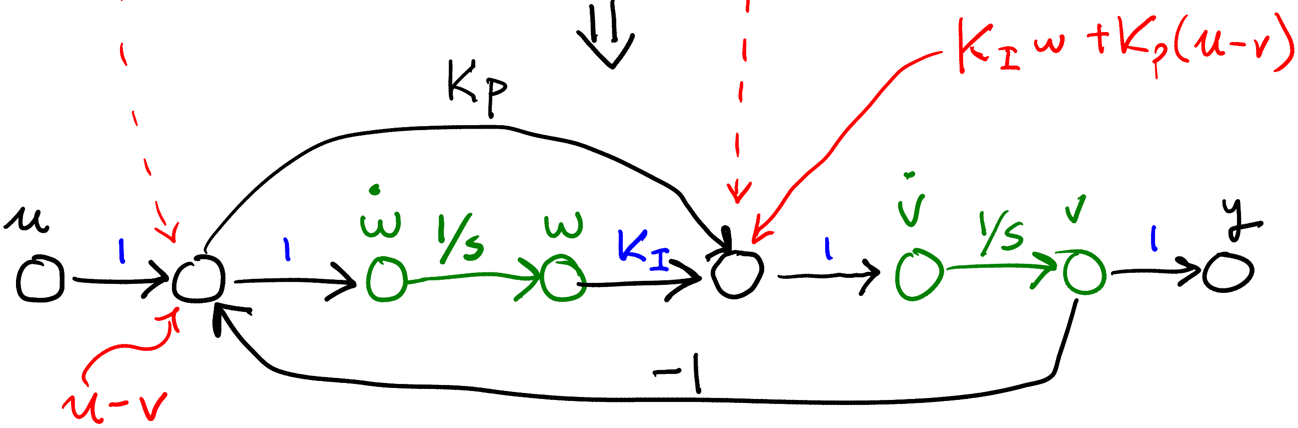

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$$\begin{pmatrix} \dot{w} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ K_I & -K_p \end{pmatrix} \begin{pmatrix} w \\ v \end{pmatrix} + \begin{pmatrix} 1 \\ K_p \end{pmatrix} u$$

$$y = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} w \\ v \end{pmatrix}$$

Working this out shows what PI control really does:

$$\dot{w} = u - v$$ integrates the error

$$\dot{v} = K_p(u-v) + K_I w$$

So we see that if $u > v$ then w increases. Then $\dot{v}$ increases proportional to K_I times w.

II. Diagonal Form

If a system has distinct poles, you can diagonalize the A matrix. This can be done directly from the transfer function.

Example

$$\frac{Y(s)}{R(s)} = \frac{s+1}{s^2+5s+6} = \frac{s+1}{(s+2)(s+3)}$$

distinct poles!

Rewrite this using partial fractions:

$$\frac{s+1}{(s+2)(s+3)} = \frac{K_1}{s+2} + \frac{K_2}{s+3}$$

at $s=-2$ we get

$$K_1 = \left.\frac{s+1}{s+3}\right|_{s=-2} = -1$$

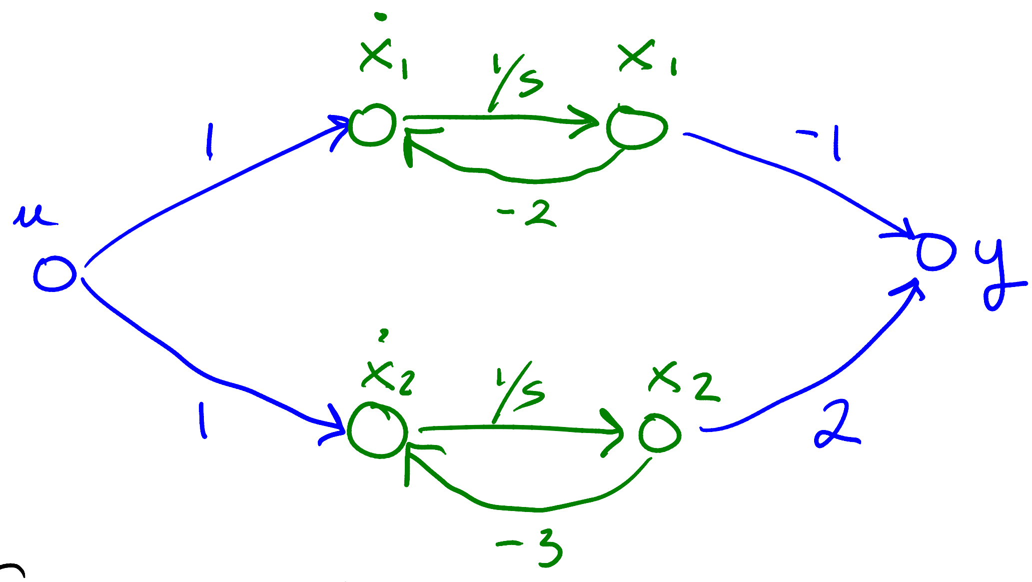
at $s=-3$ we get

$$K_2 = \left.\frac{s+1}{s+2}\right|_{s=-3} = 2$$

So that

$$\frac{Y(s)}{R(s)} = \frac{-1}{s+2} + \frac{2}{s+3}$$

We can then associate a state with each term



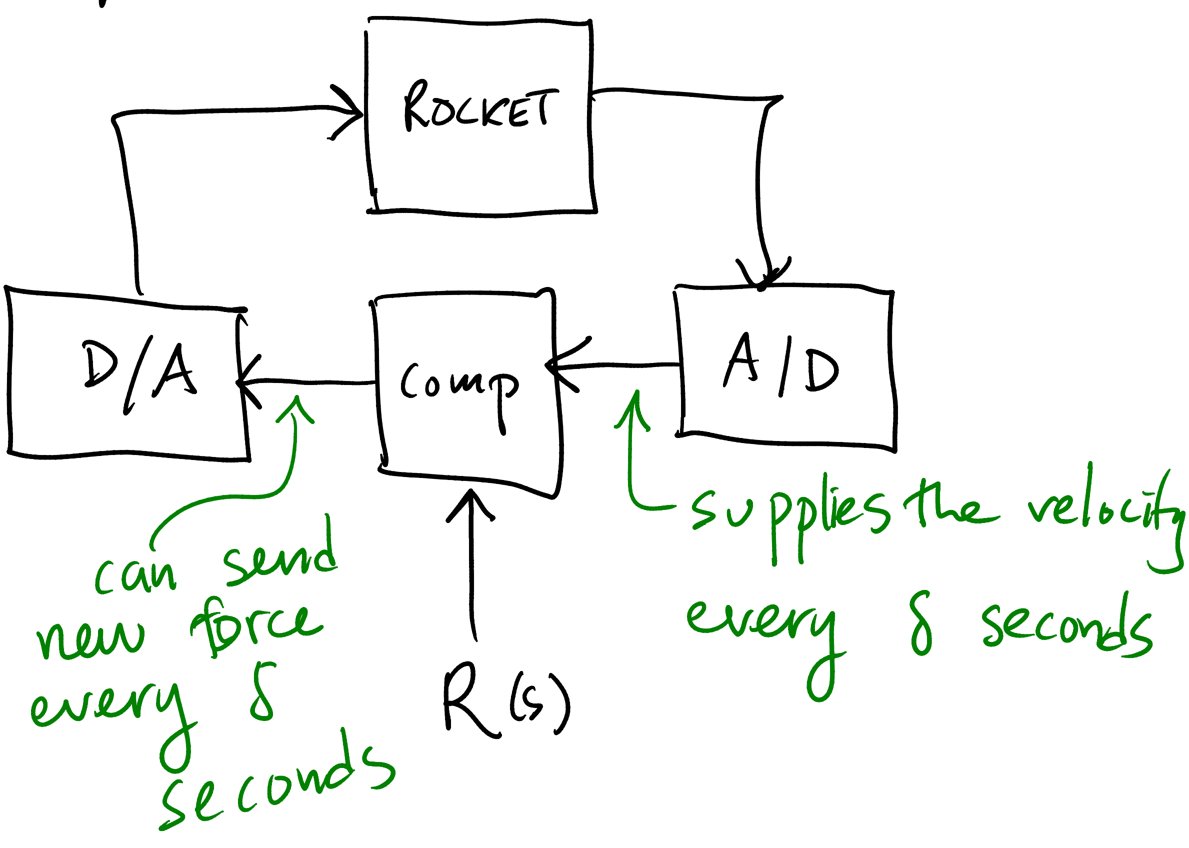
So we get

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u$$

$$y = \begin{pmatrix} -1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

III. An Application of State Space

Suppose we wanted to impliment a control system for the rocket. Let's assume we have a computer and an I/O card in the following setup:



Then we need to write some code for the computer that updates every δ seconds. One way is to use Euler's method:

$$\dot{x} = \lim_{\delta \to 0} \frac{x(t+\delta) - x(t)}{\delta}$$

$$\approx \frac{x(t+\delta) - x(t)}{\delta}$$

so that

$$x(t+\delta) = x(t) + \delta \dot{x} \quad ,$$

or

$$x_{new} = x_{old} + \delta \dot{x}$$

For example:

$$\dot{w} = r - v$$

$$\dot{v} = \underbrace{K_p(r-v) + K_I w}_{= u}$$

becomes

$$\omega_{new} = \omega_{old} + \delta(r - v)$$

current input

current measurement

$$u = K_p(r-v) + K_I \omega_{new}$$

In pseudocode:

assume this happens every δ seconds,

Init: $\omega := 0$

Interrupt: when a new v is read

$v :=$ read from A/D

$r :=$ current user input

$\omega := \omega + \delta(r-v)$

$u := K_p(r-v) + K_I\omega$

output (u) to the D/A

End