Lecture 5b: Signal Flow Graphs and Mason's Rule

...in which we determine a general method for determing the transfer function from a complex network.

I. Signal-Flow Graphs

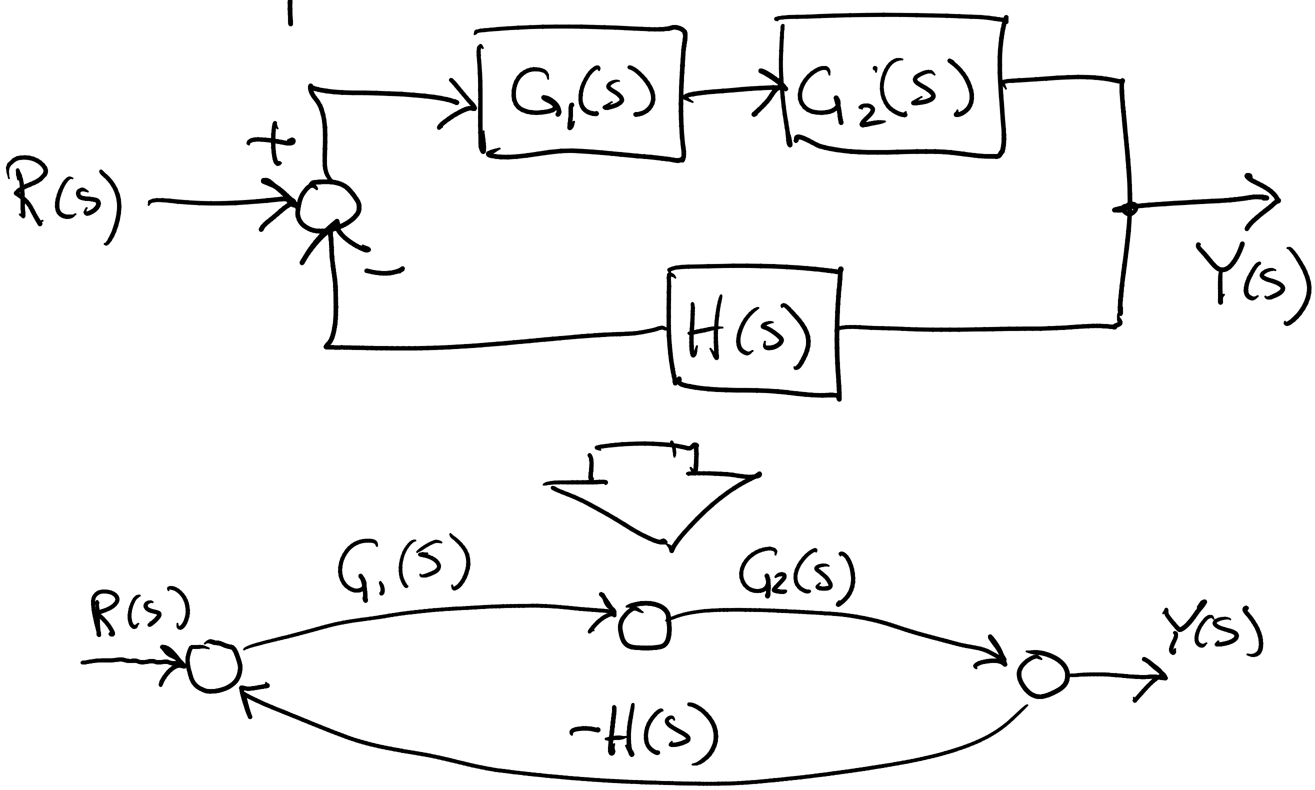
A signal-flowgraph is similar to a block diagram, but it more clearly shows the loops in the system.

The signal-flowgraph will also be useful later when we find multiple ways to find state space representations for systems.

To transform a block diagram into a signal-flow graph you

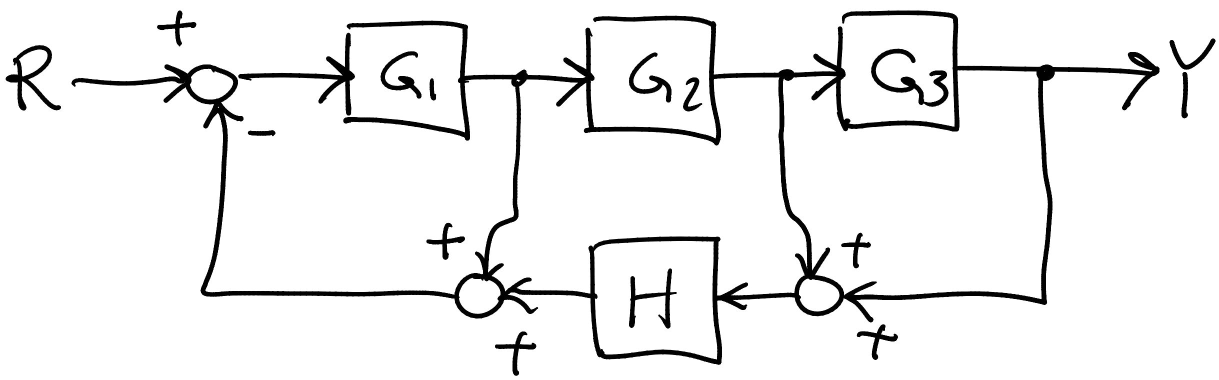
1. Make blocks into edges representing signals
2. Make edges into nodes

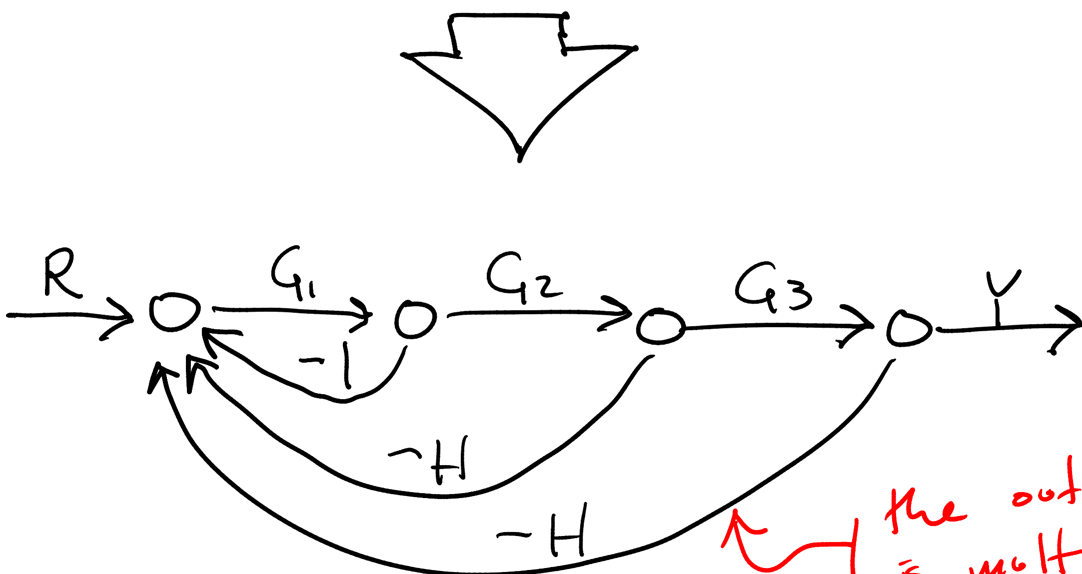
Example:



: note minus sign

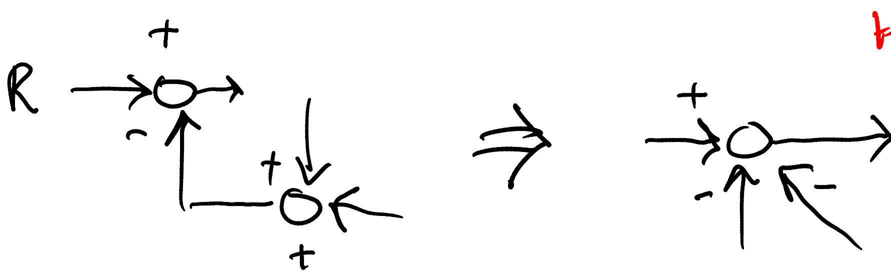
Example:





the output Y is multiplied by the gain H and fed back.

Note:



II. Mason's Rule

The paths and loops in a signal flow graph tell you how to get a transfer function for the system.

Definitions

Loop: A loop is a path of edges starting and ending at the same node.

Loop Gain: The product of the signals, or gains, appearing on each edge of the loop.

Forward Path: A path from $R(s)$ to $Y(s)$. There may be multiple such paths.

Forward Path Gain: The product of the gains appearing on a given forward path.

Non-touching Loops: Two loops with no nodes in common.

Non-touching Loop Gain: The product of the loop gains from a set of non-touching loops.

Masons Loop Rule is:

$$\boxed{\frac{Y(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta}}$$

where

K = the number of forward paths

T_k = the forward path gain of the kth forward path

$\Delta = 1 - \sum$ loop gains

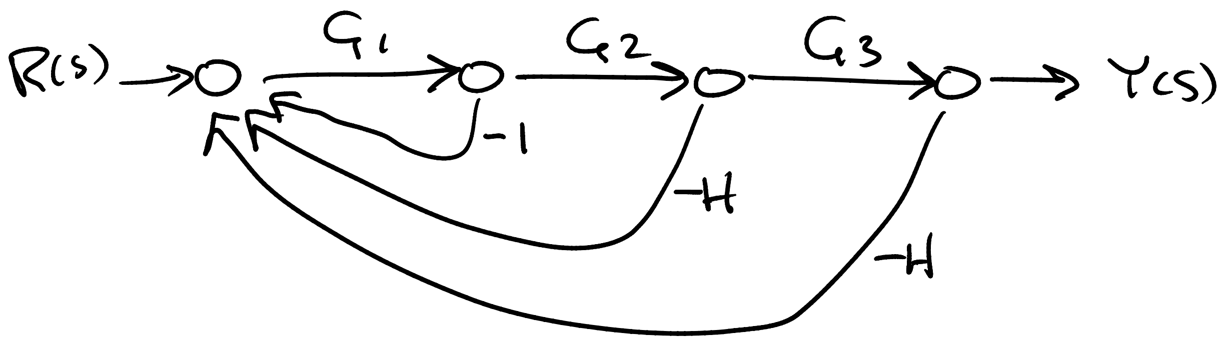
$+ \sum$ non-touching loop gains taken two at a time

$- \sum$ non-touching loop gains taken 3 at a time

$+ \ldots$

$\Delta_k = \Delta - \sum$ terms in Δ coming from loops that touch T_k

Example: Consider again the sfg.



Forward Paths (only one)

$$T_1 = G_1 G_2 G_3$$

Loops

$$L_1 = -G_1$$
$$L_2 = -G_1 G_2 H$$
$$L_3 = -G_1 G_2 G_3 H$$

There are no non-touching loops

$$\Delta = 1 - (L_1 + L_2 + L_3) + 0 - 0 + 0 - 0 \ldots$$
$$= 1 + G_1 + G_1 G_2 H + G_1 G_2 G_3 H$$

$$\Delta_1 = \Delta - \underbrace{(\Delta - 1)} = 1$$

this is how to write all loop terms that touch T_1.

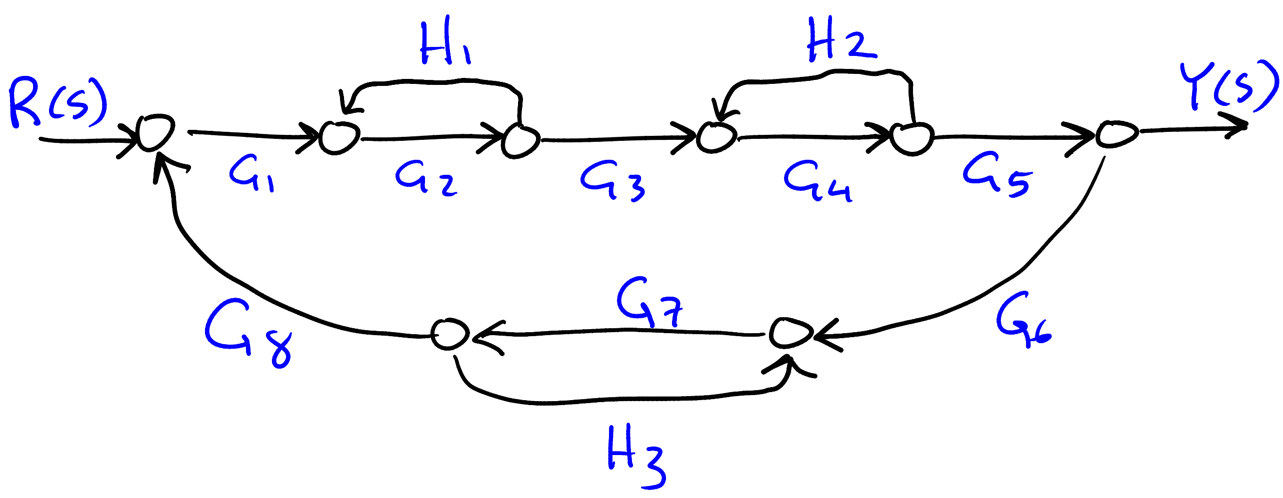
Thus

$$\frac{Y(s)}{R(s)} = \frac{T_1(s)\Delta_1(s)}{\Delta(s)}$$

compare to lecture 4a

$$= \frac{G_1 G_2 G_3}{1 + G_1 + G_1 G_2 H + G_1 G_2 G_3 H}$$

Example:



Forward Path

$$T_1 = G_1 G_2 G_3 G_4 G_5$$

Loops

$$L_1 = G_2 H_1$$

$$L_2 = G_4 H_2$$

$$L_3 = G_7 H_3$$

$$L_4 = G_2 G_3 G_4 G_5 G_6 G_7 G_8$$

Non-touching pairs

$$L_{12} = L_1 \cdot L_2 = G_2 G_4 H_1 H_2$$

$$L_{13} = L_1 L_3 = G_2 G_7 H_1 H_3$$

$$L_{23} = L_2 L_3 = G_4 G_7 H_2 H_3$$

Non-touching triple

$$L_{123} = L_1 L_2 L_3 = G_2 G_4 G_7 H_1 H_2 H_3$$

$$\Delta = 1 - (L_1 + L_2 + L_3)$$
$$+ (L_{12} + L_{13} + L_{23})$$
$$- L_{123}$$

$\Delta_1 = \Delta -$ everything except the initial 1 and L_3

$$= \Delta - (\Delta - L_3 - 1)$$
$$= 1 - L_3$$

So

$$\frac{Y(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4 G_5 (1 - L_3)}{1 - L_1 - L_2 - L_3 + L_{12} + L_{13} + L_{23} - L_{123}}$$

= a pretty complicated T.F.!