Models for the fluid/solid transition in gravity driven shallow water flows

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We consider a gravity driven flow of granular mass over a topography. The material is described by the visco-plastic model of Drucker-Prager [3]:

$$\begin{cases} \partial_t u + u \cdot \nabla u + \mathrm{div} P = -\vec{g}, \quad \mathrm{div} u = 0, \\ P = p\mathrm{Id} - \nu Du - k\frac{Du}{||Du||}, \quad Du = \frac{\nabla u + (\nabla u)^t}{2}, \\ k = \sqrt{2}\lambda p, \end{cases} \tag{1}$$

with u the velocity, $\vec{g}$ the gravity, p the pressure, ν the viscosity ans λ a friction coefficient. We consider a two dimensional domain delimited by a fixed bottom and a free surface transported by the material. The boundary conditions are the following: vanishing surface tension and no slip condition at the bottom.
We propose an asymptotic expansion of the model in the spirit of [1, 2], under a thin layer assumption, that enables to describe the coexistence of a fluid subdomain where $Du \neq 0$ and a solid subdomain where $Du = 0$.

A toy model that is able to describe the main features of the thin layer approximation is as follows: find a function $u(t, Z)$ and an interface $b(t)$ such that

$$\partial_t u + S - \nu \partial_Z^2 u = 0 \text{ for } b(t) < Z < h, \qquad \partial_Z U \geq 0, \tag{2}$$

where the source term $S(t, Z)$ and $h > 0$ are given, with the boundary conditions

$$\partial_Z U(t, h) = 0, \quad U(t, b(t)) = 0, \quad \partial_Z U(t, b(t)) = 0. \tag{3}$$

We analyze the problem (2)-(3) and show that the evolution of the fluid/solid interface $b(t)$ is driven by $b^\star(t)$ the zero of S, i.e. satisfying $S(t, b^\star(t)) = 0$.

References

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