CSC380 **Homework 1** **Fall 2024**

This homework is due on Friday, September 13th at 11.59pm. Late submissions are not accepted.

Submission Guidelines

- You must make two submissions:
  - Your complete homework as a SINGLE PDF file by the stated deadline to the gradescope. **Include your code and output of the code as texts in the PDF.**
  - Your codes to a separate submission: a single notebook file including codes for questions 2, 3, 4, and 5.
- For your PDF submissions:
  - Select the page number for the answer to each question in the Gradescope.
  - You may submit typed or handwritten/scanned answers. If you decide to submit handwritten answers then please ensure that it is easily readable.
  - You can easily scan and upload your answers as a PDF using the Gradescope mobile app.

1. Assume that we roll two fair six-sided dice. Let E be the event that the two dice's outcomes sum to 8. What is the probability of E?

2. Continuing with question 1: Initialize the random seed to 2024 using `numpy.random.seed`. Using `numpy.random.randint`, simulate 1,000 throws of two fair six-sided dice. Paste your code here. From these simulations, what is the empirical frequency of E (i.e., the percentage of times this event occurred in simulation)?

3. Continuing with question 2: Reset the random seed to 2024 and repeat the above simulation a total of 10 times and report the empirical frequency of E for each of the 10 runs. Paste your code here. The empirical frequency of E from each simulation will differ. Why do these numbers differ? Yet, the probability of E is fixed and was calculated in part (a) above. Why does the probability disagree with the empirical frequencies?

4. Recall that A, B are independent if $P(A \cap B) = P(A)P(B)$. Consider tossing a fair die. Let $A = \{2, 4, 6\}$, $B = \{1, 2, 3, 4\}$. Simulate draws from the sample space and verify whether the frequencies verify independence of A, B or not. Paste your code. Also, verify whether the events are independent or not theoretically.

5. Repeat question 4 for $A = \{2, 3, 4, 6\}$, $B = \{1, 2, 3, 4\}$.

6. Use Bayes' Theorem to solve the following problem.

   Suppose that 80 percent of all computer scientists are shy, whereas only 15 percent of all data scientists are shy. Suppose also that 80 percent of the people at a large gathering are computer scientists and the other 20 percent are data scientists. If you meet a shy person at random at the gathering, what is the probability that the person is a computer scientist?

7. Suppose that two players Alice and Bob take turns rolling a pair of balanced dice and that the winner is the first player who obtains the sum of 6 on a given roll of the two dice. If Alice rolls first, what is the probability that Bob will win?

8. Suppose that a box contains 8 red balls and 2 blue balls. If five balls are selected at random, without replacement, determine the probability mass function of the number of red balls that will be obtained.

9. Given that the pdf of X for some constant c is:

$$f(x) = \begin{cases} cx^2 & for \;\; 1 \leq x \leq 2 \\ 0 & otherwise \end{cases}$$

   (a) Find c.

   (b) Find $P(X > 1.75)$.

10. Given that the pdf of X is:

$$f(x) = \begin{cases} 1/4 & for \;\; 0 < x < 1 \\ 3/8 & for \;\; 3 < x < 5 \\ 0 & otherwise \end{cases}$$

    Find the cdf of X.