# CVE154 Exam 2

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This exam is primarily aimed to assess your ability to model an engineering problem as solving for $N$ unknowns from $M$ linear equations, *i.e.*,

$$\underbrace{\begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} & \cdots & A_{1,N} \\ A_{2,1} & A_{2,2} & A_{2,3} & \cdots & A_{2,N} \\ A_{3,1} & A_{3,2} & A_{3,3} & \cdots & A_{3,N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{M,1} & A_{M,2} & A_{M,3} & \cdots & A_{M,N} \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_N \end{bmatrix}}_{\boldsymbol{x}} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_M \end{bmatrix}}_{\boldsymbol{b}},$$

where $\boldsymbol{x}$ are the unknowns. Intuitively, the *impulses* $\boldsymbol{b}$ applied to a physical system cause some *responses* $\boldsymbol{x}$ from the system, according to the *design parameters* $\boldsymbol{A}$ of the system. We typically want to solve for $\boldsymbol{x}$ given $\boldsymbol{b}$ and $\boldsymbol{A}$. It will suffice us for now to deal with real numbers, and to leave complex-valued representations to tomorrow's imaginations.

While the use of AI tools to answer this exam is not prohibited, it is nevertheless of ethical interest to disclose such use. This is in line with the MSU Policy on the Fair and Ethical Use of AI and Its Applications. As such, please include a brief statement indicating the AI tools you used and how you used them in answering this exam.

Write your answers in an A4-sized document saved as a PDF file with a filename following the pattern `EXM-02_<Class number>_<ID number>`. (For example, if your ID number is 2013-0024 and you are enrolled to the W45M456 class, then your answers should be in the file `EXM-02_W45M456_2013-0024.pdf`). To submit your answers, upload the PDF file to the Google Drive folder you created in Exam 1. Lastly, send a link to the uploaded PDF file via email with the subject `CVE154 Exam 2`.

The deadline for submitting your answers is 4 November 2024 at 1159 hours. You may, of course, submit at a later time, but your overall score will be computed as $0.80^t r$, where $r$ is your raw score (*i.e.*, assuming you submitted no later than the deadline), and $t$ is the continuous-valued number of hours elapsed since the deadline.

## 1 Truss issues

Consider the 1D truss system shown in Figure 1. The nominal dimensions of the members are summarized in Table 1.

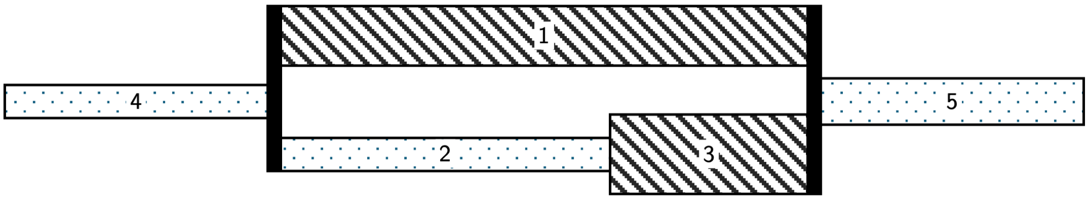


**Figure 1** Member numbers are as shown. Dot-coloured members are made of a material having a 9-GPa modulus of elasticity, while the other members are made of a material with 900 MPa.

**Table 1** Nominal dimensions for the members in the 1D truss system of Figure 1.

| Member | Length (m) | Cross-sectional area (mm$^2$) |
|---|---|---|
| 1 | 8.0 | 90.0 |
| 2 | 5.0 | 50.0 |
| 3 | 3.0 | 120.0 |
| 4 | 4.0 | 50.0 |
| 5 | 4.0 | 70.0 |

**10 marks** Assuming that the members are made of linearly elastic materials, and that their masses are negligible, show that the net external forces applied at the junctions ($p_i$'s) are linearly related to the axial deformations of the junctions ($\delta_i$'s) as follows:

$$\underbrace{\begin{bmatrix} K_{1,1} & K_{1,2} & K_{1,3} & K_{1,4} & K_{1,5} \\ K_{2,1} & K_{2,2} & K_{2,3} & K_{2,4} & K_{2,5} \\ K_{3,1} & K_{3,2} & K_{3,3} & K_{3,4} & K_{3,5} \\ K_{4,1} & K_{4,2} & K_{4,3} & K_{4,4} & K_{4,5} \\ K_{5,1} & K_{5,2} & K_{5,3} & K_{5,4} & K_{5,5} \end{bmatrix}}_{\boldsymbol{K}} \underbrace{\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \end{bmatrix}}_{\boldsymbol{\delta}} = \underbrace{\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{bmatrix}}_{\boldsymbol{p}}, \tag{1}$$

where $\boldsymbol{K}$ is the *(global) stiffness matrix*. Indicate the junction numbering you used.

**5 marks** You are considering a case study wherein:

- the left end of Member 4 is fixed to a rigid support;
- a 350-N leftward force is applied to the junction connected to the right end of Member 1;
- a 100-N leftward force is applied to the junction connecting Members 2 and 3;
- a 400-N rightward force is applied to the junction connected to the left end of Member 5;
- a 700-N rightward force is applied to the right end of Member 5; and
- a 325-N rightward force acts on the junction connecting Members 2 and 3.

Show and reason how Equation (1) must be modified. Identify the unknown quantities and describe how to solve for these.

**5 marks** You now modify the above case study by removing the force applied to the right end of Member 5, and fixing that end to a rigid support. Discuss what becomes of Equation (1), and how to solve for the resulting unknown quantities.

## 2 Mix and match

A production facility owned by White Party Industries has a network of five mixing vessels and connecting pipes. Each vessel is equipped with at least one inlet, at least one outlet, and agitating means, all configured to maintain a steady concentration of baby oil. Table 2 summarizes the designed flow rates to and fro the vessels.

**Table 2** Nominal flow rates to and fro the mixing vessels of Section 2.

| From | To | Flow rate ($m^3/s$) |
|---|---|---|
| Vessel 1 | Vessel 2 | 7.00 |
| Vessel 2 | Vessel 3 | 6.00 |
| Vessel 3 | Vessel 4 | 3.00 |
| Vessel 4 | Vessel 5 | 2.00 |
| Vessel 5 | Vessel 4 | 4.00 |
| Vessel 4 | Vessel 3 | 5.00 |
| Vessel 3 | Vessel 2 | 2.00 |
| Vessel 2 | Vessel 1 | 4.00 |
| External 20-mg/$m^3$ source | Vessel 1 | 4.00 |
| External 15-mg/$m^3$ source | Vessel 5 | 2.00 |
| Vessel 1 | Vessel 3 | 1.00 |
| Vessel 3 | External | 6.00 |

**10 marks** Assuming that steady-state mass balance holds, express the task of determining the vessel concentrations from the nominal flow rates as solving for $\boldsymbol{\kappa}$ in the linear system

$$\boldsymbol{M}\boldsymbol{\kappa} = \boldsymbol{i}. \tag{2}$$

**5 marks** Industrial control systems typically allow monitored quantities to deviate within acceptable ranges from their nominal values. Suppose that the flow rates have tolerances of 3 % from their nominal values. Set up $\boldsymbol{M}$ and $\boldsymbol{i}$ for when it is desired to determine the vessel concentrations corresponding to the minimum allowable flow rates.

**10 marks** During an emergency where the flow rates peaked their allowable values, D. Dee made a judgment call to close the valves at both ends of the pipe connecting Vessels 1 and 3. Set up the linear system that must be solved to estimate the vessel concentrations, and comment on the quality of the solution based on the condition number of the coefficient matrix.

## 3 Bonus

**3 marks** In your stay in the university, what do you consider to be the most profound thing you've heard or learned (so far) from a teacher?