CVE154 Exam 2, Part 3

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Referring to Figure 1, the rigid tripod assembly is subject to a force $\boldsymbol{F} = 42\mathbf{i} + 17\mathbf{j} - 69\mathbf{k}$ N, supported by a ball-and-socket joint at A and by rollers at B and C.

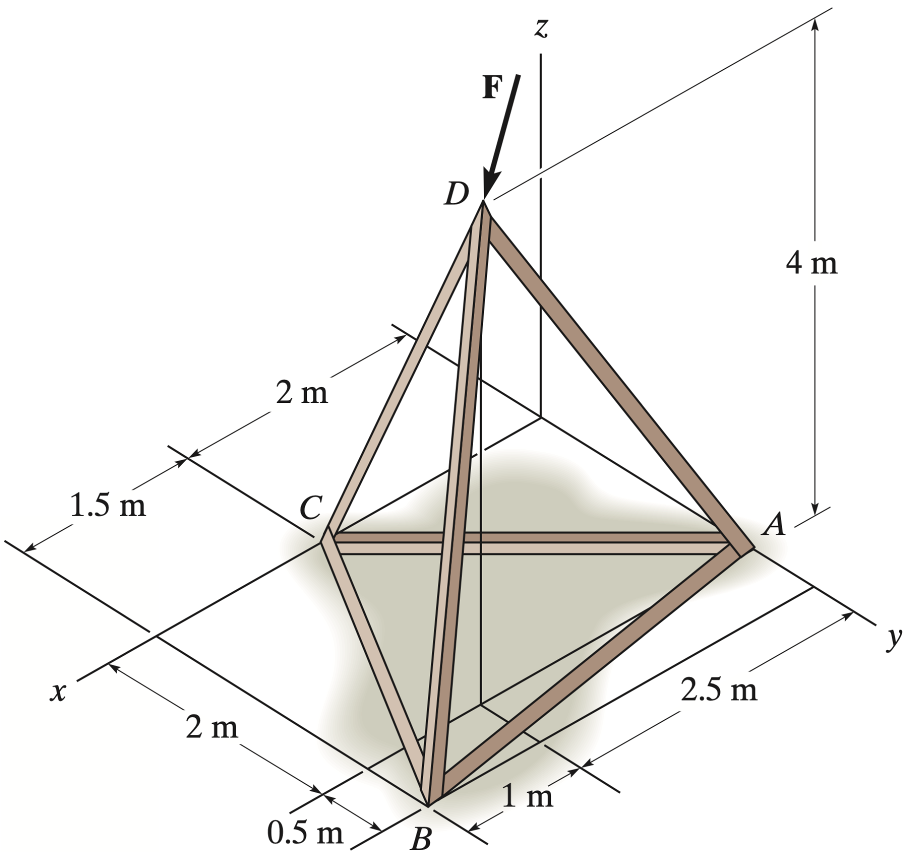

Figure 1 The tripod assembly is supported by a ball-and-socket joint and rollers. The image is a screenshot of the accompanying figure for Problem 4-61 of *Engineering Mechanics: Statics and Dynamics (14th ed.)*, the authorship and copyright of which belong to R. C. Hibbeler.

P1 (15 pt.) Derive a system of linear equations $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$ where $\boldsymbol{x}$ collects the unknown z-components of the reaction forces. Comment on the solvability of the linear system by making observations on $\boldsymbol{A}$.

Let $\boldsymbol{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{z}$ N. Introduce A_z, B_z, and C_z as the z-components of the reaction forces at A, B, and C, respectively. Force balance implies

$$A_z + B_z + C_z + F_z = 0. \tag{1}$$

Moment balance about the x-axis gives

$$2A_z + 2.5B_z - 4F_y + 2F_z = 0, \tag{2}$$

and about the y-axis,

$$3.5B_z + 2C_z - 4F_x + 2.5F_z = 0. \tag{3}$$

(Note that it is possible to set up other moment balance equations, depending on where the moment axis is taken.) Therefore, with $F_x = 42$, $F_y = 17$, and $F_z = -69$,

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2.5 & 0 \\ 0 & 3.5 & 2 \end{bmatrix} \begin{bmatrix} A_z \\ B_z \\ C_z \end{bmatrix} = \begin{bmatrix} F_z \\ 4F_y - 2F_z \\ 4F_x - 2.5F_z \end{bmatrix} = \begin{bmatrix} -69 \\ 206 \\ 340.5 \end{bmatrix}. \tag{4}$$

The coefficient matrix has a determinant of 8 and is nonsingular. Hence, Equation (4) is solvable; specifically yielding

$$A_z = -110.90625 \text{ N}, \quad B_z = 171.125 \text{ N}, \quad \text{and} \quad C_z = -129.21875 \text{ N}. \tag{5}$$

P2 (10 pt.) Solve for $\boldsymbol{x}$ via Gaussian elimination. You may use any pivoting strategy.

The following sequence of elementary row operations,

$$R_2 \leftarrow R_2 - 2R_1, \tag{6a}$$
$$R_3 \leftarrow R_3 - 7R_2, \tag{6b}$$

are sufficient to transform the augmented matrix corresponding to Equation (4) into its reduced row echelon form,

$$\begin{bmatrix} 1 & 1 & 1 & -69 \\ 0 & 0.5 & -2 & 344 \\ 0 & 0 & 16 & -2067.5 \end{bmatrix}, \tag{7}$$

and thence obtain via backward substitution the values in Equation (5).

P3 (15 pt.) Solve for $\boldsymbol{x}$ via LU decomposition. You may use any pivoting strategy.

One can infer from Equations (6) and (7) that the coefficient matrix of Equation (4) can be LU-factored as

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 7 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0.5 & -2 \\ 0 & 0 & 16 \end{bmatrix}. \tag{8}$$

So, solving for $\boldsymbol{x}$ requires the steps of solving for an intermediate vector $\boldsymbol{y}$ in the lower-triangular system

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 7 & 1 \end{bmatrix} \boldsymbol{y} = \begin{bmatrix} -69 \\ 206 \\ 340.5 \end{bmatrix}, \tag{9}$$

and then solving the upper-triangular system

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0.5 & -2 \\ 0 & 0 & 16 \end{bmatrix} \begin{bmatrix} A_z \\ B_z \\ C_z \end{bmatrix} = \boldsymbol{y}, \tag{10}$$