

CVE154 Exam 2, Part 3

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Referring to Figure 1, the rigid tripod assembly is subject to a force $\mathbf{F} = 42\mathbf{i} + 17\mathbf{j} - 69\mathbf{k}$ N, supported by a ball-and-socket joint at A and by rollers at B and C .

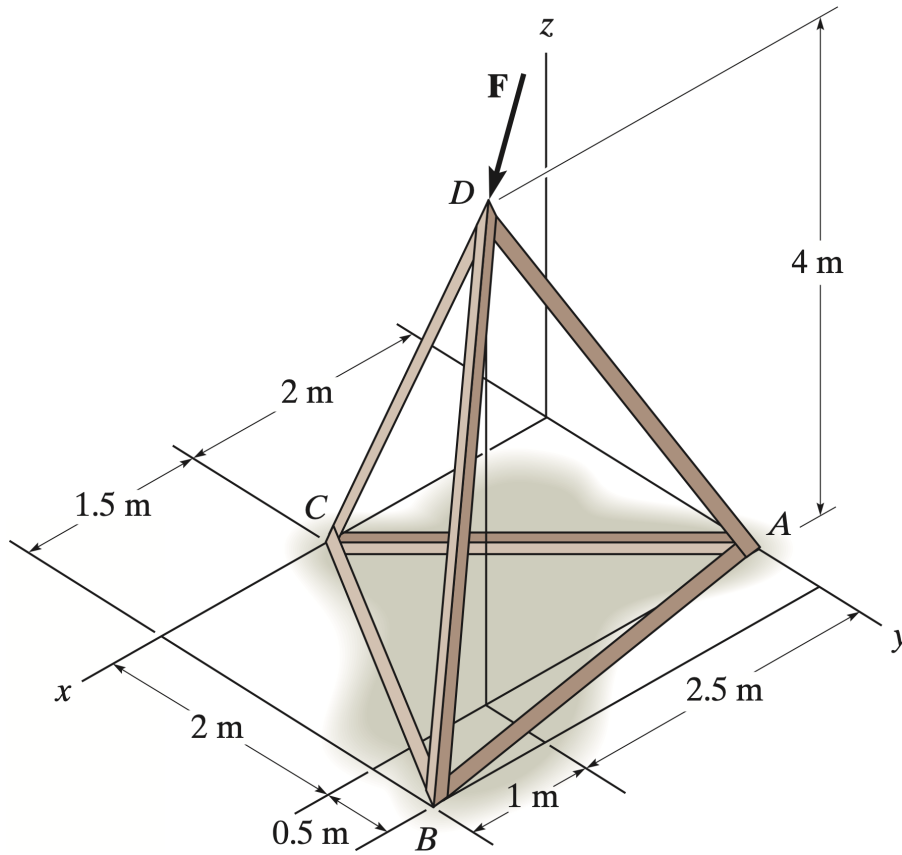


Figure 1 The tripod assembly is supported by a ball-and-socket joint and rollers. The image is a screenshot of the accompanying figure for Problem 4-61 of *Engineering Mechanics: Statics and Dynamics* (14th ed.), the authorship and copyright of which belong to R. C. Hibbeler.

P1 (15 pt.) Derive a system of linear equations $\mathbf{Ax} = \mathbf{b}$ where \mathbf{x} collects the unknown z -components of the reaction forces. Comment on the solvability of the linear system by making observations on \mathbf{A} .

Let $\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$ N. Introduce A_z , B_z , and C_z as the z -components of the reaction forces at A , B , and C , respectively. Force balance implies

$$A_z + B_z + C_z + F_z = 0. \quad (1)$$

Moment balance about the x -axis gives

$$2A_z + 2.5B_z - 4F_y + 2F_z = 0, \quad (2)$$

and about the y -axis,

$$3.5B_z + 2C_z - 4F_x + 2.5F_z = 0. \quad (3)$$

(Note that it is possible to set up other moment balance equations, depending on where the moment axis is taken.) Therefore, with $F_x = 42$, $F_y = 17$, and $F_z = -69$,

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2.5 & 0 \\ 0 & 3.5 & 2 \end{bmatrix} \begin{bmatrix} A_z \\ B_z \\ C_z \end{bmatrix} = \begin{bmatrix} -F_z \\ 4F_y - 2F_z \\ 4F_x - 2.5F_z \end{bmatrix} = \begin{bmatrix} 69 \\ 206 \\ 340.5 \end{bmatrix}. \quad (4)$$

The coefficient matrix has a determinant of 8 and is nonsingular. Hence, Equation (4) is solvable; specifically yielding

$$A_z = -24.65625 \text{ N}, \quad B_z = 102.125 \text{ N}, \quad \text{and} \quad C_z = -8.46875 \text{ N}. \quad (5)$$

P2 (10 pt.) Solve for \mathbf{x} via Gaussian elimination. You may use any pivoting strategy.

The following sequence of elementary row operations,

$$R_2 \leftarrow R_2 - 2R_1, \quad (6a)$$

$$R_3 \leftarrow R_3 - 7R_2, \quad (6b)$$

are sufficient to transform the augmented matrix corresponding to Equation (4) into its reduced row echelon form,

$$\begin{bmatrix} 1 & 1 & 1 & 69 \\ 0 & 0.5 & -2 & 68 \\ 0 & 0 & 16 & -135.5 \end{bmatrix}, \quad (7)$$

and thence obtain via backward substitution the values in Equation (5).

P3 (15 pt.) Solve for \mathbf{x} via LU decomposition. You may use any pivoting strategy.

One can infer from Equations (6) and (7) that the coefficient matrix of Equation (4) can be LU-factored as

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 7 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0.5 & -2 \\ 0 & 0 & 16 \end{bmatrix}. \quad (8)$$

So, solving for \mathbf{x} requires the steps of solving for an intermediate vector \mathbf{y} in the lower-triangular system

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 7 & 1 \end{bmatrix} \mathbf{y} = \begin{bmatrix} 69 \\ 206 \\ 340.5 \end{bmatrix}, \quad (9)$$

and then solving the upper-triangular system

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0.5 & -2 \\ 0 & 0 & 16 \end{bmatrix} \begin{bmatrix} A_z \\ B_z \\ C_z \end{bmatrix} = \mathbf{y}, \quad (10)$$