CVE154 Exam 2, Part 2

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2024-11-06

1 Blank space

Consider the linear system $\boldsymbol{Ax} = \boldsymbol{b}$ of N equations and N unknowns.

1. If $\boldsymbol{A}$ is upper-triangular, $\boldsymbol{x}$ is directly obtained using ___ substitution.
2. The ___ matrix is formed by appending $\boldsymbol{b}$ as a new column to $\boldsymbol{A}$.
3. The system has a unique solution if the determinant of $\boldsymbol{A}$ is ___.
4. If $\boldsymbol{A}$ is lower-triangular, $\boldsymbol{x}$ is directly obtained using ___ substitution.
5. If the inverse of the coefficient matrix, $\boldsymbol{A}^{-1}$, exists, then $\boldsymbol{x}$ is computed as ___.
6. $\boldsymbol{A}$ is well-conditioned if its condition number of $\boldsymbol{A}$ is not significantly greater than ___.

Consider direct methods for solving the above linear system.

7. ___ pivoting interchanges rows of the augmented matrix to find the pivot element.
8. ___ involves factoring $\boldsymbol{A}$ into a product of a lower- and an upper-triangular matrix, with the requirement that $\boldsymbol{A}$ is nonsingular.
9. The ___ method extends Gaussian elimination by transforming the system into an equivalent form $\boldsymbol{Dx} = \boldsymbol{d}$ where $\boldsymbol{D}$ is diagonal.
10. In the ___ variant of LU decomposition, the lower-triangular factor of $\boldsymbol{A}$ has 1s on the diagonal.
11. ___ pivoting looks for the pivot element across rows and columns of the augmented matrix.
12. In the ___ variant of LU decomposition, the diagonal elements of the upper-triangular factor of $\boldsymbol{A}$ are 1s.
13. ___ involves factoring $\boldsymbol{A}$ as a product of a lower- and an upper-triangular matrix, each being a transpose of the other.

2 Million reasons

3 pt. In the linear system $\boldsymbol{Ax} = \boldsymbol{b}$ of N equations and N unknowns, the coefficient matrix has a determinant of -1709. Is the system solvable? Why do you think so?

4 pt. Suppose for a physical problem you derived two equivalent linear system representations: $\boldsymbol{Ax} = \boldsymbol{a}$ and $\boldsymbol{By} = \boldsymbol{b}$, where $\boldsymbol{x}$ and $\boldsymbol{y}$ are the vectors containing unknown quantities. $\boldsymbol{A}$ and $\boldsymbol{B}$ have condition numbers of 2.024 and 2024, respectively. Which linear system should you use?

5 pt. Discuss how Gaussian elimination can be used to compute the inverse of a square, nonsingular matrix $\boldsymbol{A}$.

3 Look at those cavemen go

10 pt. Say you have a robot equipped with distance sensors so that you can collect Cartesian coordinates of N points — *i.e.*, (x_1, y_1, z_1), (x_2, y_2, z_2), $\ldots$, (x_N, y_N, z_N) — on the roof of a cave. Hoping to simplify the estimation of the cave's dimensions, you hypothesize that the cave is adequately modelled as a portion of a sphere $(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2 = \rho^2$. Here, the problem is determining from collected data the parameters α, β, γ, and ρ. Show that this problem can be modelled as a system of N linear equations in four unknowns.

Bonus

3 pt. Estimate the number of pedestrian crossings inside the campus. Explain your thought process.