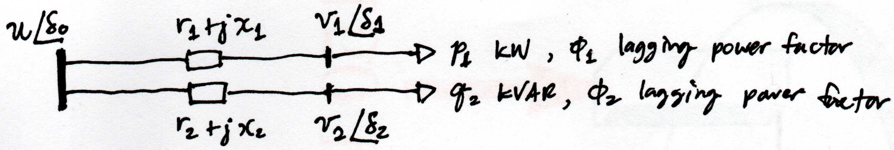
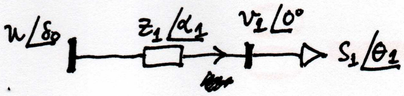


Introduce the following intermediate phasors:

- $z_1\angle\alpha_1 = r_1 + jx_1$ [ohms]
- $z_2\angle\alpha_2 = r_2 + jx_2$ [ohms]
- $s_1\angle\theta_1 = \dfrac{p_1}{1000\,\phi_1}\angle\cos^{-1}(\phi_1)$ [mega volt-amperes]
- $s_2\angle\theta_2 = \dfrac{q_2}{1000\sin(\cos^{-1}(\phi_2))}\angle\cos^{-1}(\phi_2)$ [mega volt-amperes]

Consider the circuit to Bonnie, and suppose (w/o loss of generality) $\delta_1 = 0°$. Then, we have by elementary circuit theory and analysis:



$$u\angle\delta_0 = v_1\angle 0 + \underbrace{\left(\frac{s_1}{v_1}\angle -\theta_1\right)}_{\text{current phasor [kiloamperes] drawn by Bonnie}}\left(z_1\angle\alpha_1\right)$$

$$\Rightarrow u\angle\delta_0 = v_1\angle 0 + \frac{s_1 z_1}{v_1}\angle\alpha_1-\theta_1$$

$$\begin{cases} \Rightarrow u^2 = \left[v_1 + \dfrac{s_1 z_1}{v_1}\cos(\alpha_1-\theta_1)\right]^2 + \left[\dfrac{s_1 z_1}{v_1}\sin(\alpha_1-\theta_1)\right]^2 & [1] \\ \Rightarrow \tan\delta_0 = \dfrac{s_1 z_1}{v_1}\sin(\alpha_1-\theta_1) \Big/ \left[v_1 + \dfrac{s_1 z_1}{v_1}\cos(\alpha_1-\theta_1)\right] & [2] \end{cases}$$

Recall that u is known at this point; so v_1 can be solved from [1]. Also, δ_0 can be solved from [2] once v_1 is determined. With some trivial manipulation of [1], we can say that v_1 is the root of the following univariate function:

$$f(v) = v^4 + \left[2s_1 z_1\cos(\alpha_1-\theta_1) - u^2\right]v^2 + (s_1 z_1)^2 \qquad [3]$$

We'll ~~consider similar~~ consider the circuit to Clyde similarly: assuming for a moment that $\delta_2 = 0°$, and denoting the ~~voltage~~ phase angle of the substation bus by $\bar{\delta}_0$. As such,

$$u\angle\bar{\delta}_0 = v_2\angle 0 + \frac{s_2 z_2}{v_2}\angle\alpha_2-\theta_2 \qquad [4] \qquad \Rightarrow \begin{cases} u^2 = \left[v_2 + \dfrac{s_2 z_2}{v_2}\cos(\alpha_2-\theta_2)\right]^2 + \left[\dfrac{s_2 z_2}{v_2}\sin(\alpha_2-\theta_2)\right]^2 & [5] \\ \tan\bar{\delta}_0 = \dfrac{s_2 z_2}{v_2}\sin(\alpha_2-\theta_1)\Big/\left[v_2 + \dfrac{s_2 z_2}{v_2}\cos(\alpha_2-\theta_2)\right] & [6] \end{cases}$$

Observe that [5] is sufficient for us to solve for v_2. Therefore, we can say that v_2 is the root of

$$g(v) = v^4 + \left[2s_2 z_2\cos(\alpha_2-\theta_2) - u^2\right]v^2 + (s_2 z_2)^2 \qquad [7]$$

Once v_2 is determined, $\bar{\delta}_0$ is given by [6]. Now, the substation bus in the circuit to Bonnie is the same substation bus in the circuit to Clyde. This means $\bar{\delta}_0$ has to be shifted by $\Delta\delta_0$ so as to equal δ_0: $(\bar{\delta}_0 + \Delta\delta_0 = \delta_0)$. In symbols, this is just multiplying both sides of [4] by $1\angle\Delta\delta_0$:

$$u\angle\bar{\delta}_0+\Delta\delta_0 = v_2\angle 0+\Delta\delta_0 + \frac{s_2 z_2}{v_2}\angle\alpha_2-\theta_2+\Delta\delta_0,$$

which tells us that our prior assumption that $\delta_2 = 0°$ has to be ~~change~~ corrected, i.e., $\delta_2 = \Delta\delta_0 = \delta_0 - \bar{\delta}_0$. Finally, the desired phase-angle difference is just

$$\delta_1 - \delta_2 = -\Delta\delta_0.$$