Part I-A

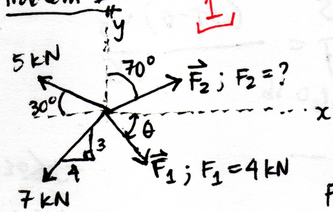
1. G	6. I
2. H	7. C
3. F	8. B
4. J	9. E
5. A	10. D

Part I-B

1. a	6. a
2. a	7. c
3. d	8. a
4. c	9. b
5. c	10. c

Problem 1

1



For static equilibrium to hold, the algebraic sums of the forces along x and along y are zero.

0.75 0.5

$$\begin{cases} 4\cos\theta + F_2\sin(70^\circ) - 5\cos(30^\circ) - 7(4/5) = 0 & [1] \\ -4\sin\theta + F_2\cos(70^\circ) + 5\sin(30^\circ) - 7(3/5) = 0 & [2] \end{cases}$$

0.75 0.5

From [2], $F_2 = \dfrac{4\sin\theta + 1.7}{\cos(70^\circ)}$ [3]

Substituting [3] onto [1], $4\cos\theta + 4\tan(70^\circ)\sin\theta + 1.7\tan(70^\circ) - 5\cos(30^\circ) - 5.6 = 0$ [3]

Note that [3] is a root-finding problem suitably solved by standard scientific calculators. A "trick" that proved to be helpful to my calculator is to express θ in radians, thereby giving $\boxed{\theta \approx 0.117370862 \text{ rad.} = 6.72485503^\circ}$ From [3], we have $\boxed{F_2 \approx 6.339995772 \text{ kN.}}$

Alternatively, we can venture more algebraically by finding a way to "eliminate" θ first from [1] and [2], thus solving for F_2. Getting the $\cos\theta$ (resp., $\sin\theta$) term on one side of [1] (resp., [2]) and squaring both sides of the resulting equation,

$$16\cos^2\theta = \sin^2(70^\circ)F_2^2 - 2\sin(70^\circ)\left(5\cos(30^\circ)+5.6\right)F_2 + \left(5\cos(30^\circ)+5.6\right)^2 \quad [4]$$

$$16\sin^2\theta = \cos^2(70^\circ)F_2^2 + 2\cos(70^\circ)\left(5\sin(30^\circ)-4.2\right)F_2 + \left(5\sin(30^\circ)-4.2\right)^2 \quad [5]$$

Adding the corresponding sides of [4] and [5],

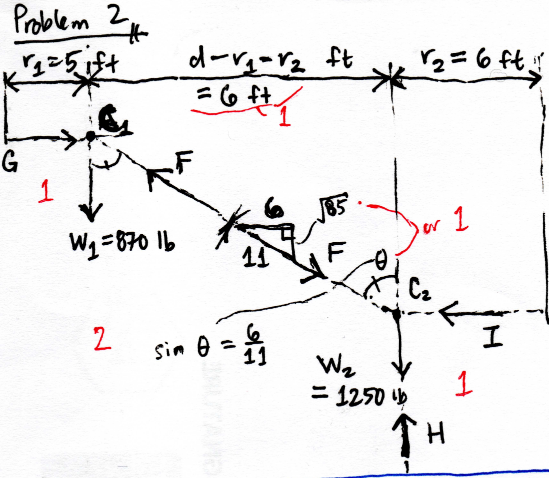
$$F_2^2 + \underbrace{\left[10\cos(70^\circ)\sin(30^\circ) - 8.4\cos(70^\circ) - 10\sin(70^\circ)\cos(30^\circ) - 11.2\sin(70^\circ)\right]}_{b} F_2 + \underbrace{\left[5\cos(30^\circ)+5.6\right]^2 + \left[5\sin(30^\circ)-4.2\right]^2 - 16}_{c} = 0 \quad [7]$$

which means we can solve for F_2 from a quadratic equation. Therefore,

$\boxed{F_2 \approx 13.48540688 \text{ kN or } 6.339995772 \text{ kN}}$ 1, respectively corresponding to

$\boxed{\theta \approx 133.275145^\circ \text{ or } 6.724855031^\circ}$ 1 according to [1].

Problem 2



Geometric inspection tells us that the distance from C_1 to C_2 is $r_1 + r_2 = 11$ ft.

Static equilibrium implies the following force balance equations for the smaller cylinder:

$$\begin{cases} G = F\left(\frac{6}{11}\right) & \text{(along horizontal) [1]} \\ W_1 = \frac{\sqrt{11^2-6^2}}{11} F & \text{(along vertical) [2]} \end{cases}$$

1

From [2], $\boxed{F = \frac{11}{\sqrt{85}} W_1 \approx 1038.012291 \text{ lb}}$

Then, from [1], $\boxed{G = \frac{6}{\sqrt{85}} W_1 \approx 566.1884949 \text{ lb.}}$ 1

Similarly invoking force balance for the larger cylinder:

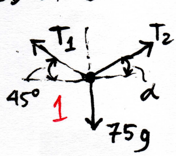
$$\begin{cases} \frac{6}{11} F = I & [3] \quad \Rightarrow \quad \boxed{I = G \approx 566.1884949 \text{ lb.}} \; 1 \\ \frac{\sqrt{11^2-6^2}}{11} F + W_2 = H & [4] \quad \Rightarrow \quad \boxed{H = W_1 + W_2 \doteq 2120 \text{ lb.}} \; 1 \end{cases}$$

It is also acceptable (and quicker) to obtain H by treating the cylinders as a composite object and doing force balance analysis along the vertical direction. Such an approach makes it clear why G and I are equal in magnitude.

Problem 3

T_1 : tension in AB
T_2 : tension in BC
T_3 : tension in CD

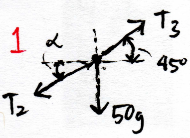
FBD at B:



Force balance equations:

$$\Rightarrow \begin{cases} T_1 \cos(45^\circ) = T_2 \cos\alpha & 1 \quad [1] \\ T_1 \sin(45^\circ) + T_2 \sin\alpha = 75g & 1 \quad [2] \end{cases}$$

FBD at C:



Force balance equations:

$$\Rightarrow \begin{cases} T_2 \cos\alpha = T_3 \cos(45^\circ) & 1 \quad [3] \\ T_2 \sin\alpha + 50g = T_3 \sin(45^\circ) & 1 \quad [4] \end{cases}$$

1

Observe that [1] and [3] imply $T_1 = T_3$, and that [2] and [4] lead to

$$T_2 \sin\alpha = 75g - T_1 \sin(45^\circ) = \underbrace{T_3}_{T_1!} \sin(45^\circ) - 50g \Rightarrow T_1 = T_3 = \frac{125g}{\sqrt{2}} \approx 866.79359 \text{ N}$$

1

From [1] and [2],

$$\frac{T_2 \sin\alpha}{T_2 \cos\alpha} = \tan\alpha = \frac{75g - T_1 \sin(45^\circ)}{T_1 \cos(45^\circ)} \Rightarrow \boxed{\alpha \approx 11.30993247^\circ} \; 1$$

$$\Rightarrow \boxed{T_2 \approx 625.0537464 \text{ N}} \; 1$$

Problem 4# Coordinates of A: $(12, 8, 0)$ in, of B: $(0, 4, 18)$ in, and of D: $(0, 8, 0)$ in.

Position vector from A to B: $\vec{r}_{AB} = \langle -12, -4, 18 \rangle$ in $\Rightarrow r_{AB} = 22$ in.
1

It is given that $r_{AC} = r_{CA} = 11$ in, which means C is the midpoint of the rod AB, and so has coordinates that are average of the coordinates of A and of B: $C(6, 6, 9)$ in. Then, the position vector from C to D is $\vec{r}_{CD} = \langle -6, 2, -9 \rangle$ in, with magnitude $r_{CD} = 11$ in. Therefore, the cord force vector is 1

$$\vec{F}_{CD} = 3\,\frac{\vec{r}_{CD}}{r_{CD}}\ \text{lb} = \left\langle \frac{-18}{11}, \frac{6}{11}, \frac{-27}{11} \right\rangle \text{lb} \approx \left\langle \begin{matrix} -1.636363636, \\ 0.5454545455, \\ -2.454545455 \end{matrix} \right\rangle \text{lb.}$$
1

The component of $\vec{F}_{CD}$ parallel to rod AB is:

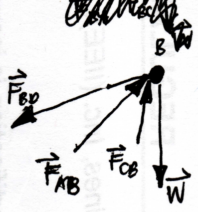
$$\boxed{\vec{G}_{/\!/} = \underbrace{\left(\vec{F}_{CD} \cdot \frac{\vec{r}_{AB}}{r_{AB}} \right)}_{\approx -1.214876033\ \text{lb}} \frac{\vec{r}_{AB}}{r_{AB}} \approx \left\langle \begin{matrix} 0.6626596544, \\ 0.2208865515, \\ -0.9939894816 \end{matrix} \right\rangle \text{lb}}$$
1

Then, by definition, the perpendicular component of $\vec{F}_{CD}$ would be

$$\vec{G}_{/\!/} + \vec{G}_{\perp} = \vec{F}_{CD} \Rightarrow \boxed{\vec{G}_{\perp} = \vec{F}_{CD} - \vec{G}_{/\!/} \approx \left\langle \begin{matrix} -2.299023291, \\ 0.324567994, \\ -1.460555973 \end{matrix} \right\rangle \text{lb}}$$
1

Observe that $\vec{F}_{CD} \cdot \frac{\vec{r}_{AB}}{r_{AB}}$ is negative, i.e., the component of $\vec{F}_{CD}$ parallel to the rod acts in the direction from B to A. Therefore, the bead will tend to slide **toward A.**
5

Problem 5#



1

Coordinates: $A(2, 0, 0)$ m, $B(0, 4, 4)$ m, $C(-2, 0, 0)$ m, $D(0, -5.6, 0)$ m

Relevant position vectors: $\vec{r}_{AB} = \langle -2, 4, 4 \rangle$ m, $(r_{AB} = 6\ \text{m})$

$\vec{r}_{CB} = \langle 2, 4, 4 \rangle$ m $(r_{CB} = 6\ \text{m})$

$\vec{r}_{BD} = \langle 0, -9.6, -4 \rangle$ m $(r_{BD} = 10.4\ \text{m})$

Force vectors:

$$\vec{F}_{BD} = F_{BD}\left(\frac{\vec{r}_{BD}}{r_{BD}}\right) = F_{BD}\left\langle 0, -\frac{12}{13}, -\frac{5}{13} \right\rangle \text{kN}$$

$$\vec{F}_{AB} = F_{AB}\left(\frac{\vec{r}_{AB}}{r_{AB}}\right) = F_{AB}\langle -1/3, 2/3, 2/3 \rangle \text{kN}$$

$$\vec{F}_{CB} = F_{CB}\left(\frac{\vec{r}_{CB}}{r_{CB}}\right) = F_{CB}\langle 1/3, 2/3, 2/3 \rangle \text{kN}$$

$$\vec{W} = W\langle 0, 0, -1 \rangle \text{kN}$$

Static equilibrium requires that $\vec{F}_{AB} + \vec{F}_{CB} + \vec{F}_{BD} + \vec{W} = 0$, which expands to the following per-component sums:

Problem 5, cont'd.

(x-components) 1 $\quad -\left(\tfrac{1}{3}\right)F_{AB} + F_{CB}\left(\tfrac{1}{3}\right) = 0 \Rightarrow \underbrace{F_{AB} = F_{CB}}_{1} \quad [1]$

(y-components) 1 $\quad -\left(\tfrac{12}{13}\right)F_{BD} + \left(\tfrac{2}{3}\right)F_{AB} + \left(\tfrac{2}{3}\right)F_{CB} = 0 \xRightarrow{\text{w/ [1]}} \underbrace{F_{BD} = \tfrac{13}{9}F_{AB} = \left(\tfrac{13}{9}\right)F_{CB}}_{1} \quad [2]$

(z-components) 1 $\quad -\left(\tfrac{5}{13}\right)F_{BD} + \left(\tfrac{2}{3}\right)F_{AB} + \left(\tfrac{2}{3}\right)F_{CB} - W = 0$

$$\xRightarrow{\text{via [1] \& [2]}} \left(\tfrac{7}{9}\right)F_{AB} = \left(\tfrac{7}{9}\right)F_{CB} = W \Rightarrow \underbrace{F_{AB} = F_{CB} = \left(\tfrac{9}{7}\right)W}_{1} \quad [3]$$

If the force limits of the members (AB & CB) are reached, $F_{AB} = F_{CB} = 2.5$ kN, and from [2] we have $F_{BD} \approx 3.61111$ kN, which is beyond the design limit for the cable. Therefore, we 1 peg F_{BD} to the design limit, i.e., $F_{BD} = 3$ kN and infer F_{AB} and F_{CB} from [2]; $F_{AB} = F_{CB} = 27/13$ kN ≈ 2.07692 kN. 1 This is acceptable because F_{AB} and F_{CB} do not exceed the 2.5-kN limit. Now, the corresponding maximum catch weight is ~~given by~~ obtained via [3]:

$$\boxed{W = \left(\tfrac{7}{9}\right)\left(\tfrac{27}{13}\right)\text{ kN} = 21/13 \text{ kN} \approx 1.61538 \text{ kN}.}$$ 1