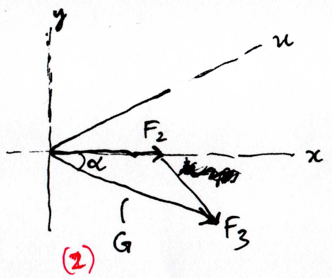
Problem 1-a



(2)

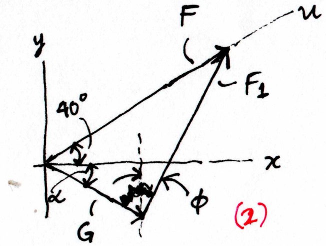
Let G be the magnitude of the resultant of $\vec{F_2}$ & $\vec{F_3}$. [redacted] By cosine law,

$$G^2 = \underbrace{F_2^2}_{0.5\ \text{kip}} + \underbrace{F_3^2}_{5\ \text{kip}} - 2F_2F_3\cos\left[90^\circ + \tan^{-1}(3/5)\right]$$

$\Rightarrow G \cong \underline{5.274701771393112\ \text{kip}}$ (1)

By sine law, $\dfrac{F_3}{\sin\alpha} = \dfrac{G}{\sin[90^\circ + \tan^{-1}(0.6)]}$

$\Rightarrow \alpha \cong \underline{54.37389567435551^\circ}$ (1)



(2)

Let F be the magnitude of the net force. [redacted] By inspection, its minimum value is zero, i.e., when $\boxed{F_1 = G \cong 5.274701771393112\ \text{kip}}$ (2)

and $\boxed{\phi = 270^\circ + \alpha \cong 324.3738956743555^\circ}$ (2)

Problem 1-b

$$10\langle\cos(40^\circ), \sin(40^\circ)\rangle = F_1\langle\sin\phi, \cos\phi\rangle + 0.5\langle 1, 0\rangle + 5\left\langle\frac{3}{\sqrt{34}}, -\frac{5}{\sqrt{34}}\right\rangle$$

This implies that

$$\begin{cases} F_1\sin\phi = 10\cos(40^\circ) - 0.5 - \dfrac{15}{\sqrt{34}} & (3) \\ F_1\cos\phi = 10\sin(40^\circ) + \dfrac{25}{\sqrt{34}} & (3) \end{cases}$$

So,

$$\tan\phi = \frac{10\cos(40^\circ) - 0.5 - 15/\sqrt{34}}{10\sin(40^\circ) + 25/\sqrt{34}} \Rightarrow \boxed{\phi \cong 23.179058081189645^\circ}$$

(2)

and therefore,

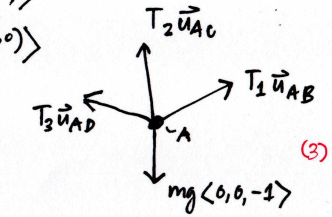
$$\boxed{F_1 \cong 11.656241062400024\ \text{kips}}$$

(2)

Problem 2

Let T_1, T_2, and T_3 be the tensions in AB, AC, and AD; and m be the mass of the pot. Observe that [redacted]

- $\vec{u}_{AB} = \langle 0, \cos(40^\circ), \sin(40^\circ)\rangle$
- $\vec{u}_{AC} = \langle -\sin(20^\circ), -\cos(50^\circ), \sin(50^\circ)\rangle$
- $\vec{u}_{AD} = \langle \sin(30^\circ), -\cos(50^\circ), \sin(50^\circ)\rangle$



(3)

Static equilibrium requires that

$$T_1\vec{u}_{AB} + T_2\vec{u}_{AC} + T_3\vec{u}_{AD} + mg\langle 0, 0, -1\rangle = \langle 0,0,0\rangle$$

$$\begin{cases} 0T_1 - T_2\sin(20^\circ) + T_3\sin(30^\circ) = 0 & (3) \\ T_1\cos(40^\circ) - T_2\cos(50^\circ) - T_3\cos(50^\circ) = 0 & (3) \\ T_1\sin(40^\circ) + T_2\sin(50^\circ) + T_3\sin(50^\circ) - mg = 0 & (2) \end{cases}$$

If AB fails, $T_1 = 60$, and
$T_2 \cong 42.46051364$ N < 50 N (ok)
$T_3 \cong 29.04470192$ N < 35 N (ok)
$m \cong 9.51838085$ kg
(2)

If AC fails, $T_2 = 50$, and
$T_1 \cong 70.65387917$ N > 60 N (not ok)
$T_3 \cong 34.20201433$ N < 35N ok)
$m \cong 11.20850885$ kg
(2)

If AD fails, $T_3 = 35$, and
$T_1 \cong 72.30234298$ N > 60N (not ok)
$T_2 \cong 51.166577$ N > 50 N (not ok)
$m \cong 11.47002062$ kg
(2)

Therefore, the pot must weigh $\boxed{\text{no more than } 9.51838085\ \text{kg}}$.

(2)

Problem 3

(a) The resultant force is $\left\langle \frac{4}{5}(0.1),\ -0.15-2-0.5-\frac{3}{5}(0.1) \right\rangle$ (3) (3)

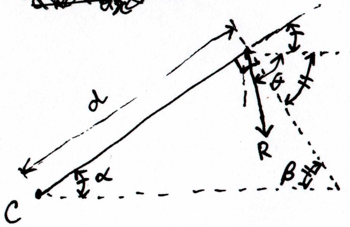
$= \boxed{\langle 0.08, -2.71\rangle \text{ kip or } 2.711180554666177 \text{ kips, at } -88.30910273704966^\circ}$ (3) (3)

The couple moment about C is

$\underbrace{0.15(1.5)}_{(1)} + \underbrace{2(3)}_{(1)} + \underbrace{0.5(4.5)}_{(1)} + \underbrace{\frac{3}{5}(0.1)(6)}_{(1)} + \underbrace{\frac{4}{5}(0.1)(8)}_{(1)} = \boxed{9.475 \text{ kip-ft, clockwise}}$ (2) (2)

(b) In order for the resultant force to cause the same moment about C, whilst intersecting segment BC, the intersection must be above and to the right of C. Moreover, let the magnitude of the resultant be denoted by R and its direction clockwise from the x-axis be denoted by θ. By geometry,

$\alpha = 90^\circ - \tan^{-1}(0.75)$ and $\beta = \tan^{-1}(0.75)$



Therefore, by the definition of moment (i.e., d as the moment arm),

$R\cos(\theta-\beta)\, d = 9.475 \Rightarrow d \cong \boxed{5.606508875739645 \text{ ft}}$ (5)

Problem 4

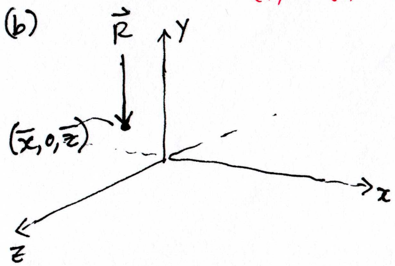
Observe that each column load is 3.85 m from O.

(a) The equivalent force-couple system at O comprises a resultant force $\vec{R}$ and a couple moment $\vec{M}_O$. By inspection,

- $\vec{R} = (150+110+200+40)\langle 0,-1,0\rangle \Rightarrow \boxed{\vec{R} = 500\langle 0,-1,0\rangle \text{ kN}}$ (3)
- $\vec{C} = \langle \underbrace{110(3.85)-200(3.85)}_{\text{net moment (ccw) about x-axis}},\ 0,\ \underbrace{150(3.85)-40(3.85)}_{\text{net moment (ccw) about z-axis}} \rangle$ kN

$\Rightarrow \boxed{\vec{C} = \langle -346.5, 0, 423.5\rangle \text{ kN}\cdot\text{m}}$ (4) (4) (4)

(b)



In order for $\vec{R}$ to cause a clockwise moment about the x-axis and at the same time a counter clockwise moment about the z-axis, it has to hit point $(\bar{x}, 0, \bar{z})$ where $\bar{x}, \bar{z} < 0$.

Hence, $|\vec{R}||\bar{z}| = 346.5$ and $|\vec{R}||\bar{x}| = 423.5$ $\Rightarrow$ $\bar{z} = -0.693$ m, $\bar{x} = -0.847$ m

Therefore the single-force equivalent of the column loads intersects the mat at the point $\boxed{(-0.847, 0, -0.693) \text{ m}}$.

(2) (1) (2)