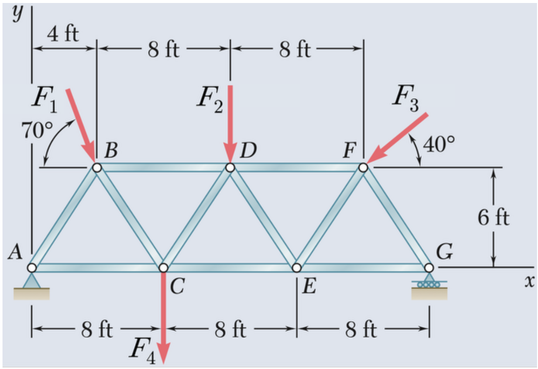
Solutions to Quiz 3

for the ENS161 B5-1 and B6 classes *offered in* A.Y. 2025-2026 S1



Problem 1. One needs to compute the resultant of the loads to get their force-couple equivalents. The horizontal and vertical components of such resultant are computed as

$$R_x = F_1\cos(70°) - F_3\cos(40°) \text{ and } R_y = -F_1\sin(70°) - F_2 - F_3\sin(40°) - F_4$$

from which the magnitude R and direction R_θ are trivial to determine. Then, in the case of a force-couple equivalent at C, the (counterclockwise) couple moment is computed as

$$M_C = F_1\sin(70°)\,(1.2192) - F_1\cos(70°)\,(1.8288) - F_2(1.2192) + F_3\cos(40°)\,(1.8288) - F_3\sin(40°)\,(3.6576)$$

Similarly, a force-couple equivalent at E will have the following (counterclockwise) couple moment:

$$M_E = F_1\sin(70°)\,(3.6576) - F_1\cos(70°)\,(1.8288) - F_2(1.2192) + F_3\cos(40°)\,(1.8288) - F_3\sin(40°)\,(1.2192) + F_4(2.4384)$$

The single-force equivalent of the loads will be the same resultant as above, just judiciously placed. To determine where the single-force equivalent load intersects the line through A and G, relative to A, one needs the total moment M_A of the loads about A (or about G, but the former would make for a more straightforward computation). Assuming M_A is counterclockwise,

$$M_A = -F_1\sin(70°)\,(1.2192) - F_1\cos(70°)\,(1.8288) - F_2(3.6576) - F_3\sin(40°)\,(6.096) + F_3\cos(40°)\,(1.8288) - F_4(2.4384)$$

Whether the point of intersection is to the left or to the right of A can then be ascertained by considering the sense of M_A. Lastly, the distance from A is the moment arm needed by the resultant to produce the magnitude of M_A, *i.e.*,

$$|R_y|d_A = |M_A|$$

The same strategy can be adapted in finding where the single-force equivalent load intersects the line through B and F, relative to F. We have the total moment M_F of the loads about F as

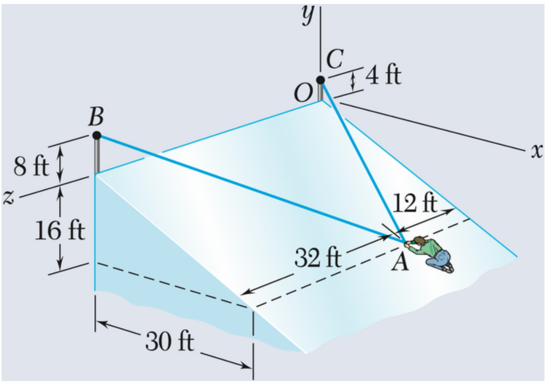
$$M_F = F_1\sin(70°)\,(4.8768) + F_2(2.4384) + F_4(3.6576)$$

which, being unequivocally counterclockwise, places the resultant intersecting the line through B and F at a distance d_F to the left of F, so that

$$|R_y|d_F = |M_F|$$

Under static equilibrium, all forces (loads and support reactions) acting on the truss must cause no tendency to translate or rotate (about any point). This means that the algebraic sum of horizontal forces is zero, the algebraic sum of vertical forces is zero, and the algebraic sum of moments, say, about C, is zero. Suppose we assume that the support reactions have positive senses, we can solve for them from a system of three linear equations:

$$A_h + F_1\cos(70°) - F_3\cos(40°) = 0$$
$$A_v - F_1\sin(70°) - F_2 - F_3\sin(40°) - F_4 = 0$$
$$A_h(0) - A_v(2.4384) + M_C + G_v(4.8768) = 0$$



Problem 2. By geometry,

$$\boldsymbol{u}_{AB} = \frac{\langle -30, 24, 32\rangle}{50}$$
$$\boldsymbol{u}_{AC} = \frac{\langle -30, 20, -12\rangle}{38}$$
$$\boldsymbol{u}_{N} = \frac{\langle 16, 30, 0\rangle}{34}$$

Let T_{AB} and T_{AC} be the respective tensions in ropes AB and AC. The weight of the person can be represented by the vector $W\langle 0,-1,0\rangle$. The force N exerted by the surface on the person is just the normal force acting along $\boldsymbol{u}_N$. Absent its dimensions, the person is treated as a particle under static equilibrium, for which the requisite condition is for the force vectors add up to zero:

$$T_{AB}\boldsymbol{u}_{AB} + T_{AC}\boldsymbol{u}_{AC} + N\boldsymbol{u}_N + W\langle 0,-1,0\rangle = \langle 0,0,0\rangle$$

which translates into a system of three equations with the rope tensions and the normal force as unknowns.

The moment of the person's weight about the line joining B and O is given by

$$[(\boldsymbol{r}_{BA} \times \langle 0,-W,0\rangle)\cdot \boldsymbol{u}_{BO}]\boldsymbol{u}_{BO}$$

where

$$\boldsymbol{r}_{BA} = 0.3048\langle 30,-24,-32\rangle$$
$$\boldsymbol{u}_{BO} = \frac{\langle 0,-4,-44\rangle}{\sqrt{1952}}$$

Similarly, the moment of the normal force about the same line is

$$[(\boldsymbol{r}_{BA} \times N\boldsymbol{u}_N)\cdot \boldsymbol{u}_{BO}]\boldsymbol{u}_{BO}$$