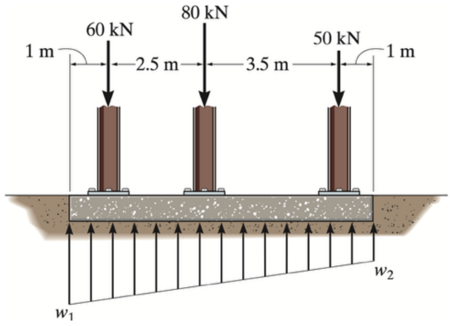
Solutions to Quiz 2

for the ENS161 B5-1 and B6 classes *offered in* A.Y. 2025-2026 S1

Problem 1. A concrete footing is designed to transmit loads from three steel columns down to the soil. It is assumed that the soil exerts a distributed upward reaction whose intensity varies linearly as shown.



Solutions. Regardless of where the force-couple equivalent of the column loadings is at, the resultant force is always a 190-kN downward force. However, when the force-couple equivalent is at the left edge of the footing, the couple moment is the total moment due to the column loadings about the left edge: 690 kN-m, clockwise. Similarly, when the force-couple equivalent is at the right edge of the footing, the couple moment is the total moment due to the column loadings about the right edge: 830 kN-m, counterclockwise.

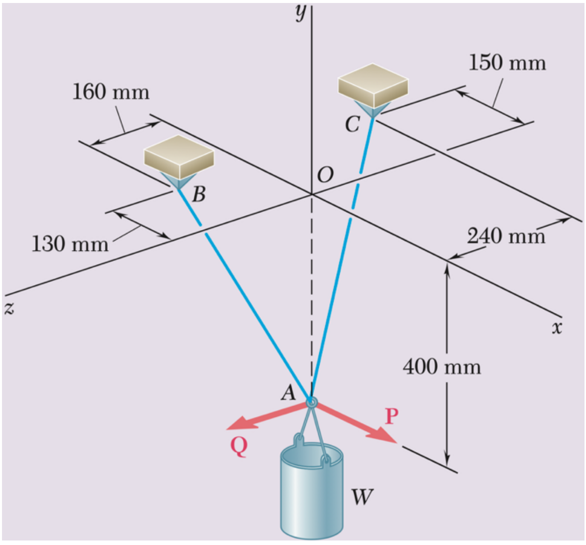
Moreover, the single-force equivalent of the column loadings is a 190-kN downward resultant force that causes a 830-kN-m counterclockwise moment about the right edge of the footing, i.e., the resultant is approximately 4.36842 m from the right edge.

Now, if the soil supports the loaded footing, then the distributed upward reaction must counter (i) the downward resultant of the column loadings, and (ii) the moment due thereto (e.g., about the right edge of the footing). Such balances of forces and of moments translate to two equations:

- $4(w_1 - w_2) + 8w_2 = 190$
- $\frac{64}{3}(w_1 - w_2) + 32w_2 = 830$

whence the values for w_1 and w_2 are solved to be 30.3125 and 17.1875 kN/m, respectively.

Problem 2. The container is supported by cable BAC that passes through a frictionless ring at A, and two forces P and Q taken to act at the ring.



Solutions.

The moment of the 69-N container weight about B is given by

$$\boldsymbol{r}_{BA} \times \boldsymbol{W} = \langle 0.13, -0.4, -0.16 \rangle \times \langle 0, -69, 0 \rangle = \langle -11.04, 0, -8.97 \rangle \text{ N-m.}$$

The moment of the 42-N container weight about the axis through B and C is given by

$$\{(\boldsymbol{r}_{CA} \times \boldsymbol{W}) \cdot \boldsymbol{u}_{BC}\}\boldsymbol{u}_{BC} \cong \langle -0.28908, 0, -5.78155 \rangle \text{ N-m.}$$

If the container weighs 421 N, force balance at the ring requires

$$\boldsymbol{W} + \boldsymbol{P} + \boldsymbol{Q} + T\frac{\boldsymbol{r}_{AB}}{r_{AB}} + T\frac{\boldsymbol{r}_{AC}}{r_{AC}} = \langle 0,0,0 \rangle$$

where:

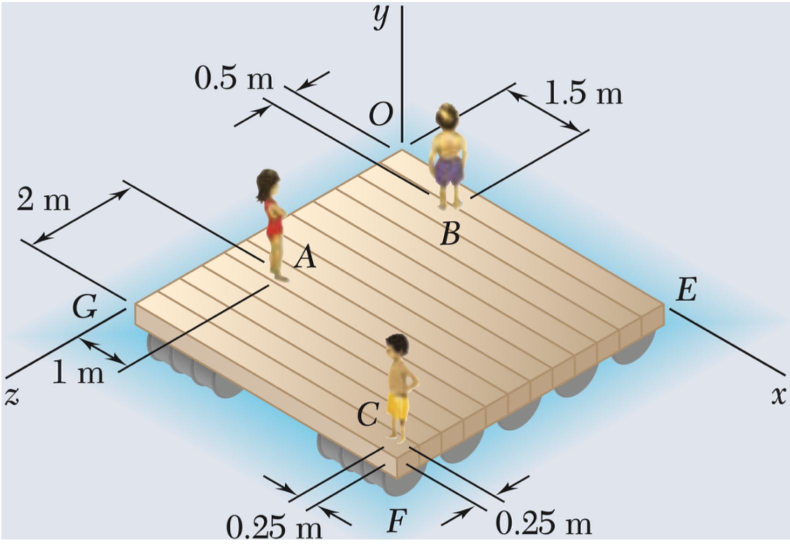
- $\boldsymbol{W} = \langle 0, -421, 0 \rangle$,
- $\boldsymbol{P} = P\langle 1,0,0 \rangle$,
- $\boldsymbol{Q} = Q\langle 0,0,1 \rangle$,
- $\boldsymbol{r}_{AB} = \langle -0.13, 0.4, 0.16 \rangle$, and
- $\boldsymbol{r}_{AC} = \langle -0.15, 0.4, -0.24 \rangle$.

This then expands into a system of three linear equations in P, Q, and T, i.e.,

$$P - \frac{0.13}{0.45}T - \frac{0.15}{0.49}T = 0$$
$$-421 + \frac{0.4}{0.45}T + \frac{0.4}{0.49}T = 0$$
$$Q + \frac{0.16}{0.45}T - \frac{0.24}{0.49}T = 0$$

which give $T \cong 246.88963$ N, $P \cong 146.90213$ N, and $Q \cong 33.14255$ N.

Problem 3. A rectangular raft is to be analyzed when loaded with multiple persons, approximating the weights to be concentrated forces.



Solutions.

For the three-weight loading, the equivalent force-couple and single-force systems both share a common resultant: a 1035-N downward force. It should be straightforward to verify that for the equivalent force-couple system at corner *F*, the couple moment is $\langle -1760, 0, 3780 \rangle$ N-m. Furthermore, the equivalent single-force system can be obtained by moving the 1035-N resultant away from corner *F* such that its moment about *F* is $\langle -1760, 0, 3780 \rangle$ N-m. Looking at the *xy*- and the *yx*-planes should lead to the resultant acting at the point $(3.34783, 0, 2.29952)$ m.

Consider now the case of the four-weight loading. The resultant – now a 1460-N downward force passing through the center of the raft – induces a moment of $\langle 2920, 0, -5110 \rangle$ N-m about the origin. Suppose that the fourth weight is at $(\bar{x}, 0, \bar{z})$ m. Then, we have the following.

- $375(2) + 260(0.5) + 400(3.75) + 425\bar{z} = 2920$, ergo, $\bar{z} \cong 1.27059$ m
- $-375(1) - 260(1.5) - 400(6.75) - 425\bar{x} = -5110$, ergo, $\bar{x} \cong 3.87059$ m