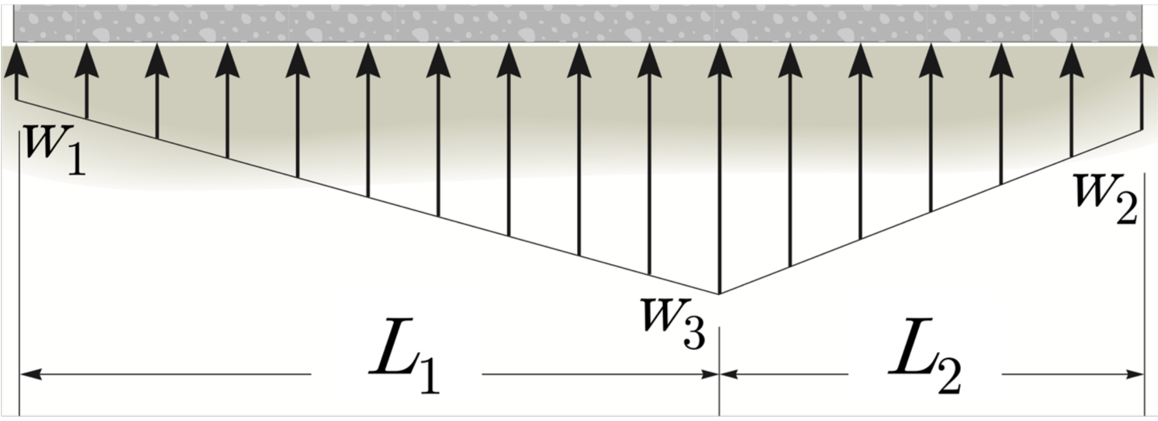
Solutions to Problem Set 3

for the ENS161 B5-1 and B6 classes *offered in* A.Y. 2025-2026 S1



Solutions for Problem 1. The resultant of the distributed, upward force has a magnitude given by

$$w_1L_1 + \frac{(w_3 - w_1)L_1}{2} + w_2L_2 + \frac{(w_3 - w_2)L_2}{2}$$

When the equivalent force-couple system is to be at the left end, the couple moment is counterclockwise and has a magnitude given by

$$\frac{w_1(L_1)^2}{2} + \frac{(w_3 - w_1)(L_1)^2}{3} + \frac{w_2L_2(2L_1 + L_2)}{2} + \frac{(w_3 - w_2)L_2(3L_1 + L_2)}{6}$$

On the other hand, when the equivalent force-couple system is to be at the right end, the couple moment is clockwise with a magnitude given by

$$\frac{w_1L_1(L_1 + 2L_2)}{2} + \frac{(w_3 - w_1)L_1(L_1 + 3L_2)}{6} + \frac{w_2(L_2)^2}{2} + \frac{(w_3 - w_2)(L_2)^2}{3}$$

It is a little less straightforward when the equivalent force-couple system is to be at the midspan, for it lies within the left distribution (since $L_1 > L_2$). Therefore, the couple moment is computed as

$$-\frac{w_1L_1L_2}{2} - \frac{(w_3 - w_1)L_1(-L_1 + 3L_2)}{12} + \frac{w_2L_1L_2}{2} + \frac{(w_3 - w_2)L_2(3L_1 - L_2)}{12}$$

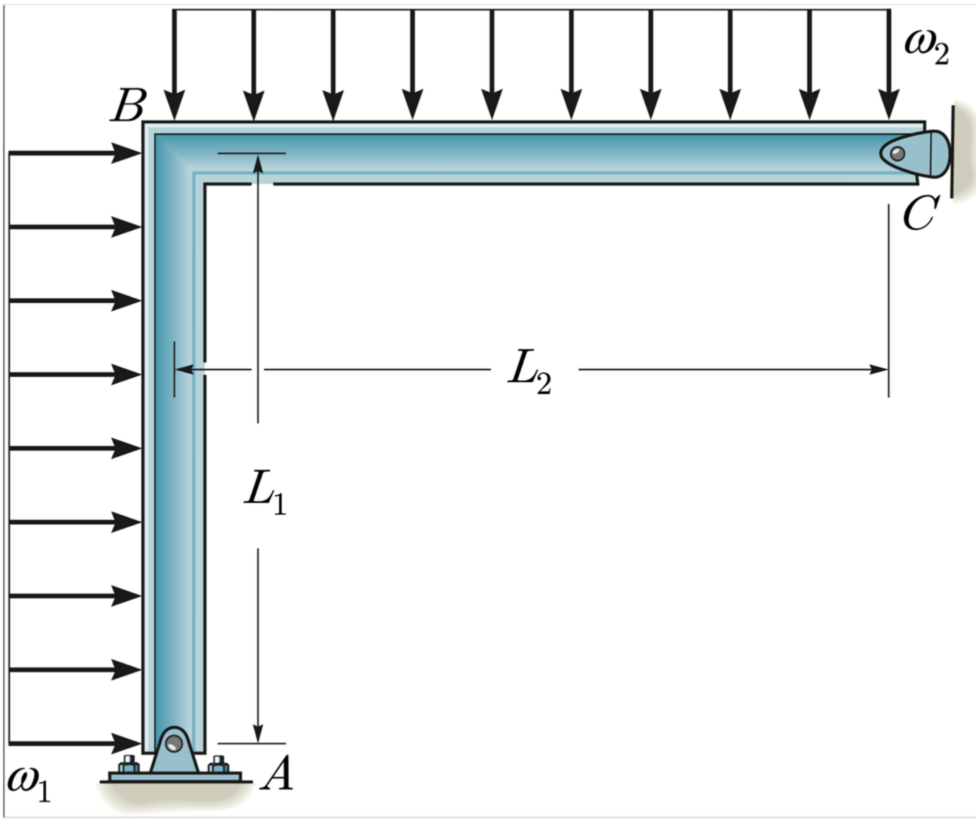
the positive sign meaning counterclockwise, and the negative otherwise.

The single-force equivalent is just the same resultant computed above and located d from the left end, with

$$\left[w_1L_1 + \frac{(w_3 - w_1)L_1}{2} + w_2L_2 + \frac{(w_3 - w_2)L_2}{2}\right]d$$
$$= \frac{w_1(L_1)^2}{2} + \frac{(w_3 - w_1)(L_1)^2}{3} + \frac{w_2L_2(2L_1 + L_2)}{2} + \frac{(w_3 - w_2)L_2(3L_1 + L_2)}{6}$$

The counteracting steel ball must weigh as the magnitude of the resultant of the distributed, upward force, and must be placed where the single-force equivalent acts, *i.e.*, $L_1 + L_2 - d$ from the right end of the beam.

Note: The above computations imply appropriate unit conversions.



Solutions to Problem 2. Any equivalent force-couple system, regardless of its location, will have a resultant force whose horizontal and vertical components are $\omega_1 L_1$ and $-\omega_2 L_2$, i.e., a magnitude of

$$\sqrt{(\omega_1 L_1)^2 + (-\omega_2 L_2)^2}$$

For an equivalent force-couple system at *A*, the couple moment is computed as

$$-\frac{\omega_1 (L_1)^2}{2} - \frac{\omega_2 (L_2)^2}{2}$$

while an equivalent force-couple system at *B* will have a couple moment computed as

$$\frac{\omega_1 (L_1)^2}{2} - \frac{\omega_2 (L_2)^2}{2}$$

and an equivalent force-couple system at *C* will have a couple moment given by

$$\frac{\omega_1 (L_1)^2}{2} + \frac{\omega_2 (L_2)^2}{2}$$

The single-force equivalent intersecting the horizontal member is the same resultant force determined above, but it must induce the same couple moment about *C* as the couple moment of the equivalent force-couple system thereat. This occurs when the resultant is d_x units to the left of *C*, provided that

$$\omega_2 L_2 d_x = \frac{\omega_1 (L_1)^2}{2} + \frac{\omega_2 (L_2)^2}{2}$$

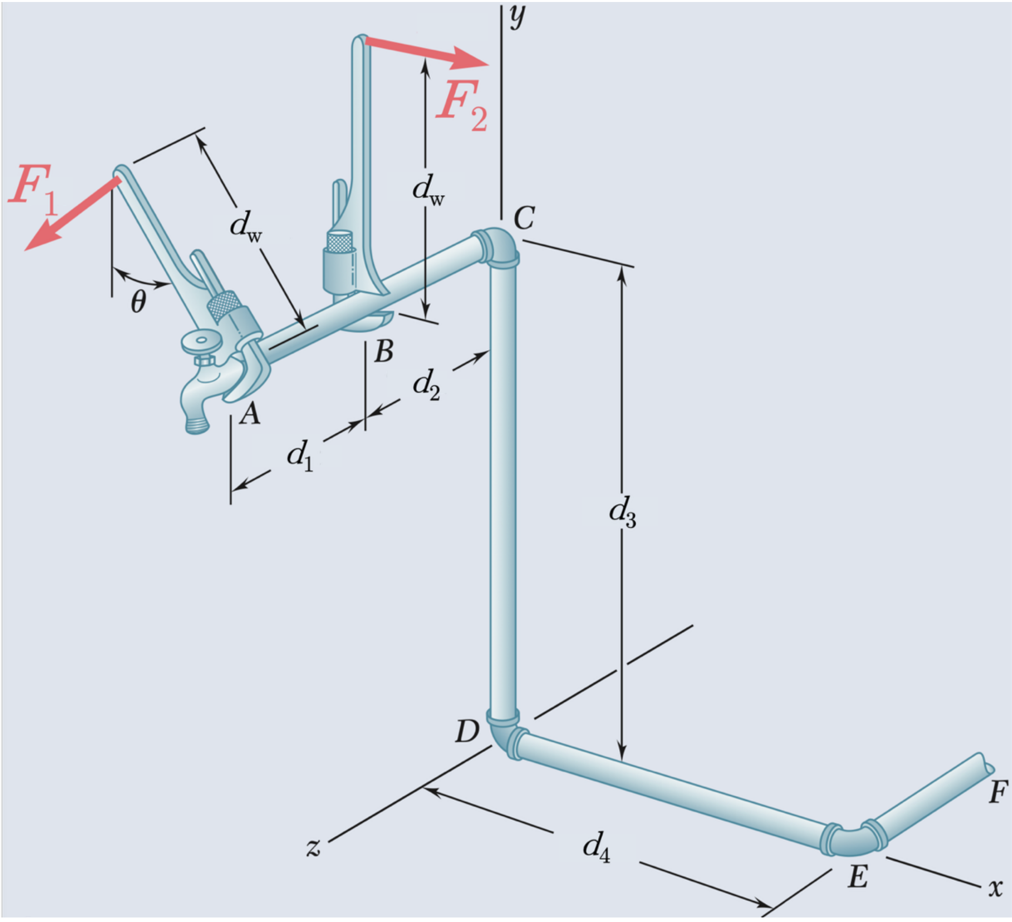
Following the same reasoning, the single-force equivalent intersecting the vertical member is the same resultant force positioned d_y above *A* such that the induced moment about *A* is the same couple moment in an equivalent force-couple system at *A*. In symbols,

$$-\omega_1 L_1 d_y = -\frac{\omega_1 (L_1)^2}{2} - \frac{\omega_2 (L_2)^2}{2}$$

The rocker and the pin provide support (whence ***C***, ***H***, and ***V*** are called support reactions) so that the frame is in static equilibrium under the distributed loadings. Therefore, the frame must experience zero net force and zero moment (about any point), to wit:

- a balance of vertical forces, $\boldsymbol{V} - \omega_2 L_2 = 0$;
- a balance of horizontal forces, $\omega_1 L_1 + \boldsymbol{C} + \boldsymbol{H} = 0$; and
- a balance of moments (say, about A), $-\boldsymbol{C}\frac{1}{2}L_1 - \frac{1}{2}\omega_1 (L_1)^2 - \frac{1}{2}\omega_2 (L_2)^2 = 0$.

Here, we make an initial assume that the support reactions have positive senses. The magnitudes and senses are ultimately determined by solving the three equations.



Solutions to Problem 3. The directions of $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are respectively represented by the unit vectors

$$\boldsymbol{u}_1 = \langle -\sin(90-\theta), -\cos(90-\theta), 0\rangle$$
$$\boldsymbol{u}_2 = \langle 1,0,0\rangle$$

The moment of $\boldsymbol{F}_1$ about E is computed as $\boldsymbol{r}_{EG} \times \boldsymbol{F}_1$, where G is the point upon which $\boldsymbol{F}_1$ acts on the left wrench, *i.e.*, at the following coordinates

$$(-d_\mathrm{w}\sin\theta\,, \quad d_3 + d_\mathrm{w}\sin\theta\,, \quad d_1 + d_2)$$

Similarly, the moment of $\boldsymbol{F}_2$ about E is computed as $\boldsymbol{r}_{EH} \times \boldsymbol{F}_2$, where H is the point upon which $\boldsymbol{F}_2$ acts on the right wrench, i.e., at the following coordinates

$$(0, \quad d_3 + d_\mathrm{w}, \quad d_2)$$

Therefore, the total moment of the forces about E is

$$(\boldsymbol{r}_{EG} \times \boldsymbol{F}_1) + (\boldsymbol{r}_{EH} \times \boldsymbol{F}_2)$$

The moment of $\boldsymbol{F}_1$ about the line joining B and E is given by

$$\left[(\boldsymbol{r}_{EG} \times \boldsymbol{F}_1) \cdot \frac{\boldsymbol{r}_{BE}}{|\boldsymbol{r}_{BE}|}\right] \frac{\boldsymbol{r}_{BE}}{|\boldsymbol{r}_{BE}|}$$

Similarly, the moment of $\boldsymbol{F}_2$ about the line joining E and A is given by

$$\left[(\boldsymbol{r}_{EH} \times \boldsymbol{F}_2) \cdot \frac{\boldsymbol{r}_{EA}}{|\boldsymbol{r}_{EA}|}\right] \frac{\boldsymbol{r}_{EA}}{|\boldsymbol{r}_{EA}|}$$

and the moment of $\boldsymbol{F}_2$ about the line connecting C and E is

$$\left[(\boldsymbol{r}_{EH} \times \boldsymbol{F}_2) \cdot \frac{\boldsymbol{r}_{CE}}{|\boldsymbol{r}_{CE}|}\right] \frac{\boldsymbol{r}_{CE}}{|\boldsymbol{r}_{CE}|}$$

Let M be the midpoint between A and E, *i.e.*,

$$\left(\frac{d_4}{2}, \frac{d_3}{2}, \frac{d_1 + d_2}{2}\right)$$

Then, the equivalent force-couple system at M consists of the resultant of the wrench forces,

$$\boldsymbol{F}_1 + \boldsymbol{F}_2$$

and the couple moment about M due to the wrench forces,

$$(\boldsymbol{r}_{MG} \times \boldsymbol{F}_1) + (\boldsymbol{r}_{MH} \times \boldsymbol{F}_2)$$