Anticipatory Power Flow

Formulation, Computation, and Applications

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The steady-state grid as a weighted graph $(\mathcal{N}, \mathcal{E})$

N buses as nodes, E branches as edges,

- $\mathcal{N}$: key locations (*e.g.*, substation)
- $\mathcal{E}$: bus-to-bus interfaces (*e.g.*, lines, cables, transformers)

Branch admittances act as edge weights

- How well a branch admits power flow

Bus admittance matrix $\boldsymbol{Y} = \boldsymbol{G} + \mathrm{j}\boldsymbol{B}$ act as graph Laplacian

- Sparsity reflects grid topology
- Elements computed from admittances

☞ $\boldsymbol{Y}$ encodes grid intrinsics

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Buses are entry and exit points for power

- Demand units draw $\boldsymbol{p}_{\mathsf{d}} + \mathrm{j}\boldsymbol{q}_{\mathsf{d}} \in \mathbb{C}^{U}$
- Supply units inject $\boldsymbol{p}_{\mathsf{u}} + \mathrm{j}\boldsymbol{q}_{\mathsf{u}} \in \mathbb{C}^{D}$
- $\boldsymbol{C}_{\mathsf{u}}$, $\boldsymbol{C}_{\mathsf{d}}$: connection matrices
- $\boldsymbol{d} := (\boldsymbol{p}_{\mathsf{d}}, \boldsymbol{q}_{\mathsf{d}})$ and $\boldsymbol{c} := (\boldsymbol{p}_{\mathsf{u}}, \boldsymbol{q}_{\mathsf{u}})$

☞ Net nodal injections: $\boldsymbol{C}_{\mathsf{u}} (\boldsymbol{p}_{\mathsf{u}} + \mathrm{j}\boldsymbol{q}_{\mathsf{u}}) - \boldsymbol{C}_{\mathsf{d}} (\boldsymbol{p}_{\mathsf{d}} + \mathrm{j}\boldsymbol{q}_{\mathsf{d}})$

Bus voltages $\boldsymbol{v}\underline{/\boldsymbol{\delta}} \equiv \boldsymbol{v}\mathrm{e}^{\mathrm{j}\boldsymbol{\delta}}$ are state variables

- For a grid with $\boldsymbol{Y}$, its state is computable given $\boldsymbol{s} := (\boldsymbol{v}, \boldsymbol{\delta})$

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Power flow equations (PFE)

The standard model of steady-state grid physics

Nodal power balance

$$\mathrm{Diag}(\boldsymbol{v}\underline{/\boldsymbol{\delta}})\,\mathrm{conj}(\boldsymbol{Y}\boldsymbol{v}\underline{/\boldsymbol{\delta}}) = \boldsymbol{C}_{\mathsf{u}}\left(\boldsymbol{p}_{\mathsf{u}} + \mathrm{j}\boldsymbol{q}_{\mathsf{u}}\right) - \boldsymbol{C}_{\mathsf{d}}\left(\boldsymbol{p}_{\mathsf{d}} + \mathrm{j}\boldsymbol{q}_{\mathsf{d}}\right)$$

or, equivalently, as a system of $2N$ nonlinear equations,

$$\begin{bmatrix} \mathrm{Diag}(\boldsymbol{v} \odot \cos\boldsymbol{\delta}) & \mathrm{Diag}(\boldsymbol{v} \odot \sin\boldsymbol{\delta}) \\ \mathrm{Diag}(\boldsymbol{v} \odot \sin\boldsymbol{\delta}) & -\mathrm{Diag}(\boldsymbol{v} \odot \cos\boldsymbol{\delta}) \end{bmatrix} \begin{bmatrix} \boldsymbol{G} & -\boldsymbol{B} \\ \boldsymbol{B} & \boldsymbol{G} \end{bmatrix} \begin{bmatrix} \boldsymbol{v} \odot \cos\boldsymbol{\delta} \\ \boldsymbol{v} \odot \sin\boldsymbol{\delta} \end{bmatrix} = \begin{bmatrix} \boldsymbol{C}_{\mathsf{u}}\boldsymbol{p}_{\mathsf{u}} - \boldsymbol{C}_{\mathsf{d}}\boldsymbol{p}_{\mathsf{d}} \\ \boldsymbol{C}_{\mathsf{u}}\boldsymbol{q}_{\mathsf{u}} - \boldsymbol{C}_{\mathsf{d}}\boldsymbol{q}_{\mathsf{d}} \end{bmatrix}$$

- Staple requirement in power flow analysis (PFA) computations
- PFE parameters: $\boldsymbol{Y}$ as grid intrinsics, $\boldsymbol{d}$ as external stimuli
- $(\boldsymbol{c}, \boldsymbol{s})$ are power-flow feasible for $(\boldsymbol{Y}, \boldsymbol{d})$ if they satisfy the PFE for $(\boldsymbol{Y}, \boldsymbol{d})$

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Power flow manifold (PFM)

A geometric intuition for the PFE

PFE as a manifold in $(\boldsymbol{c}, \boldsymbol{s})$–space

Expressed as $\phi(\boldsymbol{c}, \boldsymbol{s}; \boldsymbol{Y}, \boldsymbol{d}) = \mathbf{0}_{2N}$, the PFE describe a manifold of all points $(\boldsymbol{c}, \boldsymbol{s})$ that are power-flow feasible for $(\boldsymbol{Y}, \boldsymbol{d})$.

- PFE parameters $(\boldsymbol{Y}, \boldsymbol{d})$ dictate the “shape” of PFM
- Power-flow feasible $(\boldsymbol{c}, \boldsymbol{s})$ for $(\boldsymbol{Y}, \boldsymbol{d}) \implies (\boldsymbol{c}, \boldsymbol{s})$ is on PFM for $(\boldsymbol{Y}, \boldsymbol{d})$
- Computation over PFE for $(\boldsymbol{Y}, \boldsymbol{d}) \implies$ finding a point on PFM for $(\boldsymbol{Y}, \boldsymbol{d})$

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Finding a point on the PFM

- Standard power flow (SPF) as completing a point on PFM
  - Fix some coordinates with slack-PV-PQ model (§2.4.1)
    - ★ Find $k \in \mathcal{N}$ with supply unit, set δ_k as reference
    - ★ Assume v_m and $p_{\mathsf{u},n}$ for all $m \in \mathcal{N}$ with supply units, and all unit n at $m \neq k$
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- Continuation power flow (CPF) as finding points through PFMs
  - Estimating stability limits: $\boldsymbol{\phi}(\boldsymbol{Y}, \boldsymbol{d}_i) = \mathbf{0}_{2N}$ with $\boldsymbol{d}_i = \lambda_i \boldsymbol{d}_{\mathrm{base}}$ [1–6]
  - Branch-outage contingency: $\boldsymbol{\phi}(\boldsymbol{Y}_i, \boldsymbol{d}) = \mathbf{0}_{2N}$ by [7–10]

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- Optimal power flow (OPF) as finding the best point on PFM
  - Best point minimizes operating cost $g(\boldsymbol{c}, \boldsymbol{s})$ subject to
    - Supply capacity: $\underline{\boldsymbol{c}} \leq \boldsymbol{c} \leq \overline{\boldsymbol{c}}$
    - Allowable state: $\boldsymbol{A}\boldsymbol{s} \leq \boldsymbol{b}$
  - Nonconvex (due to PFE) and NP-hard [11–13]

PFA computation in the online setting

- Snapshot data of past operating point: $(\widetilde{\boldsymbol{c}}, \widetilde{\boldsymbol{s}})$ in response to $\widetilde{\boldsymbol{d}}$
  - ☞ Direct or computed from measurements

- Expected conditions for upcoming dispatch
  - ▶ Updated intrinsics $\boldsymbol{Y}$ from online parameter estimation [14, 15]
  - ▶ Anticipated demand $\boldsymbol{d}$ from forecast
  - ▶ Supply limits $\underline{\boldsymbol{c}}$, $\overline{\boldsymbol{c}}$ (*e.g.*, schedule, ramp)
  - ☞ $\boldsymbol{d} \neq \widetilde{\boldsymbol{d}}$, *i.e.*, $(\widetilde{\boldsymbol{c}}, \widetilde{\boldsymbol{s}})$ not on PFM for $(\boldsymbol{Y}, \boldsymbol{d})$

- Invariant roster of supply & demand units (*i.e.*, fixed C_{u} and C_{d})

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Using the snapshot point $(\widetilde{\boldsymbol{c}}, \widetilde{\boldsymbol{s}})$, how do we find a point $(\boldsymbol{c}, \boldsymbol{x})$ on the PFM for $(\boldsymbol{Y}, \boldsymbol{d})$?

Pillars of anticipatory power flow (APF)

1. $\boldsymbol{c}$ as control, $\boldsymbol{s}$ as state, and $\boldsymbol{s}$ as an implicit function of $\boldsymbol{c}$ via PFE
   - OPF notion from '60s [16, §2.4]; used in recent works on online OPF [17, 18]
   - Compute $\boldsymbol{c}$ voltage-free, then solve for $\boldsymbol{s}$ from PFE

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2. To avoid rotational degeneracy, pick any bus $\hat{n}$ as reference and $\delta_{\hat{n}} \leftarrow \delta_{\text{ref}}$
   - PFE are sinusoids of phase angle differences, not of phase angles
   - Slack-PV-PQ is mathematically unnecessary [19, §2.2.1]
   - Non-reference voltage phase angles: $\boldsymbol{\vartheta} \in \mathbb{R}^{N-1}$

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3. To make solving PFE well-determined, add a distributed slack $\kappa \in \mathbb{R}$
   - Based on SPF notion distributed slack bus, dating back to '90s [20]
   - κ shared via slack distribution ratios $\boldsymbol{\kappa}$ based on supply limits (§3.3)
   - With $\boldsymbol{x} := \left[\boldsymbol{v}; \boldsymbol{\vartheta}; \kappa\right]$ as state variables, restate PFE as $\boldsymbol{\phi}(\boldsymbol{x}; \boldsymbol{Y}, \boldsymbol{d}, \boldsymbol{c}, \delta_{\hat{n}}) = \boldsymbol{0}_{2N}$

Solving for the anticipated supply injections

Extended economic dispatch (ED+)

$$\begin{aligned}
\min_{\boldsymbol{p},\boldsymbol{q}} \quad & \underbrace{\|\boldsymbol{p}\|_2 + \|\boldsymbol{q}\|_2}_{f_{\text{loss}}} + \underbrace{\mu_{\text{p}}\|\boldsymbol{p} - \widetilde{\boldsymbol{p}}_{\text{u}}\|_2 + \mu_{\text{q}}\|\boldsymbol{q} - \widetilde{\boldsymbol{q}}_{\text{u}}\|_2}_{f_{\text{reg}}} \\
\text{s. t.} \quad & \mathbf{1}^{\mathsf{T}}\boldsymbol{p} = p_{\text{need}}, \quad \mathbf{1}^{\mathsf{T}}\boldsymbol{q} = q_{\text{need}}, \quad \underline{\boldsymbol{p}_{\text{u}}} \le \boldsymbol{p} \le \overline{\boldsymbol{p}_{\text{u}}}, \quad \underline{\boldsymbol{q}_{\text{u}}} \le \boldsymbol{q} \le \overline{\boldsymbol{q}_{\text{u}}}, \\
\text{with} \quad & p_{\text{need}} = \operatorname{clip}\left(\mathbf{1}^{\mathsf{T}}\underline{\boldsymbol{p}_{\text{u}}}, \mathbf{1}^{\mathsf{T}}\boldsymbol{p}_{\text{d}} + p_{\text{h}} + p_{\text{o}}, \mathbf{1}^{\mathsf{T}}\overline{\boldsymbol{p}_{\text{u}}}\right), \quad q_{\text{need}} = \operatorname{clip}\left(\mathbf{1}^{\mathsf{T}}\underline{\boldsymbol{q}_{\text{u}}}, \mathbf{1}^{\mathsf{T}}\boldsymbol{q}_{\text{d}} + q_{\text{h}} + q_{\text{o}}, \mathbf{1}^{\mathsf{T}}\overline{\boldsymbol{q}_{\text{u}}}\right), \\
& p_{\text{h}} + \mathrm{j}q_{\text{h}} = \operatorname{shunt}\left(\widetilde{\boldsymbol{v}}, \widetilde{\boldsymbol{\delta}}; \boldsymbol{Y}\right), \quad \text{and} \quad p_{\text{o}} + \mathrm{j}q_{\text{o}} = \operatorname{loss}\left(\widetilde{\boldsymbol{v}}, \widetilde{\boldsymbol{\delta}}; \boldsymbol{Y}\right).
\end{aligned}$$

- Vanilla ED but considers reactive powers
  - Supply regularization $\mu_{\text{p}}, \mu_{\text{q}} \ge 0$
  - Adjust for non-demand consumption: $\operatorname{shunt}(\cdot)$ (§3.2.1) and $\operatorname{loss}(\cdot)$ (§3.2.2)

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$$\min_{\boldsymbol{p},\boldsymbol{q}} \quad \underbrace{\|\boldsymbol{p}\|_2 + \|\boldsymbol{q}\|_2}_{f_{\text{loss}}} + \underbrace{\mu_{\text{p}}\|\boldsymbol{p} - \widetilde{\boldsymbol{p}}_{\text{u}}\|_2 + \mu_{\text{q}}\|\boldsymbol{q} - \widetilde{\boldsymbol{q}}_{\text{u}}\|_2}_{f_{\text{reg}}}$$

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- Vanilla ED but considers reactive powers
  - Supply regularization $\mu_{\mathsf{p}}, \mu_{\mathsf{q}} \geq 0$
  - Adjust for non-demand consumption: $\operatorname{shunt}(\cdot)$ (§3.2.1) and $\operatorname{loss}(\cdot)$ (§3.2.2)
- Convex: checked with CVX 2.2 [21], has a convex QP form (§3.2.3)
  - ☞ Lots of well-established polynomial-time algorithms

Solving for the anticipated bus voltages

APF equations (PFE+)

$$\boldsymbol{\phi}(\boldsymbol{x};\boldsymbol{Y},\boldsymbol{d},\boldsymbol{c},\delta_{\hat{n}}) := \underbrace{\boldsymbol{e}(\boldsymbol{v},\boldsymbol{\vartheta};\boldsymbol{Y},\delta_{\hat{n}}) - \kappa \begin{bmatrix} \boldsymbol{C}_{\mathsf{u}}\boldsymbol{\kappa} \\ \boldsymbol{0}_N \end{bmatrix}}_{\boldsymbol{\psi}(\boldsymbol{x};\boldsymbol{Y},\delta_{\hat{n}})} - \begin{bmatrix} \boldsymbol{C}_{\mathsf{u}}\boldsymbol{p}_{\mathsf{u}} \\ \boldsymbol{C}_{\mathsf{u}}\boldsymbol{q}_{\mathsf{u}} \end{bmatrix} - \begin{bmatrix} \boldsymbol{C}_{\mathsf{d}}\boldsymbol{p}_{\mathsf{d}} \\ \boldsymbol{C}_{\mathsf{d}}\boldsymbol{q}_{\mathsf{d}} \end{bmatrix} = \boldsymbol{0}_{2N}$$

- A $2N$–dimensional root-finding task
  - ☞ Lots of derivative-based algorithms with fast convergence guarantees
  - See §3.3.2 for the APF Jacobian $\boldsymbol{J}(\boldsymbol{x}) := \partial_{\boldsymbol{x}}\boldsymbol{\psi}(\boldsymbol{x})$

Solving for the anticipated bus voltages

APF equations (PFE+)

$$\boldsymbol{\phi}(\boldsymbol{x};\boldsymbol{Y},\boldsymbol{d},\boldsymbol{c},\delta_{\hat{n}}) := \underbrace{\boldsymbol{e}(\boldsymbol{v},\boldsymbol{\vartheta};\boldsymbol{Y},\delta_{\hat{n}}) - \kappa \begin{bmatrix} \boldsymbol{C}_{\mathsf{u}}\boldsymbol{\kappa} \\ \boldsymbol{0}_N \end{bmatrix}}_{\boldsymbol{\psi}(\boldsymbol{x};\boldsymbol{Y},\delta_{\hat{n}})} - \begin{bmatrix} \boldsymbol{C}_{\mathsf{u}}\boldsymbol{p}_{\mathsf{u}} \\ \boldsymbol{C}_{\mathsf{u}}\boldsymbol{q}_{\mathsf{u}} \end{bmatrix} - \begin{bmatrix} \boldsymbol{C}_{\mathsf{d}}\boldsymbol{p}_{\mathsf{d}} \\ \boldsymbol{C}_{\mathsf{d}}\boldsymbol{q}_{\mathsf{d}} \end{bmatrix} = \boldsymbol{0}_{2N}$$

- A $2N$–dimensional root-finding task
  - ☞ Lots of derivative-based algorithms with fast convergence guarantees
    - See §3.3.2 for the APF Jacobian $\boldsymbol{J}(\boldsymbol{x}) := \partial_{\boldsymbol{x}}\boldsymbol{\psi}(\boldsymbol{x})$
- $\boldsymbol{c}$ is a parameter of PFE+
  - ☞ Different $\boldsymbol{c}$'s give different $\boldsymbol{x}$'s
  - ☞ Compare APF points by their κ's

Some fast solvers and algorithms for APF

For solving ED+

- SeDuMi 1.3.4 [22–24]
- SDPT3 4.0 [25–27]

☞ Free and open-source

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☞ Quadratic local convergence [33, §10.3, 11.2]

☞ Ready-to-use in modern solvers

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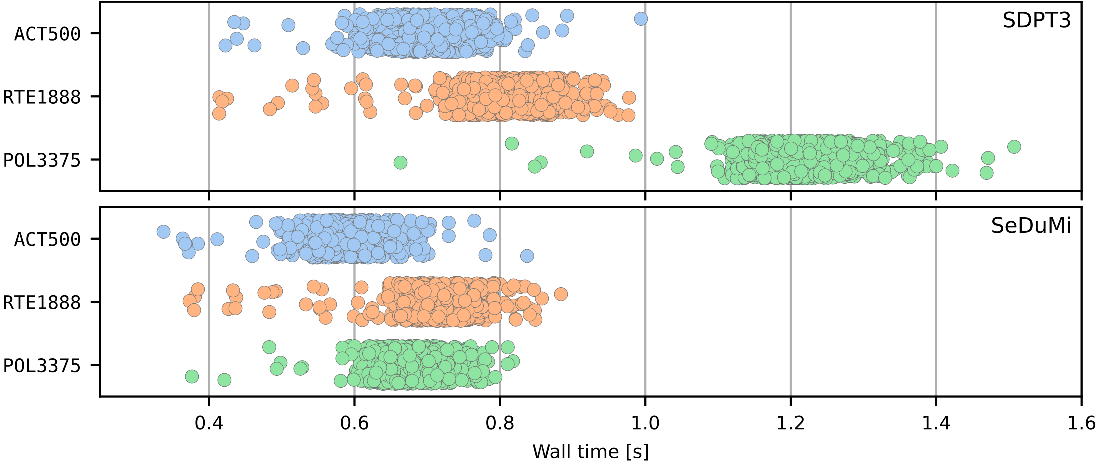
APFLib: a MATLAB library built on CVX and as an extension for MATPOWER

christian-cahig/Masterarbeit-DemoApps

Run time evaluations for solving ED+

Based on Intel Core i7-10750H CPU @ 2.60GHz w/ 16GB RAM

`ACT500`, `RTE1888`, and `POL3375` have 90, 298, and 596 supply units, respectively.



Run time evaluations for solving PFE+

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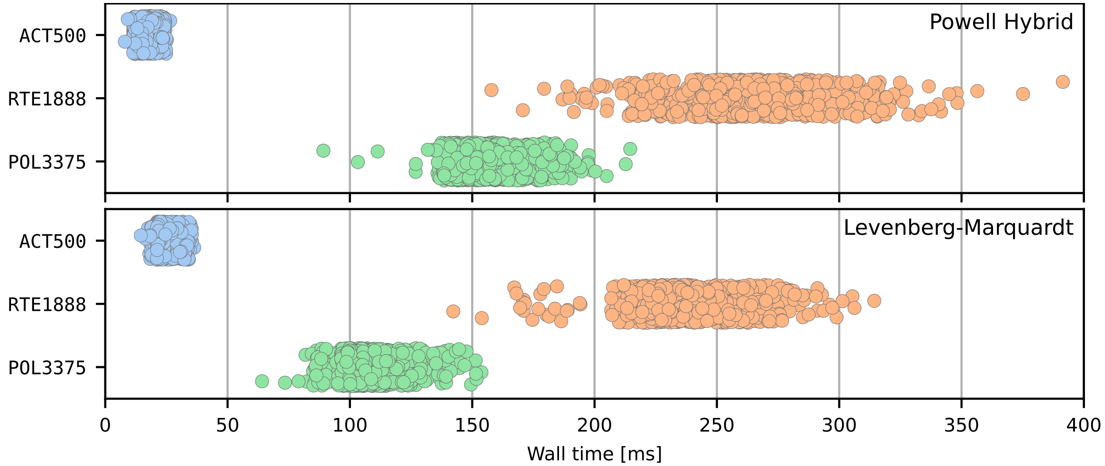
`ACT500`, `RTE1888`, and `POL3375` have 500, 1888, and 3374 buses, respectively.



Effect of supply regularization on the APF point

All else fixed, what happens to $(\boldsymbol{c}, \boldsymbol{x})$ when $(\mu_{\mathsf{p}}, \mu_{\mathsf{q}}) \in \left\{0, 10^{-4}, 1\right\} \times \left\{0, 10^{-4}, 1\right\}$?

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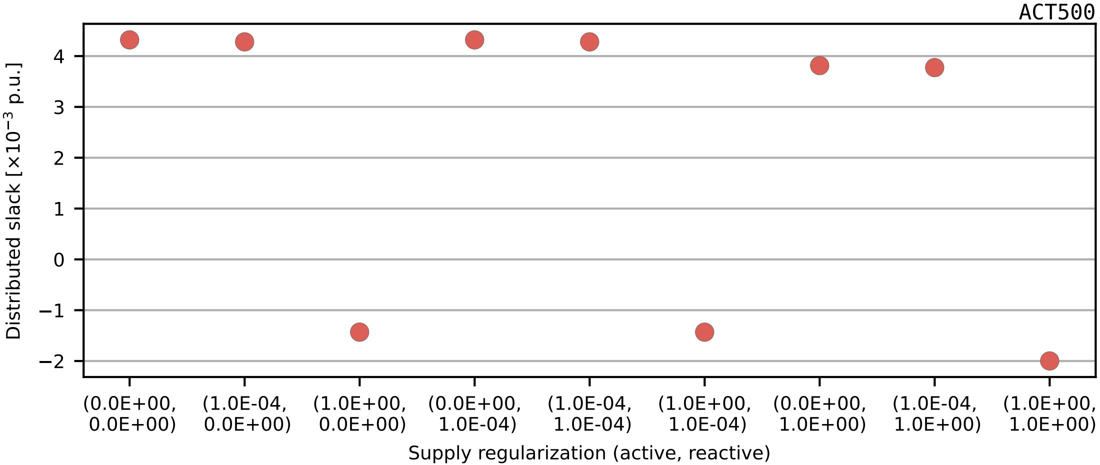

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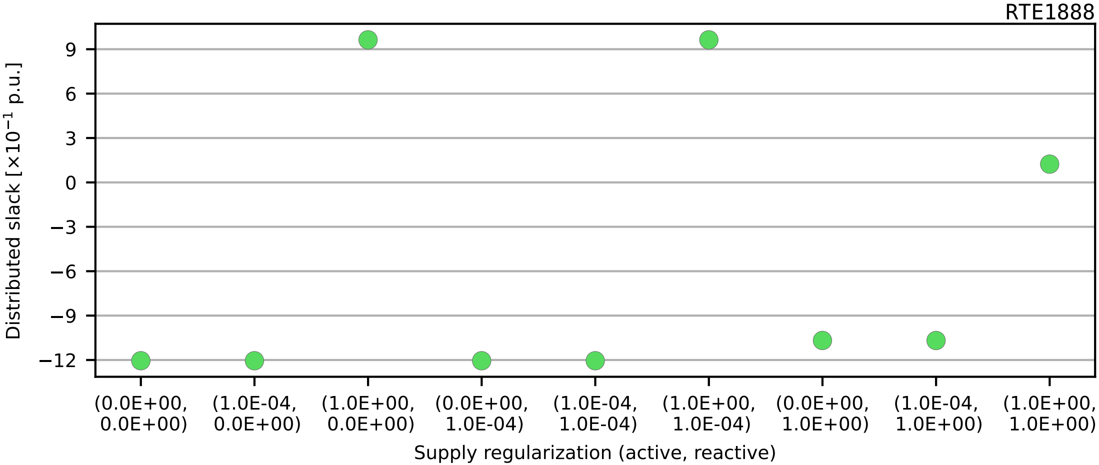

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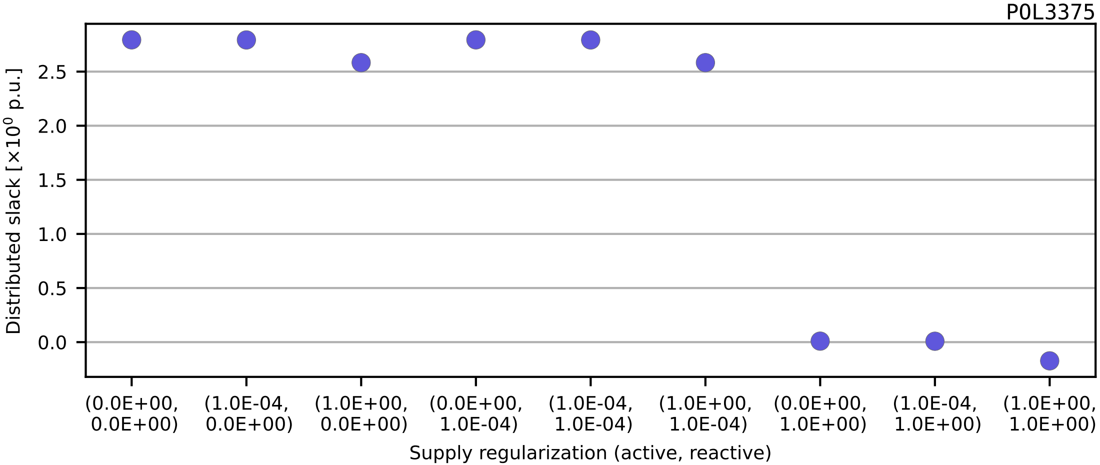

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Quadrimodal effect

The APF point $(\boldsymbol{c}, \boldsymbol{x})$ will be in one of four neighbourhoods in the upcoming-dispatch PFM.

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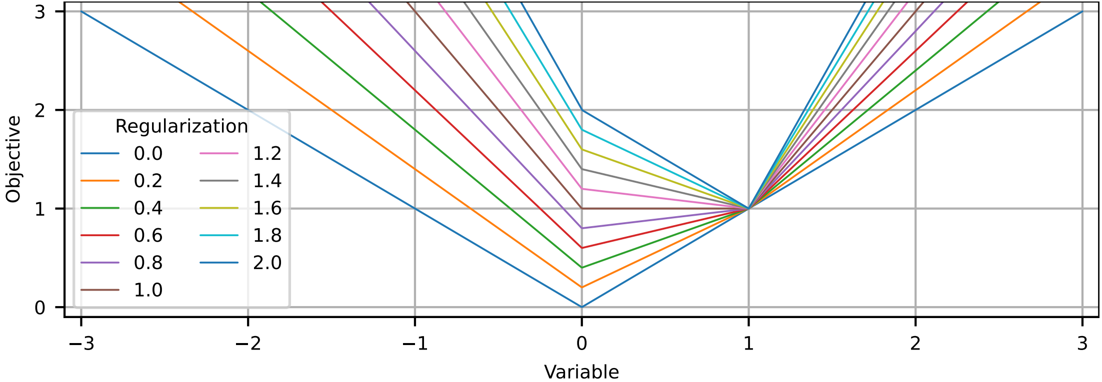
Consider ED+ in 1D: minimize $|x| + \mu|x-1|$ s.t. $-3 \leq x \leq 3$, where $\mu \geq 0$



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The APF point $(\boldsymbol{c}, \boldsymbol{x})$ will be in one of four neighbourhoods in the upcoming-dispatch PFM.

Corollary (Big-μ trick for finding four APF points)

Regularizing ED+ with $(\mu_{\mathsf{p}}, \mu_{\mathsf{q}}) \in \{0, \mu\} \times \{0, \mu\}$*, for some* $\mu \gg 0$*, yields four* $\boldsymbol{c}$*'s, and, by PFE+, four* $\boldsymbol{x}$*'s. These four independent APF instances can be run in parallel.*

APF for OPF: Providing solvers with warm-start points

OPF solvers are iterative: $(\boldsymbol{c}_{k+1}, \boldsymbol{s}_{k+1}) \leftarrow \text{update}(\boldsymbol{c}_k, \boldsymbol{s}_k)$

- User-specified starting point $(\boldsymbol{c}_0, \boldsymbol{s}_0)$
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Warm-starting a solver with APF point $(\boldsymbol{c}, \boldsymbol{x})$

- $\boldsymbol{c}_0 \leftarrow (\boldsymbol{p}_{\mathsf{u}} + \kappa\boldsymbol{\kappa}, \boldsymbol{q}_{\mathsf{u}})$
- $\boldsymbol{s}_0 \leftarrow (\boldsymbol{v}, \boldsymbol{\delta})$

☞ Search starts on PFM

APF for OPF: Providing solvers with warm-start points

Solve instances on `POL3375` via MIPS 1.4 [38]

- 408 APF instances
- Get snapshot- & warm-started optima
  - $\boldsymbol{c}^\star_\mathsf{s}$, $\boldsymbol{v}^\star_\mathsf{s}$, $\boldsymbol{\varphi}^\star_\mathsf{s}$, $g^\star_\mathsf{s}$
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At $(\mu_{\mathsf{p}}, \mu_{\mathsf{q}}) = (0, 0)$

Metric	Minimum	Maximum
ϵ_{g}	-1.7847×10^{-2}	1.90488×10^{-2}
ϵ_{v}	4.79369×10^{-9}	6.89567×10^{-5}
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ϵ_{c}	6.91804×10^{-4}	1.16527×10^{0}

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Open problems

1. Given a solver, a snapshot point S, and a APF point A, how can we tell that starting the solver at S and at A will or will not give us distinct optima?
2. How to make APF a more disciplined method of finding multiple OPF solutions?

APF for amortized OPF: Differentiating through PFE+

Neural nets to learn solution maps of OPF instances with fixed $\boldsymbol{Y}$ but varying $\boldsymbol{d}$

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- OPF-DNN: violation-based Lagrangian relaxation [39]
  - $\boldsymbol{d} \xrightarrow{\mathrm{Net}_{\boldsymbol{\theta}}(\cdot)} \boldsymbol{c}, \boldsymbol{s} \xrightarrow{\text{computations}} \ell = \cdots + \lambda \left\| \phi(\boldsymbol{c}, \boldsymbol{s}) \right\|_1$
  - ! Only encourages PFE compliance

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- DeepOPF: SPF + zeroth-order gradient estimation [40]
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  - ! Inexact gradient could hurt training

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- DC3: SPF + penalty + differentiating through SPF [41]
  - $\boldsymbol{d} \xrightarrow{\mathrm{Net}_{\boldsymbol{\theta}}(\cdot)} \hat{\boldsymbol{p}}_{\mathrm{u}}, \hat{\boldsymbol{v}} \xrightarrow{\text{SPF}} \boldsymbol{c}, \boldsymbol{s} \xrightarrow{\text{computations}} \ell = \cdots + \lambda \left\| \boldsymbol{\phi}(\boldsymbol{c}, \boldsymbol{s}) \right\|_2^2$
  - ! Differentiating through SPF is (very) complicated

APF for amortized OPF: Differentiating through PFE+

Long-term mission

Treating $\mathrm{Net}_{\boldsymbol{\theta}}(\cdot)$ as anticipating $\boldsymbol{c}$, we have $\boldsymbol{d} \xrightarrow{\mathrm{Net}_{\boldsymbol{\theta}}(\cdot)} \boldsymbol{c} \xrightarrow{\text{solve PFE+}} \boldsymbol{x} \xrightarrow{\text{computations}} \ell(\boldsymbol{c}, \boldsymbol{x}(\boldsymbol{c}))$ for any appropriate loss ℓ. Backpropagation simply follows $\mathrm{d}\ell = \big(\partial_{\boldsymbol{c}}\ell + \partial_{\boldsymbol{x}}\ell\, \partial_{\boldsymbol{c}}\boldsymbol{x}\big)\partial_{\boldsymbol{\theta}}\boldsymbol{c}\, \mathrm{d}\boldsymbol{\theta}$.

APF for amortized OPF: Differentiating through PFE+

Long-term mission

Treating $\mathrm{Net}_{\boldsymbol{\theta}}(\cdot)$ as anticipating $\boldsymbol{c}$, we have $\boldsymbol{d} \xrightarrow{\mathrm{Net}_{\boldsymbol{\theta}}(\cdot)} \boldsymbol{c} \xrightarrow{\text{solve PFE+}} \boldsymbol{x} \xrightarrow{\text{computations}} \ell(\boldsymbol{c}, \boldsymbol{x}(\boldsymbol{c}))$ for any appropriate loss ℓ. Backpropagation simply follows $\mathrm{d}\ell = \big(\partial_{\boldsymbol{c}}\ell + \partial_{\boldsymbol{x}}\ell\, \partial_{\boldsymbol{c}}\boldsymbol{x}\big)\partial_{\boldsymbol{\theta}}\boldsymbol{c}\, \mathrm{d}\boldsymbol{\theta}$.

- $\partial_{\boldsymbol{\theta}}\boldsymbol{c}$, $\partial_{\boldsymbol{c}}\ell(\cdot)$, and $\partial_{\boldsymbol{x}}\ell(\cdot)$ are trivial for modern autodiff engines

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☞ Contribution: How to differentiate through PFE+
  - Computing the backward APF Jacobian $\boldsymbol{H}(\cdot) := \partial_{\boldsymbol{c}}\boldsymbol{x}(\cdot)$
  - Computing the backward APF gradient $\boldsymbol{g}(\cdot)$, *i.e.*, $\boldsymbol{g}^{\mathsf{T}}(\cdot) \equiv \partial_{\boldsymbol{x}}\ell(\cdot)\,\boldsymbol{H}(\cdot)$

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Computing the backward APF Jacobian (§4.4.1)

Applying the implicit function theorem at an APF point $(\boldsymbol{c}, \boldsymbol{x})$, the backward APF Jacobian is the solution to $\boldsymbol{J}(\boldsymbol{x})\,\boldsymbol{H}(\boldsymbol{x}) = \mathrm{Diag}(\boldsymbol{C}_{\mathrm{u}}, \boldsymbol{C}_{\mathrm{u}})$.

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Computing the backward APF gradient (§4.4.2)

Jacobian-vector product (JVP): solve for $\boldsymbol{H}$, then $\boldsymbol{g} = \boldsymbol{H}^{\mathsf{T}}\, \nabla_{\boldsymbol{x}}\ell$

Vector-Jacobian product (VJP): solve for $\boldsymbol{u}$ from $\boldsymbol{J}^{\mathsf{T}}\boldsymbol{u} = \nabla_{\boldsymbol{x}}\ell$, then $\boldsymbol{g} = \mathrm{Diag}\big(\boldsymbol{C}_{\mathrm{u}}^{\mathsf{T}}, \boldsymbol{C}_{\mathrm{u}}^{\mathsf{T}}\big)\, \boldsymbol{u}$

APF for amortized OPF: Differentiating through PFE+

Prefer VJP to JVP

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Gradients are of ℓ in Equation (4.24). Wall times are averaged over 100 runs, based on Intel Core i7-10750H w/ 16GB RAM.

System	$\left\lVert \boldsymbol{g}_{\mathsf{jvp}} - \boldsymbol{g}_{\mathsf{vjp}} \right\rVert_{\infty}$	JVP time [s]	VJP time [s]
ACT500	3.9968×10^{-14}	3.3319×10^{-2}	4.4259×10^{-3}
RTE1888	7.3630×10^{-13}	5.4983×10^{-1}	1.8315×10^{-2}
POL3375	5.5111×10^{-12}	2.4872	3.4472×10^{-2}

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- No need to form $\boldsymbol{H}$, which is $2N \times 2U$ and dense
- With mild assumptions (§A.3.2),
  - JVP is $\mathcal{O}\left(\frac{32}{2} U N^3\right)$ flops
  - VJP is $\mathcal{O}\left(\frac{16}{3} N^3\right)$ flops

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Open problems and working ideas

1. How to solve a batch of PFE+ instances with GPU acceleration?
   - Cast as nonlinear least-squares, then use JAXOpt [42]
2. How to quantify model uncertainty?
   - Extend conformal prediction [43] into a multivariate regression case

APF in summary

- Formulation: finding power-flow feasible $(\boldsymbol{c}, \boldsymbol{s})$ for anticipated grid conditions $(\boldsymbol{Y}, \boldsymbol{d})$, using preceding snapshot values $(\widetilde{\boldsymbol{c}}, \widetilde{\boldsymbol{s}})$
  - Anticipate $\boldsymbol{c}$ by solving a convex program extended economic dispatch (ED+)
  - Compute corresponding $\boldsymbol{s}$ by solving APF equations (PFE+)
- Computation: amply handled by existing and readily available tools
  - SeDuMi and SDPT3 for ED+; Levenberg-Marquardt and Powell hybrid for PFE+
  - Sub-second run times on 3374-bus, 4161-branch, 596-generator portion of Polish grid
  - Quadrimodal effect of ED+ $\Longrightarrow$ big-μ trick for easily finding four APF points
- Applications
  1. Warm-starting OPF solvers $\Longrightarrow$ a crude method for finding multiple OPF solutions
  2. Differentiating through PFE+ $\Longrightarrow$ power flow equations as a layer in amortized OPF

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Anticipatory Power Flow

Formulation, Computation, and Applications

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