1. Basic properties of the integers

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In this lesson and onwards, we consider $\mathbb{Z}$ to be the set of integers numbers, whereas $\mathbb{Z}^+$ is the set of strictly positive integers numbers.

Definition 1.1. Let $a, b \in \mathbb{Z}$, with $a \neq 0$, then a is a divisor of b if there is an integer c such that $a \cdot c = b$. We denote this by $a \mid b$.

Remark 1. If a does not divide b, we write $a \nmid b$.

Theorem 1.1. *Let $a, b \in \mathbb{Z} \setminus \{0\}$, there is a unique positive integer d, called the **greatest common divisor of** a **and** b, satisfying*

1. *$d \mid a$ and $d \mid b$.*
2. *If $e \mid a$ and $e \mid b$ then $e \mid d$.*

Remark 2. If d is the greatest common divisor of a and b, we write $d = (a, b)$. In particular, if $(a, b) = 1$, then a and b are called coprimes.

Question 1. Why does (a, b) always exist for $a, b \in \mathbb{Z} \setminus \{0\}$?