1. Quotient Groups and Homomorphisms

1.1. Cosets and counting

Let $(G, \cdot)$ be a group.

Definition 1.1. Let $H \leq G$ and $a, b \in G$. Define $\cong_r$ over G by

$$a \cong_r b \iff ab^{-1} \in H.$$

Whenever $a \cong_r b$ we say a right is congruent to b module H. Define $\cong_l$ over G by

$$a \cong_l b \iff a^{-1}b \in H.$$

Whenever $a \cong_l b$ we say a is left congruent to b module H.

Remark 1.1.1. If G is Abelian,

$$ab^{-1} \in H \iff a^{-1}b \in H$$

for any $a, b \in G$.

Theorem 1.2. *Let $H \leq G$.*

Remark 1.2.1. Ha is a right coset of H in G

Proof. □

Corollary 1.3.

Definition 1.4 (Index)**.**

Theorem 1.5. *If K, H, G are groups with $k < H < G$ then*

$$[G : k] = [G : H] \cdot [H : k]$$

If any two of these indices are finite, so is the third.

Proof. □

Corollary 1.6 (Lagrange's theorem)**.** *If $H \leq G$, then $|G| = [G : H]|H|$. In particular, if G is finite, the order of any $a \in G$ divides $|G|$.*

Proof. □

Theorem 1.7.

Proposition 1.8.

Proof. $\square$

Proposition 1.9.