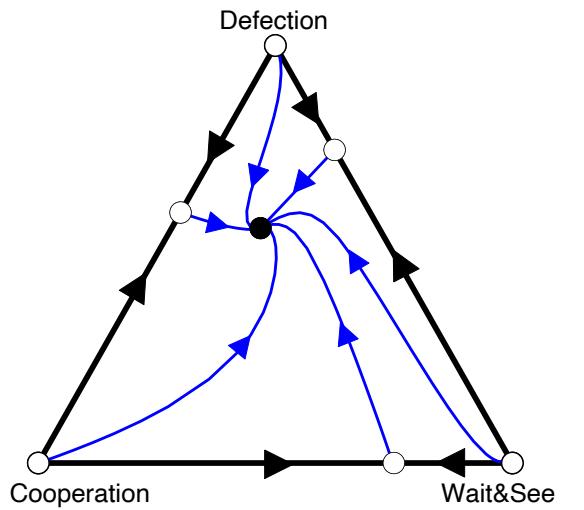


An overview

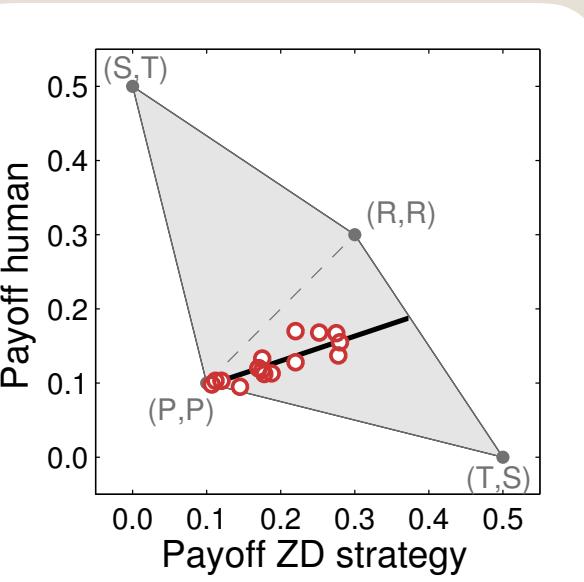
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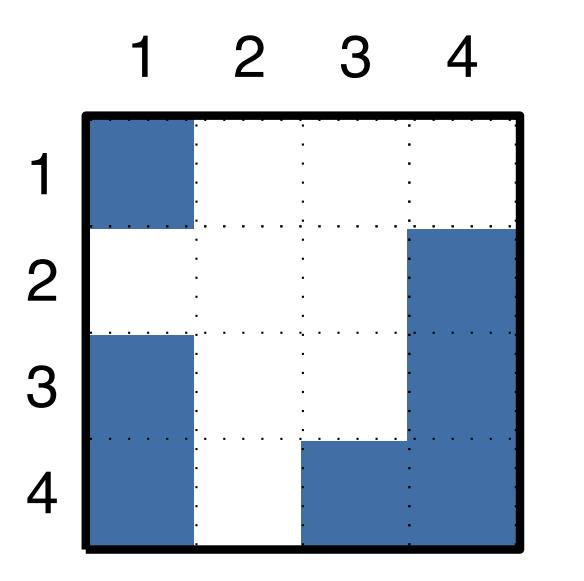
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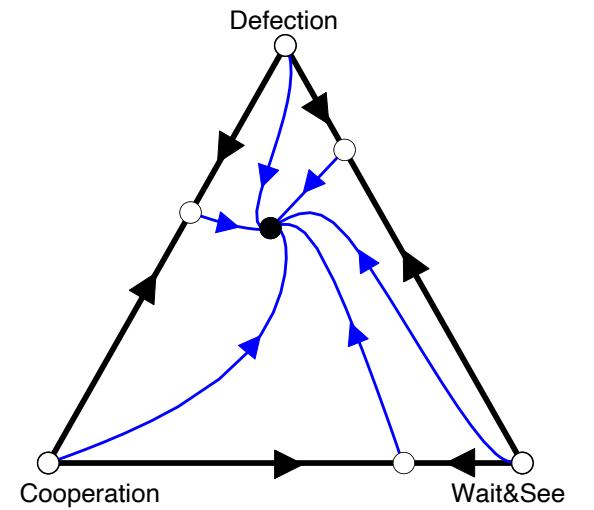
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An overview

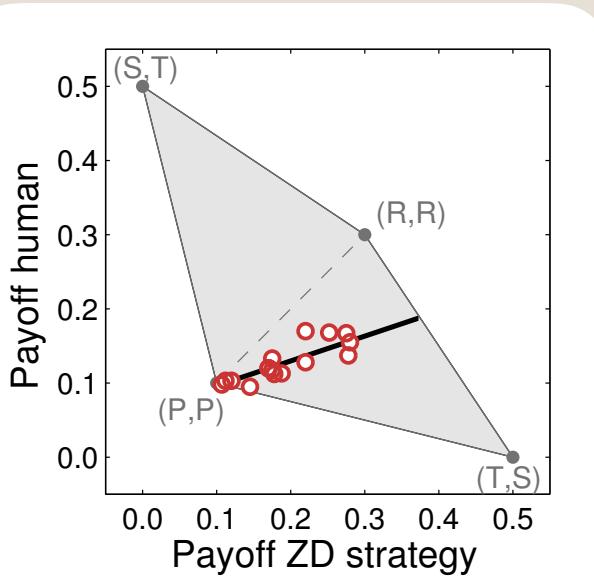
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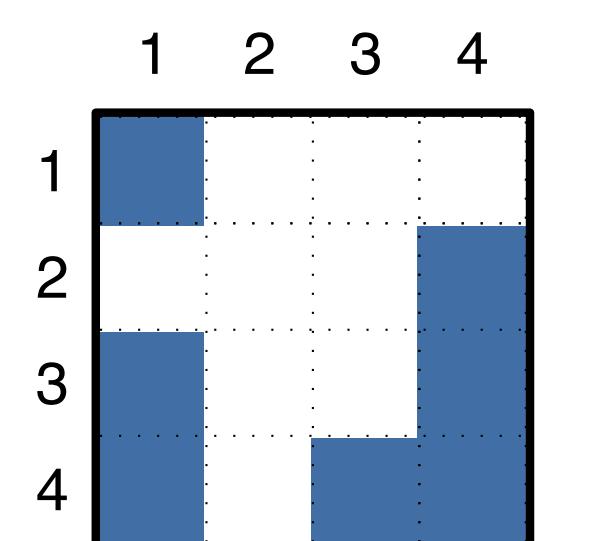
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The role of memory; the effect of changing environments; the impact of inequality



The impact of memory: Results on memory-1

Remark 4.1. Robustness of results on direct reciprocity

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Suppose I play a memory-1 strategy \mathbf{p} , and you play an arbitrary memory-k strategy \mathbf{q} , and suppose the two of us get a payoff of (π_1, π_2) as a result.

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Corollary 4.3. Checking for Nash

If \mathbf{p} is a memory-1 strategy, then (\mathbf{p}, \mathbf{p}) is a Nash equilibrium if and only if there is no profitable deviation towards another memory-1 strategy.

The impact of memory: More is different

Theorem 4.4. Checking for Nash, part II [Akin 2015]

Suppose \mathbf{p} is a *nice* memory-1 strategy (it is never the first to defect, $p_0 = p_{CC} = 1$). Then (\mathbf{p}, \mathbf{p}) is a Nash equilibrium if and only if neither a deviation towards ALLD, nor a deviation to $\mathbf{q} = (0, 0, 1, 1, 1)$ is profitable.

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The impact of memory: Reactive-n strategies

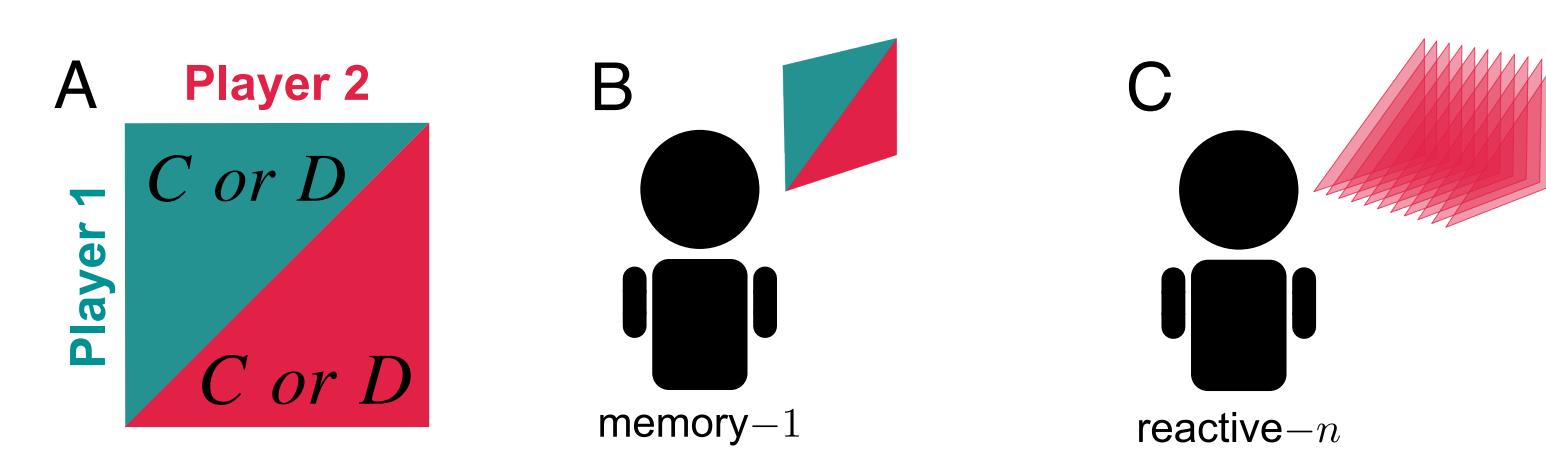
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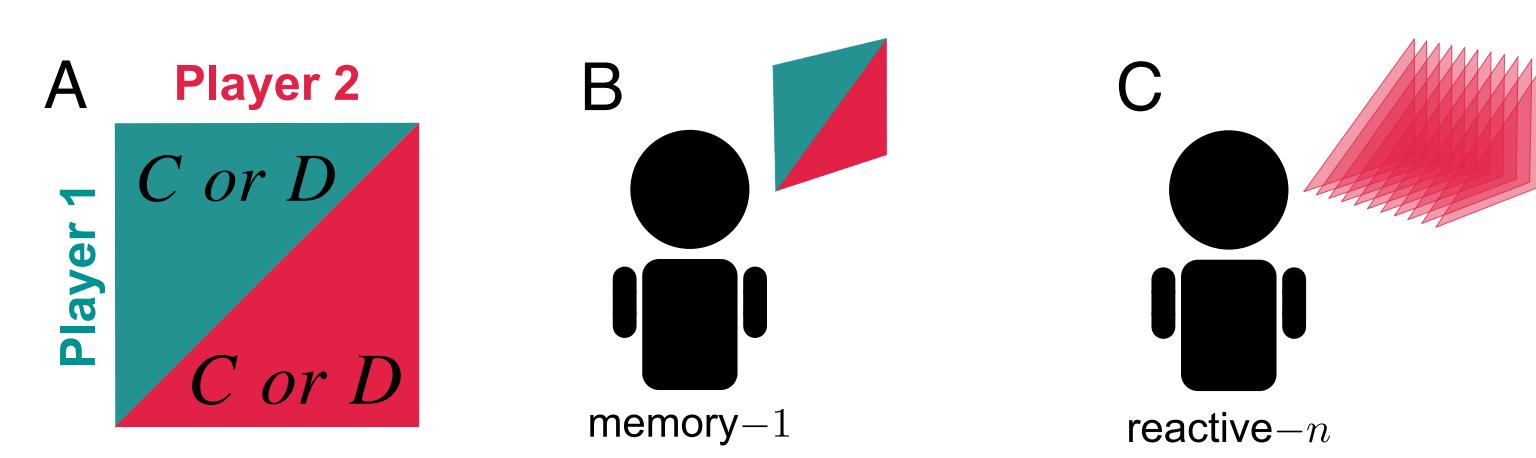
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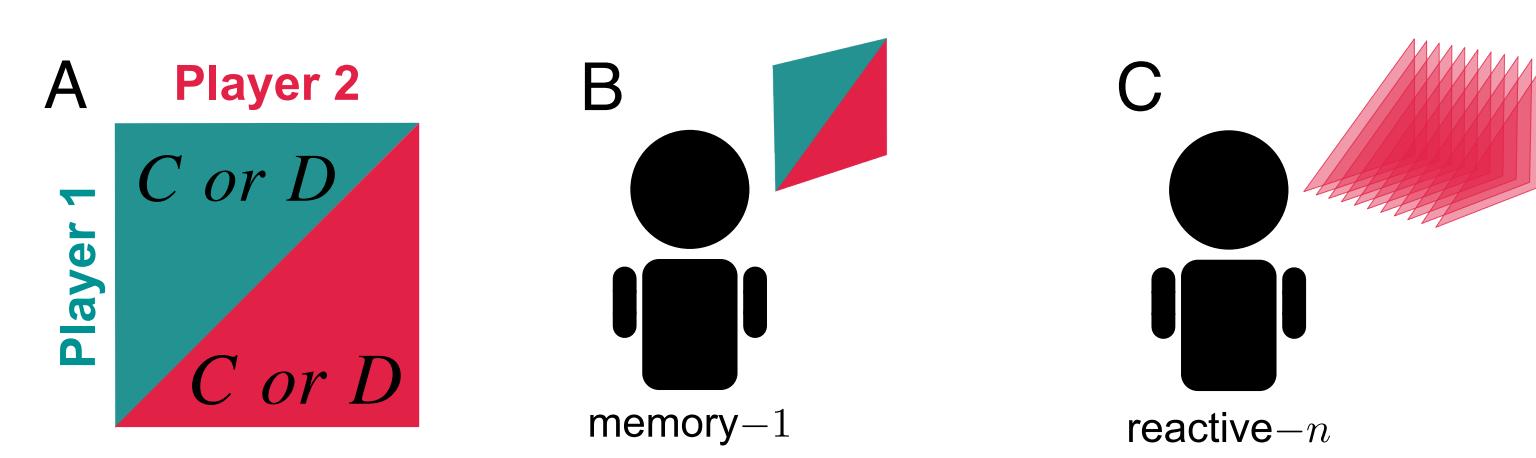
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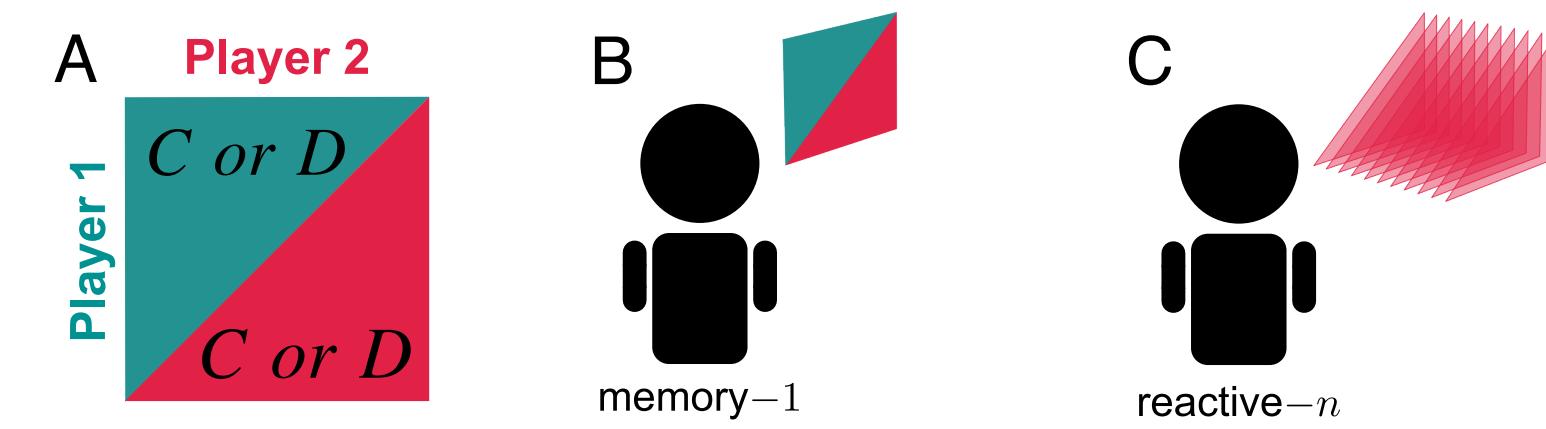
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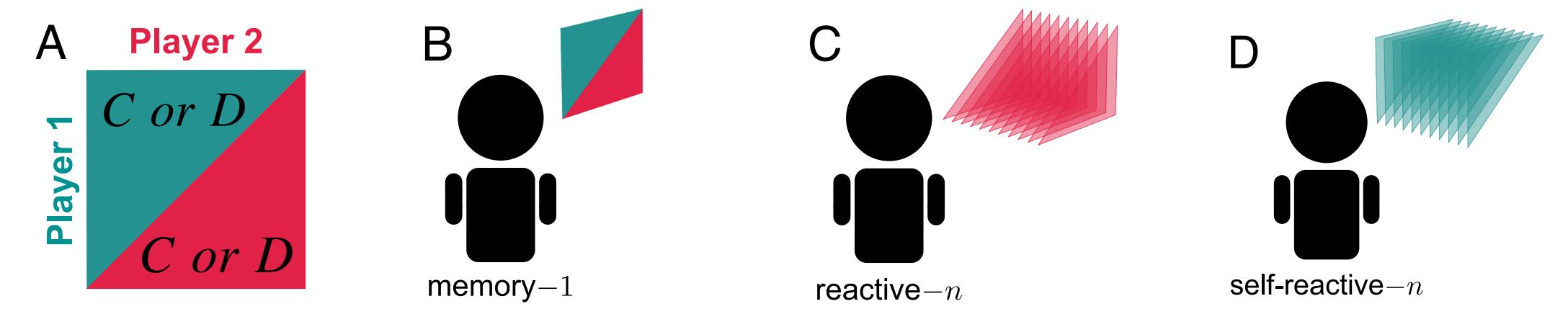
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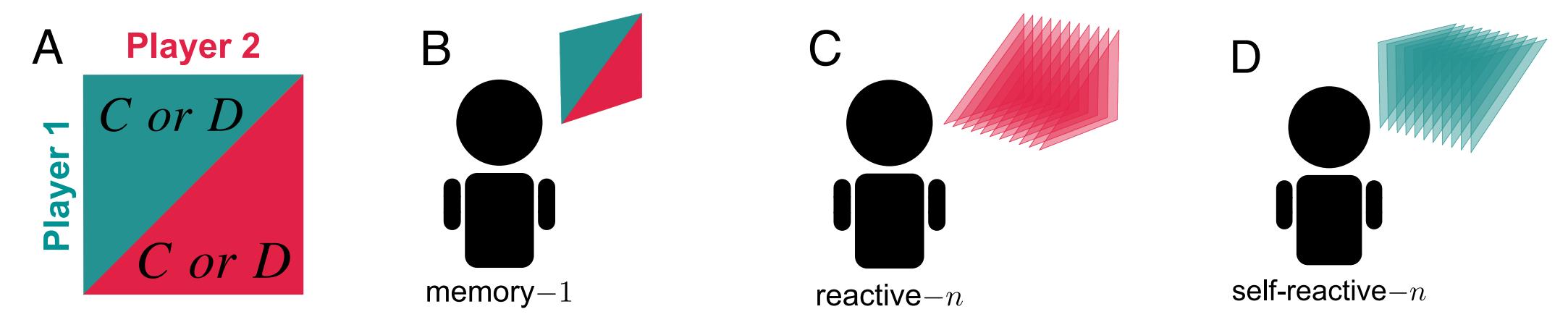
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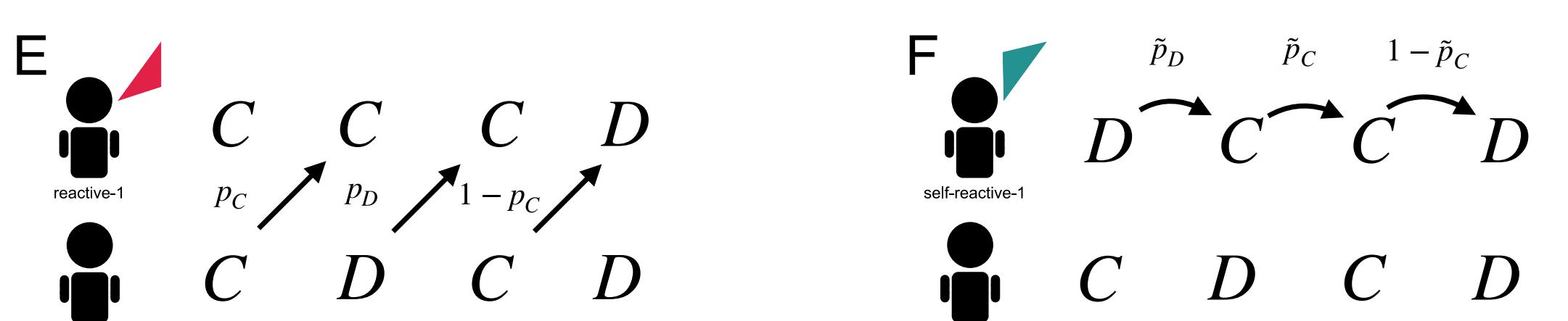
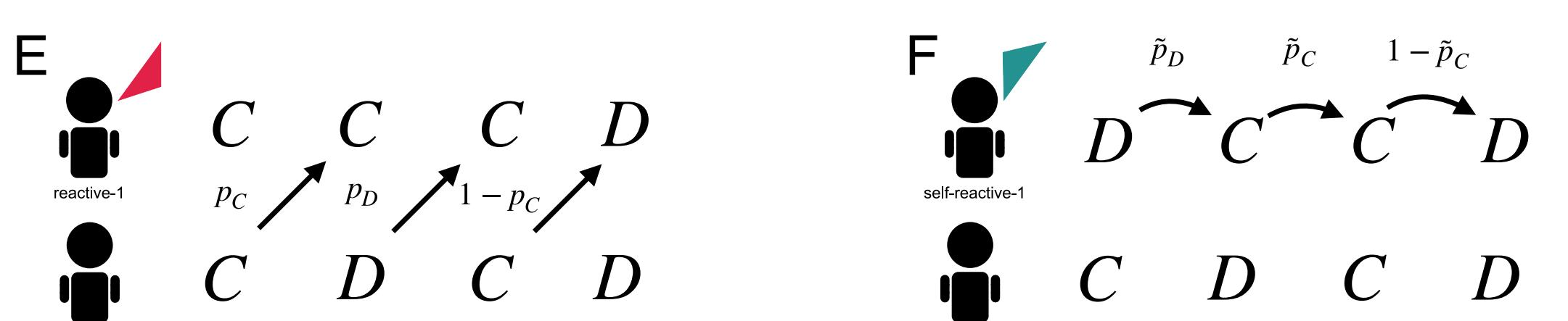
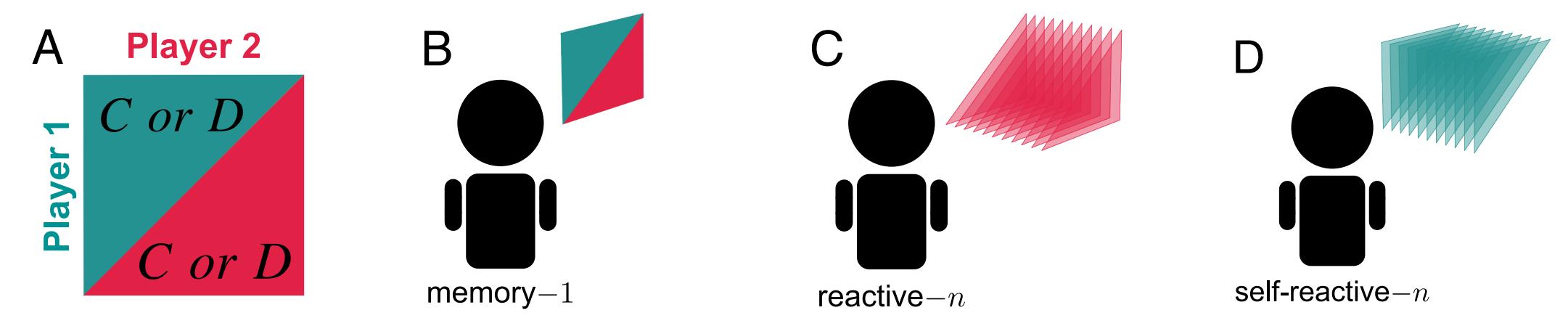
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[In fact, if the game is a donation game, there is a best response $\tilde{\mathbf{q}}$ among the pure self-reactive-(n-1) strategies.]



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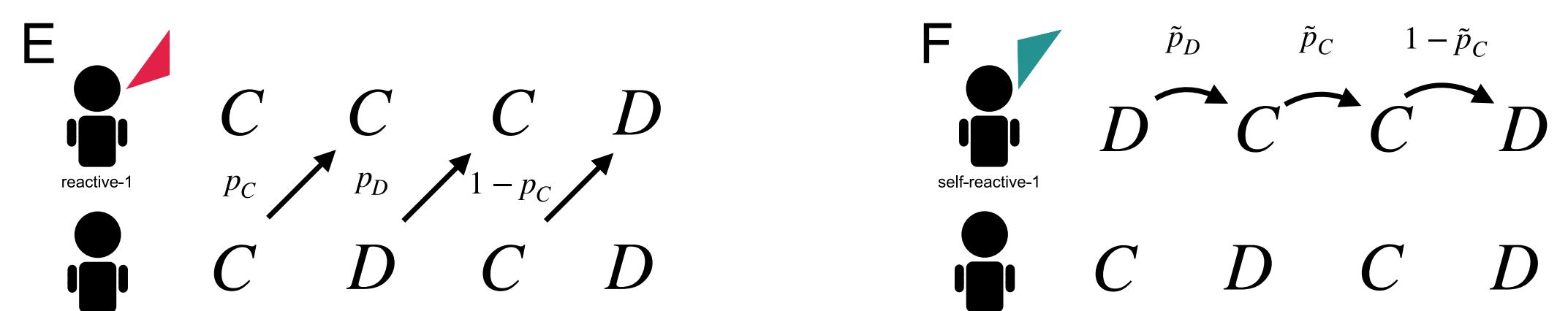
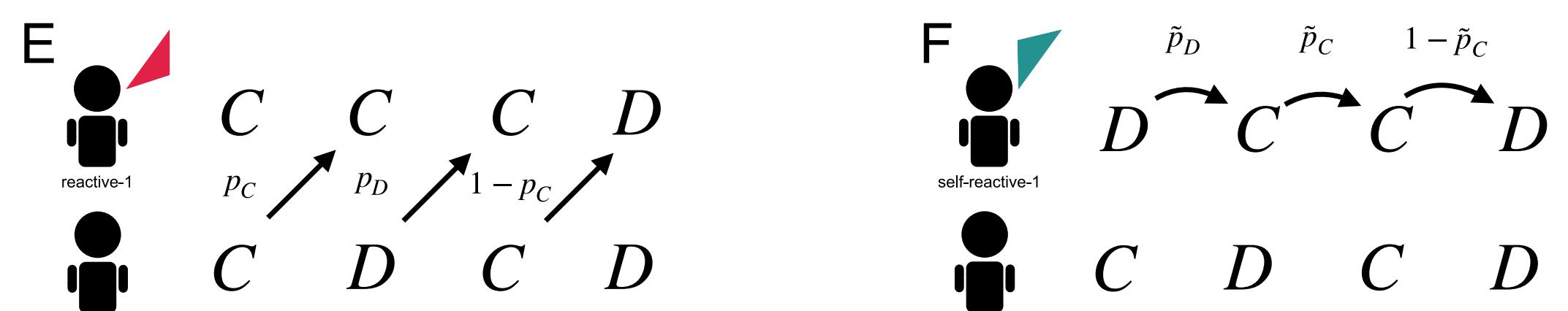
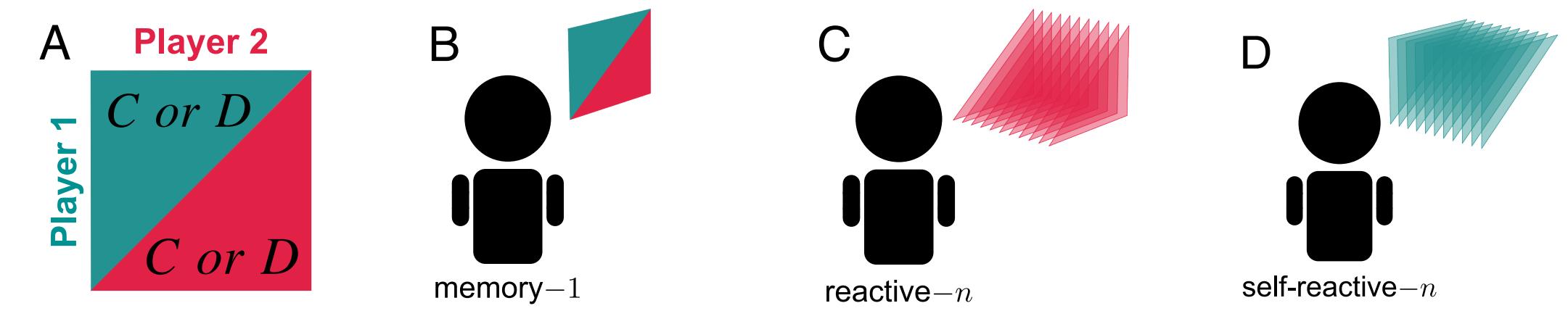
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Theorem 4.9. Best responses against reactive-n players

To any memory-n strategy \mathbf{q} , one can find a best response $\tilde{\mathbf{q}}$ among the pure self-reactive-n strategies.

[In fact, if the game is a donation game, there is a best response $\tilde{\mathbf{q}}$ among the pure self-reactive-(n-1) strategies.]



Theorem 4.10. Stable reactive-2 strategies

A nice reactive-2 strategy $\mathbf{q} = (q_{CC}, q_{CD}, q_{DC}, q_{DD})$ forms a Nash equilibrium (in the repeated donation game) if and only if

$$q_{CC} = 1, \quad \frac{q_{CD} + q_{DC}}{2} \leq 1 - \frac{1}{2} \frac{c}{b}, \quad q_{DD} \leq 1 - \frac{c}{b}.$$

The impact of memory: Reactive-n strategies

Definition 4.7. Reactive-n strategies

A memory-n strategy \mathbf{q} is called reactive, if it only depends on the co-player's last n decisions.

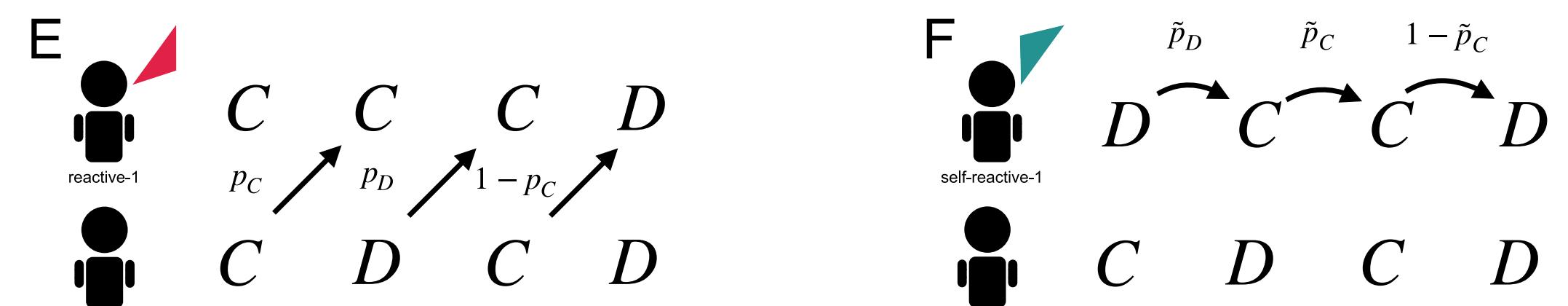
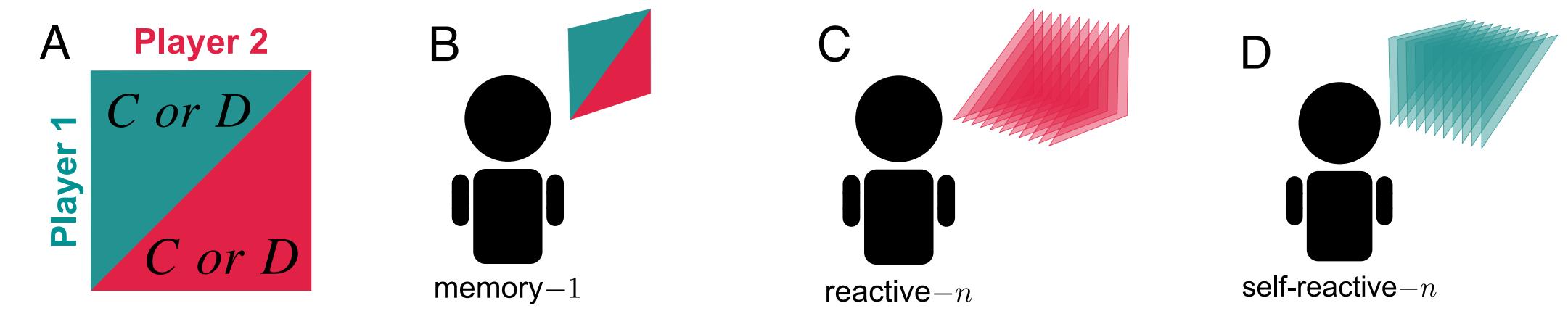
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- For every defection in memory, you reduce your cooperation probability proportionally.
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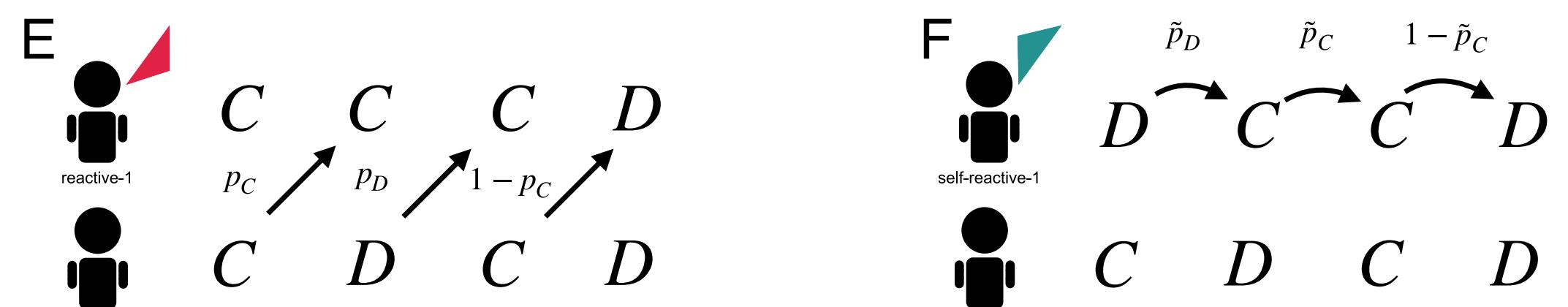
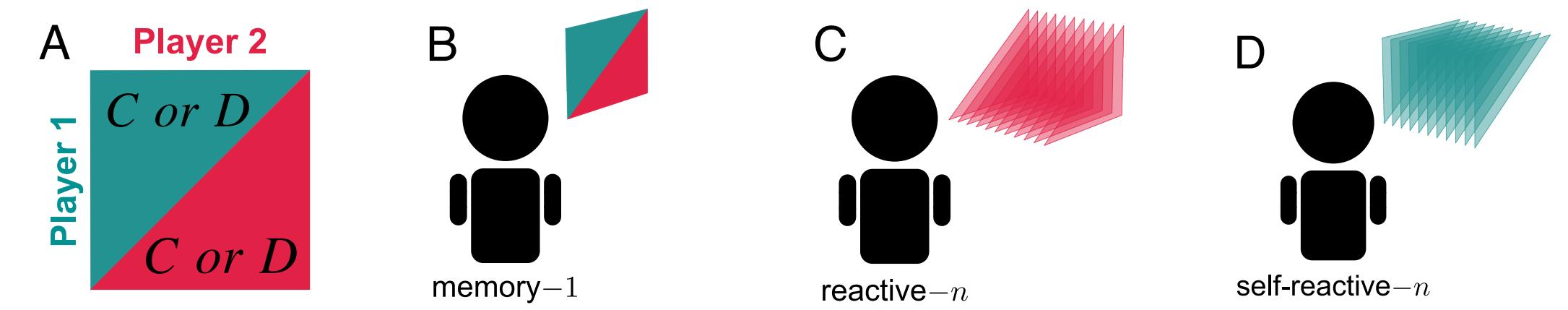
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One can derive similar conditions for reactive-3 (reactive-n).

The impact of memory: Reactive-n strategies

Remark 4.11. Evolution of Reactive-n strategies

- Consider a population of size N
- Players adopt reactive-n strategies
- They play against all other population members to get a payoff
- Strategies that yield a higher payoff are more likely to be imitated.

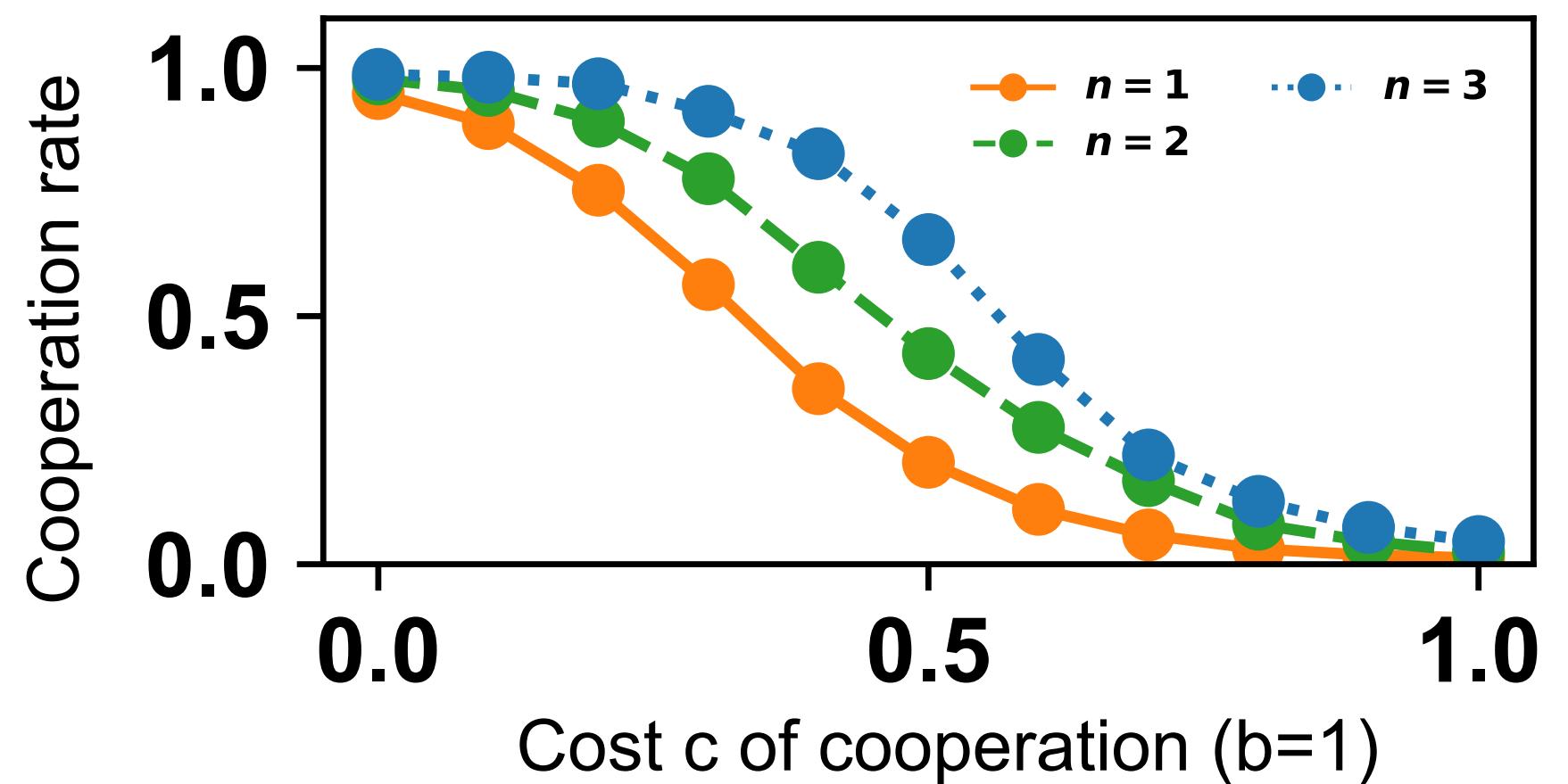
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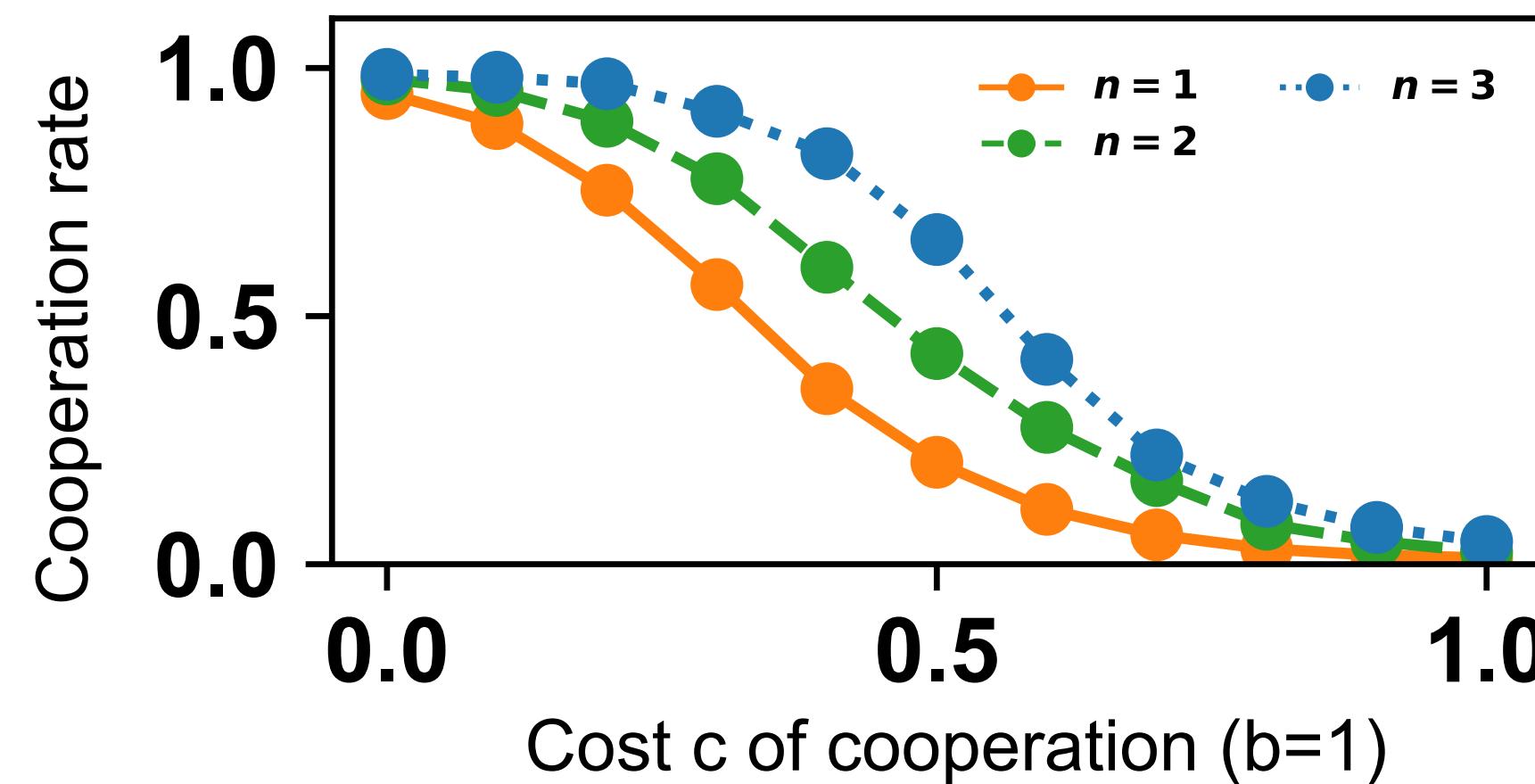
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Remark 4.12. Summary

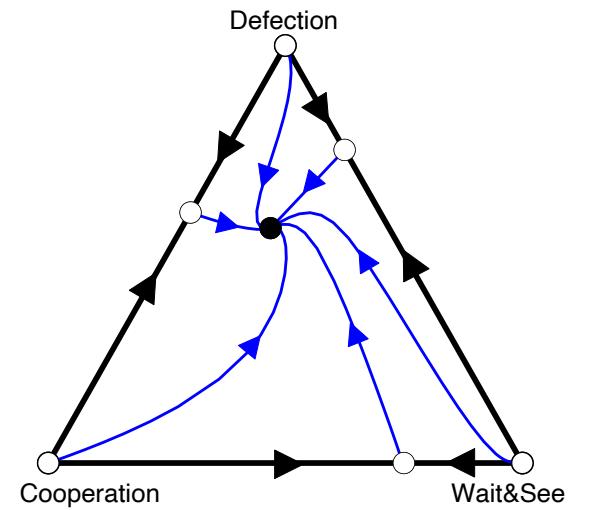
- It is interesting to explore the impact of memory capacities on direct reciprocity.
- A formal analysis can be tricky, because the size of the strategy space quickly explodes
- Still, some analytical results are feasible
- Simulations suggest that more memory helps cooperation.



An overview

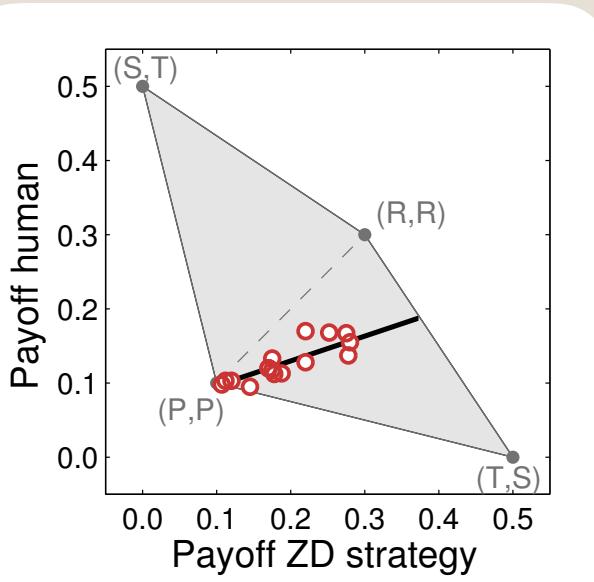
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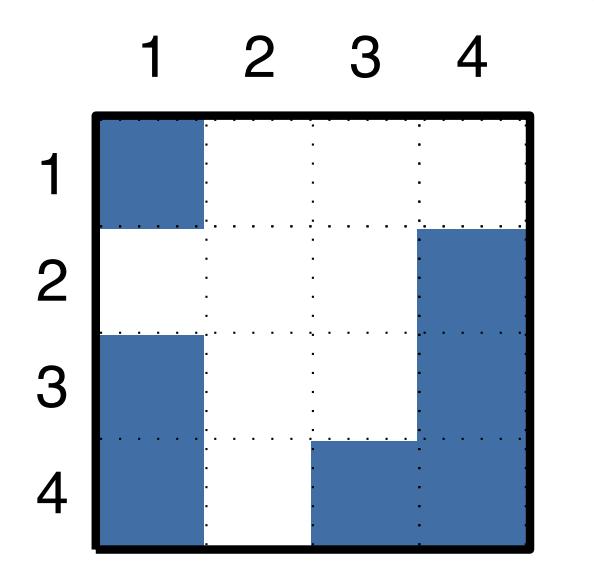
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The role of memory; the effect of changing environments; the impact of inequality



Beyond the repeated prisoner's dilemma

Remark 4.13. Is the repeated prisoner's dilemma the answer?

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	C	D
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D	9	0

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Someone defects →

← Everyone cooperates

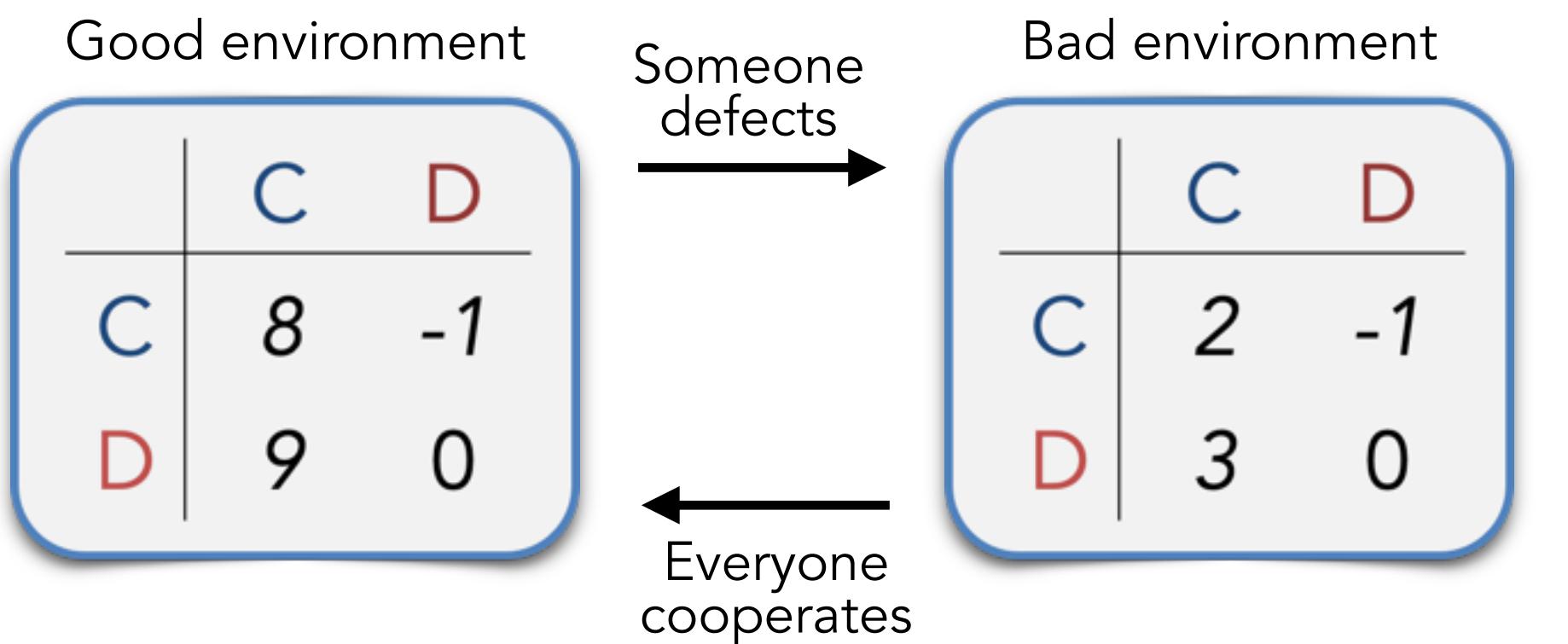
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Diagram illustrating the transition between environments:

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- A player's payoff in the stochastic game is the player's average payoff per round, $\pi_i = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \pi_i(t)$

Evolution of cooperation in stochastic games

Example 4.15. Evolution of cooperation in stochastic games

- We consider a set of N players
- Players are randomly matched in groups of n , and then play the stochastic game against each other.

Evolution of cooperation in stochastic games

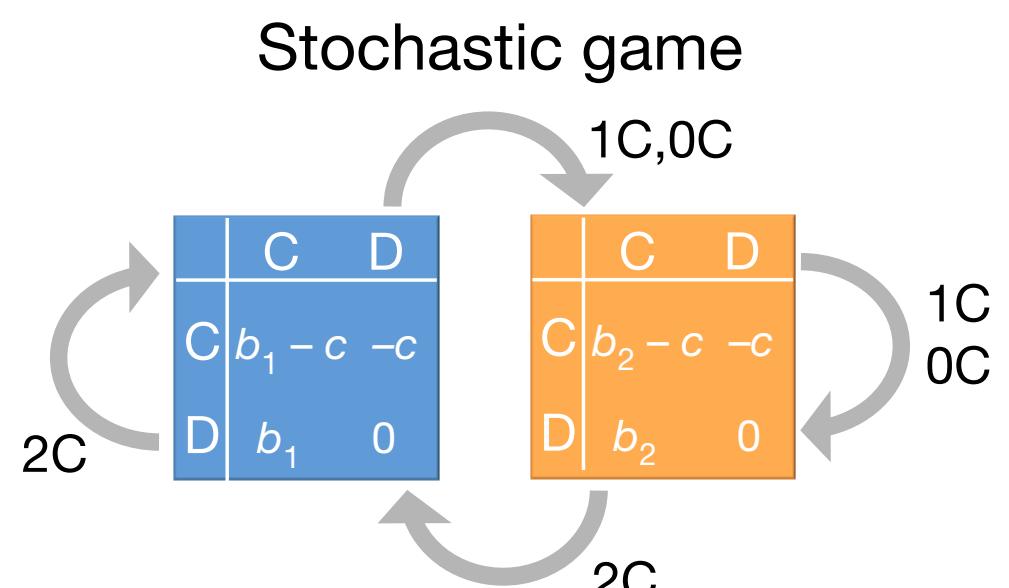
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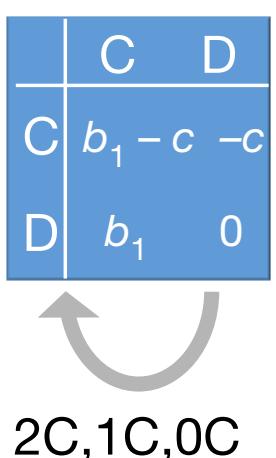
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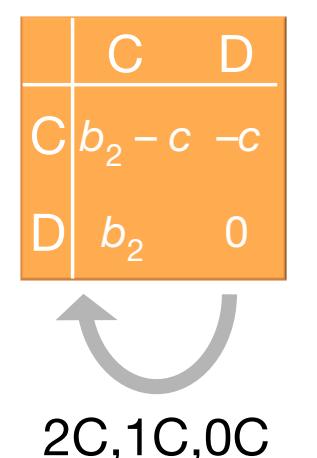
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Only game 1



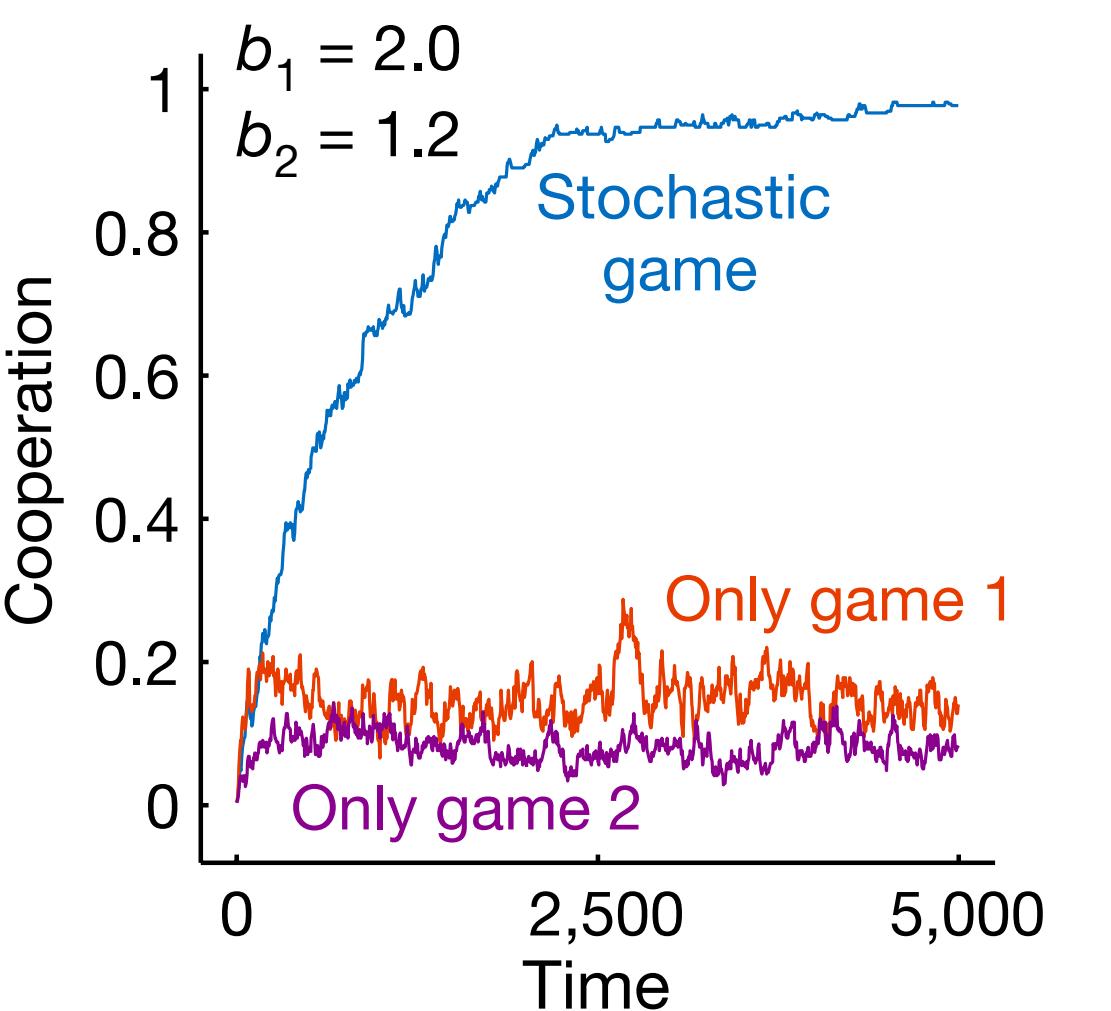
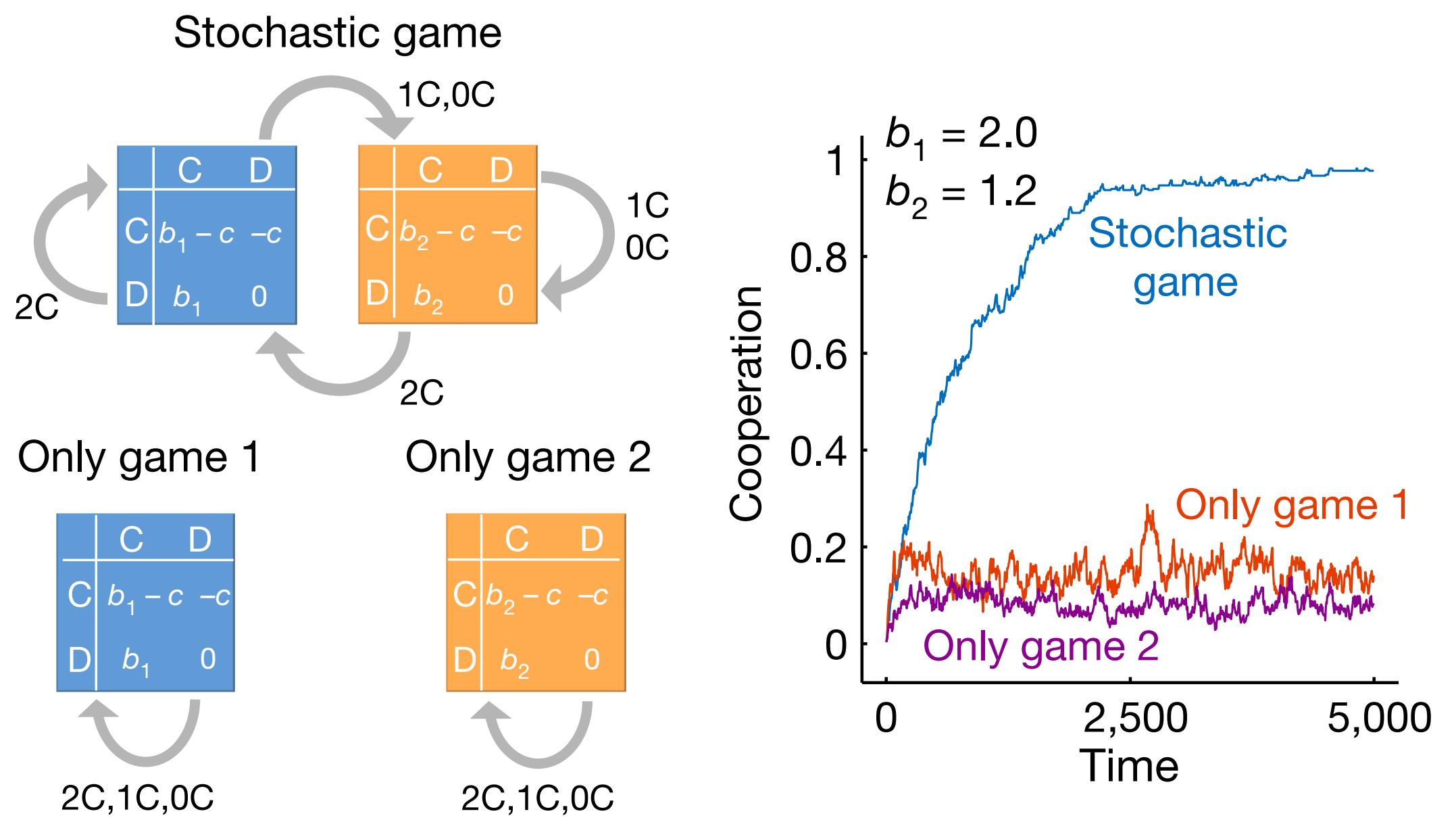
Only game 2



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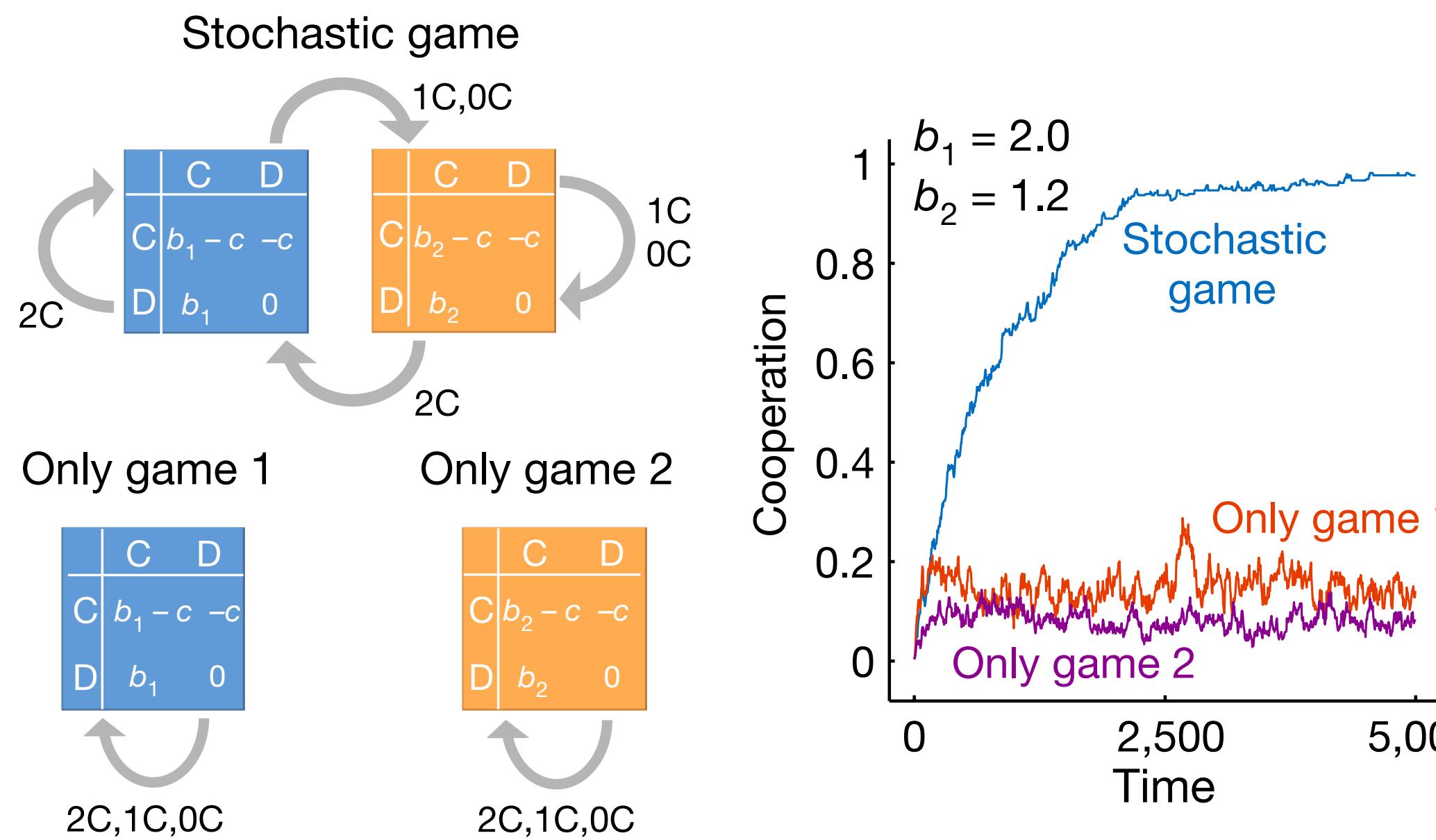
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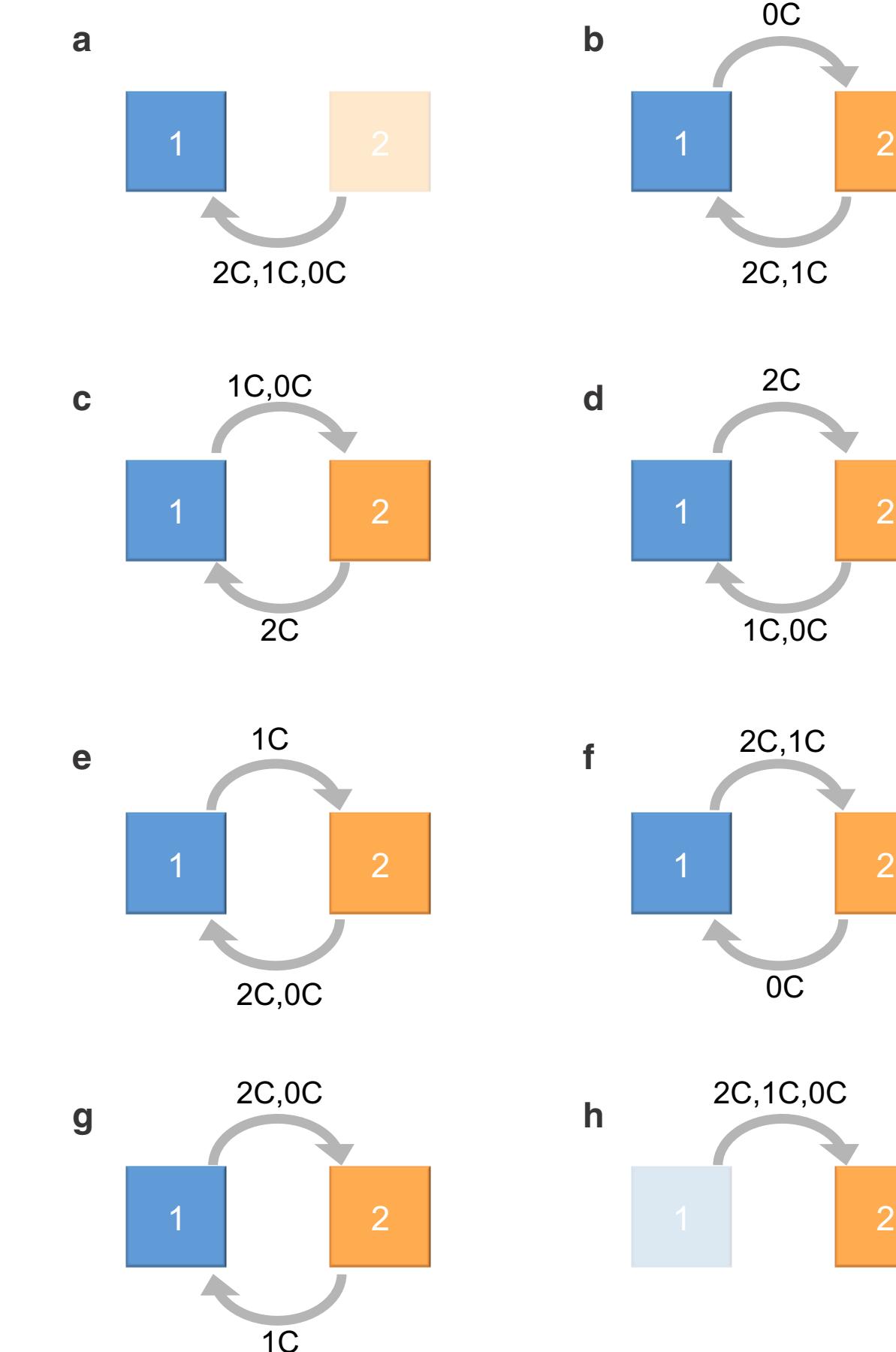
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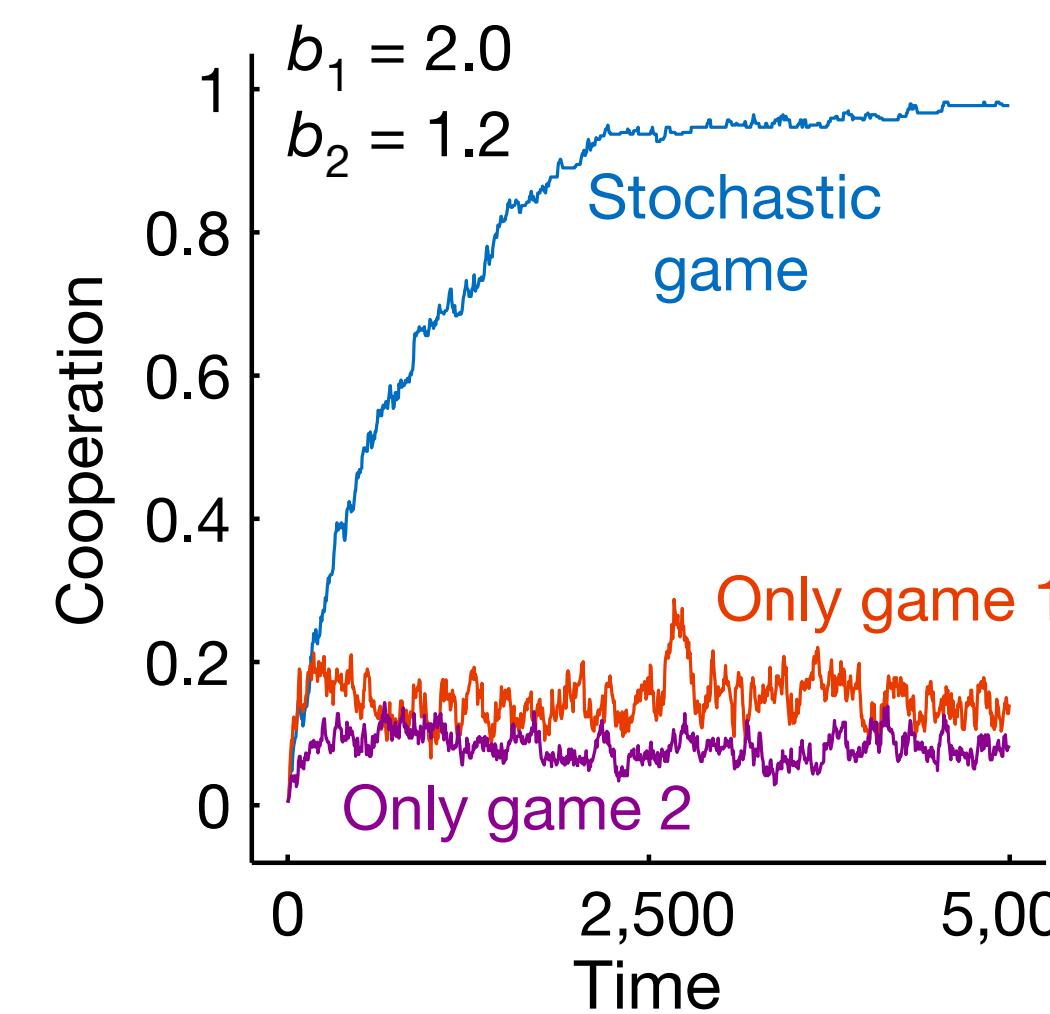
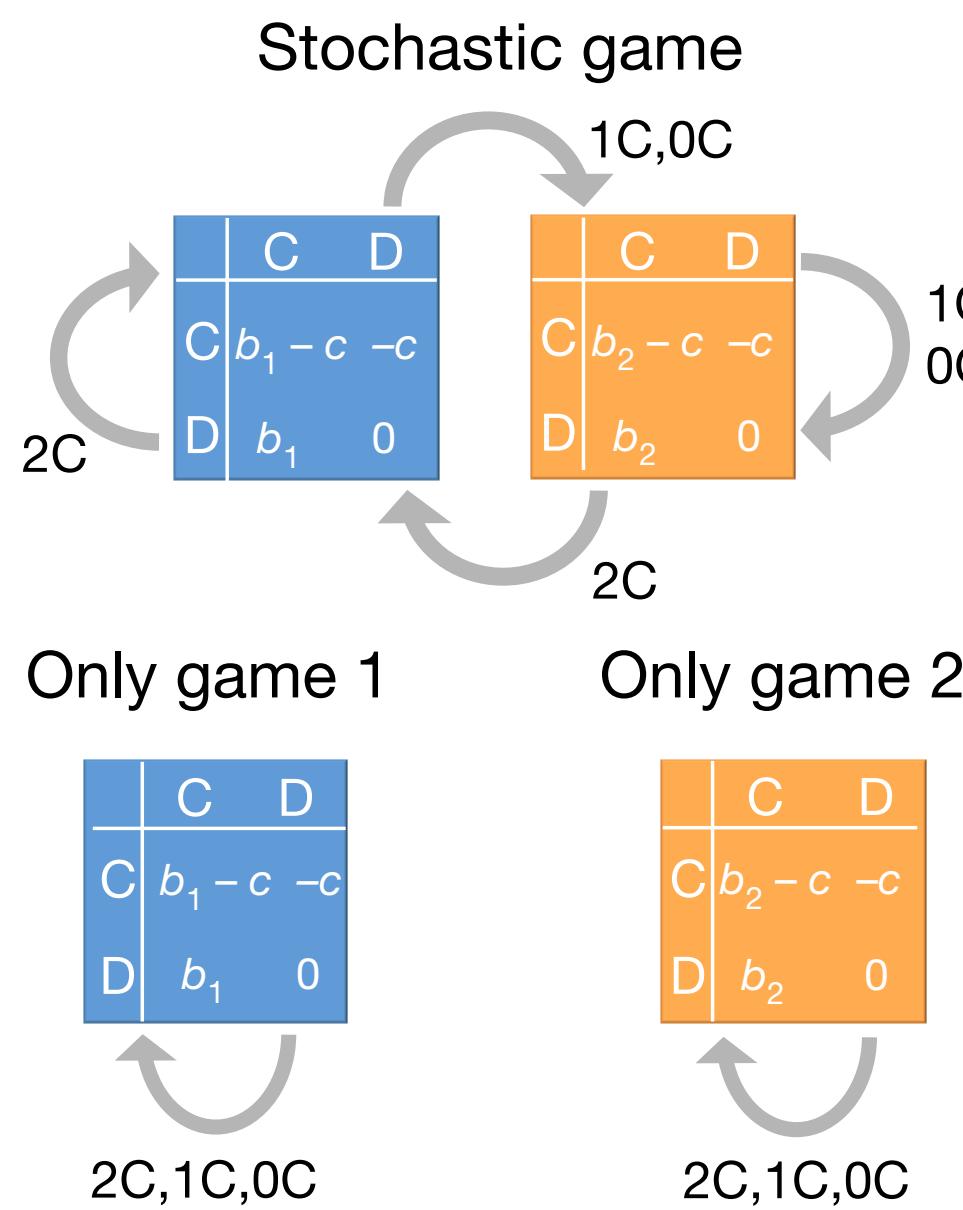
Remark 4.16. Dependence on state transitions



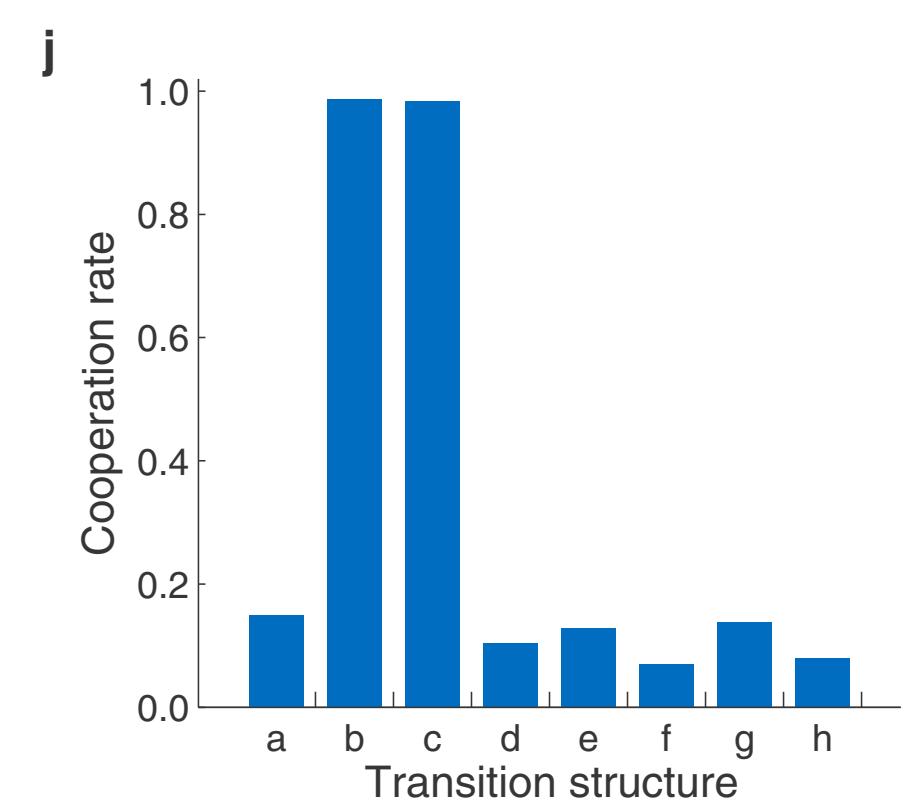
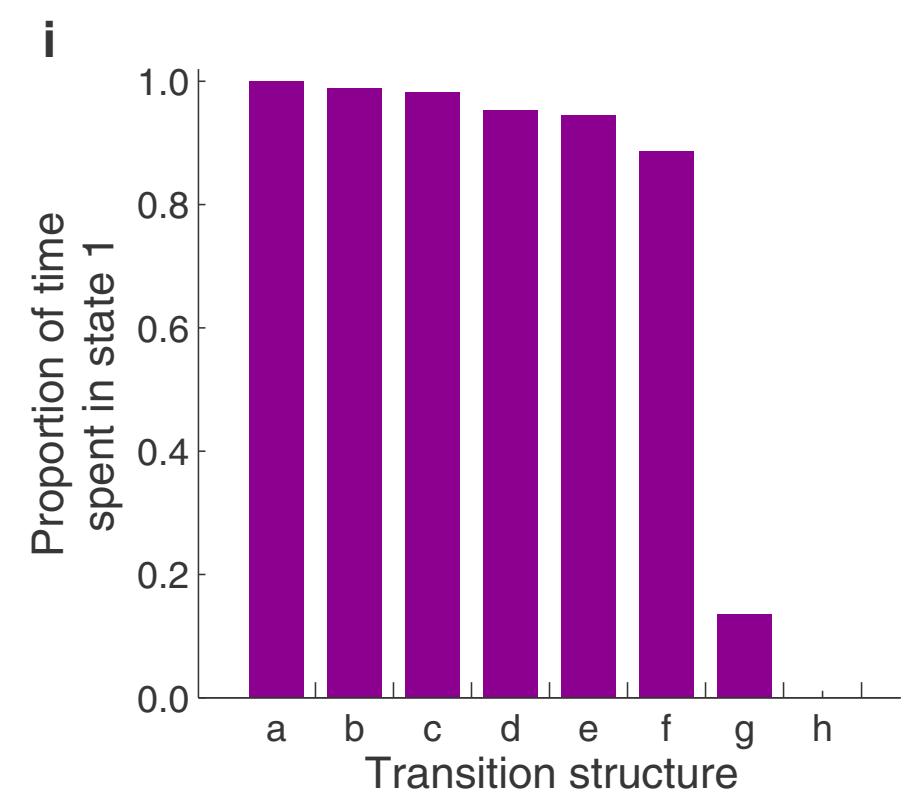
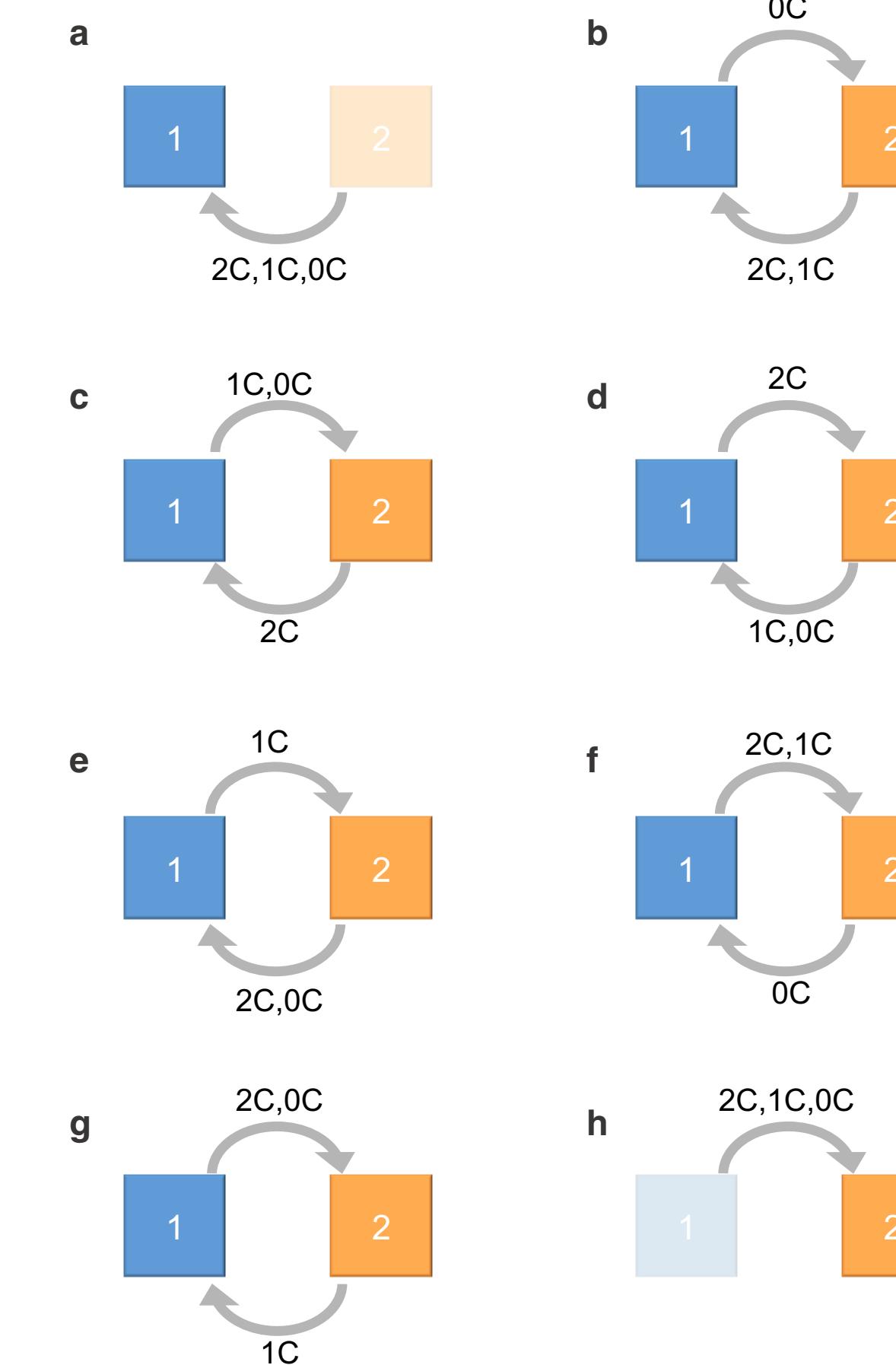
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Example 4.17. Cooperation in larger groups

- Suppose now the game is played in groups of 4 players

Evolution of cooperation in stochastic games

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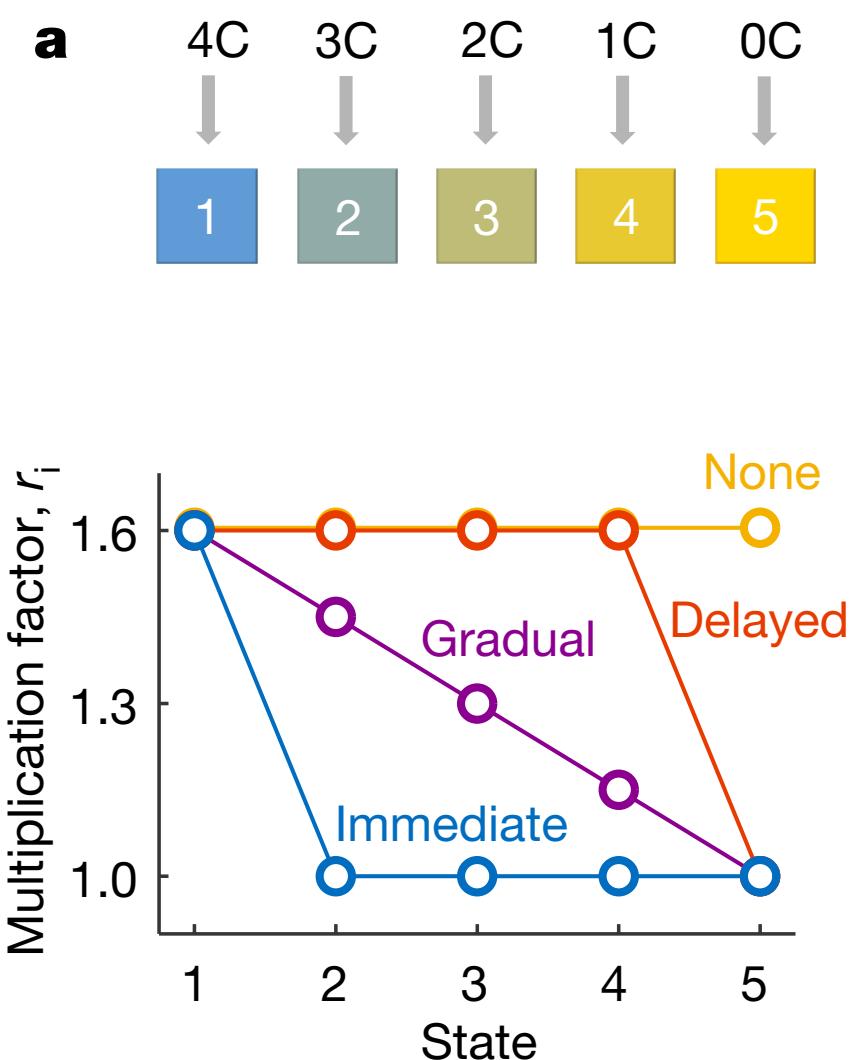
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Evolution of cooperation in stochastic games

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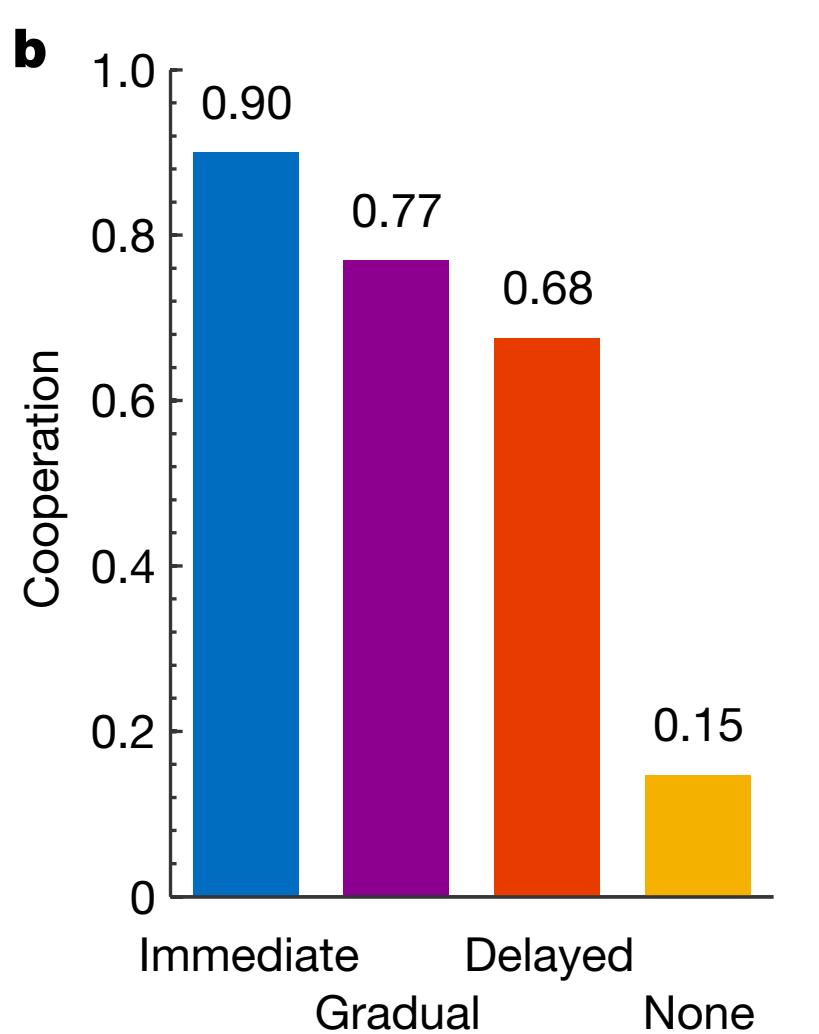
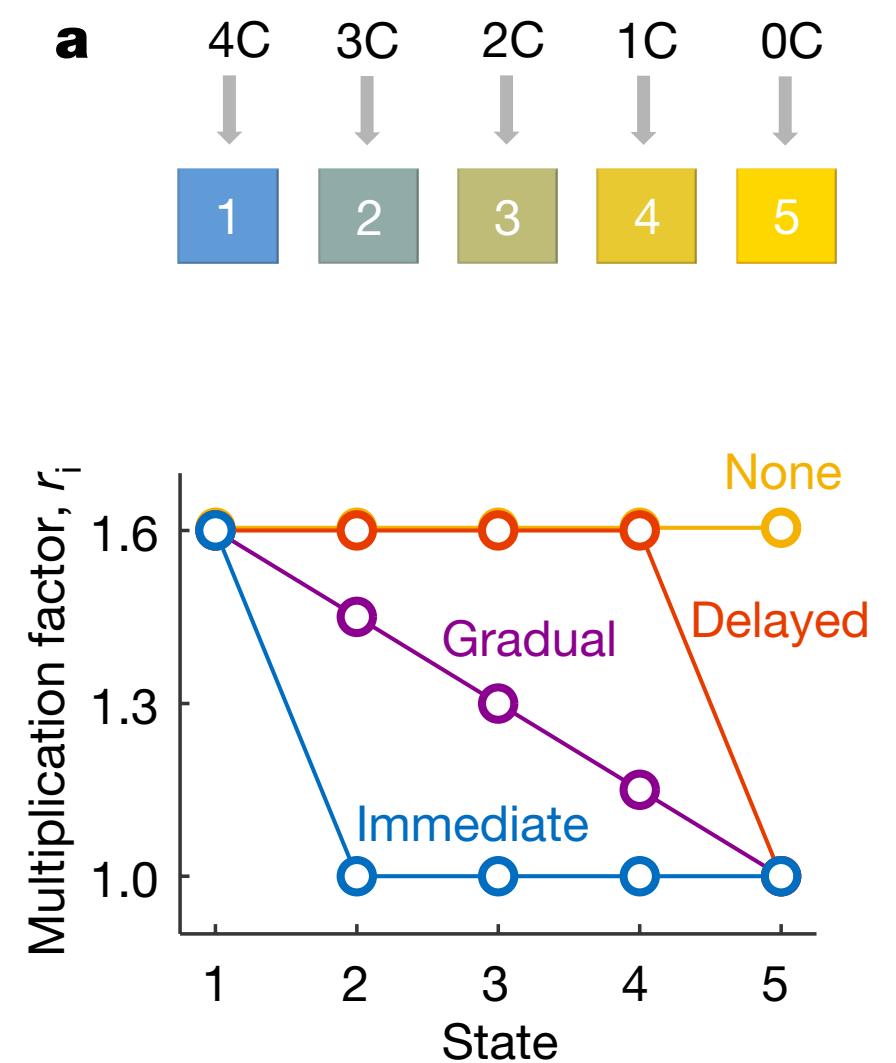
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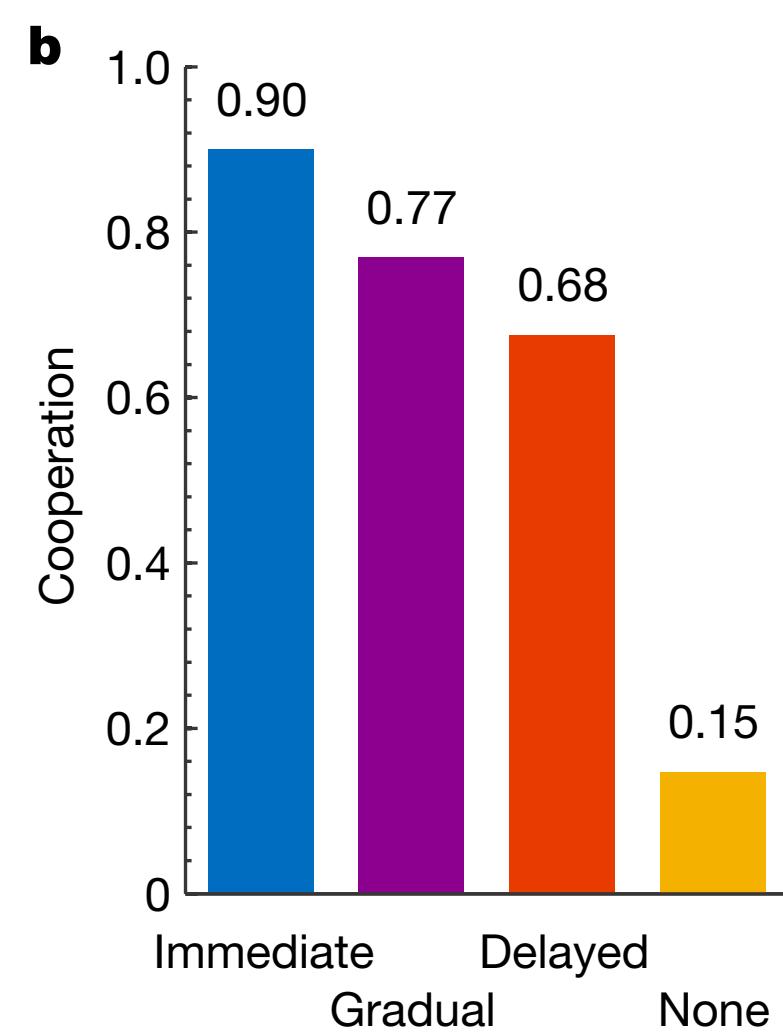
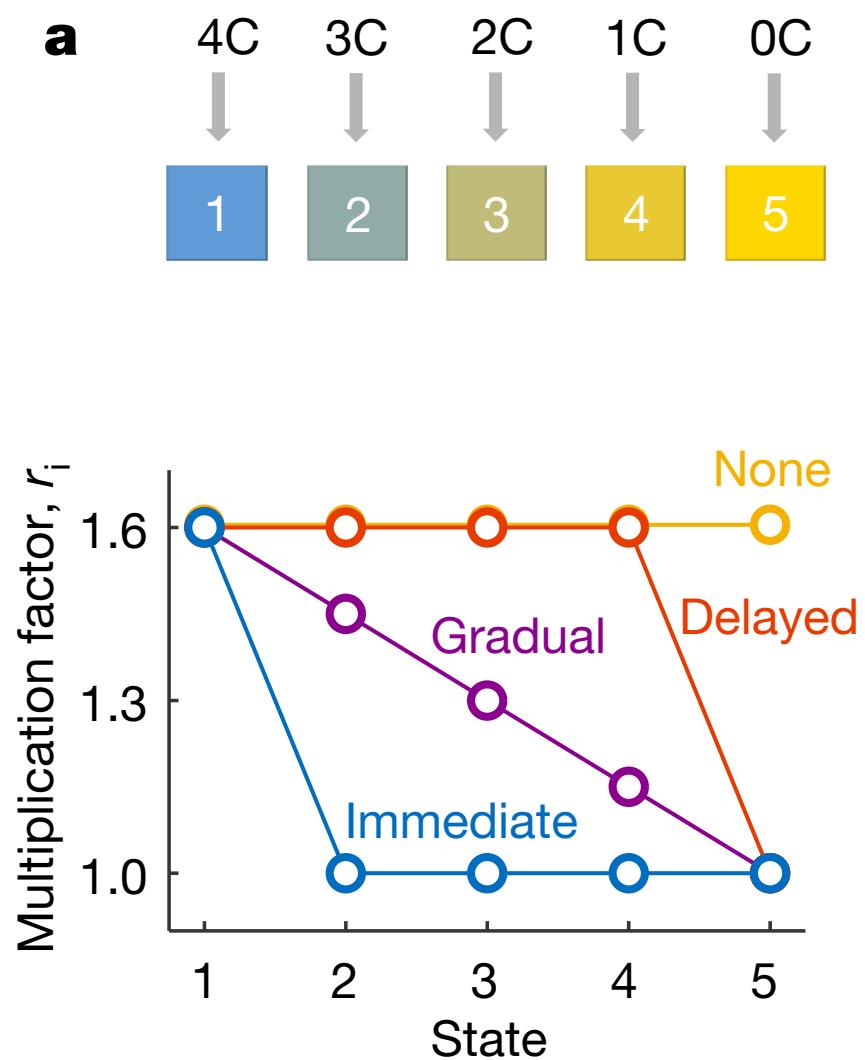
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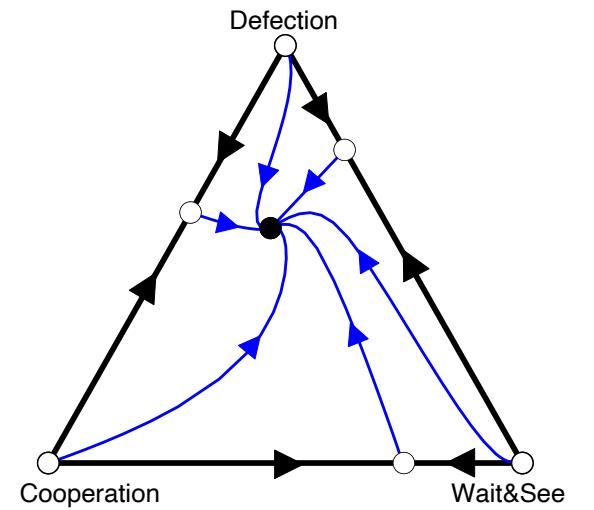
Remark 4.18. Summary

- When individual actions do not only affect payoffs, but also the players' environment, this can favour the evolution of cooperation.
- Cooperation is most favored when defection results in a quick deterioration of the environment, leading players to interact in more unprofitable games.

An overview

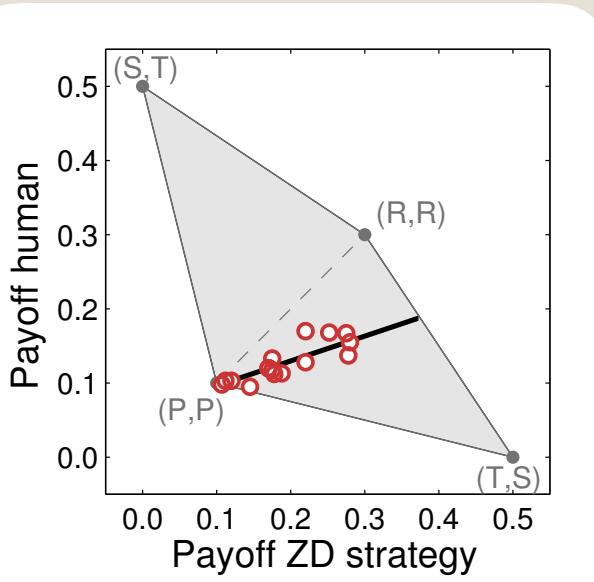
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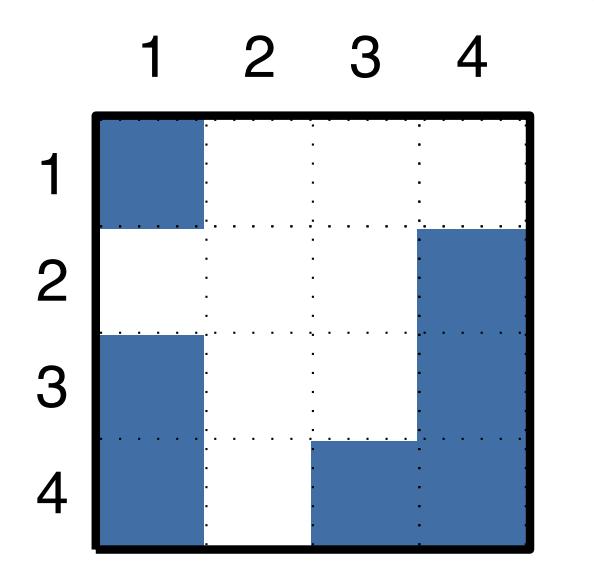
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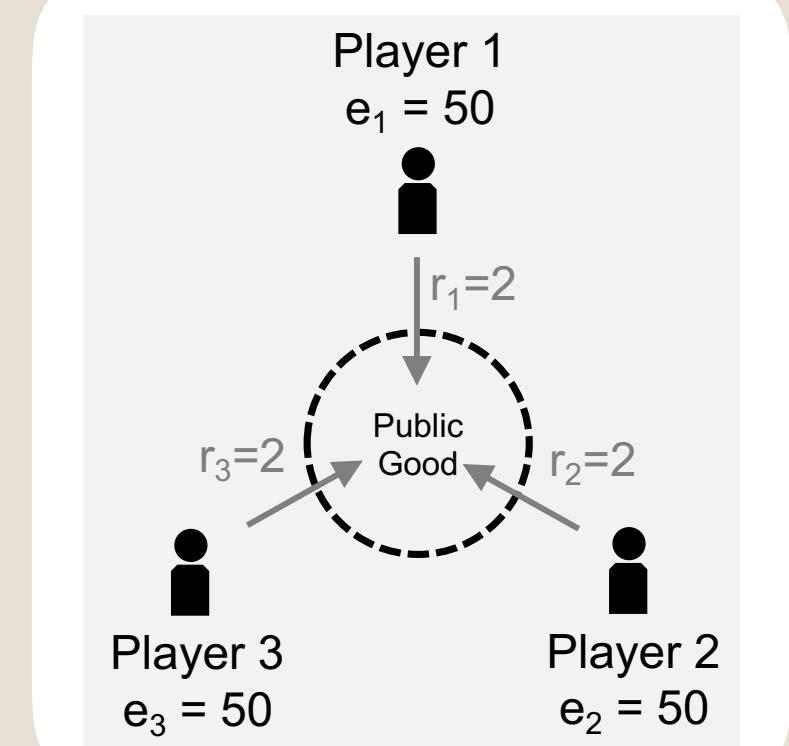
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Cooperation in asymmetric games

Remark 4.19. On the role of symmetric games

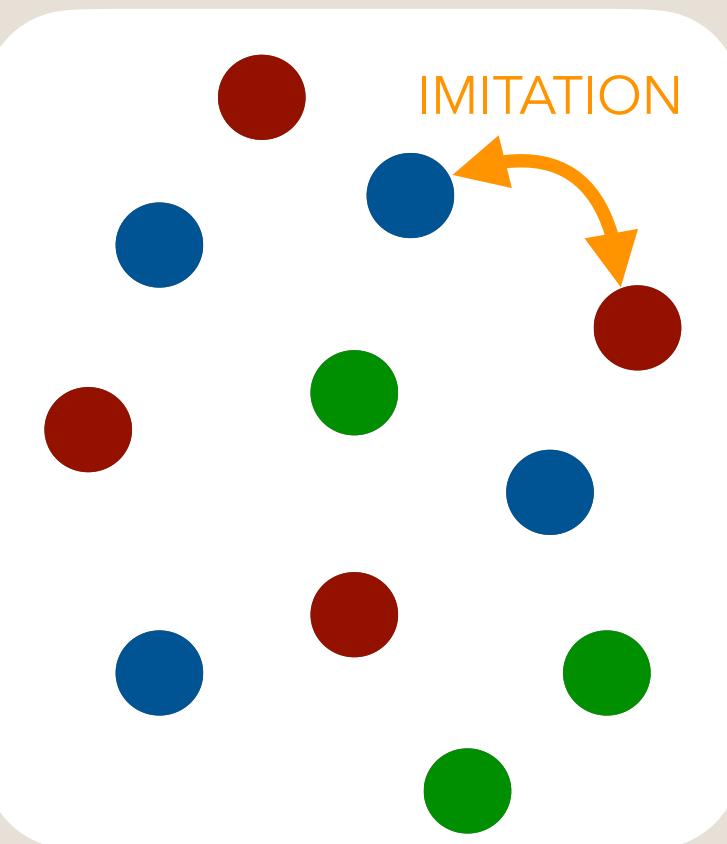
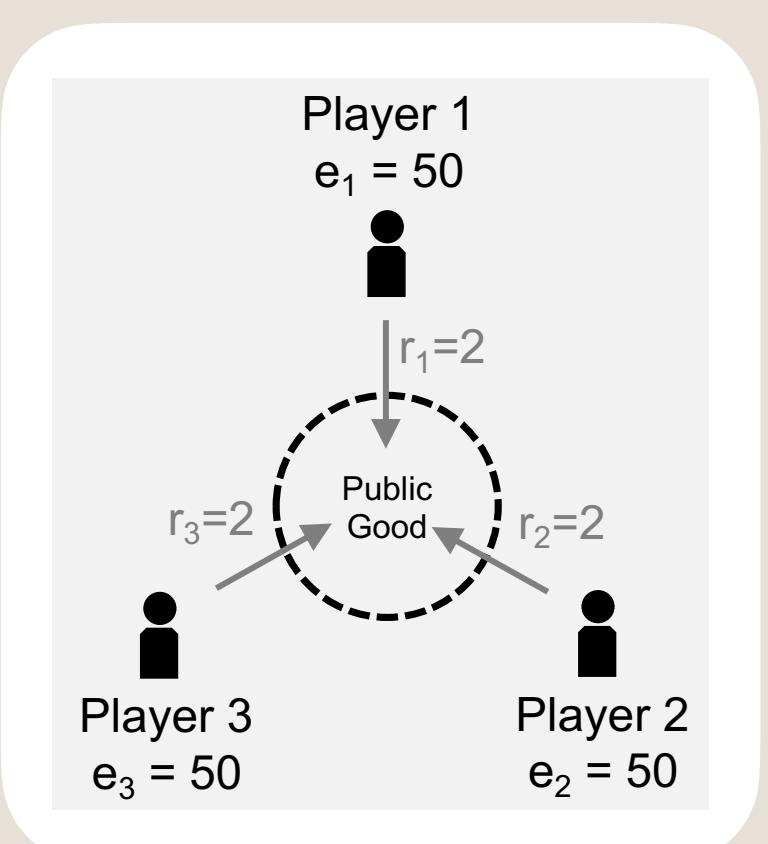
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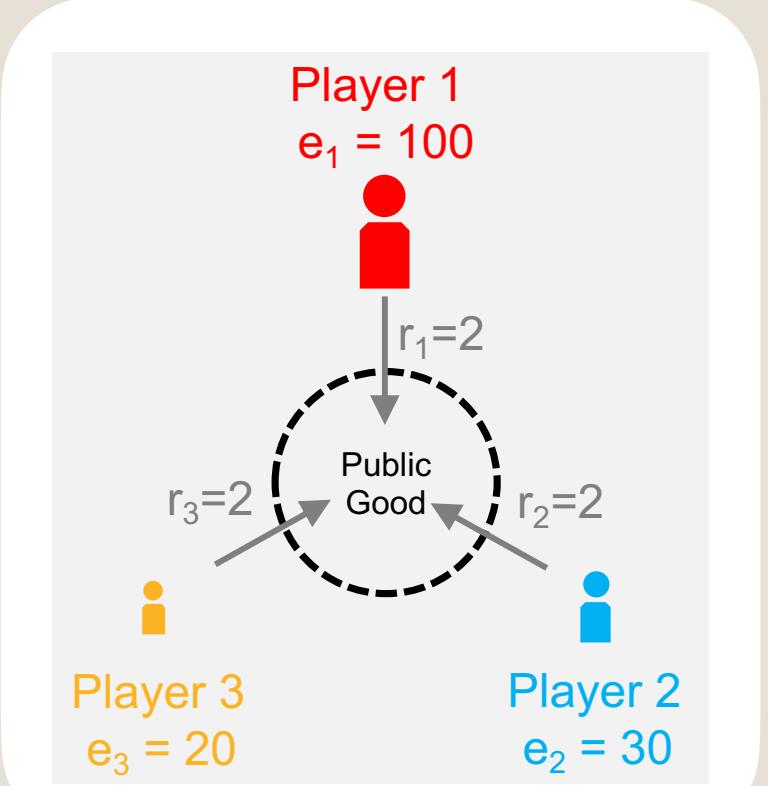
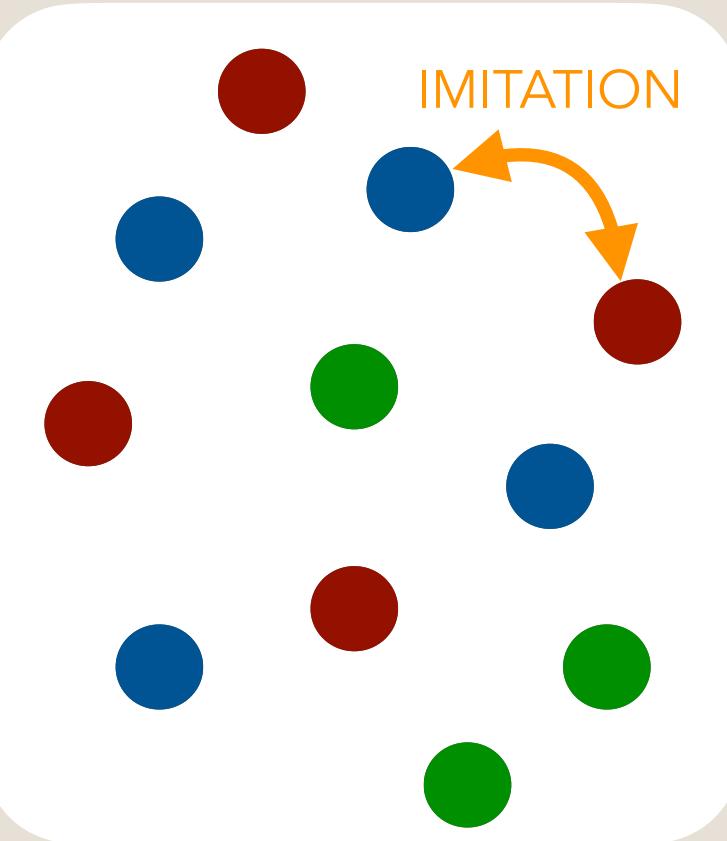
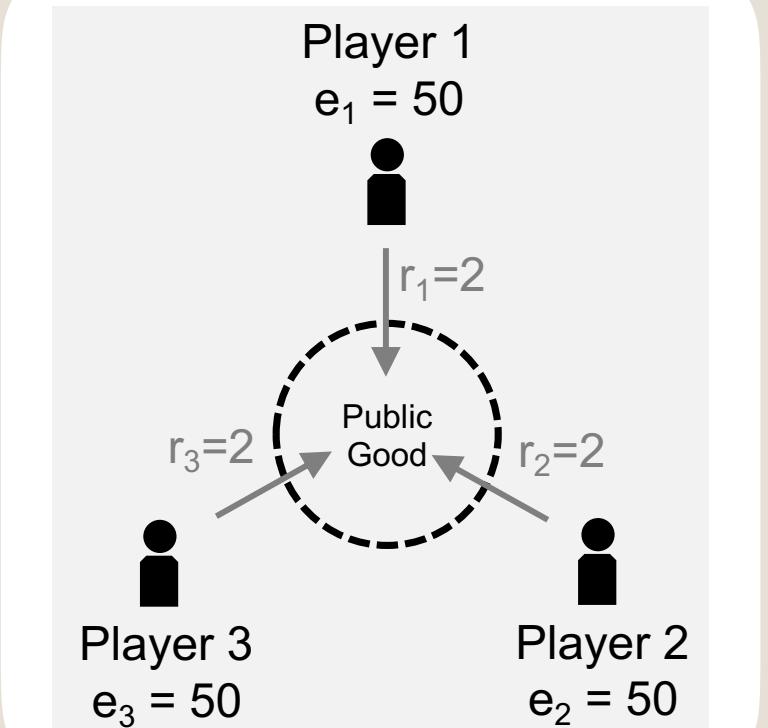
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- Many social dilemmas are asymmetric



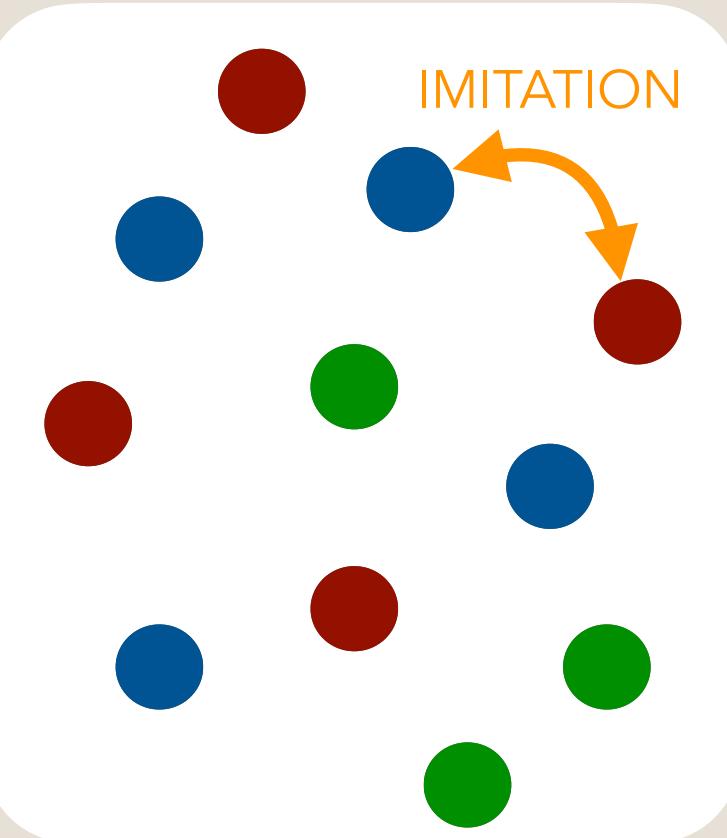
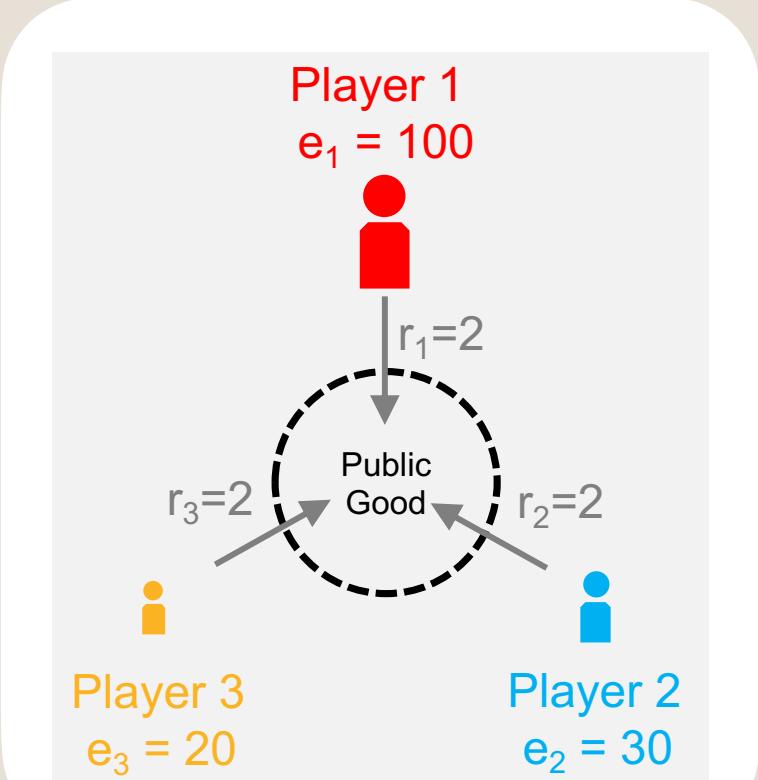
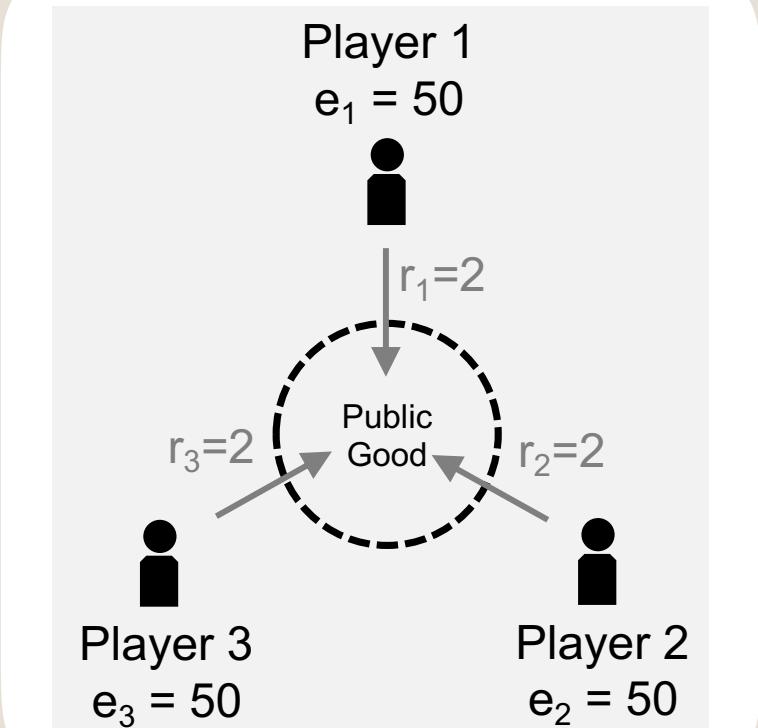
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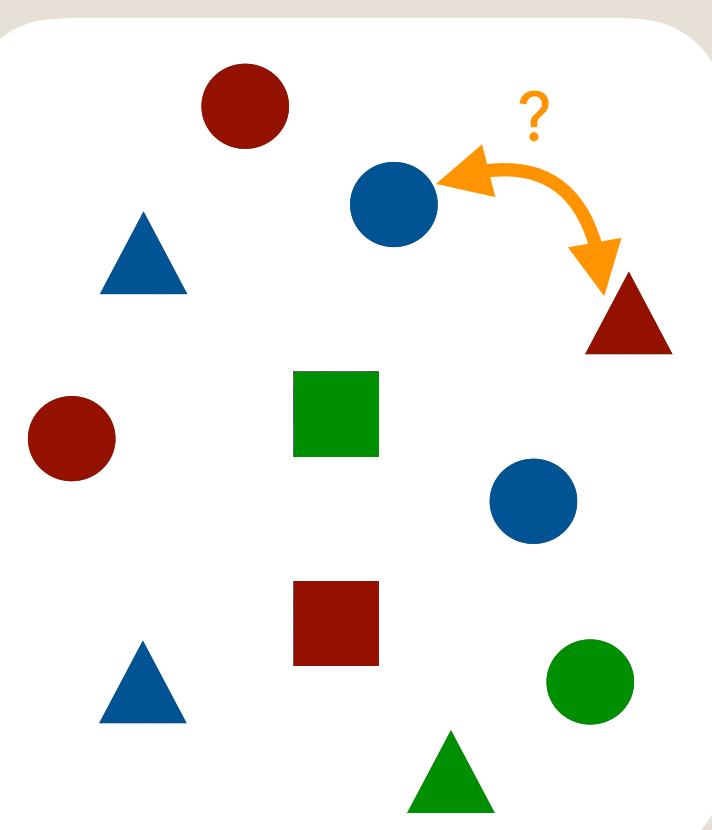
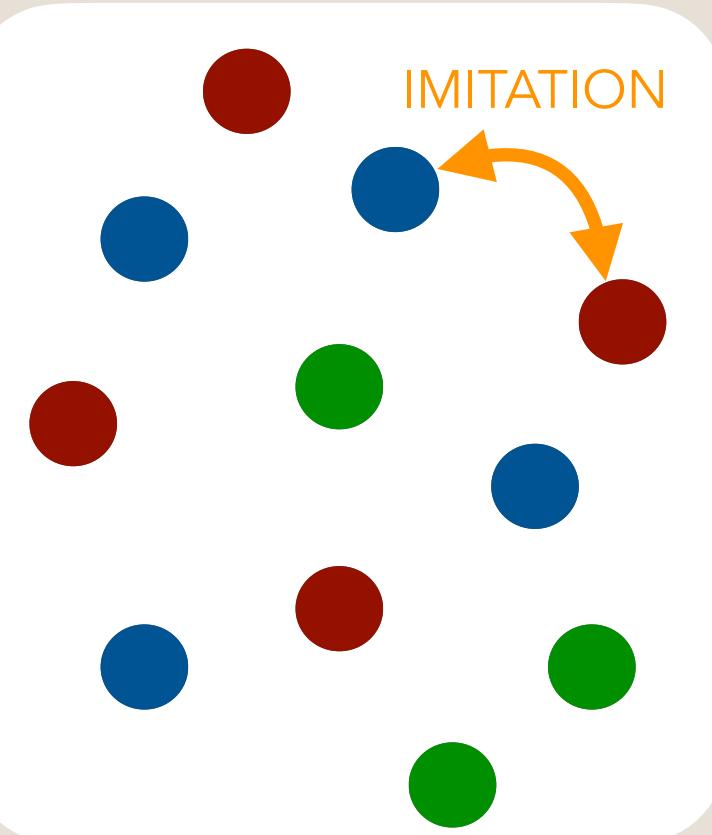
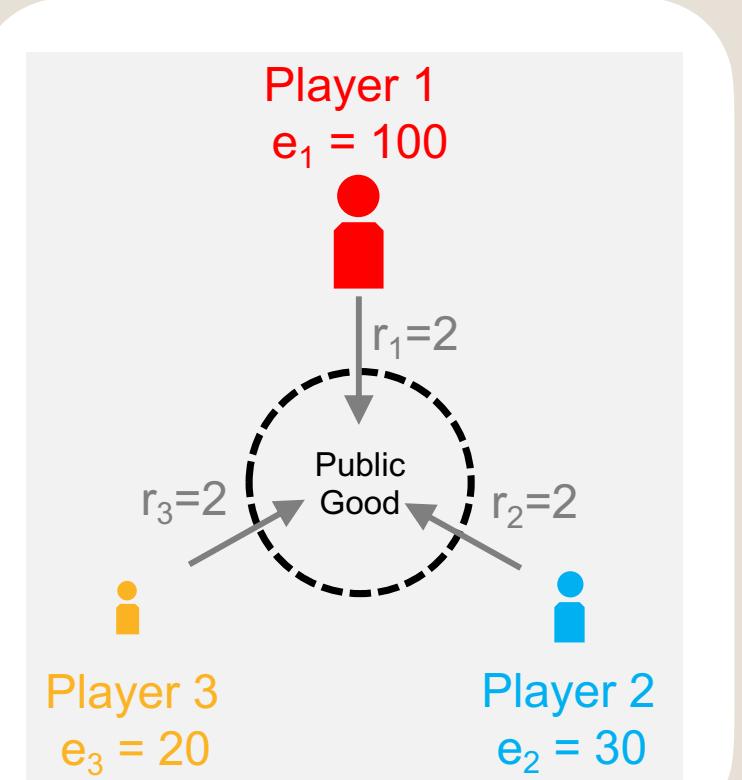
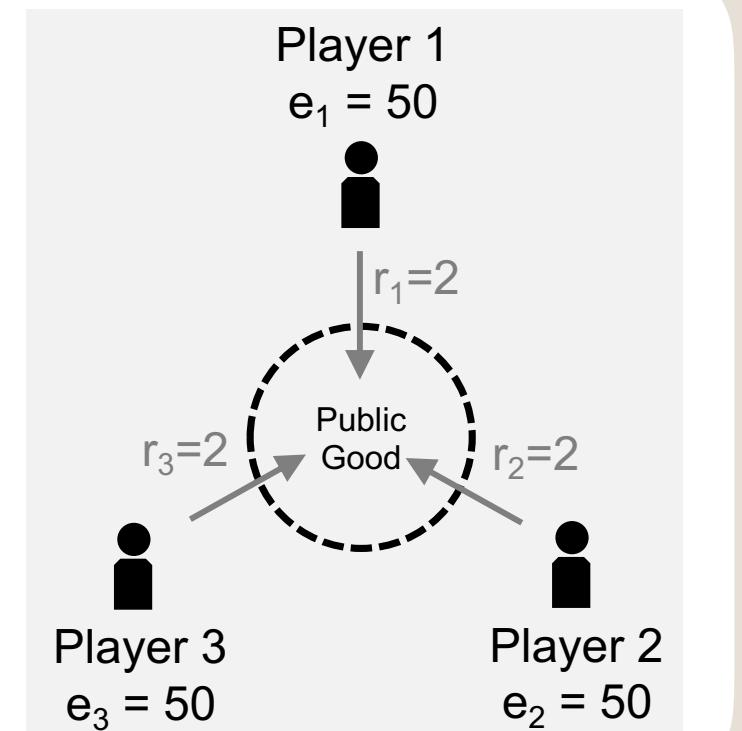
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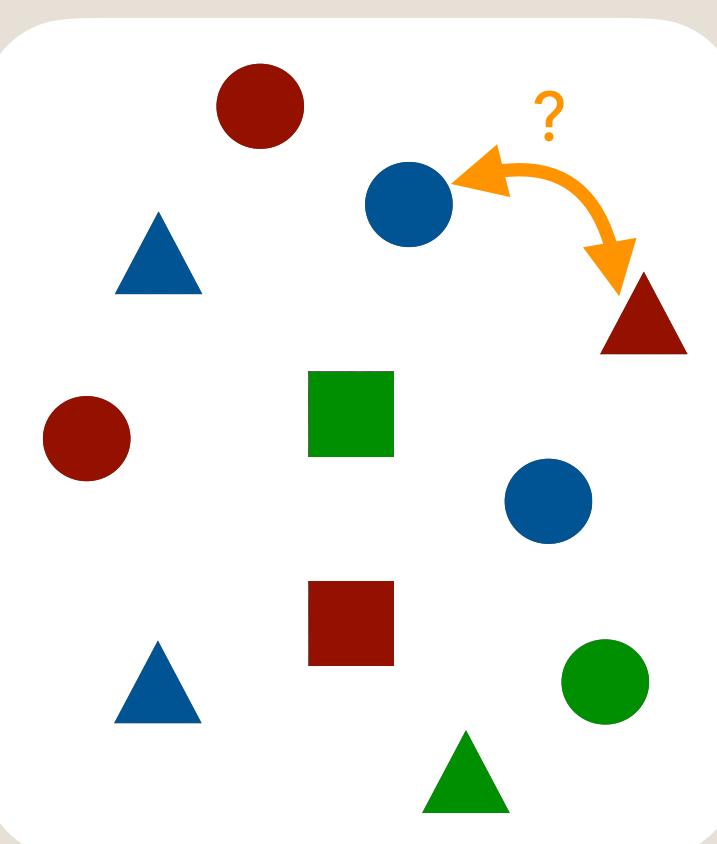
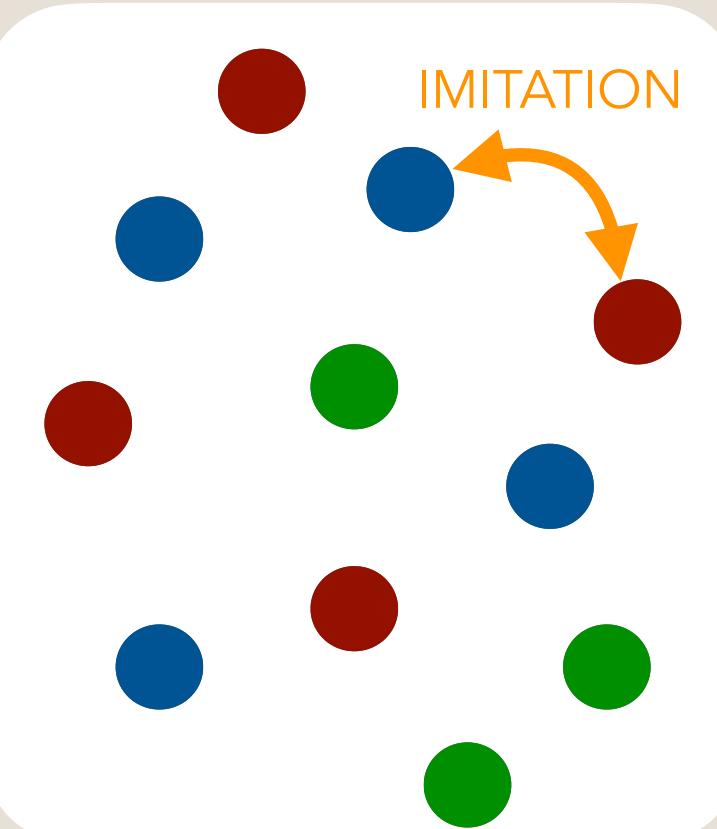
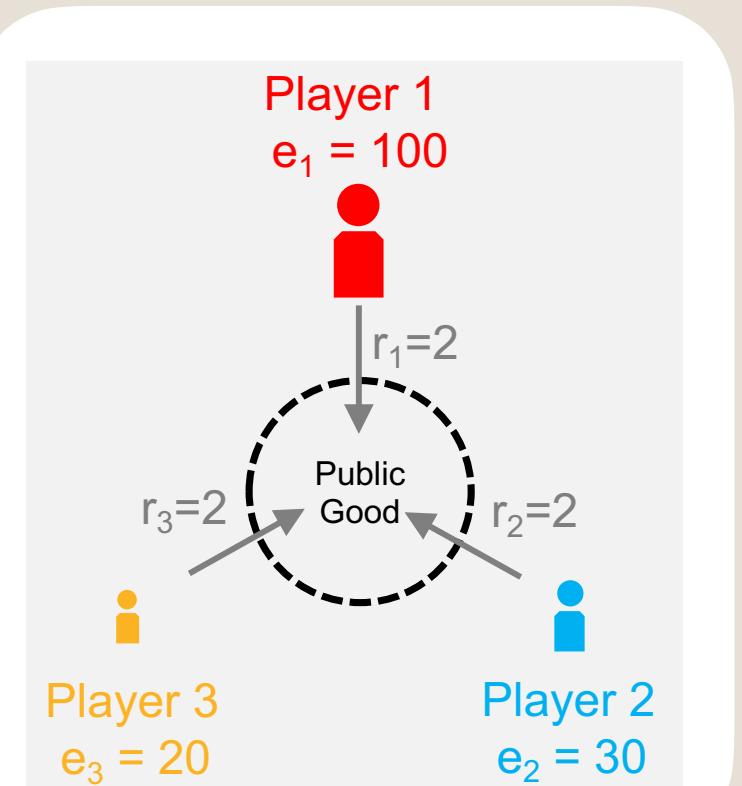
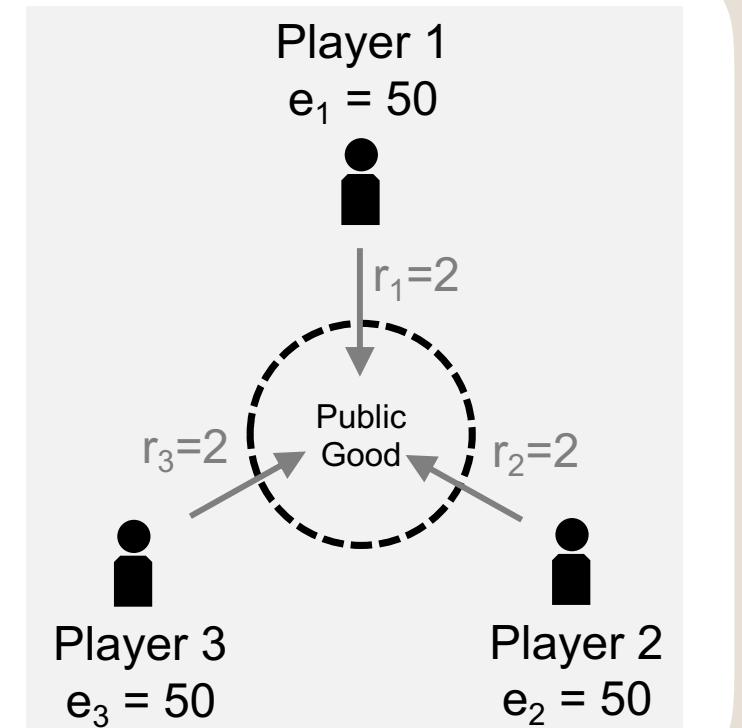
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Previous work

- Quite some work on cooperation in asymmetric social dilemmas



Climate policies under wealth inequality

Vítor V. Vasconcelos^{a,b,c}, Francisco C. Santos^{a,c}, Jorge M. Pacheco^{a,d,e}, and Simon A. Levin^{f,g,h,1}

Cooperative interaction of rich and poor can be catalyzed by intermediate climate targets
A letter

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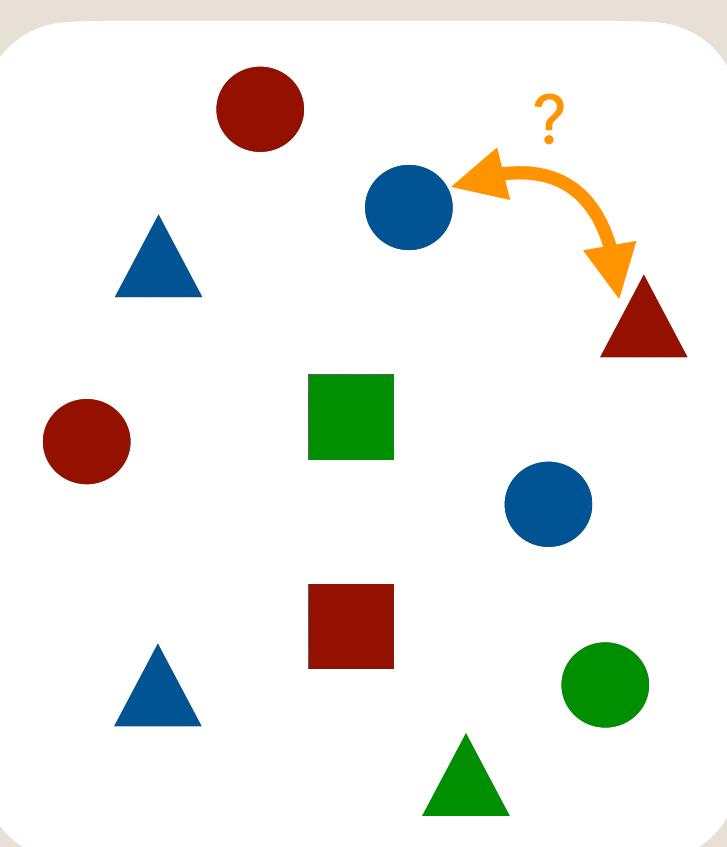
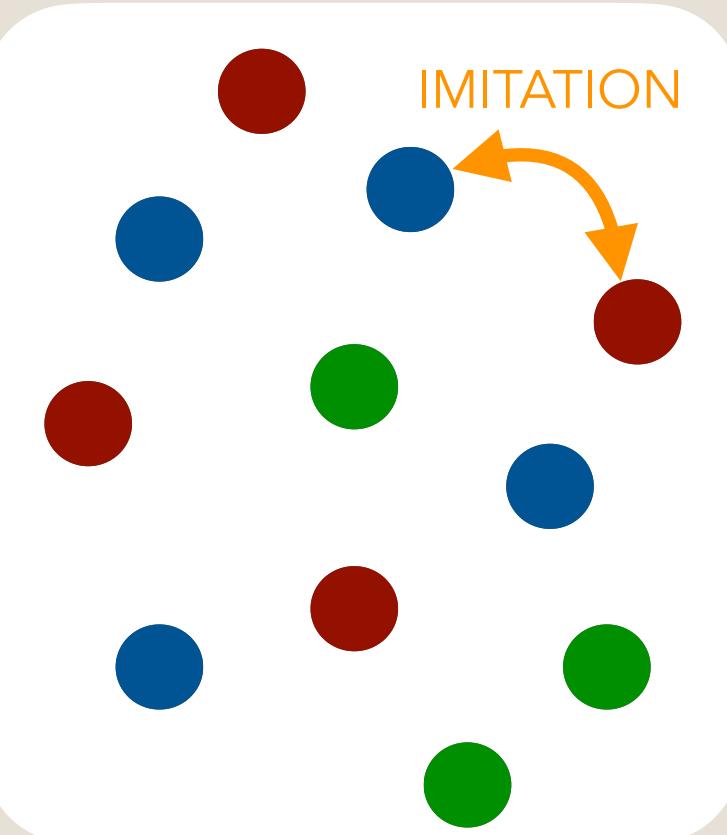
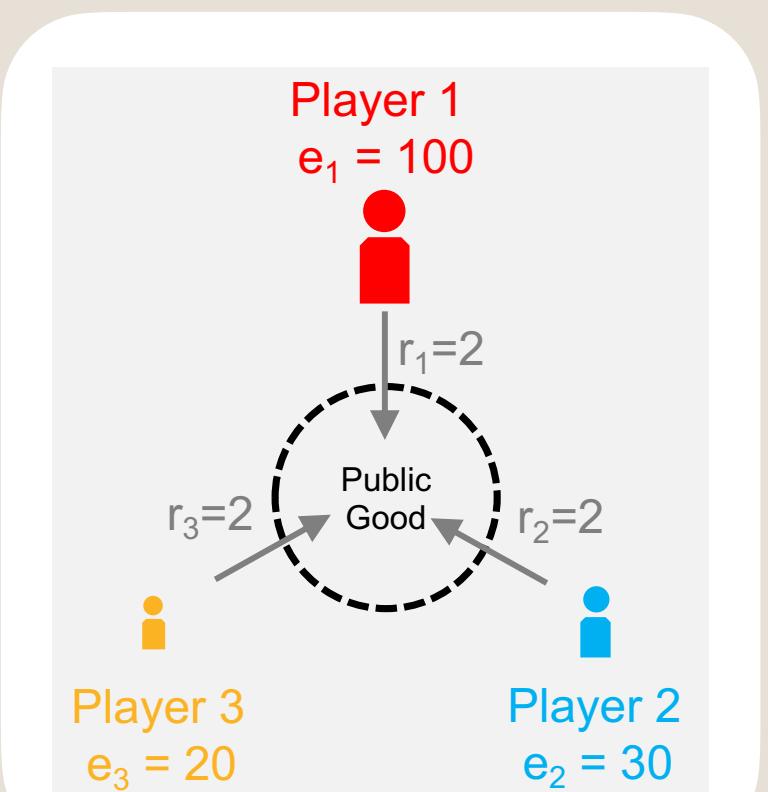
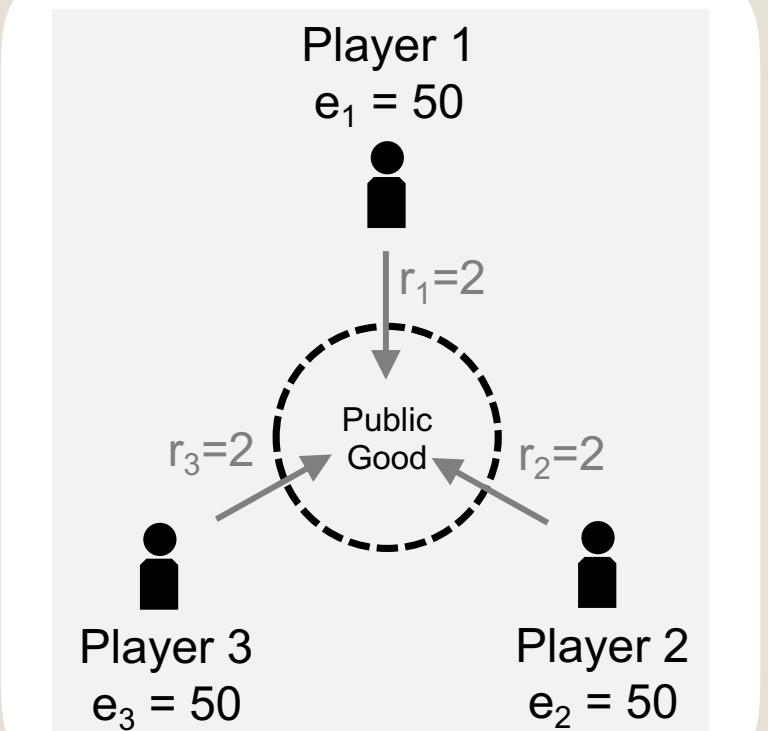
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- Is endowment inequality always detrimental?



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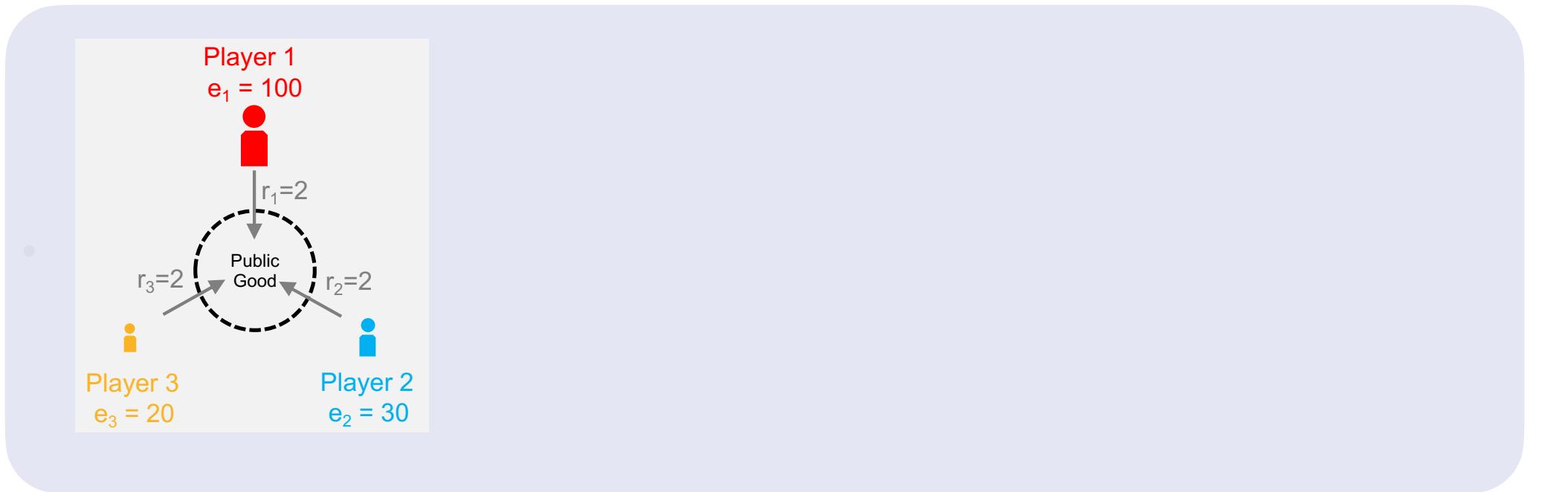
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- There is a group with n individuals who interact repeatedly
- Each round, individual i obtains an endowment e_i
- Individuals independently decide how much to contribute
- Individual i 's contribution is multiplied by r_i
- Total contributions are evenly split

Cooperation in asymmetric games

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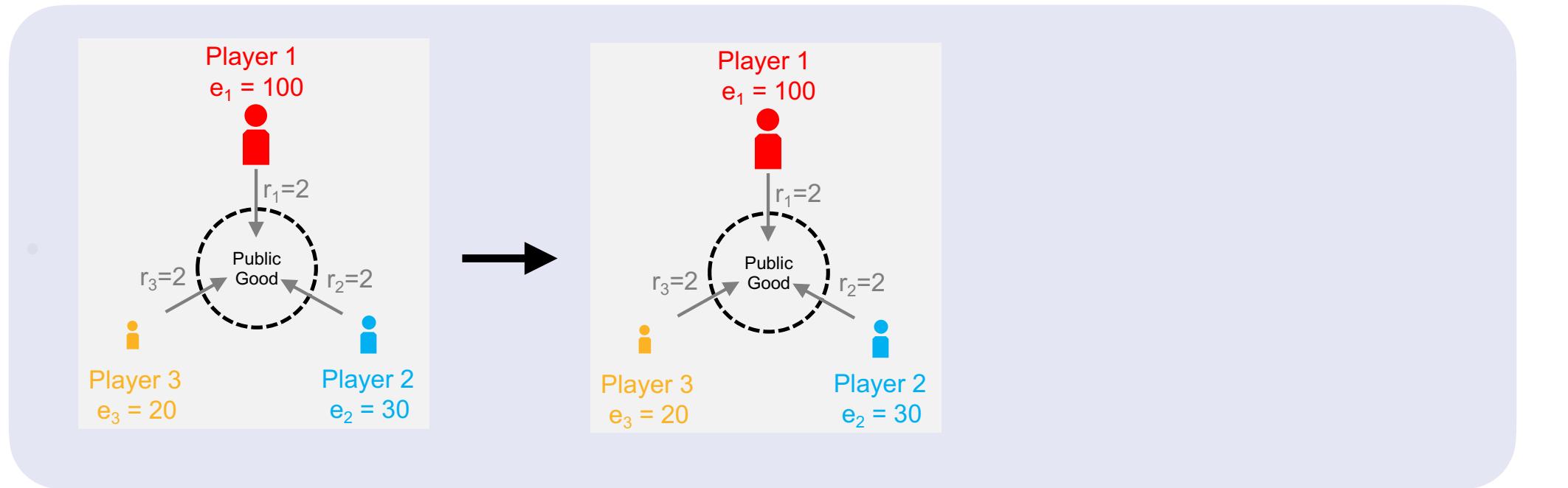
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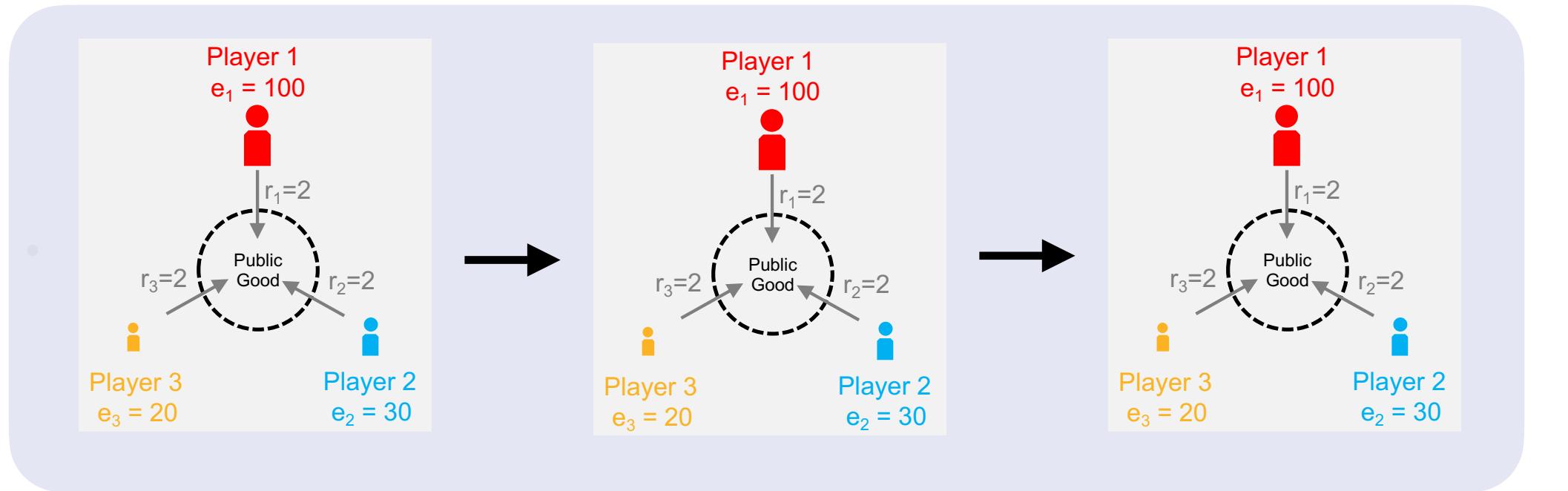
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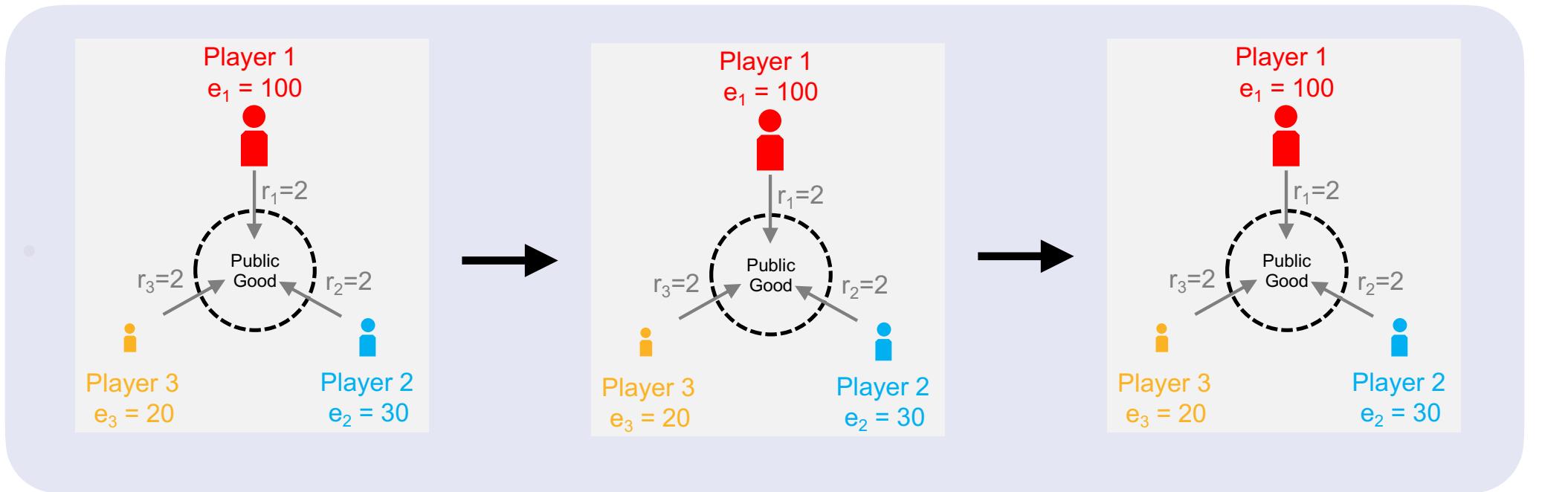
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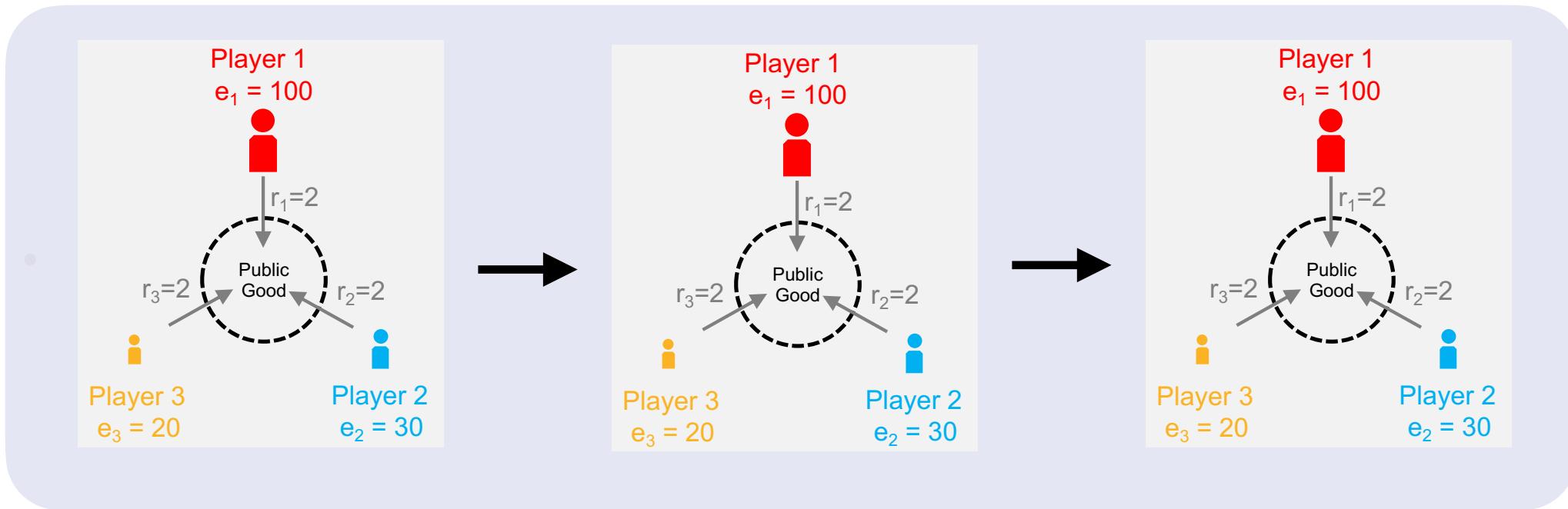
Research question

For given productivities, how should we optimally allocate endowments to maximize cooperation?

Cooperation in asymmetric games

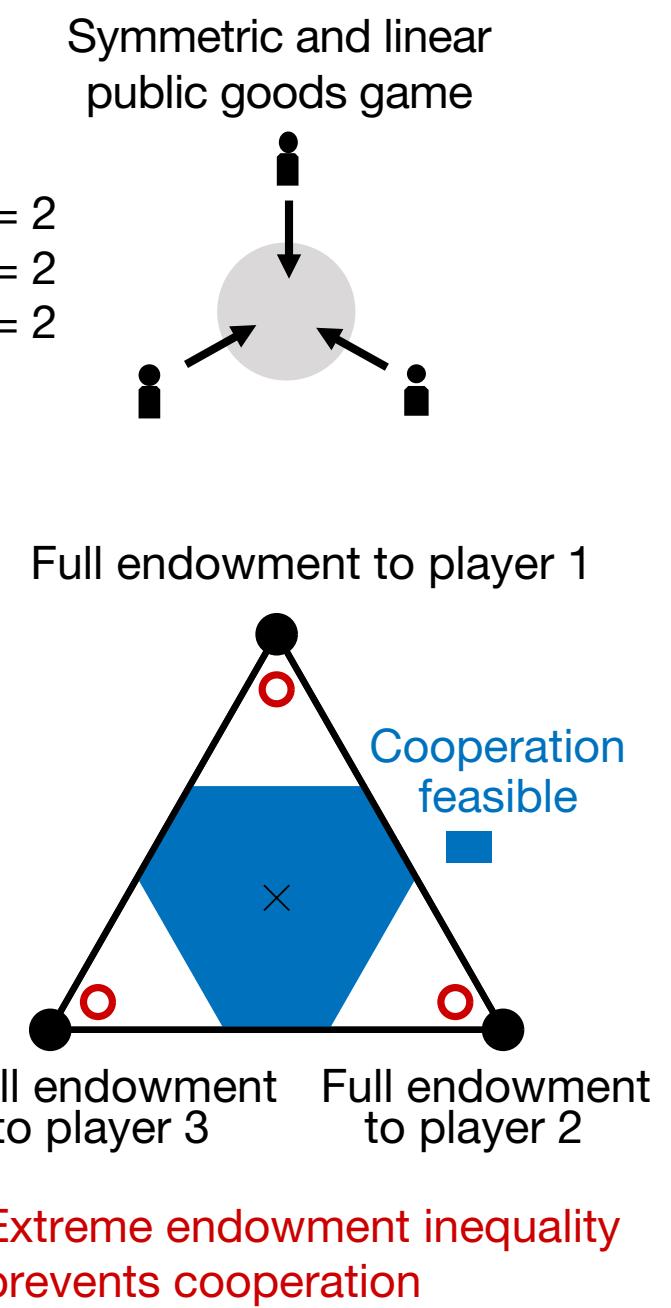
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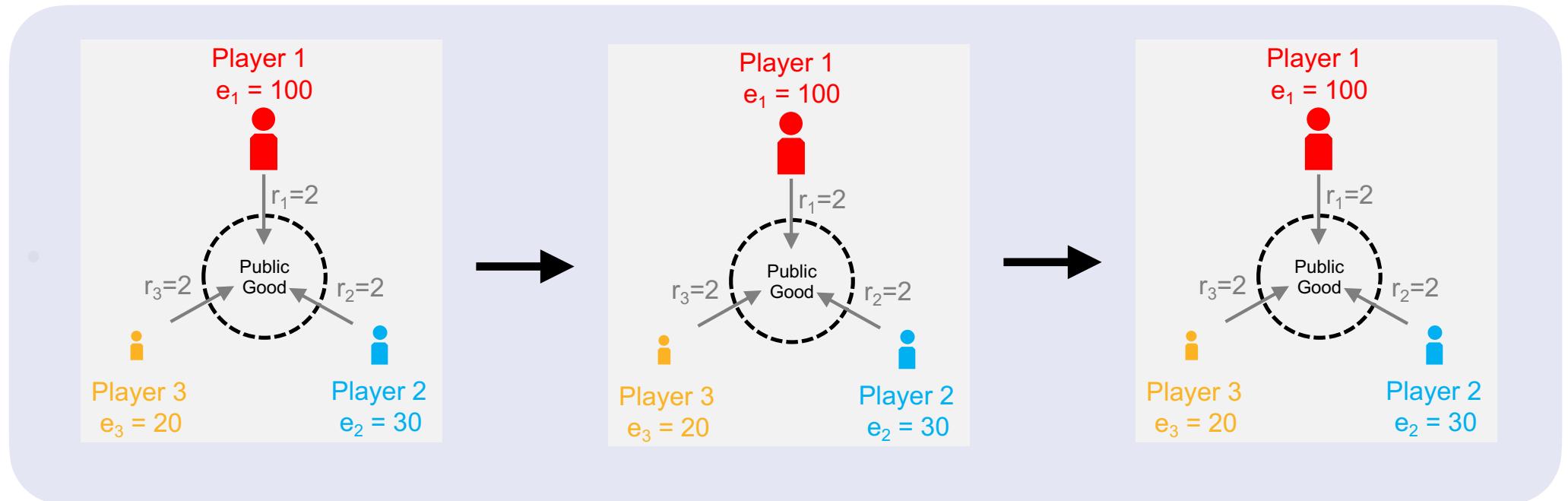
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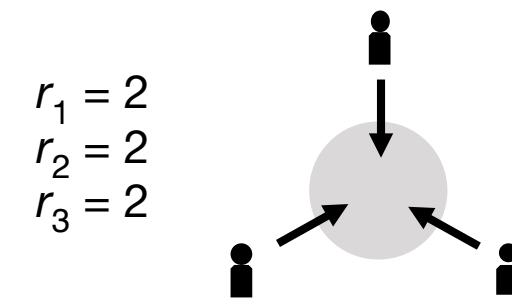
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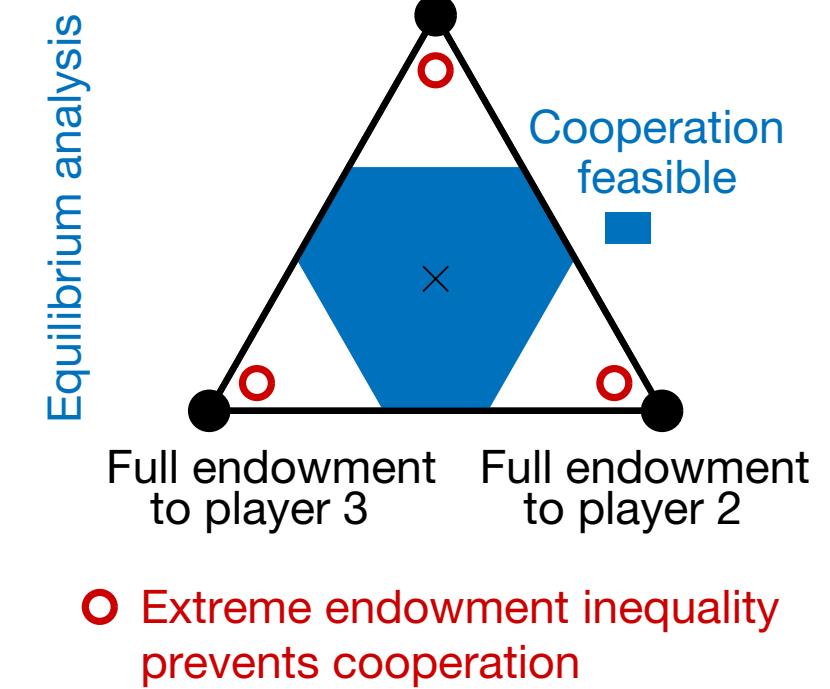
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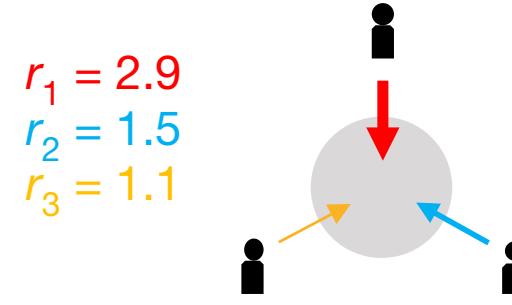
Symmetric and linear public goods game



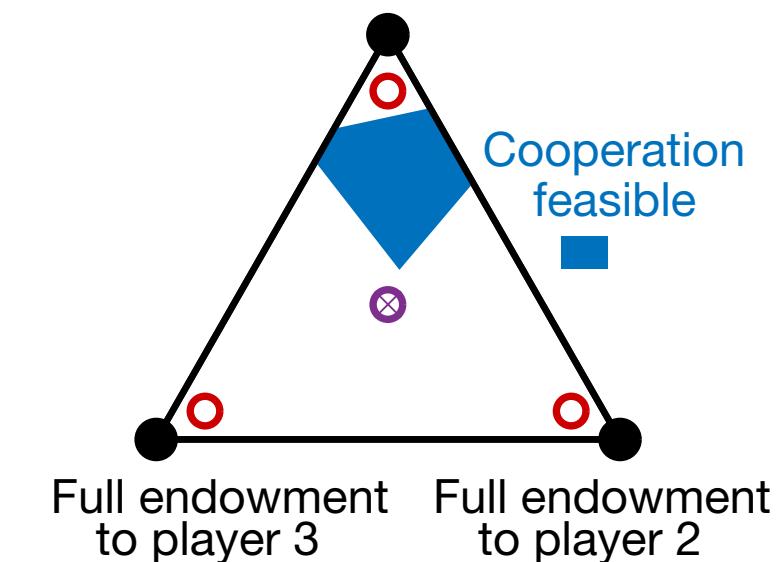
a Full endowment to player 1



Asymmetric and linear public goods game



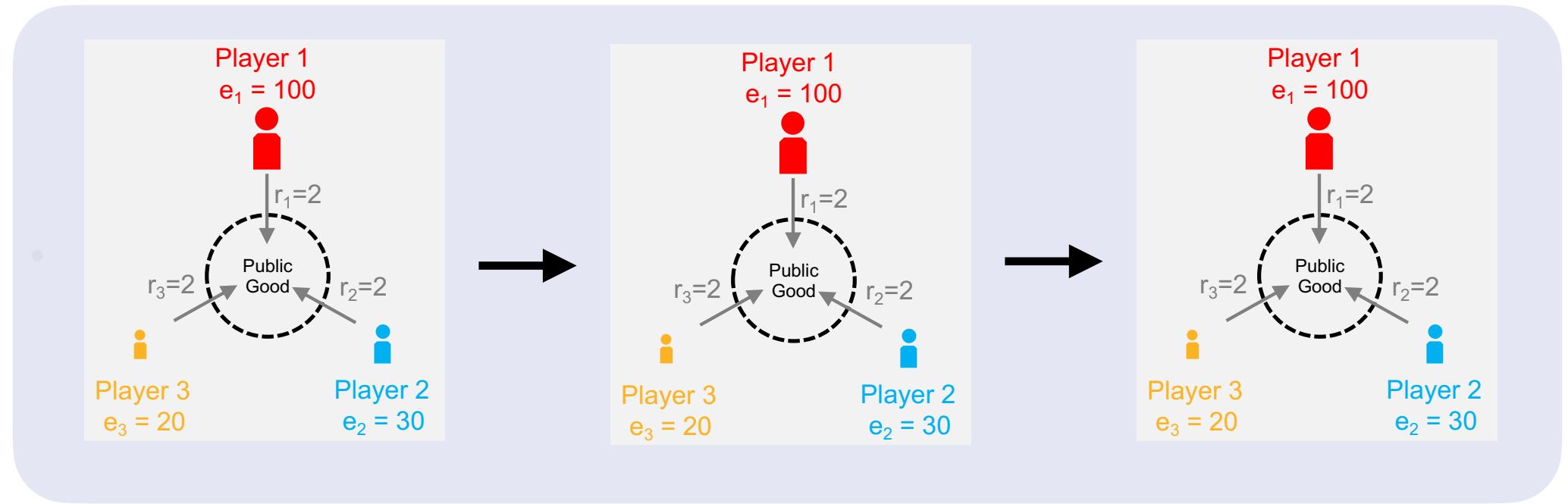
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Cooperation in asymmetric games

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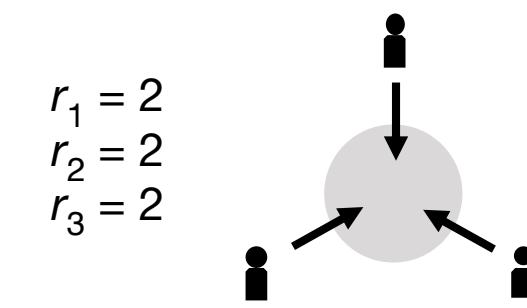
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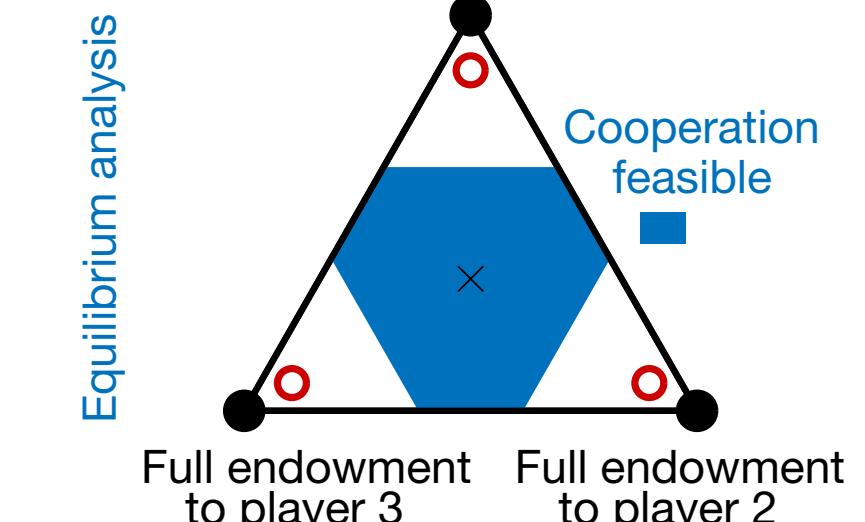
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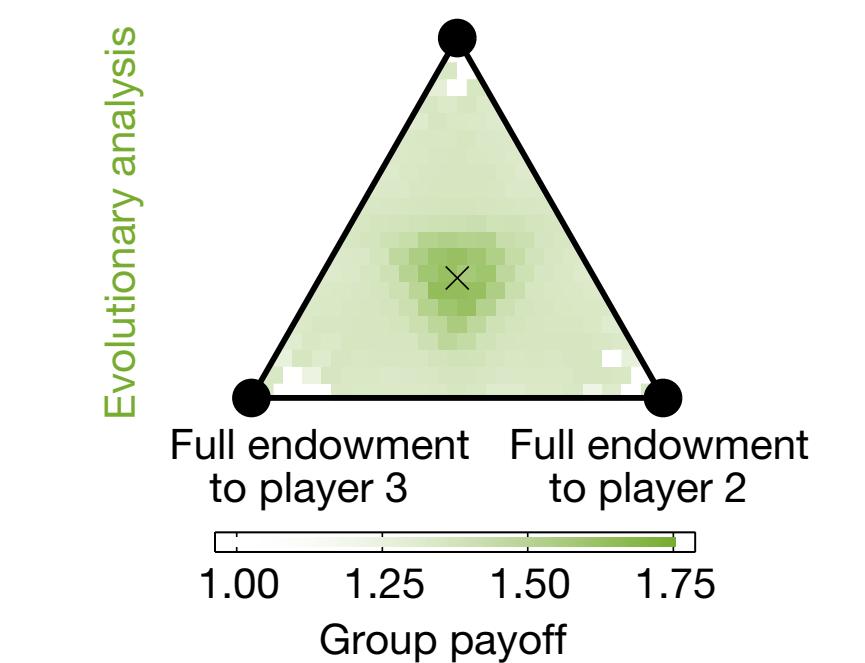


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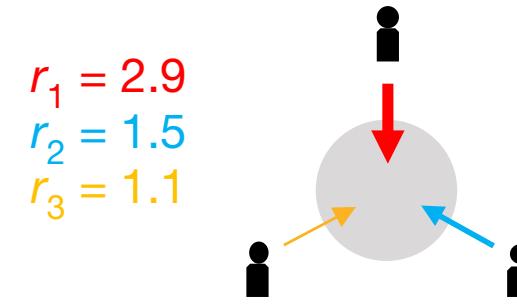


○ Extreme endowment inequality prevents cooperation

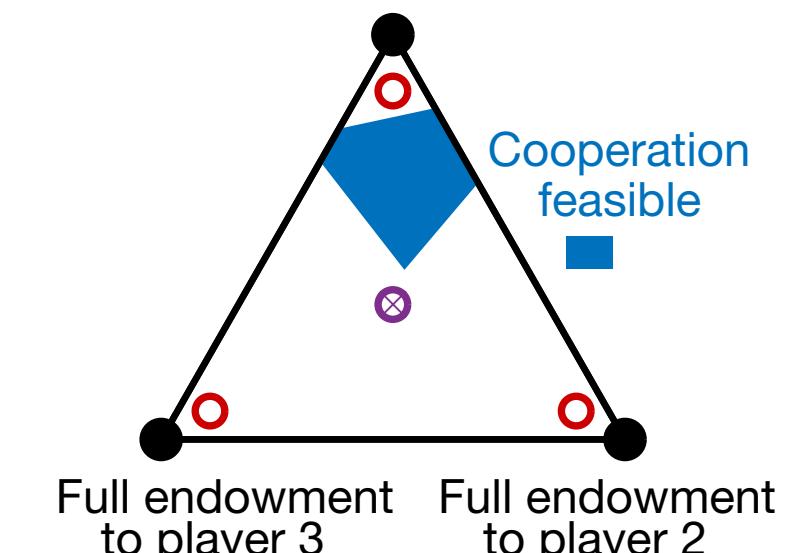
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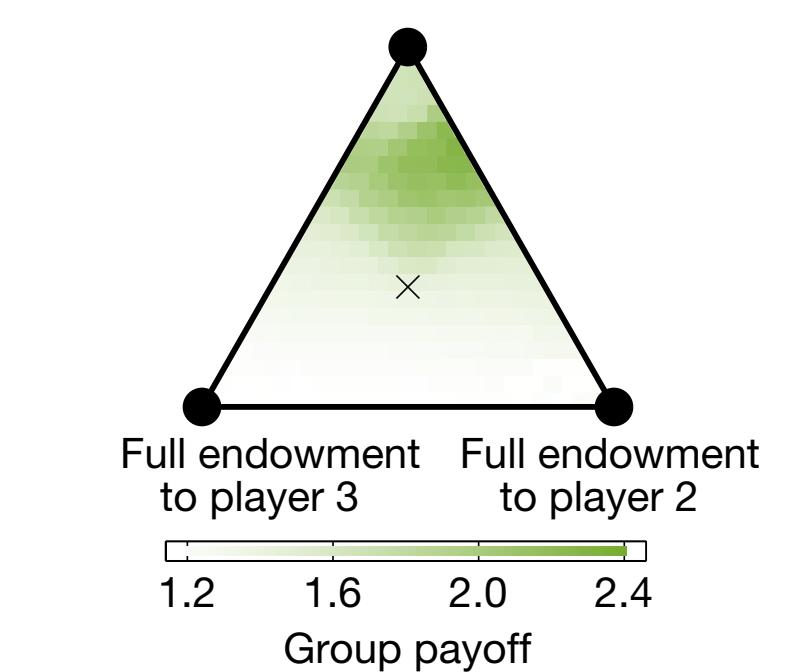
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Cooperation in asymmetric games

Remark 4.21. An experiment

- To test these qualitative predictions, we did an experiment
- ~400 participants recruited through Amazon Turk
- They play a repeated public good game in groups of two
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- Endowments can either be equal or unequal;
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		Equal productivities		Unequal productivities				
		Player 1	Player 2	Player 1	Player 2			
Equal endowments	1	Full equality	2	Productivity inequality	3			
	Endowment	50	50	Endowment	50	50		
Unequal endowments	Productivity	$\times 1.6$	$\times 1.6$	Productivity	$\times 1.9$	$\times 1.3$		
	2	Endowment inequality	4	Aligned inequality	1	2		
		Player 1	Player 2	Endowment	75	25		
		Endowment	75	25	Productivity	$\times 1.9$	$\times 1.3$	
		Productivity	$\times 1.6$	$\times 1.6$ <th data-kind="parent" data-rs="2">5</th> <td>Misaligned inequality</td> <th>1</th> <th>2</th>	5	Misaligned inequality	1	2
						Endowment	25	75
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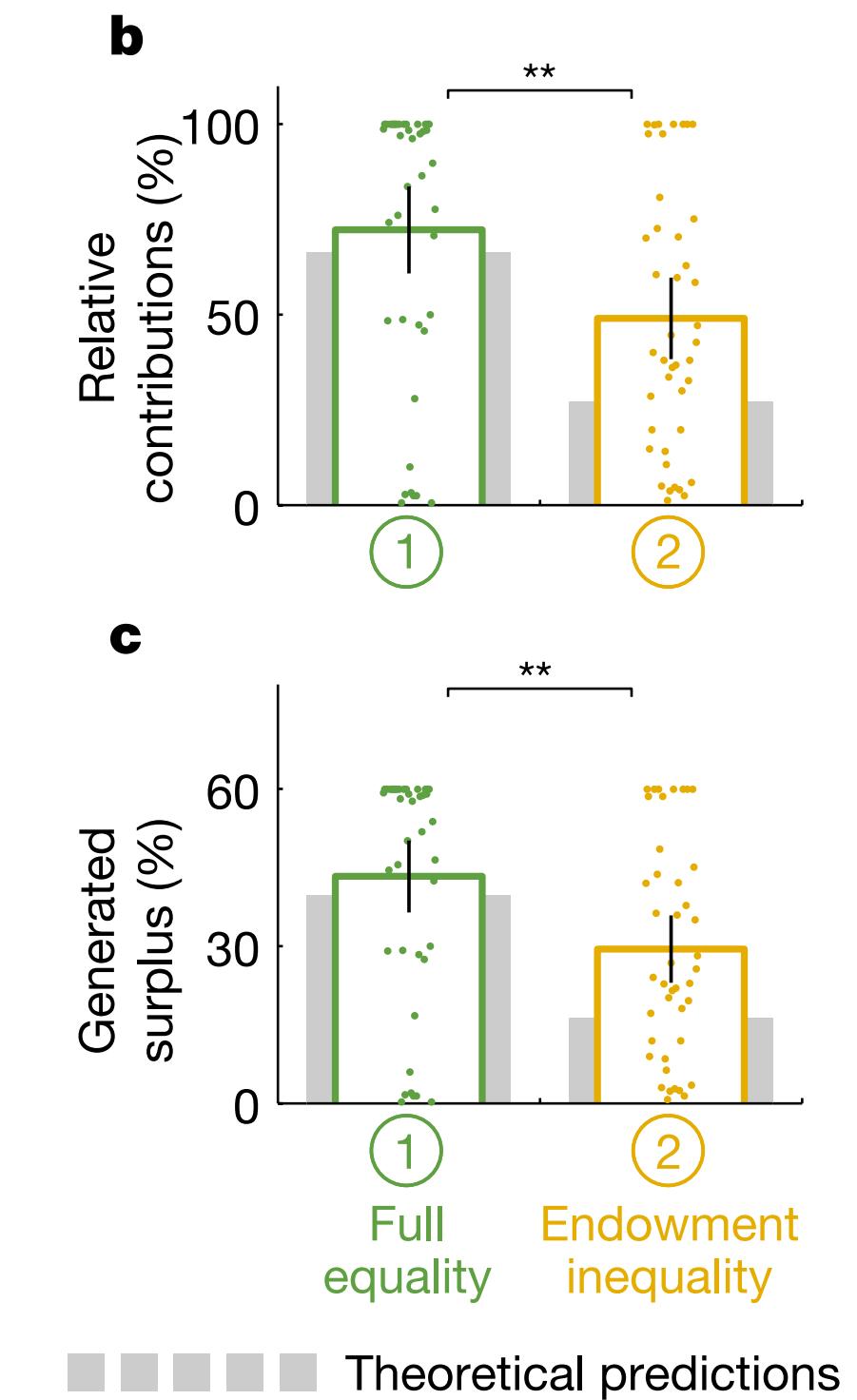
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		Player 1	Player 2	Player 1	Player 2
Equal endowments	① Full equality	Endowment	50	50	50
		Productivity	$\times 1.6$	$\times 1.6$	$\times 1.9$
Unequal endowments	② Endowment inequality	Player 1	75	25	25
		Productivity	$\times 1.6$	$\times 1.6$	$\times 1.9$
		③ Productivity inequality	Player 1	50	50
		Productivity	$\times 1.9$	$\times 1.3$	$\times 1.3$
		④ Aligned inequality	1	2	1
		Endowment	75	25	25
		Productivity	$\times 1.9$	$\times 1.3$	$\times 1.3$
		⑤ Misaligned inequality	1	2	1
		Endowment	25	75	75
		Productivity	$\times 1.9$	$\times 1.3$	$\times 1.3$



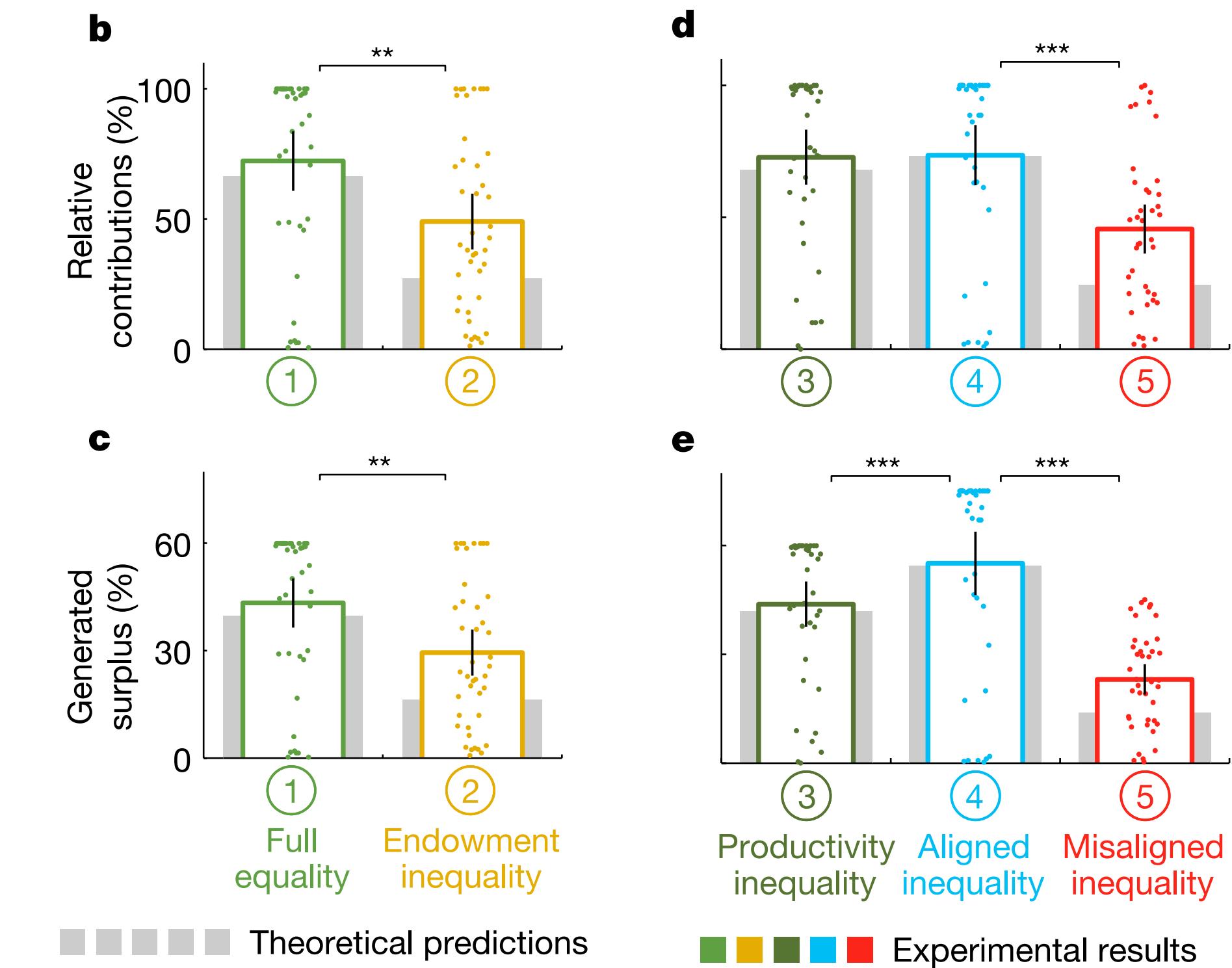
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		Player 1	Player 2	Player 1	Player 2	
Equal endowments	(1) Full equality	Endowment	50	50	Endowment	50
		Productivity	$\times 1.6$	$\times 1.6$	Productivity	$\times 1.9$
Unequal endowments	(2) Endowment inequality	Player 1	75	25	Player 1	75
		Productivity	$\times 1.6$	$\times 1.6$	Productivity	$\times 1.9$
	(4) Aligned inequality	Player 1	1	2	Player 1	1
		Endowment	75	25	Endowment	25
		Productivity	$\times 1.9$	$\times 1.3$	Productivity	$\times 1.9$
	(5) Misaligned inequality	Player 1	1	2	Player 1	25
		Endowment	25	75	Endowment	75
		Productivity	$\times 1.9$	$\times 1.3$	Productivity	$\times 1.9$

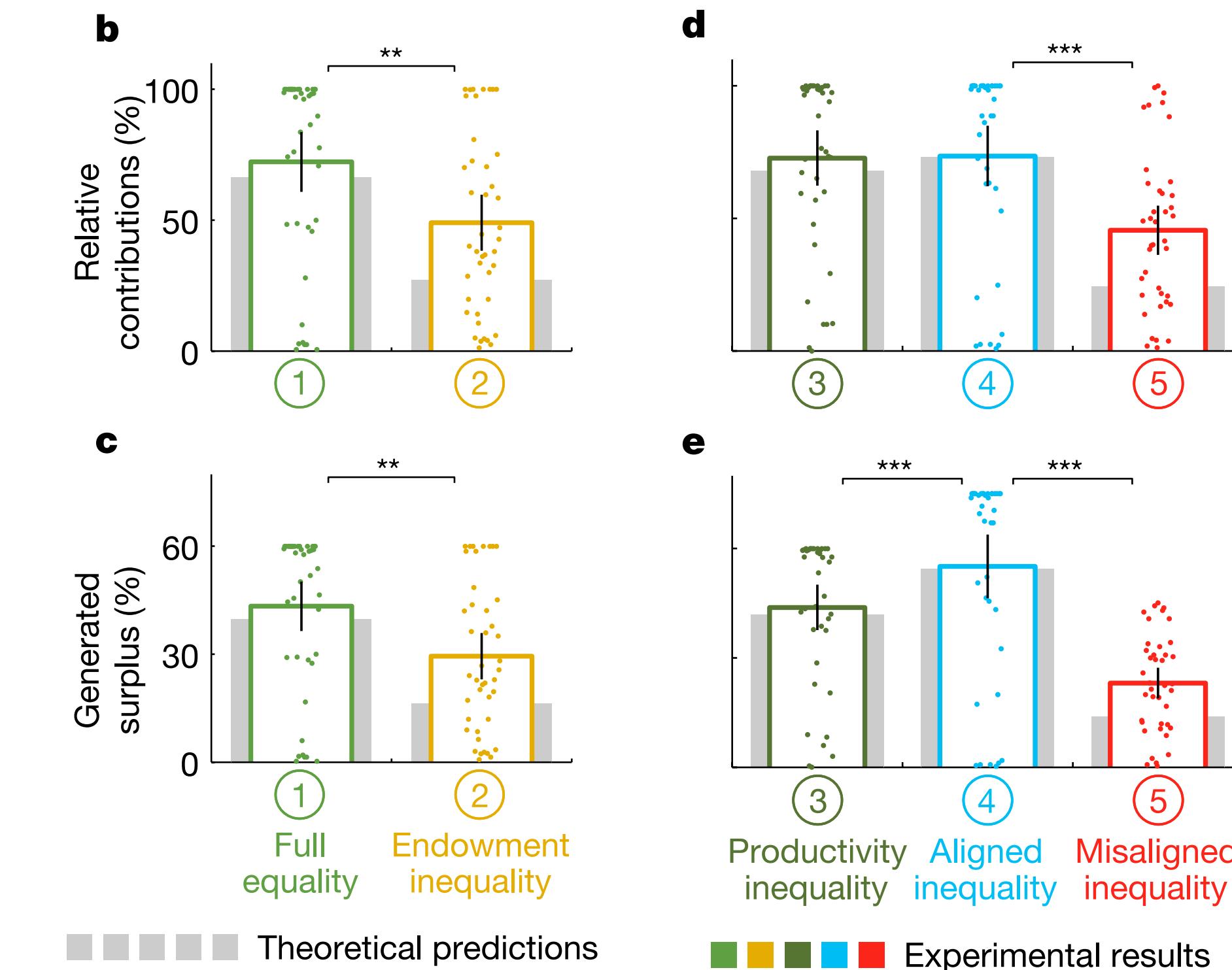


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	Endowment	75	25	Endowment	75	25	
Unequal endowments	Productivity	$\times 1.6$	$\times 1.6$	Productivity	$\times 1.9$	$\times 1.3$	
	⑤ Misaligned inequality	Player	1	2	③ Misaligned inequality	1	2
Unequal endowments	Endowment	25	75	Endowment	25	75	
	Productivity	$\times 1.9$	$\times 1.3$	Productivity	$\times 1.9$	$\times 1.3$	



Summary: How to allocate endowments to maximise cooperation?

- When players are equally productive, they should get equal contributions.
- Otherwise, more productive players should get higher endowments ('aligned inequality')

Summary

1. In my lectures, I first provided some introduction to evolutionary game theory (replicator dynamics, Moran process).
2. Then we used to these techniques to further explore the evolution of cooperation (in particular: direct and indirect reciprocity).



Summary

1. In my lectures, I first provided some introduction to evolutionary game theory (replicator dynamics, Moran process).
2. Then we used to these techniques to further explore the evolution of cooperation (in particular: direct and indirect reciprocity).
3. Thanks to the organisers, and thanks for being such an engaging audience!



Where to find more information:

<http://web.evolbio.mpg.de/social-behaviour/>

Collaborators and funding

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Josef Tkadlec, Charles University Prague

Arne Traulsen, MPI for Evolutionary Biology

Manfred Milinski, MPI for Evolutionary Biology

Laura Schmid, Nature Communications

Other Dynos, all over the world

