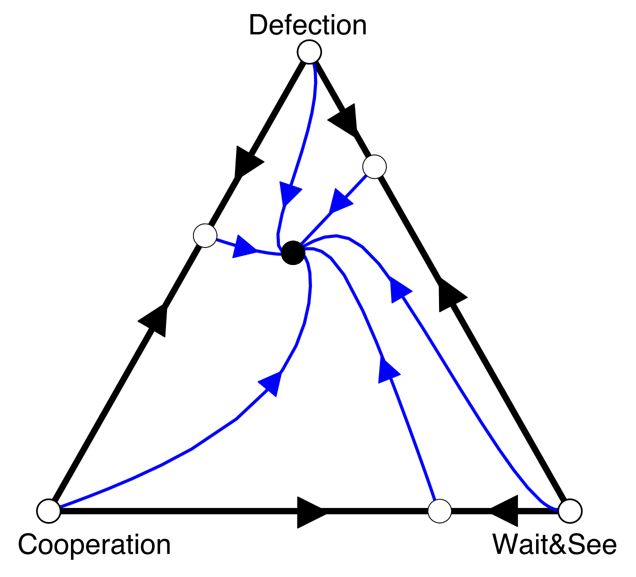
An overview

Yesterday's class (March 11, 2025)

- An introduction to evolutionary game theory
  (Replicator dynamics, games in finite populations)



Today's classes (March 12, 2025)

- Evolution of cooperation & direct reciprocity
- Social norms & indirect reciprocity

Tomorrow's class (March 13, 2025)

- Some current research: Reciprocity in complex environments

Evolution of cooperation: Motivation

Remark 2.1. Cooperation

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- Sharing food; engaging in predator inspection; taking the lead in flocks of birds; etc.

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TIT FOR TAT in sticklebacks and the evolution of cooperation

Manfred Milinski

Arbeitsgruppe für Verhaltensforschung, Fakultät f
Ruhr-Universität, Postfach 102148, 4630 Bochum

Reciprocal food sharing in the vampire bat

Gerald S. Wilkinson

Department of Biology, C-016, University of Ca
San Diego, La Jolla, California 92093, USA

Matching times of leading and following suggest cooperation through direct reciprocity during V-formation flight in ibis

Bernhard Voelkl[a,b,c,1], Steven J. Portugal[d,2], Markus Unsöld[e,f], James R. Usherwood[d], Alan M. Wilson[d], and Johannes Fritz[c,e]

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Still, one might raise the question: Why would people have these particular emotions, social norms, and institutions?

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Evolution of cooperation: Prisoner's dilemma

Remark 2.2. Cooperation in a Prisoner's dilemma

The problem of cooperation is usually illustrated with a prisoner's dilemma. Cooperation means to pay a cost c, for the co-player to get a payoff of b.

	Cooperate	Defect
Cooperate	b-c	-c
Defect	b	0

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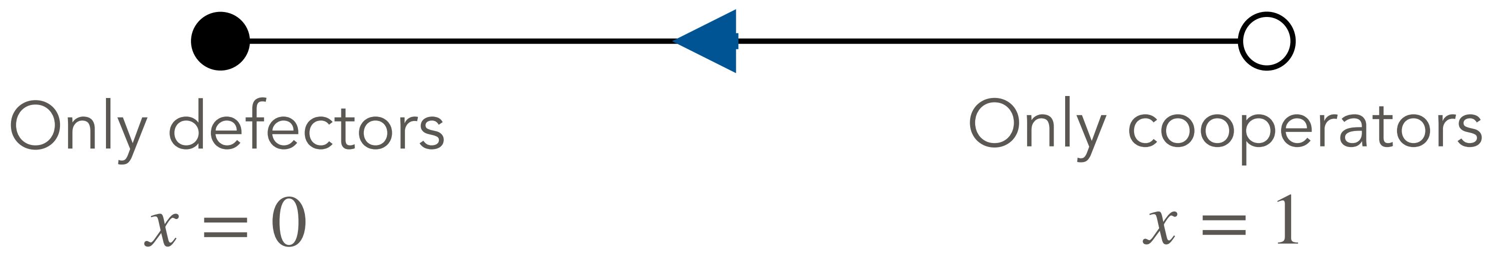
$$\dot{x} = x(1-x)(-1)$$



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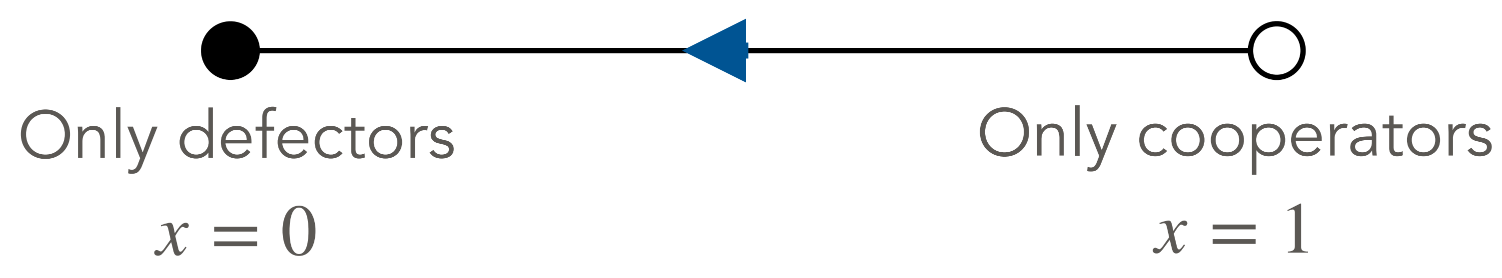

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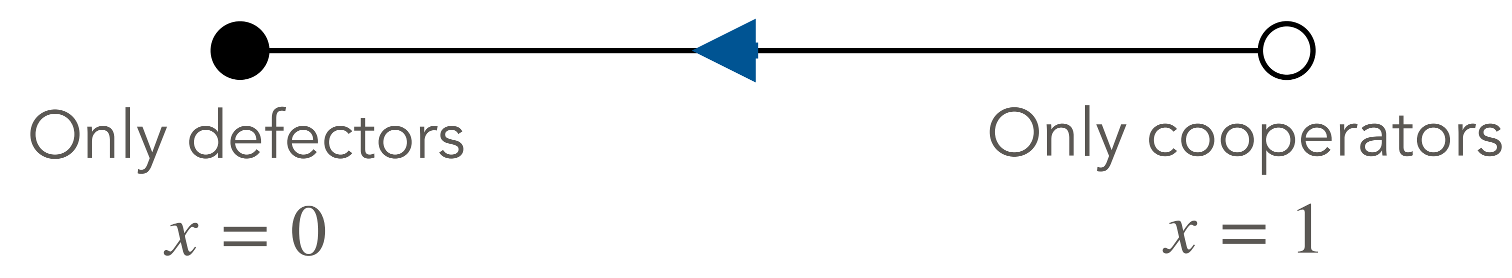

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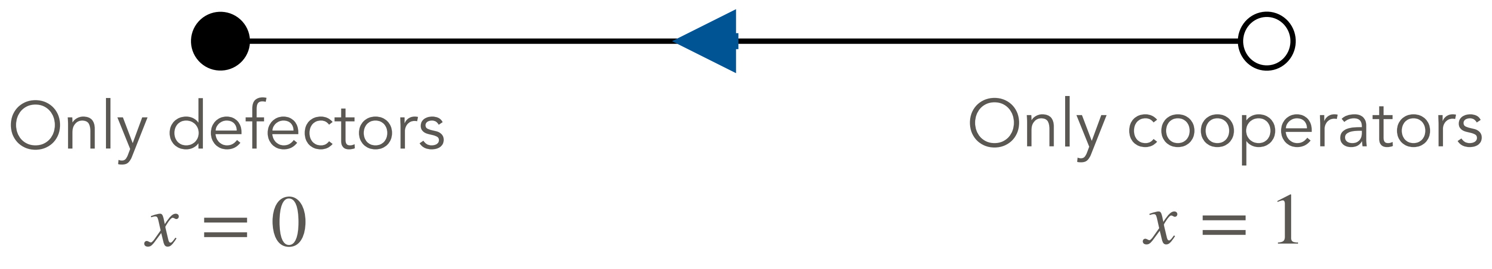

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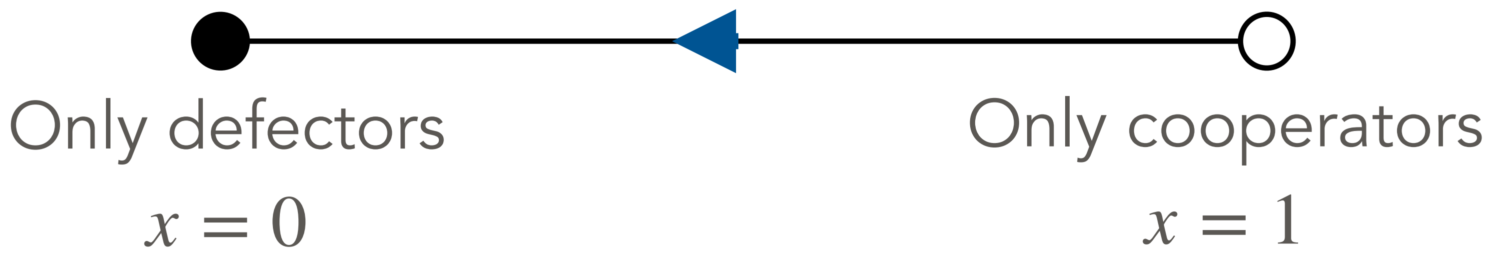

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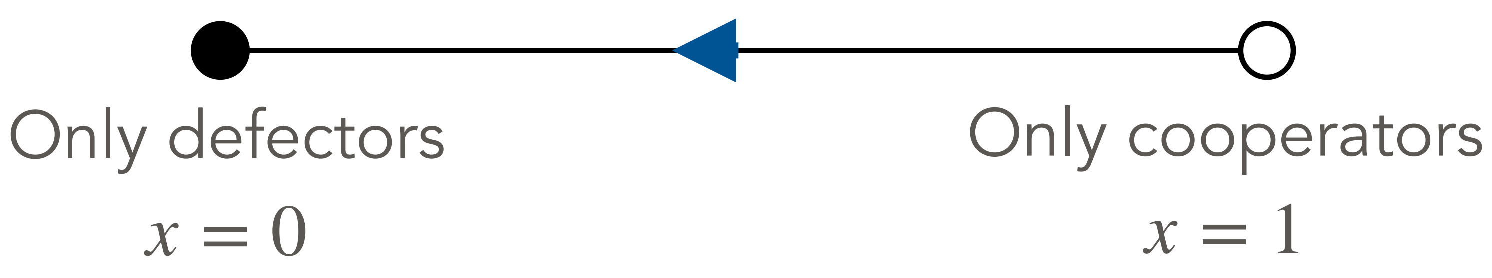

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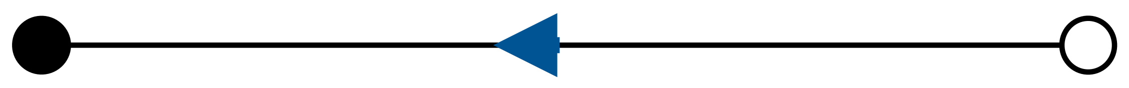

Only defectors $x = 0$

Only cooperators $x = 1$

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TARGET REVIEW

The evolution of cooperation and altruism – a general framework and a classification of models

L. LEHMANN*† & L. KELLER*

Five Rules for the Evolution of Cooperation

Martin A. Nowak

Eleven mechanisms for the evolution of cooperation

MICHAEL A. ZAGGL*

TUM School of Management, Technische Universität München, Arcisstr. 21, 80333, Munich, Germany

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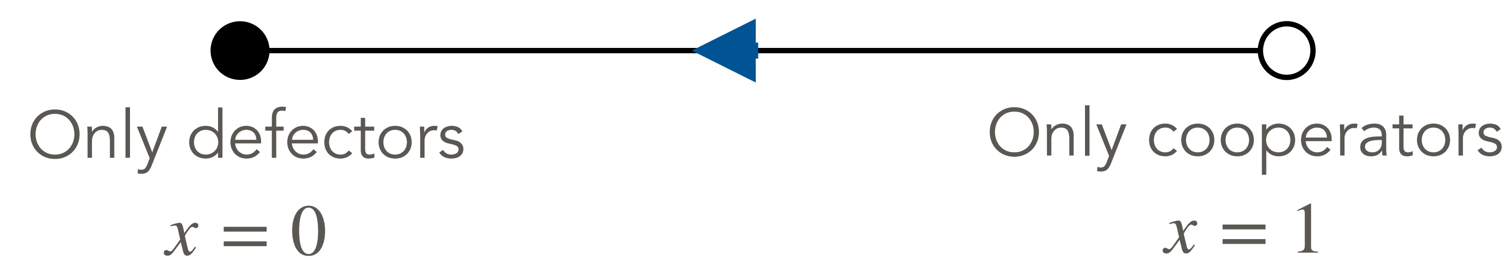

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Direct reciprocity: Repeated Prisoner's dilemma (RPD)

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To model whether cooperation can emerge when individuals may interact more than once, we consider the repeated prisoner's dilemma.

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- **Payoffs:** Payoffs $\pi_i(t)$ in each round t are the payoffs of the usual prisoner's dilemma. For the entire repeated prisoner's dilemma, we take the weighted mean:

$$\pi_i = (1-\delta)\sum_{t=1}^{\infty} \delta^t \cdot \pi_i(t)$$

[if $\delta < 1$]

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In the repeated prisoner's dilemma, strategies are more complex: In each round they need to tell the player what to do, given the outcome of the previous rounds. Formally:

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- Let $\mathbf{a}(t) = \big(a_1(t), a_2(t)\big)$ denote the outcome of round t, with $a_1(t), a_2(t) \in \{C, D\}$.

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- Let $\mathbf{a}(t) = \big(a_1(t), a_2(t)\big)$ denote the outcome of round t, with $a_1(t), a_2(t) \in \{C, D\}$.
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Direct reciprocity: Repeated Prisoner's dilemma (RPD)

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$\xrightarrow{\delta}$

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Definition 2.4. Repeated prisoner's dilemma

To model whether cooperation can emerge when individuals may interact more than once, we consider the repeated prisoner's dilemma.

- **Players:** The game takes place among two individuals
- **Actions:** The game takes place among multiple rounds (after each round, there is another round with probability $\delta > 0$). In each round, players can either cooperate or defect.
- **Order of moves:** In each round, individuals make their decision simultaneously.
- **Information:** Players know what happened in all previous rounds.
- **Payoffs:** Payoffs $\pi_i(t)$ in each round t are the payoffs of the usual prisoner's dilemma. For the entire repeated prisoner's dilemma, we take the weighted mean:

$$\pi_i = (1-\delta)\sum_{t=1}^{\infty} \delta^t \cdot \pi_i(t) \qquad \pi_i = \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \pi_i(t)$$

[if $\delta < 1$] [If $\delta = 1$]

Remark 2.5. Strategies for the repeated prisoner's dilemma

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- The set $H = \cup_{t=1}^{\infty} H_t$ is the *set of all possible histories*
- A strategy σ is now a function that takes as input the possible histories, and as output an action:

$$\sigma : H \to \{C, D\}$$

Direct reciprocity: Repeated Prisoner's dilemma (RPD)

Example 2.6. Some strategies for the RPD

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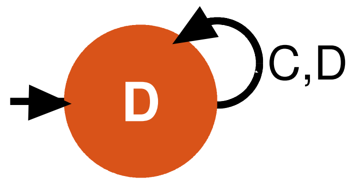

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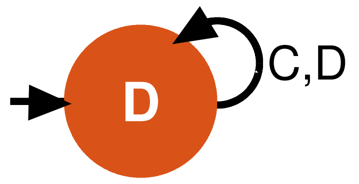

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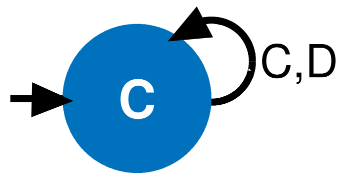

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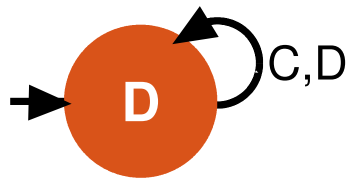

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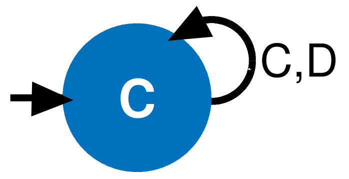

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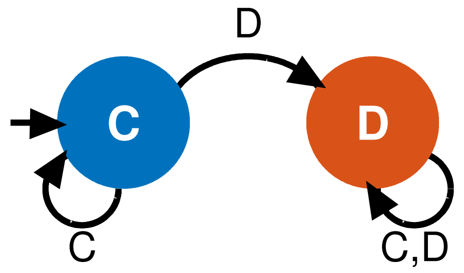

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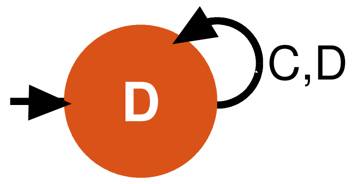

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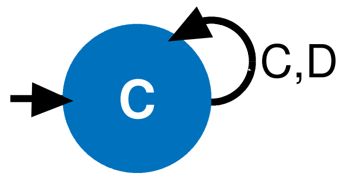

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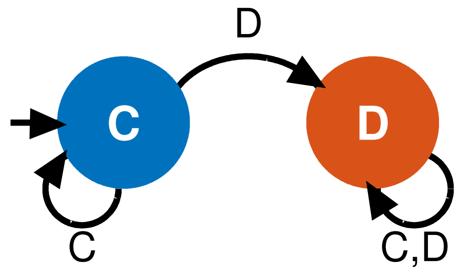

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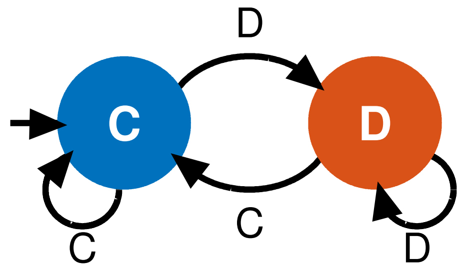

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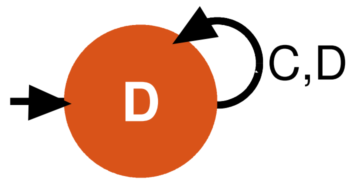

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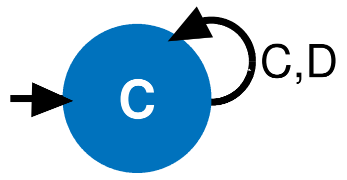

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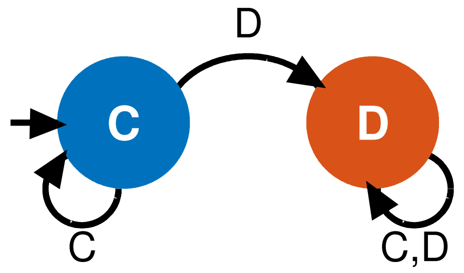

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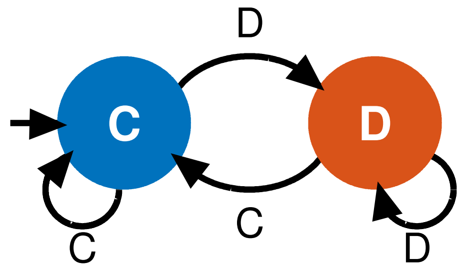

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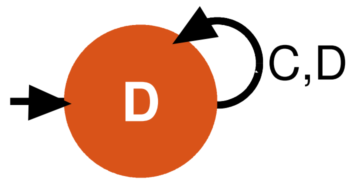

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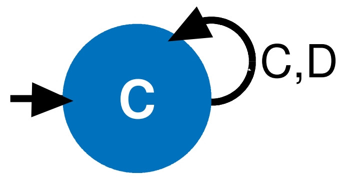

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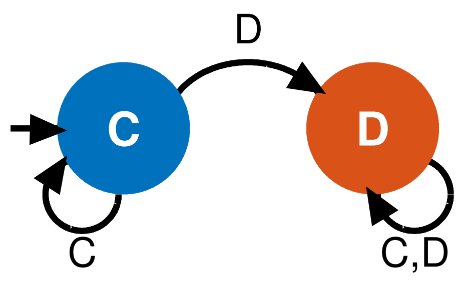

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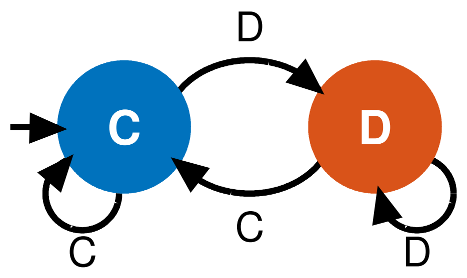

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- If the number of rounds played so far is prime, defect. Otherwise, cooperate if and only if the co-player cooperated twice as often as the focal player, across all rounds so far.

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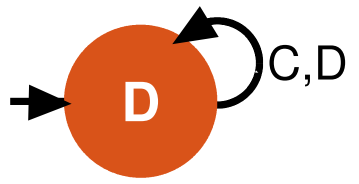

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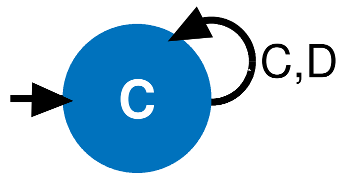

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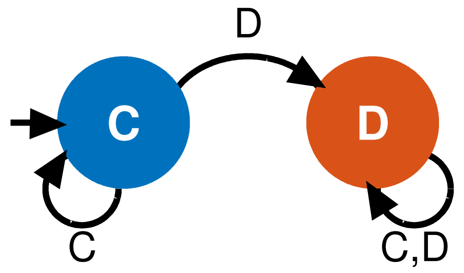

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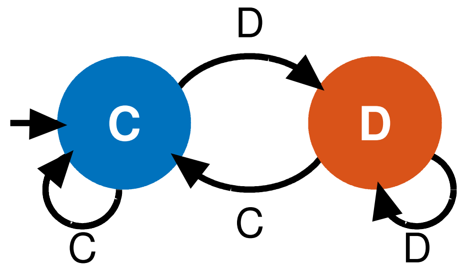

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Definition 2.7. Nash equilibrium

A strategy profile (σ_1^*, σ_2^*) is a Nash equilibrium for the repeated game if

$$\pi_1(\sigma, \sigma_2^*) \leq \pi_1(\sigma_1^*, \sigma_2^*) \quad \text{and} \quad \pi_2(\sigma_1^*, \sigma) \leq \pi_2(\sigma_1^*, \sigma_2^*) \quad \text{for all } \sigma$$

Direct reciprocity: Repeated Prisoner's dilemma (RPD)

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	C	D
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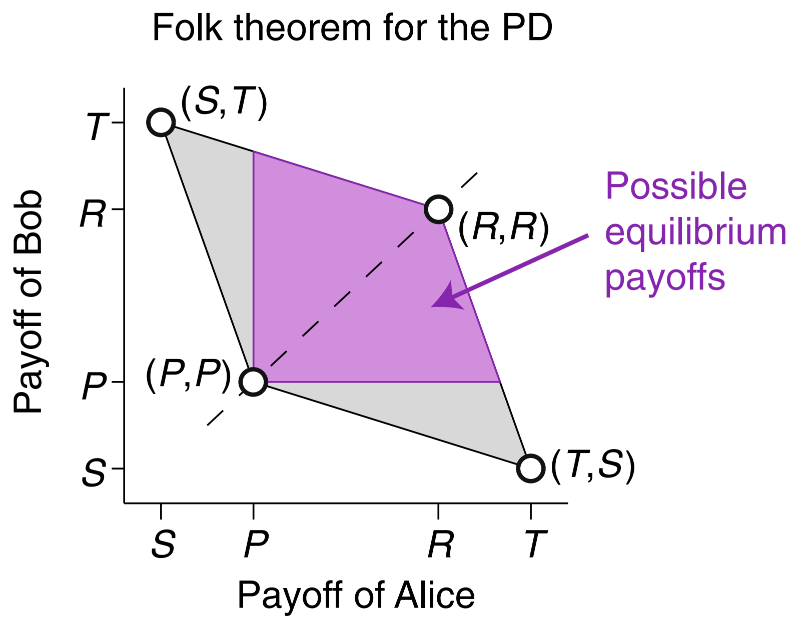

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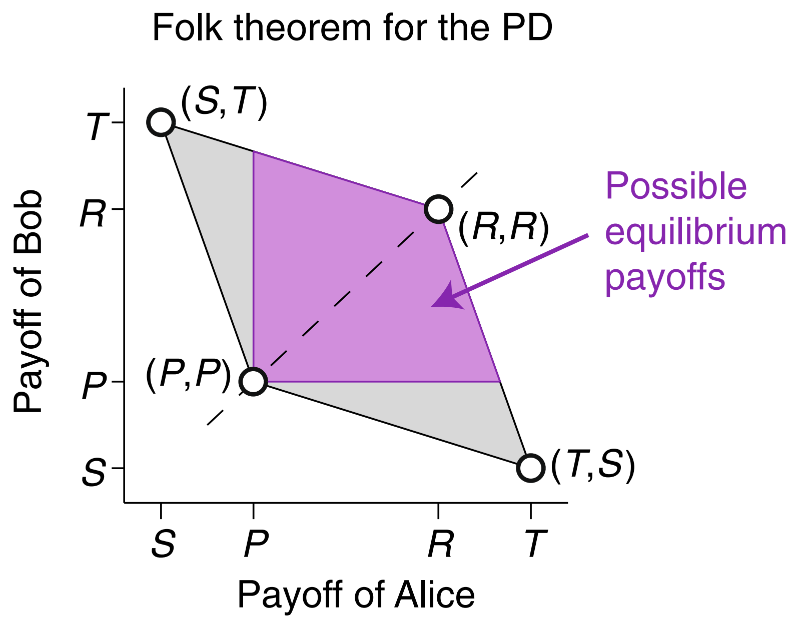

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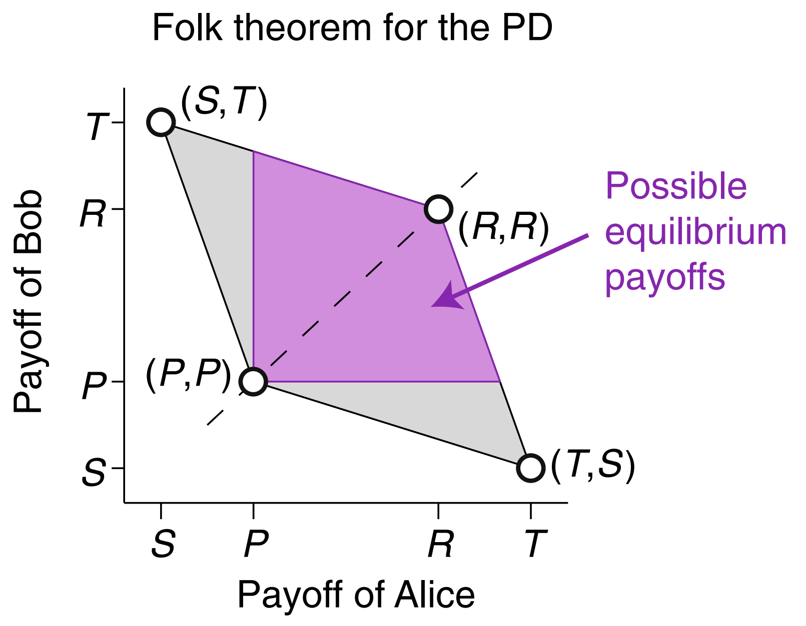

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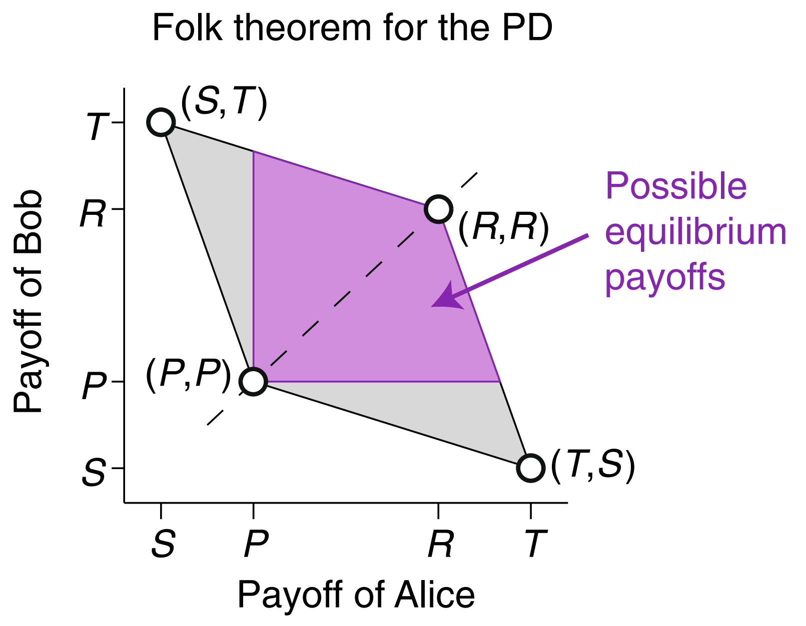

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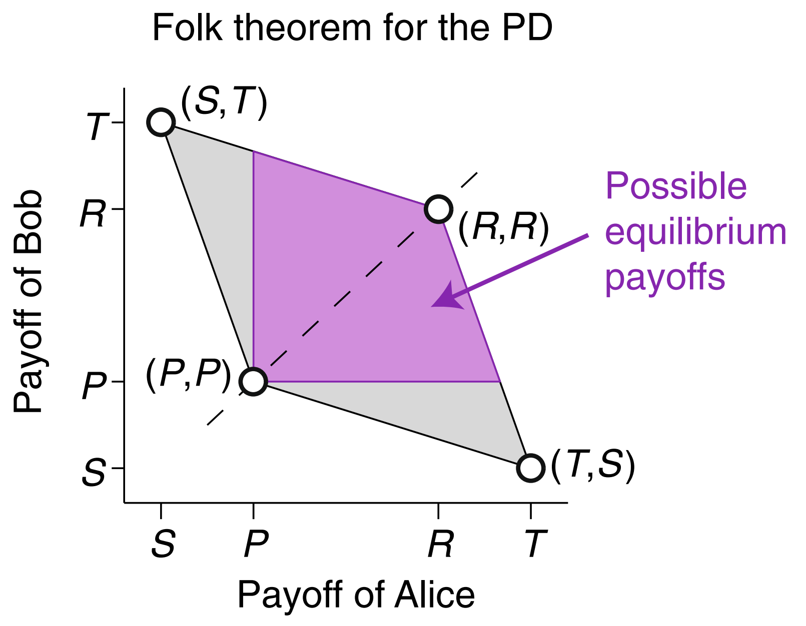

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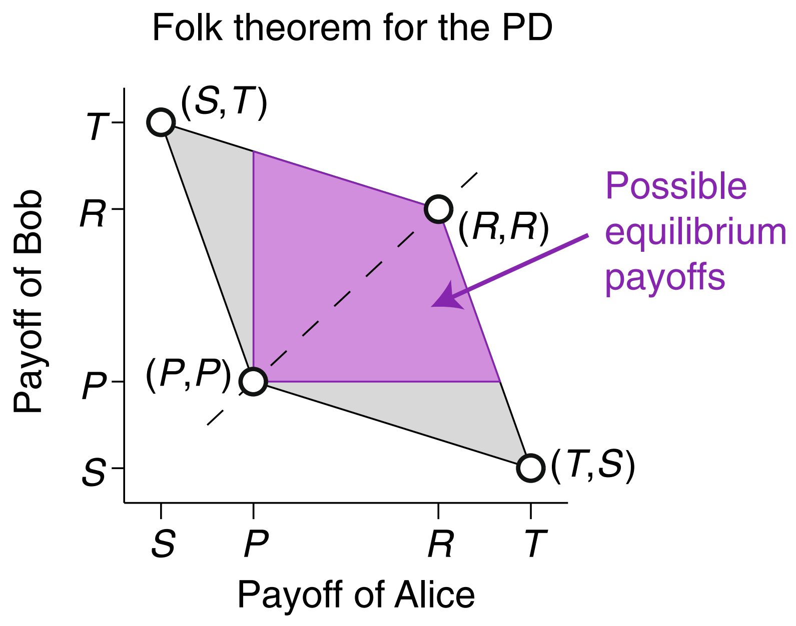

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  2. It remains unclear what strategy one should use in the repeated prisoner's dilemma in general.

Direct reciprocity: Axelrod's tournament

Remark 2.10. Axelrod & Hamilton (1981)

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The Evolution of Cooperation

Robert Axelrod and William D. Hamilton

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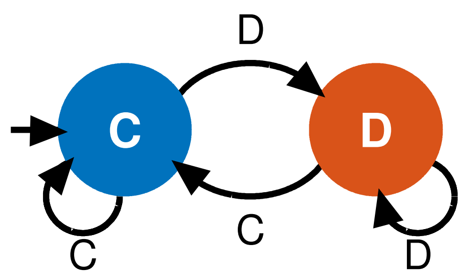

Direct reciprocity: Axelrod's tournament

The Evolution of Cooperation

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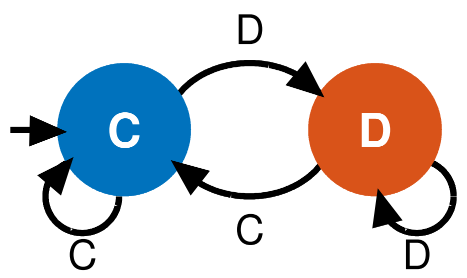

TFT

ALLD

Payoffs

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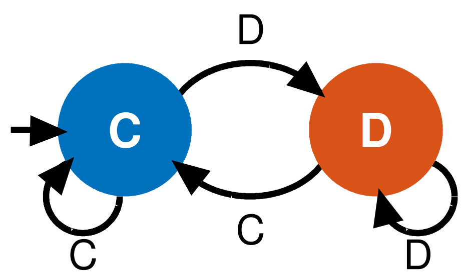

The Evolution of Cooperation

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	Round 1
TFT	C
ALLD	D
Payoffs	(-c,b)

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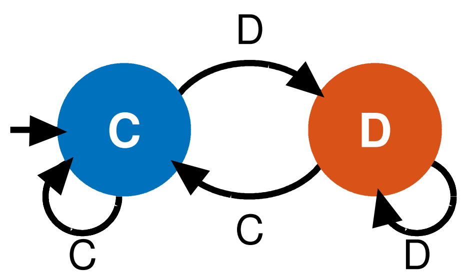

	Round 1	Round 2
TFT	C	D
ALLD	D	D
Payoffs	$(-c,b)$	$(0,0)$

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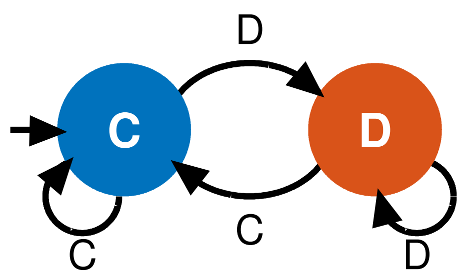

	Round 1	Round 2	Round 3
TFT	C	D	D
ALLD	D	D	D
Payoffs	(-c,b)	(0,0)	(0,0)

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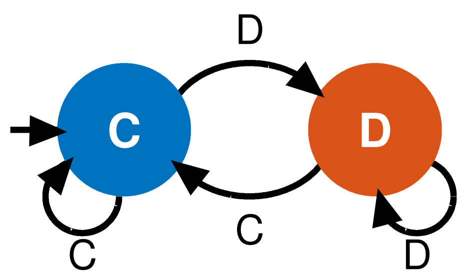

	Round 1	Round 2	Round 3	Round 4
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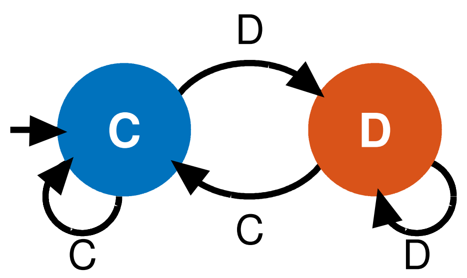

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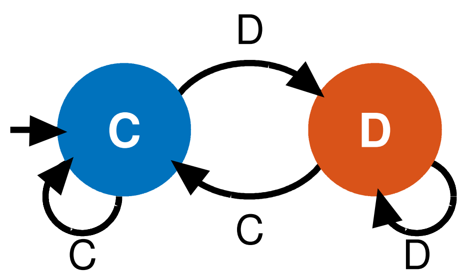

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Payoffs	(-c,b)	(0,0)	(0,0)	(0,0)	…

Average payoffs:

$$\pi_{TFT} = -(1-\delta)c$$

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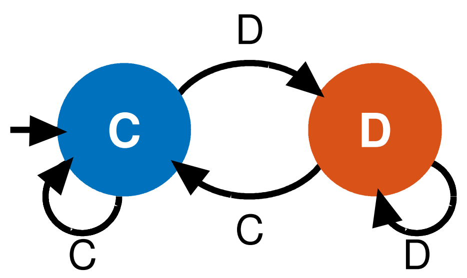

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	Round 1	Round 2	Round 3	Round 4	…
TFT	C	C	C	C	⋯
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Payoffs	(b-c,b-c)	(b-c,b-c)	(b-c,b-c)	(b-c,b-c)	…

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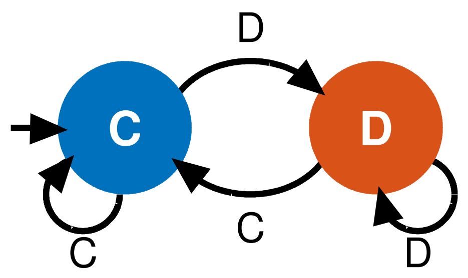

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Payoffs	(b-c,b-c)	(b-c,b-c)	(b-c,b-c)	(b-c,b-c)	…

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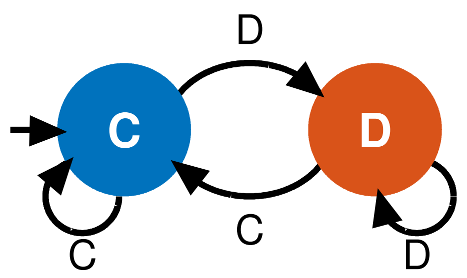

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$$\pi_{TFT} = b - c$$

Tit-for-Tat is a best response against itself if $b - c > (1-\delta)b$, or if $\delta > c/b$.

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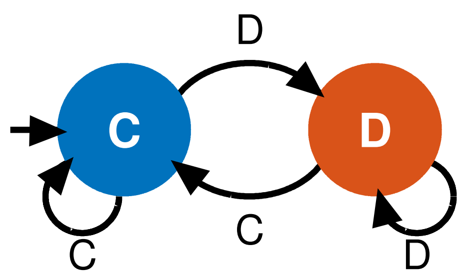

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NATURE VOL. 325 29 JANUARY 1987 LETTERS

TIT FOR TAT in sticklebacks and the evolution of cooperation

Manfred Milinski

Arbeitsgruppe für Verhaltensforschung, Fakultät für Biologie, Ruhr-Universität, Postfach 102148, 4630 Bochum 1, FRG

Reciprocal food sharing in the vampire bat

Gerald S. Wilkinson

Department of Biology, C-016, Uni San Diego, La Jolla, California 9209

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TFT	C	C	C	D	C	…

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$$M = \begin{pmatrix} p_{CC}q_{CC} & p_{CC}(1-q_{CC}) & (1-p_{CC})q_{CC} & (1-p_{CC})(1-q_{CC}) \\ p_{CD}q_{DC} & p_{CD}(1-q_{DC}) & (1-p_{CD})q_{DC} & (1-p_{CD})(1-q_{DC}) \\ p_{DC}q_{CD} & p_{DC}(1-q_{CD}) & (1-p_{DC})q_{CD} & (1-p_{DC})(1-q_{CD}) \\ p_{DD}q_{DD} & p_{DD}(1-q_{DD}) & (1-p_{DD})q_{DD} & (1-p_{DD})(1-q_{DD}) \end{pmatrix}$$

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- In particular, for any pair of memory-1 strategies $\mathbf{p}$ and $\mathbf{q}$, one can compute the players' payoffs $\pi_1(\mathbf{p},\mathbf{q})$ and $\pi_2(\mathbf{p},\mathbf{q})$.

Direct reciprocity: Evolution among Memory-1 strategies

Remark 2.13. Nowak and Sigmund (1993)

- Approach: Let evolution determine an "optimal" strategy, by running individual-based simulations.

A strategy of win–stay, lose–shift that outperforms tit-for-tat in the Prisoner's Dilemma game

Martin Nowak* & Karl Sigmund†

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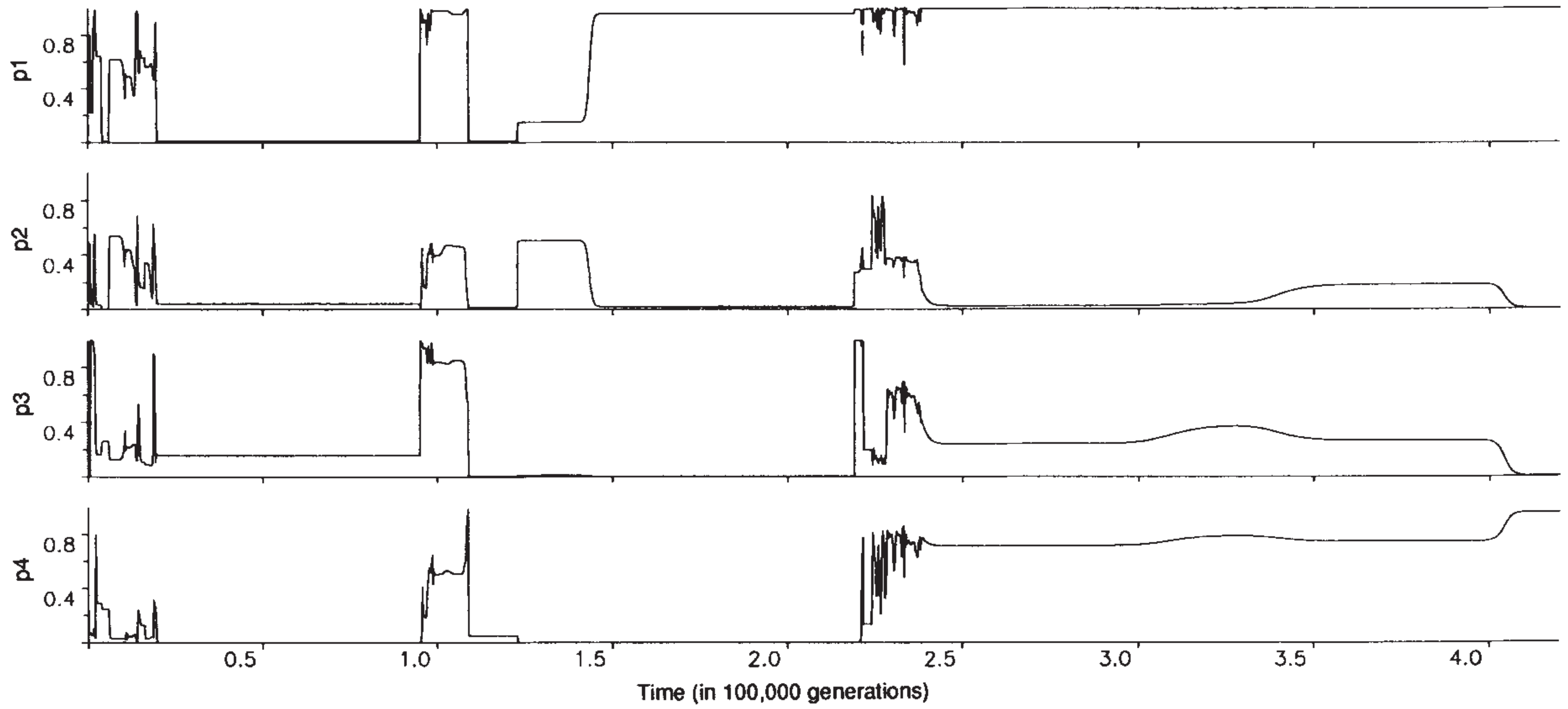

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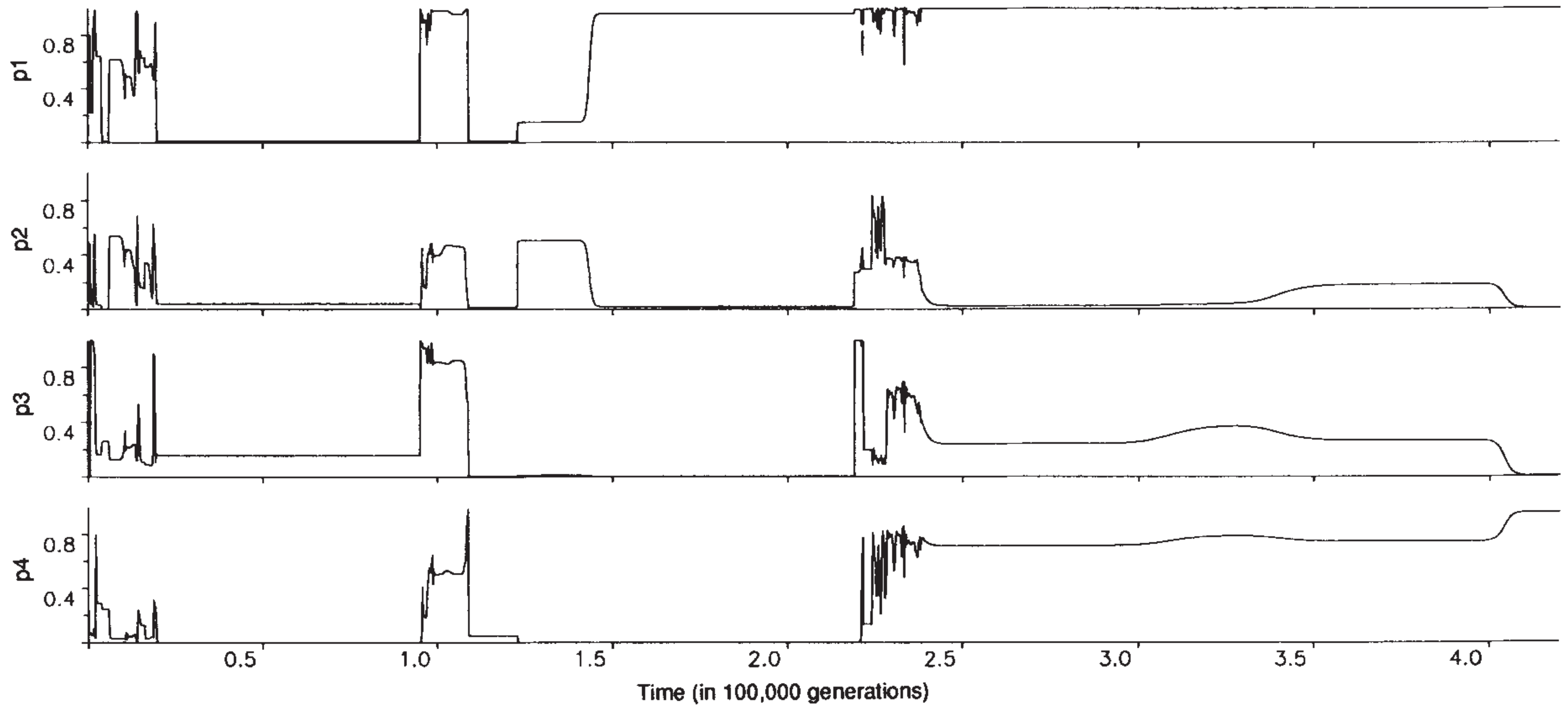

Win-stay Lose-shift WSLS = (1,1,0,0,1)
Round 1: Cooperate
All other rounds:
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Direct reciprocity: Evolution among Memory-1 strategies

Remark 2.13. Nowak and Sigmund (1993)

- Approach: Let evolution determine an "optimal" strategy, by running individual-based simulations.
- Consider a finite population of size N
- Initially, all players use random memory-1 strategies.
- Players obtain payoffs by playing the repeated prisoner's dilemma with all other population members.
- Afterwards, players reproduce proportional to their fitness. In addition, mutations may introduce new strategies.
- Result: Most simulations lead to Win-Stay Lose-Shift. This strategy is robust against errors, it can exploit unconditional cooperators, and it is a Nash equilibrium if $b>2c$.

A strategy of win–stay, lose–shift that outperforms tit-for-tat in the Prisoner's Dilemma game

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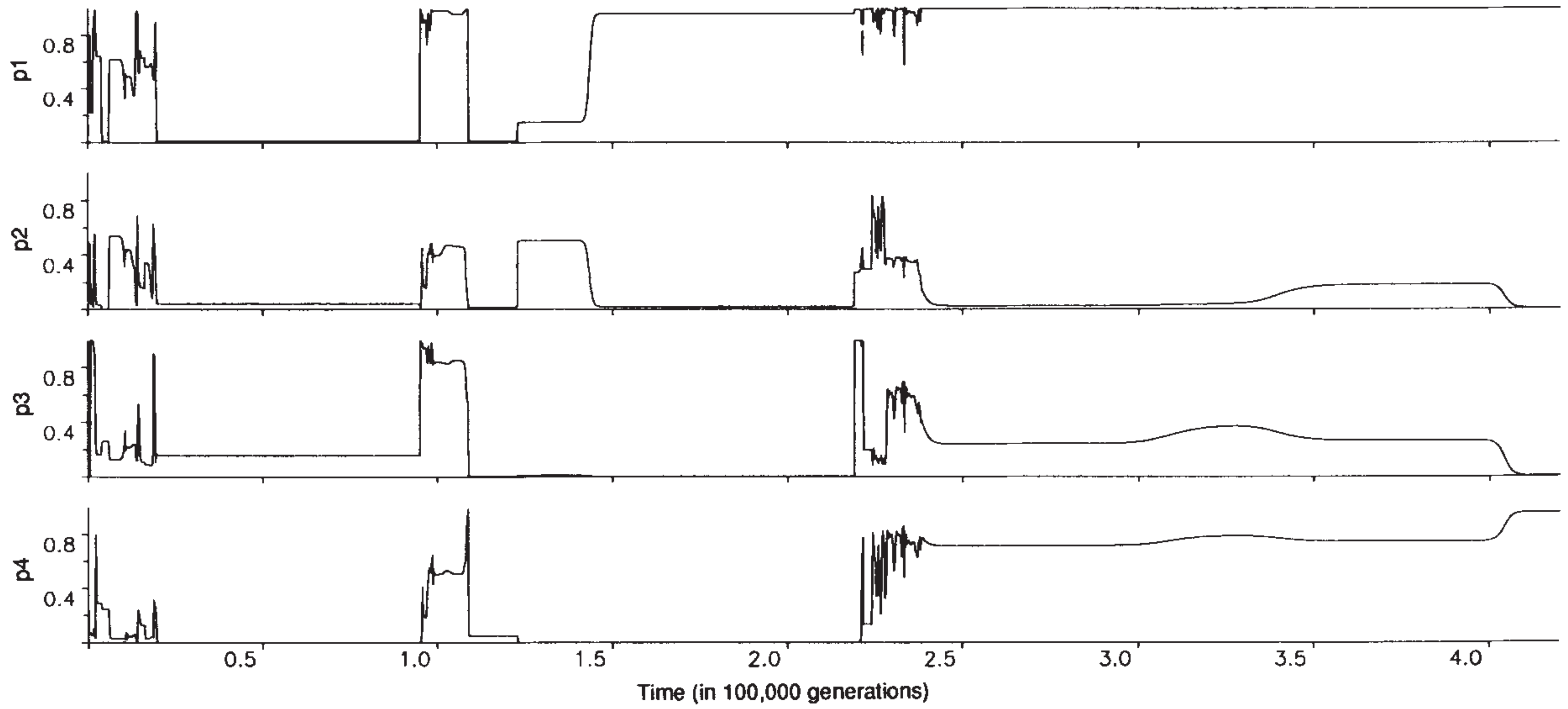

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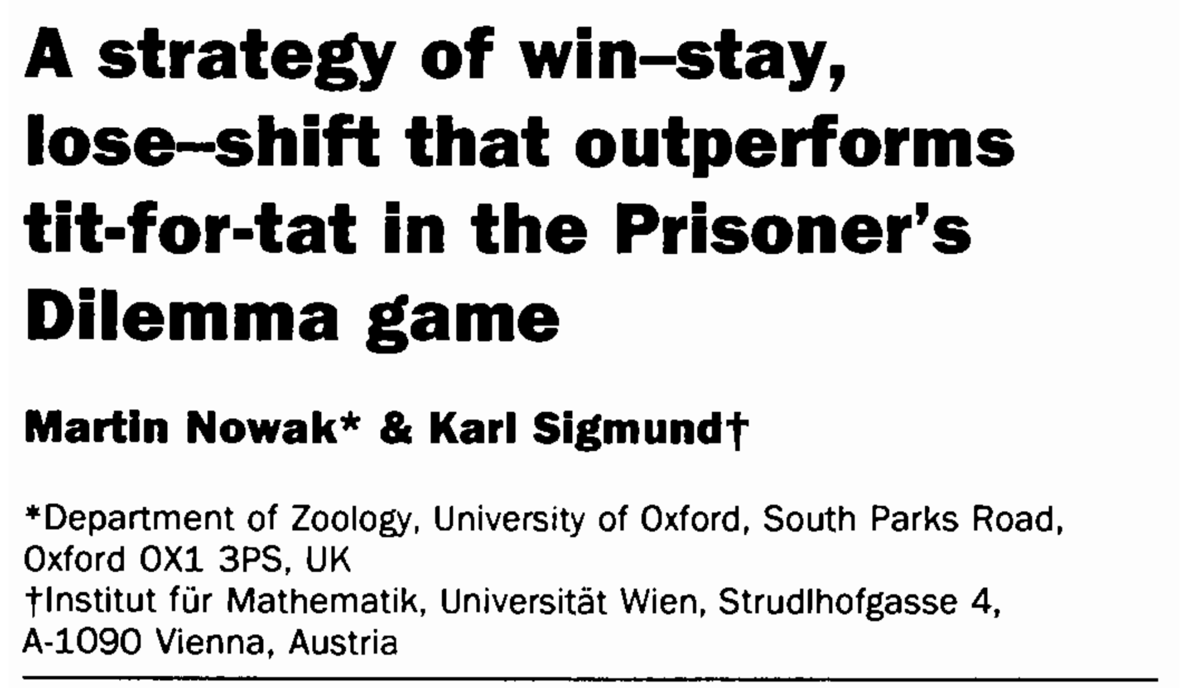
  [For memory-k strategies, one can find generalised versions of this strategy that are stable for $b > \frac{k+1}{k}c$]



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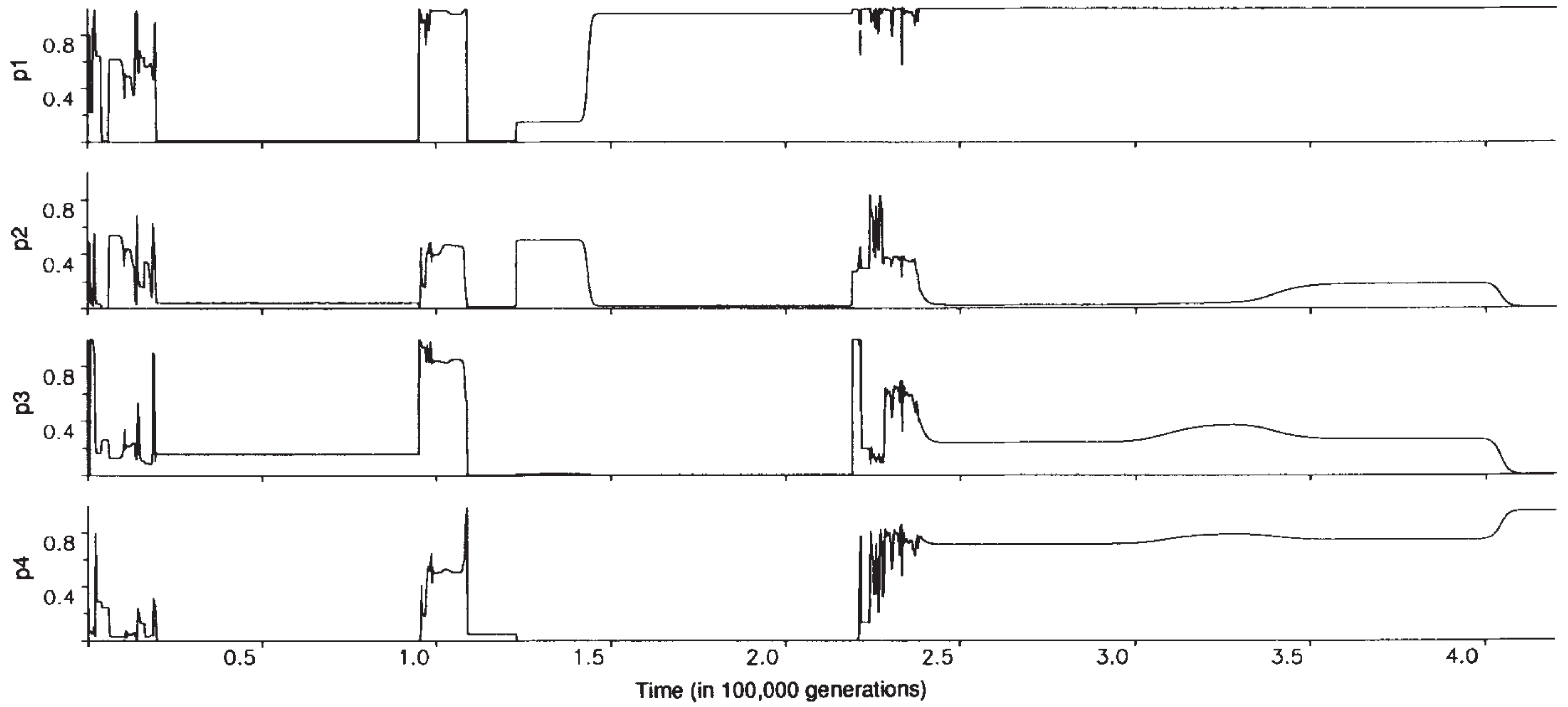

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Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent

William H. Press[a,1] and Freeman J. Dyson[b]

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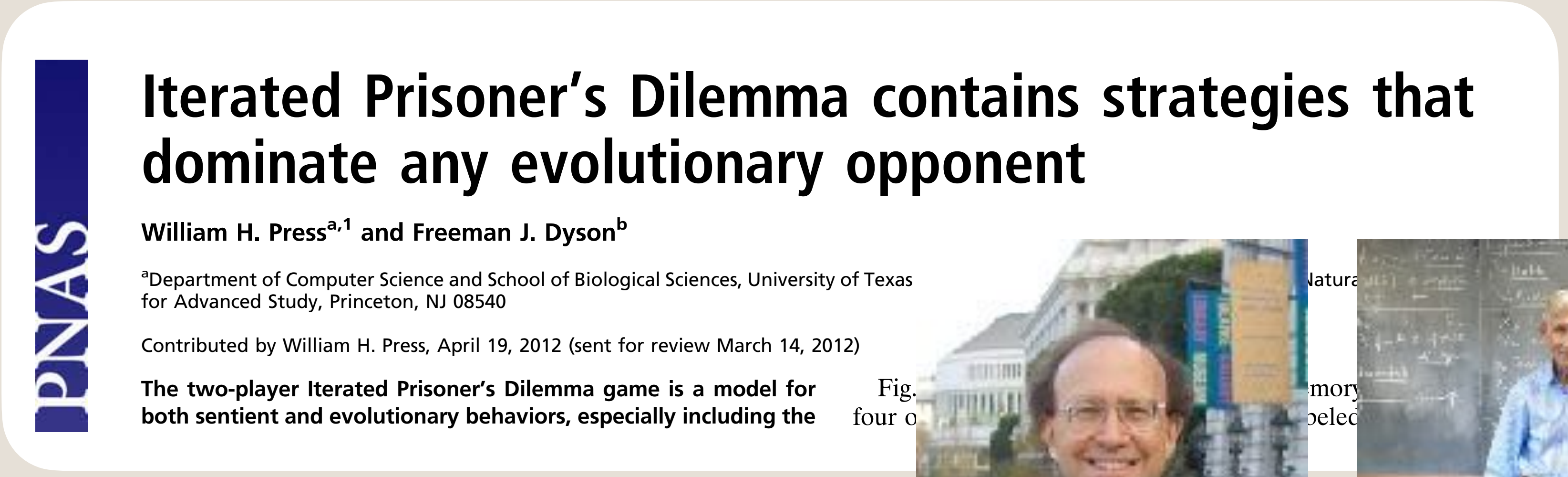

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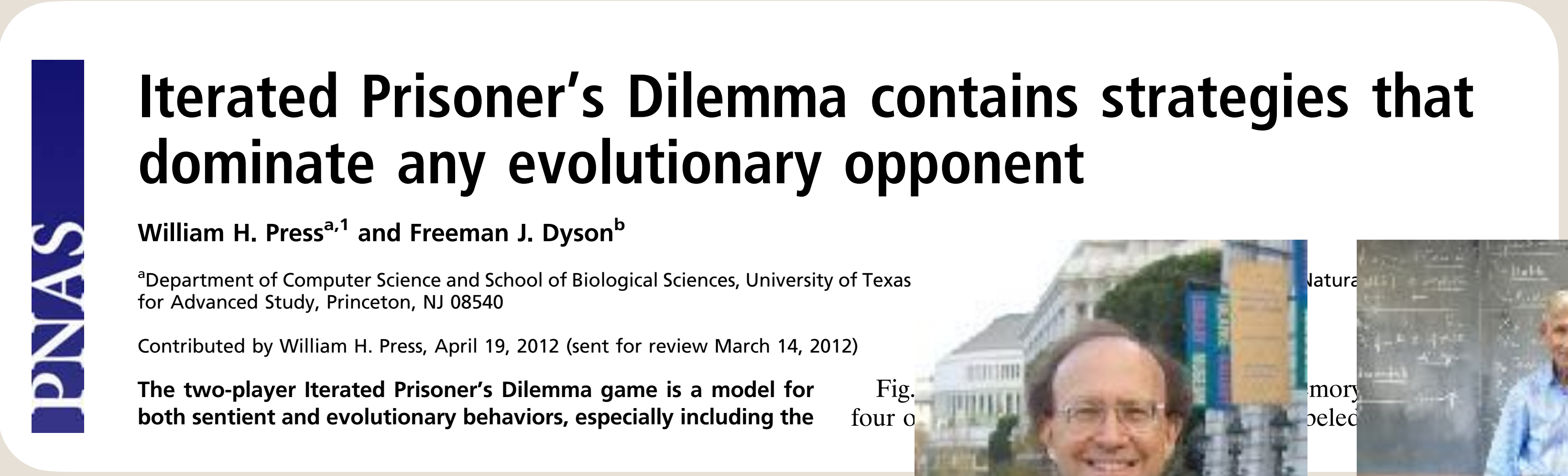

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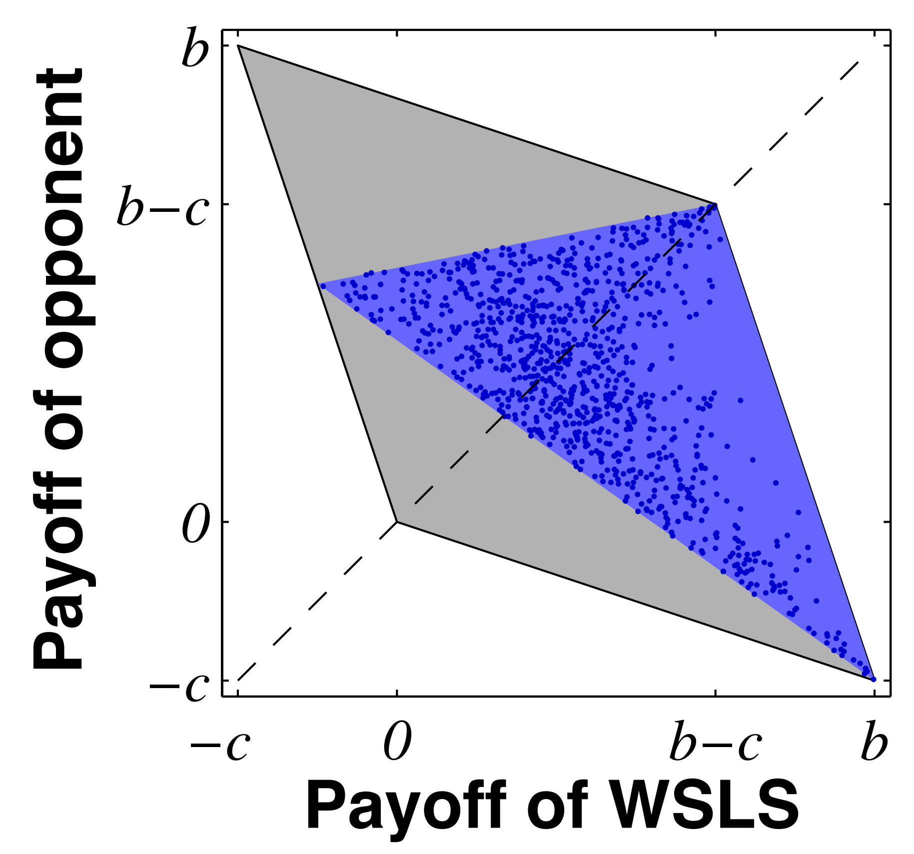

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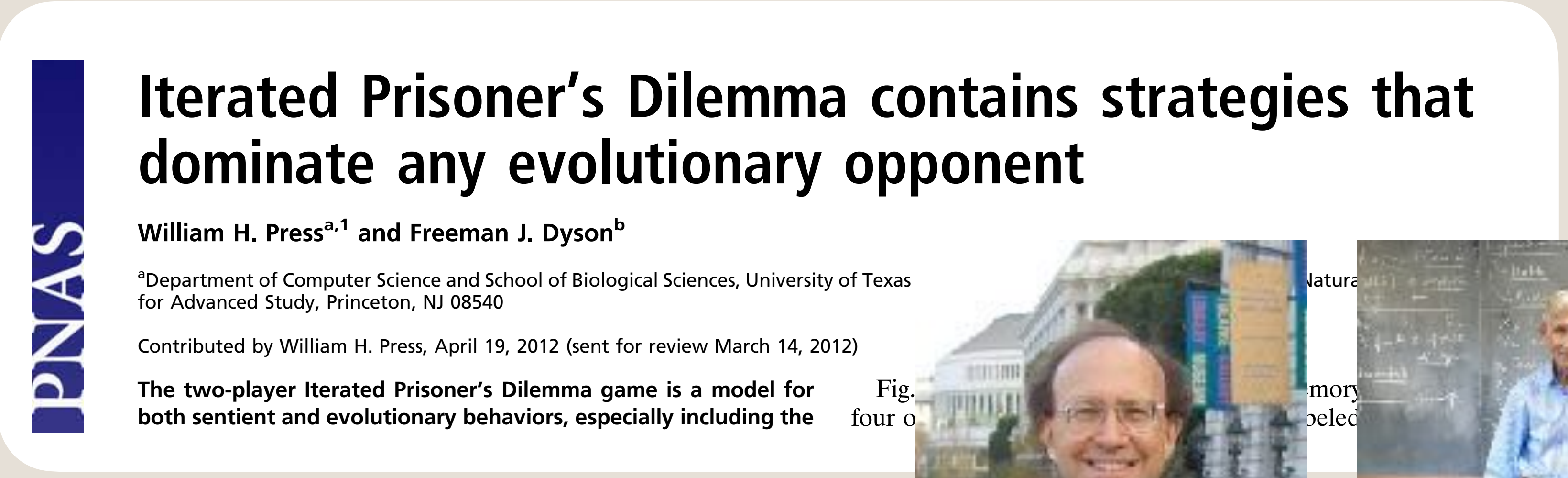
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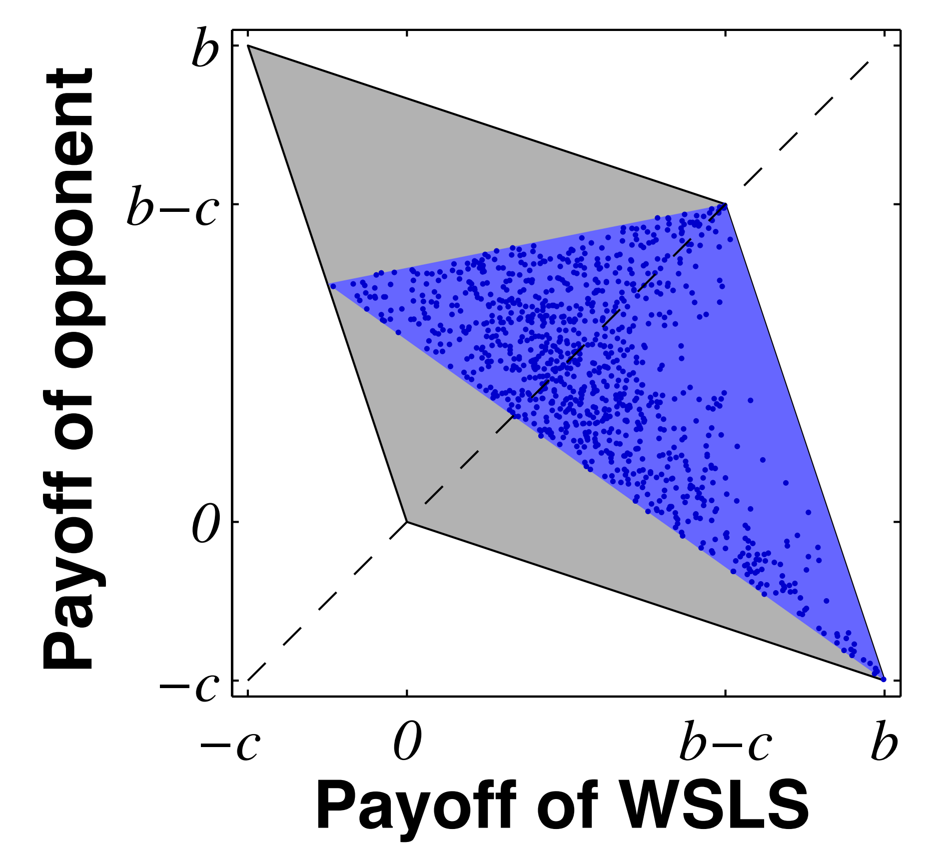

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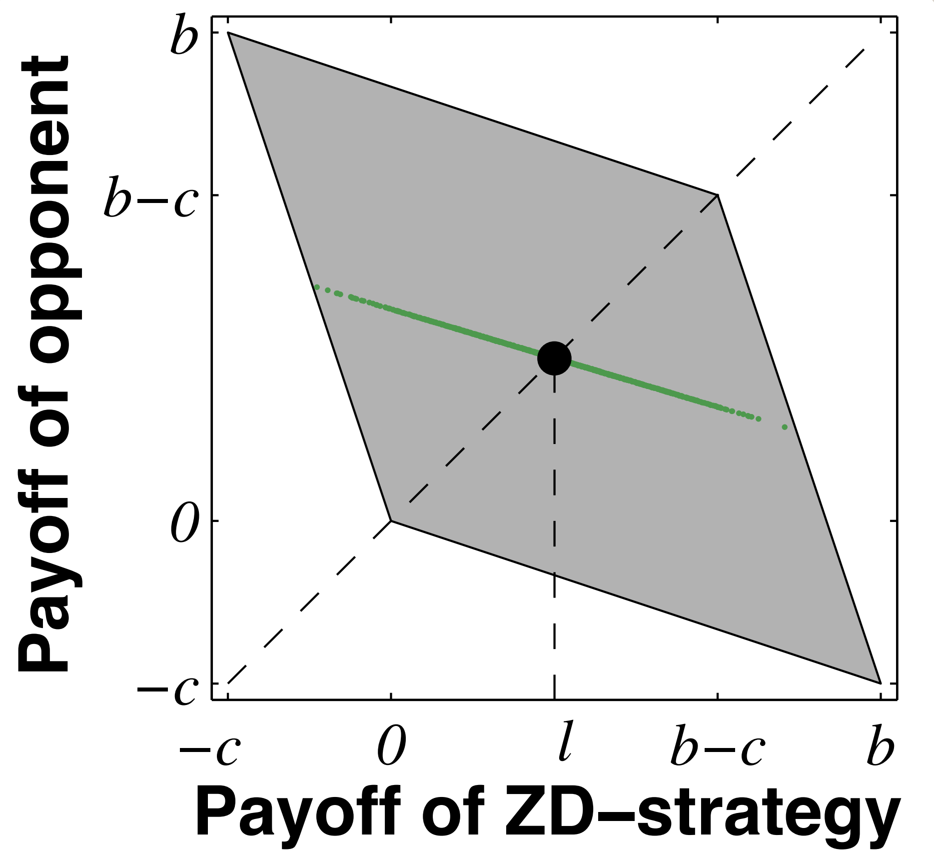

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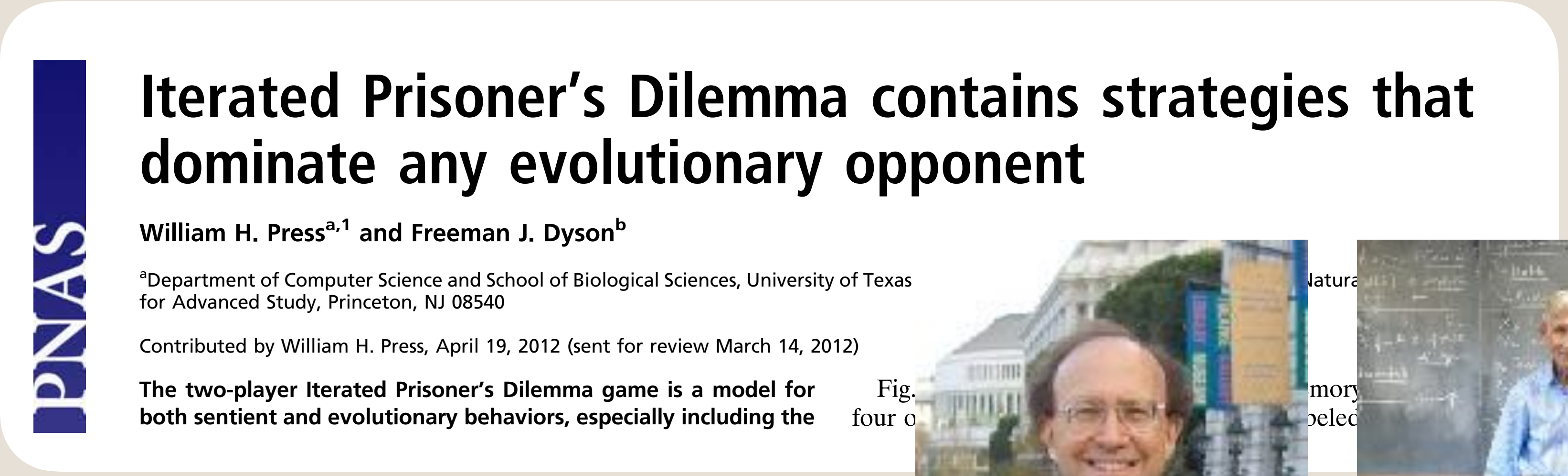
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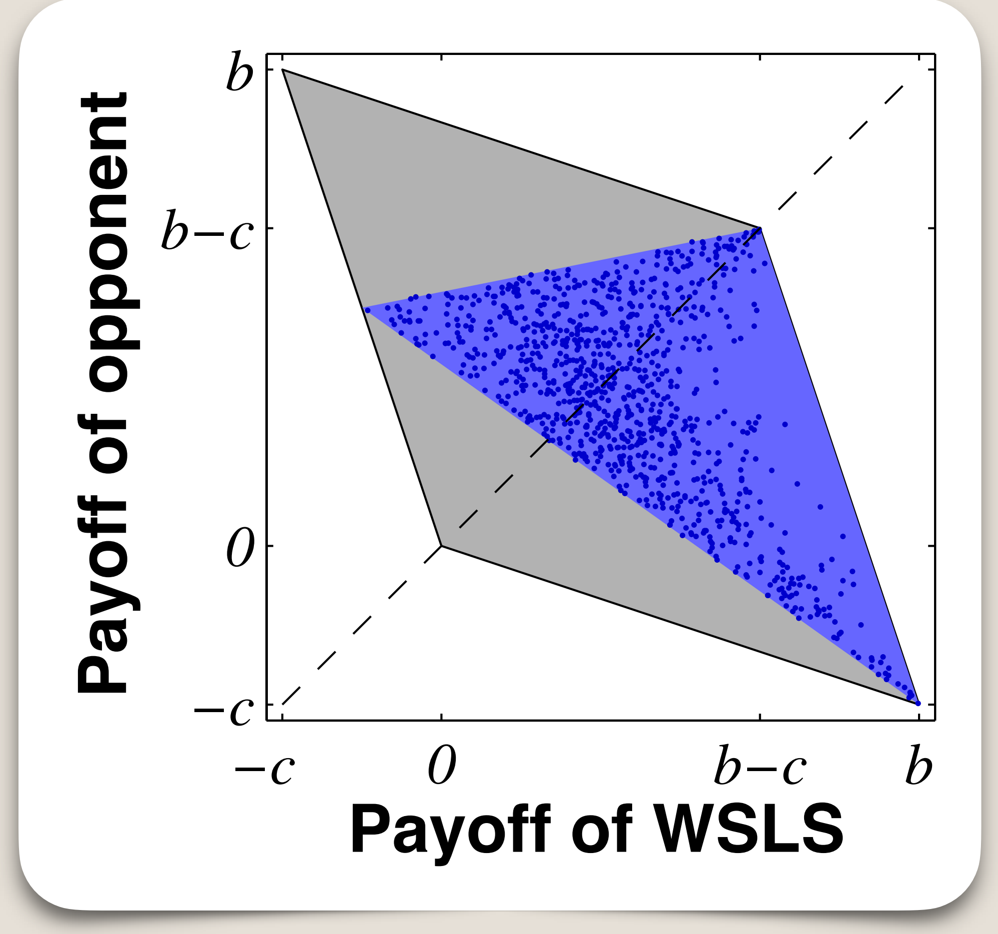

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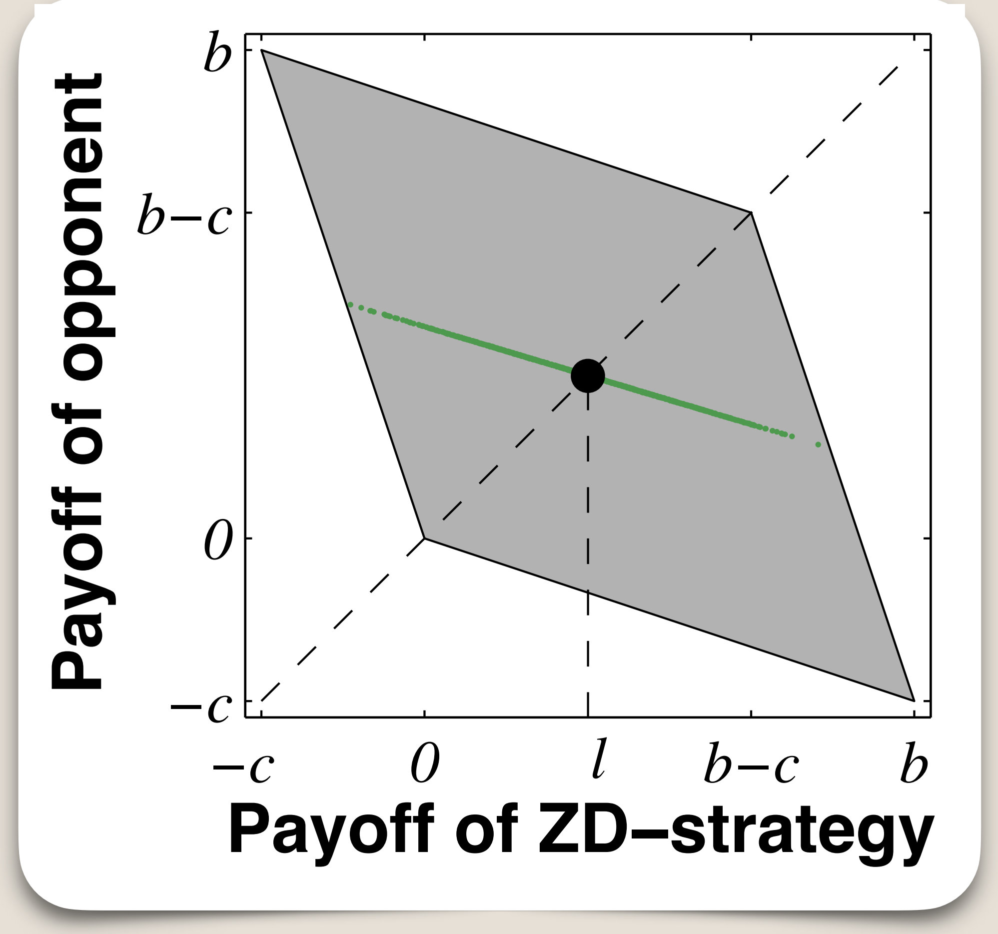

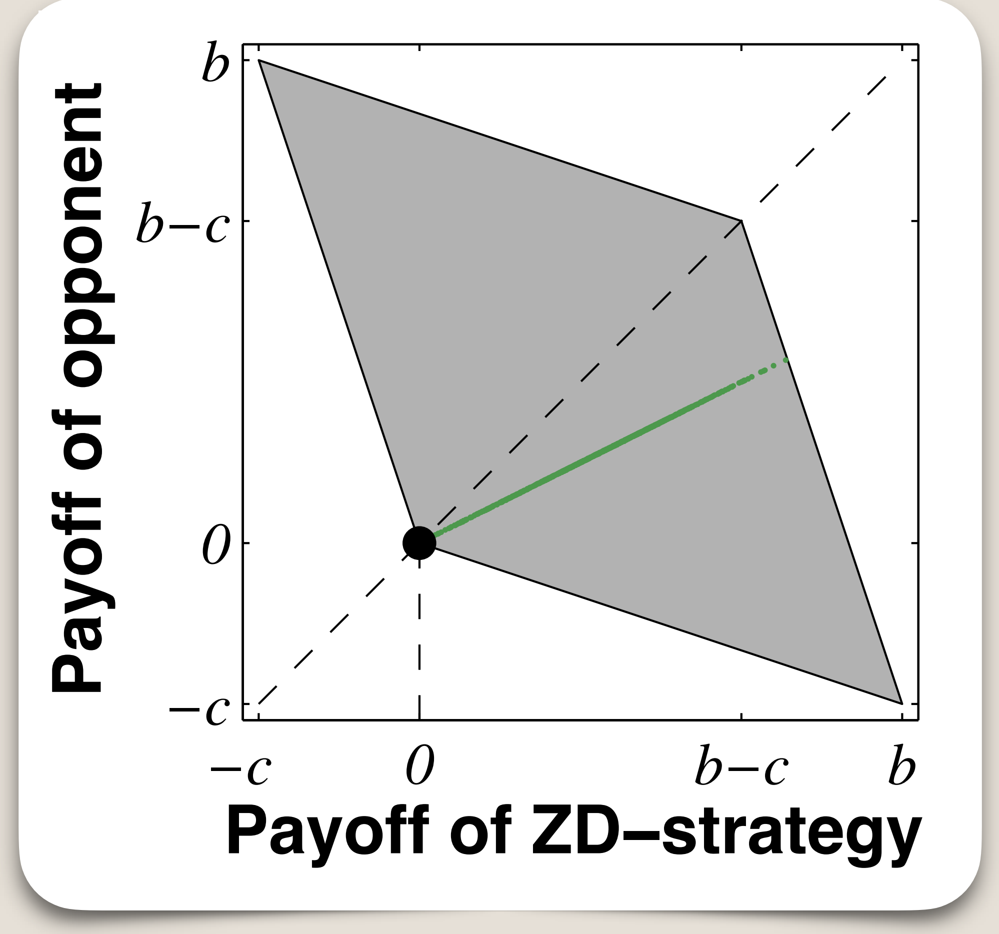

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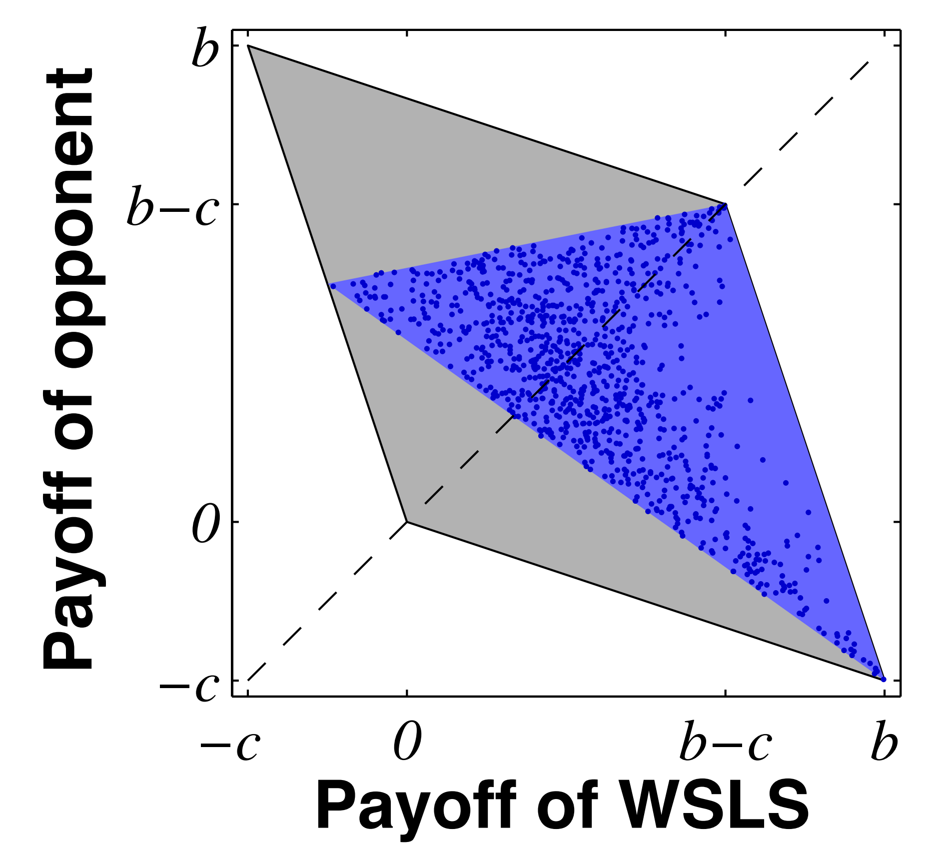

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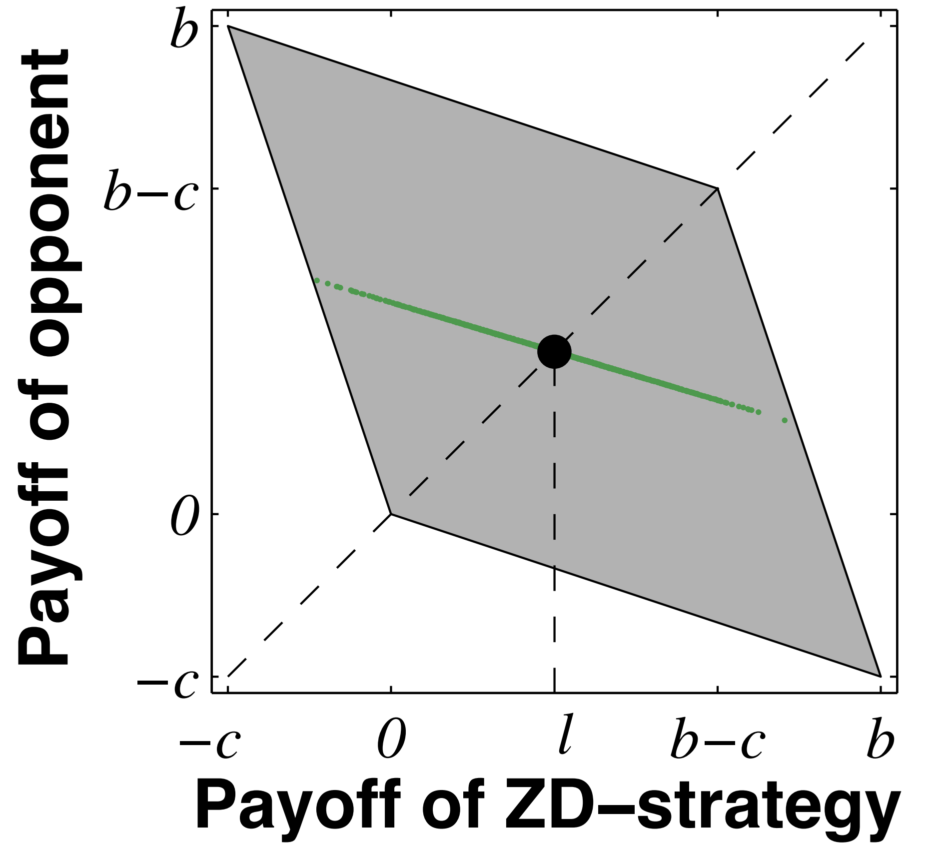
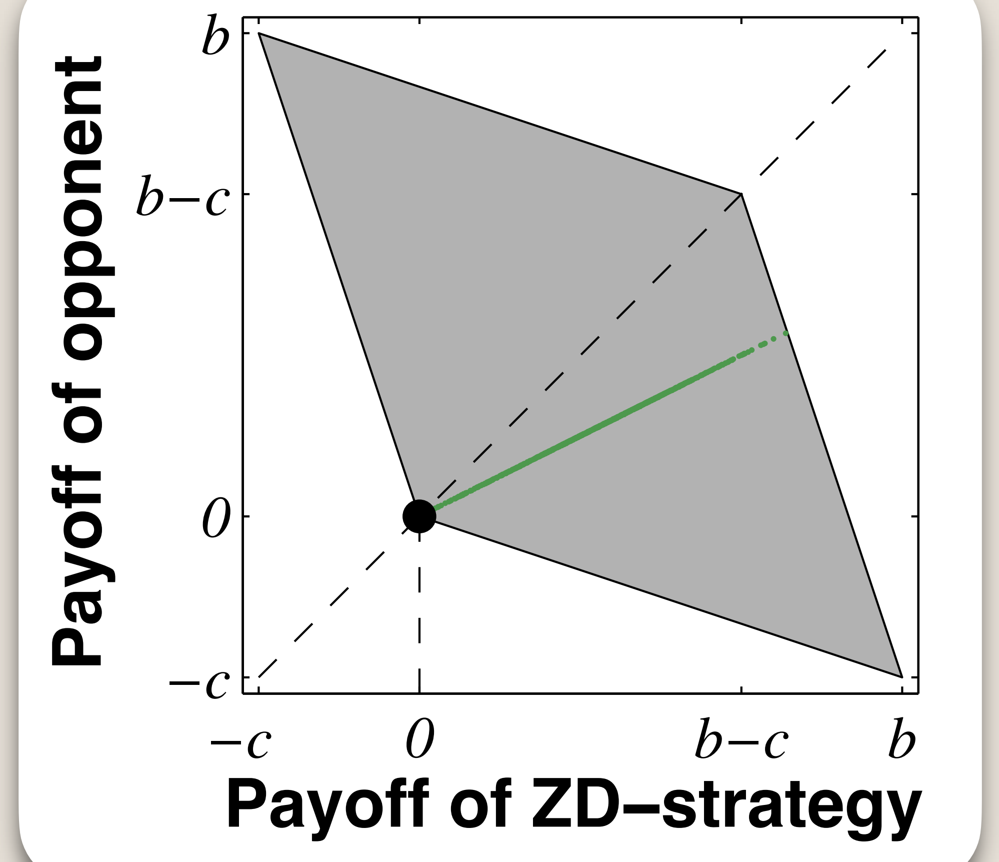


Extortionate ZD strategy



Direct reciprocity: Evolution of extortion

Evolution of extortion in Iterated Prisoner's Dilemma games

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Remark 2.15. Can extortion evolve?

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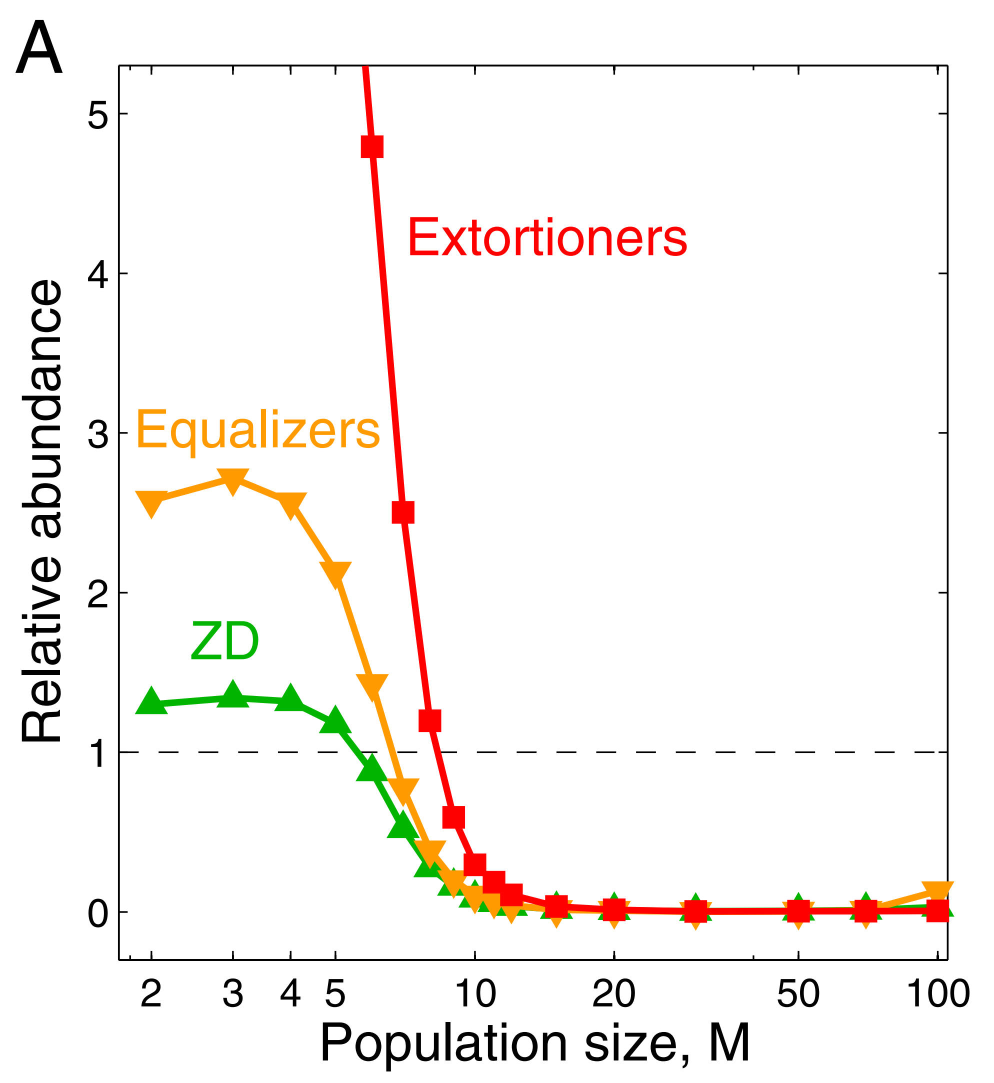

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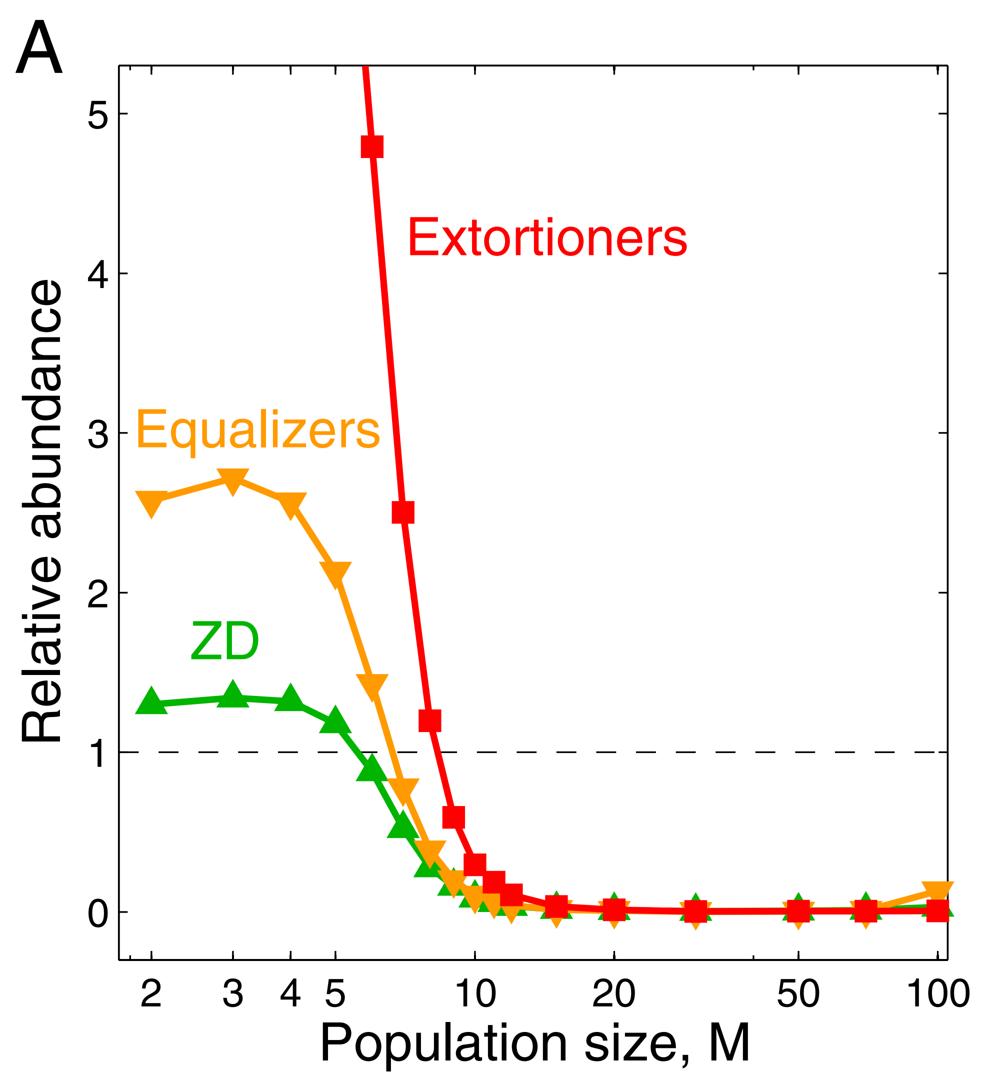

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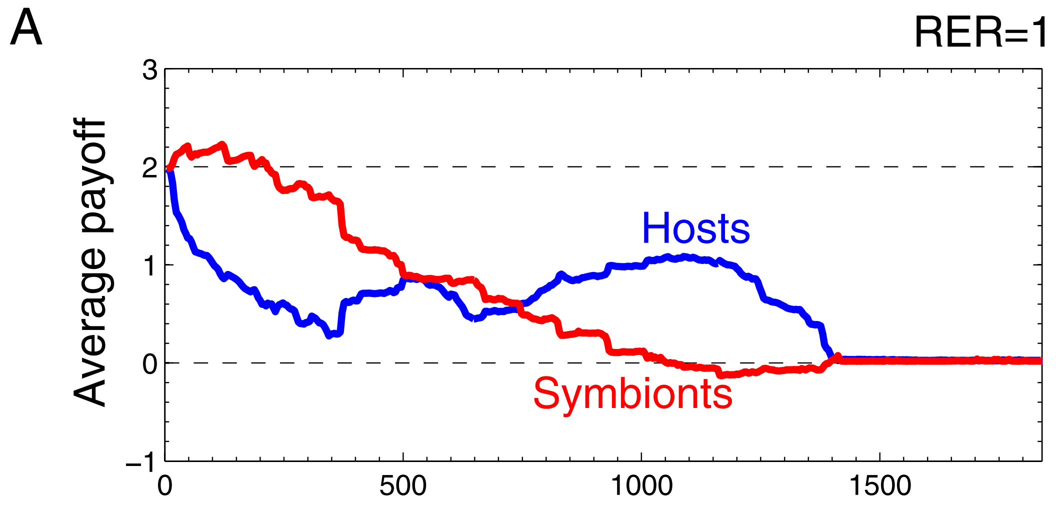

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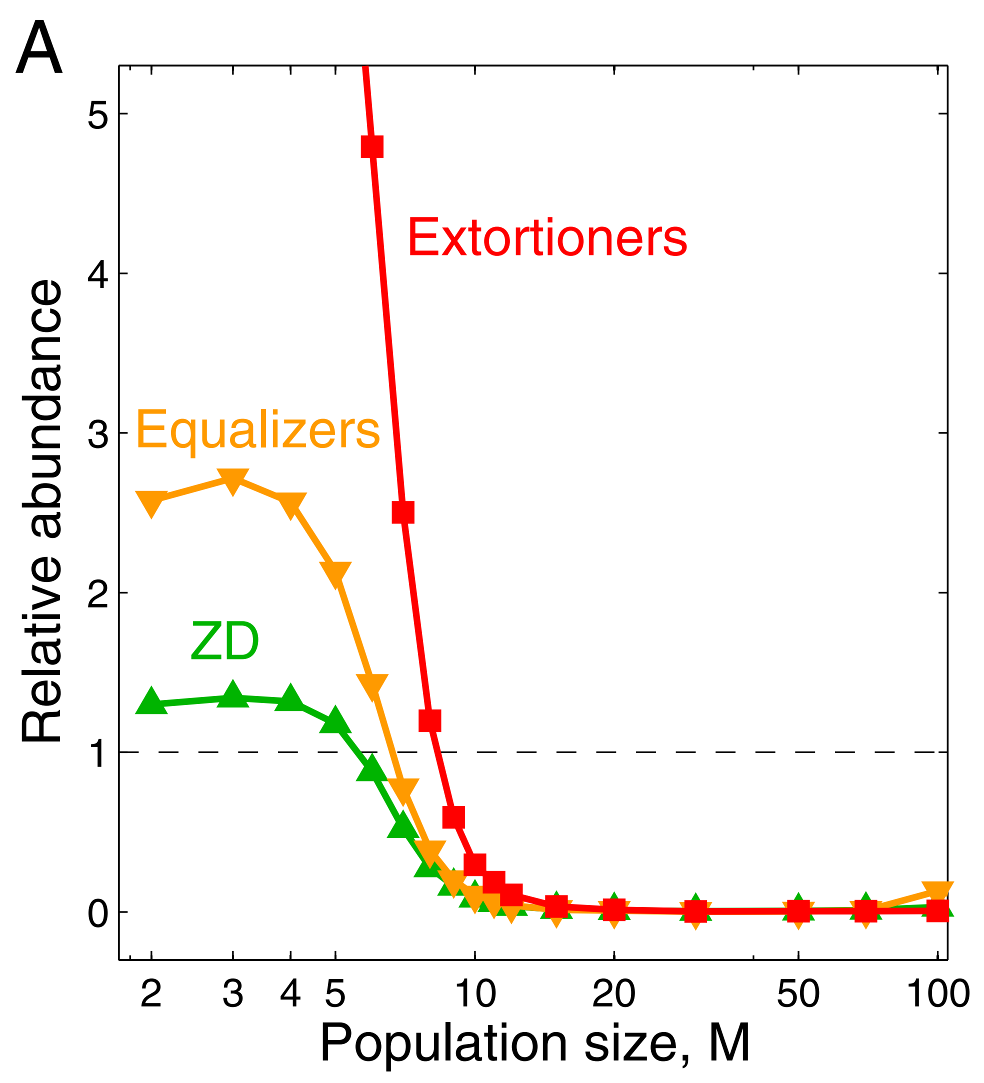

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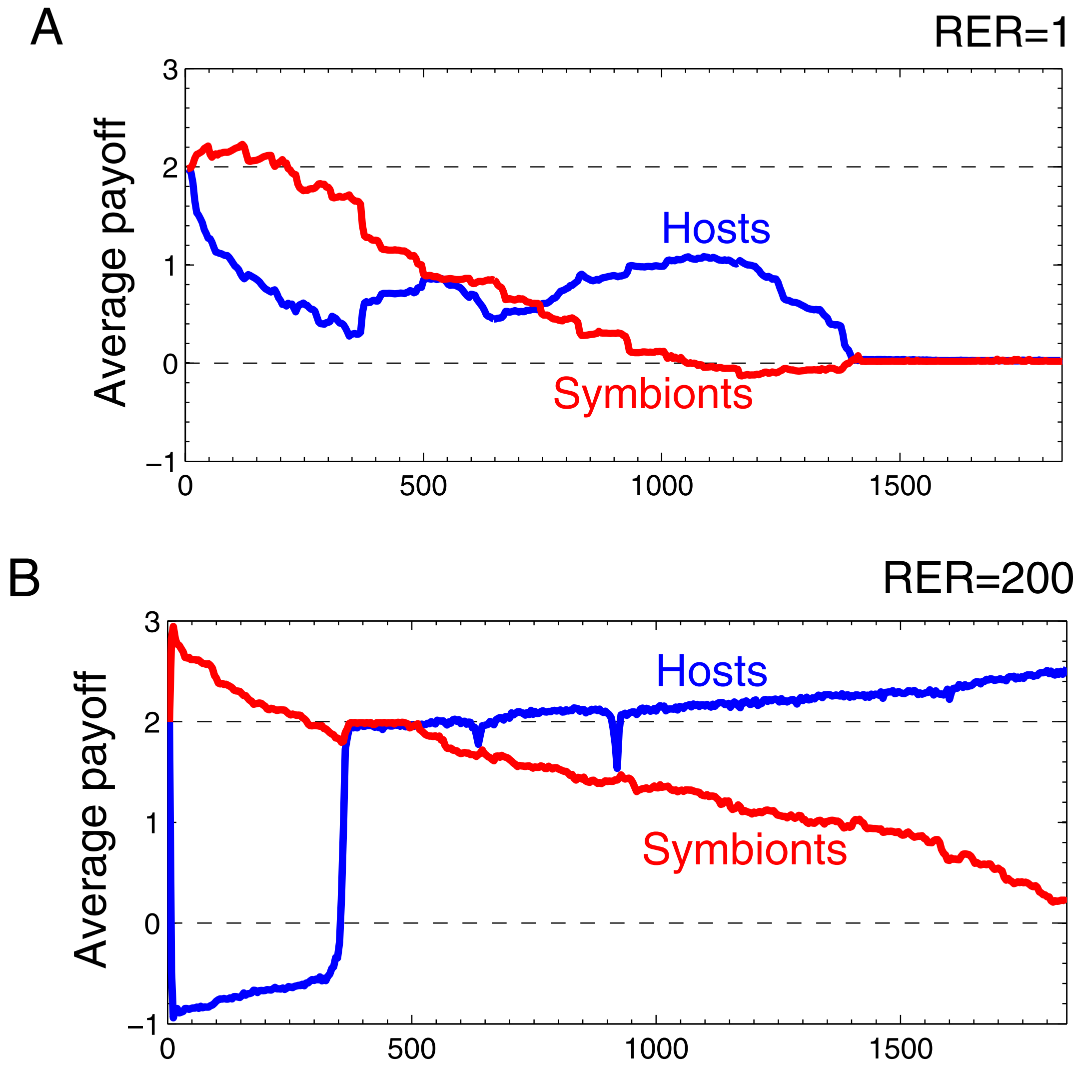

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- Result: Extortion only evolves when populations are small, or when two populations interact and one population evolves at a much slower rate ("Red king effect")

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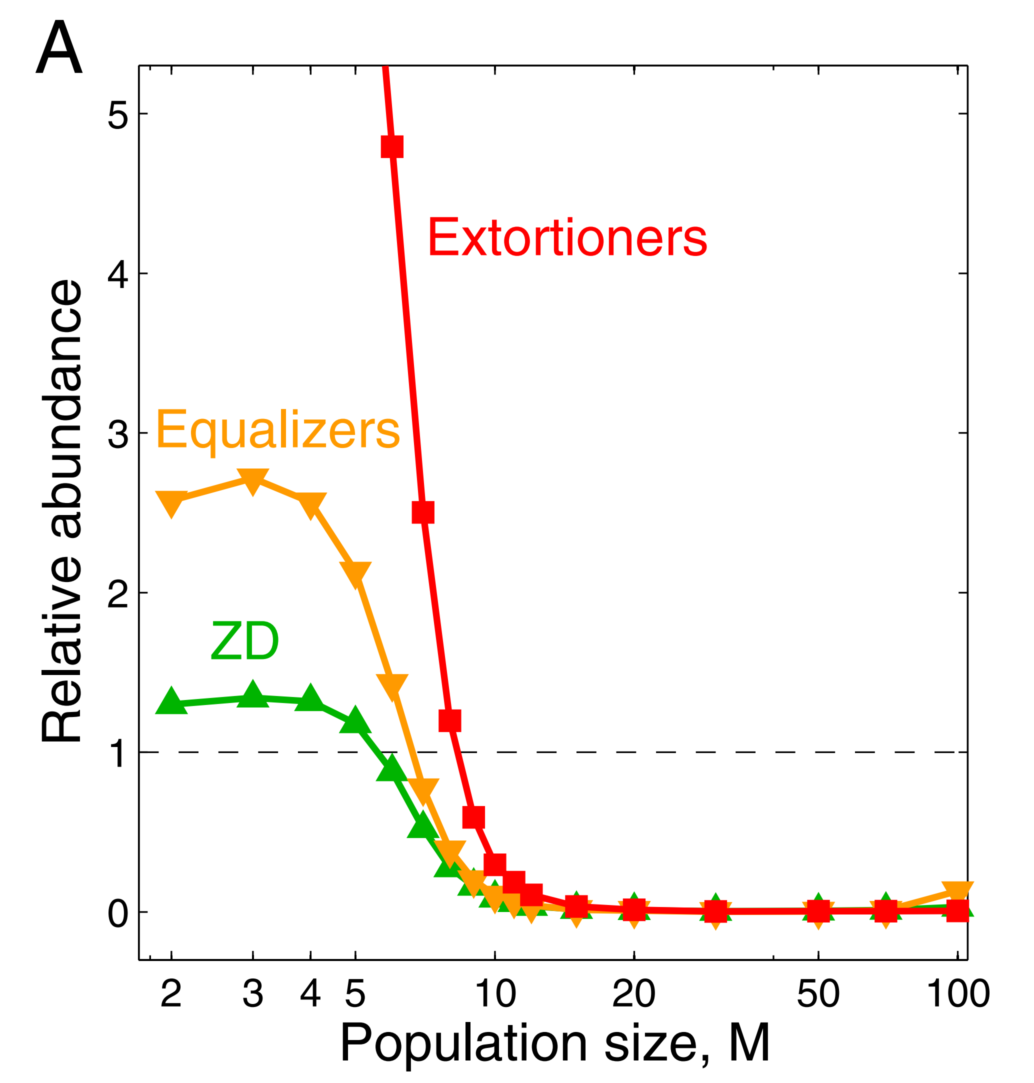

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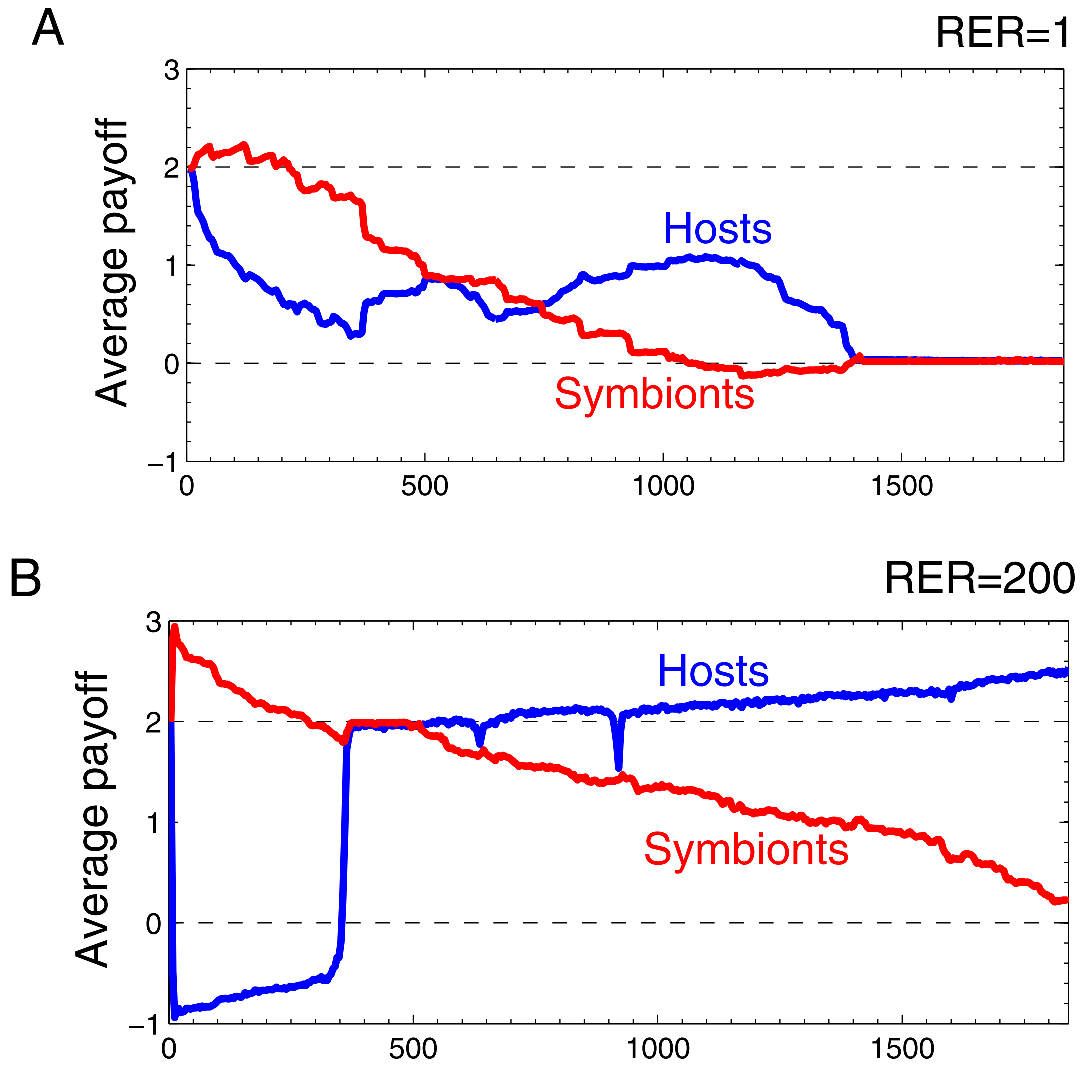

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Extortion subdues human players but is finally punished in the prisoner's dilemma

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  - 60 participants played for 60 rounds

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- Instead one could ask: How do humans react to extortionate strategies? Is it profitable to use extortion against human participants?

- Setup of the experiment:
  - 60 participants played for 60 rounds

	Cooperate	**Defect**
Cooperate	0.30 Euro	0.00 Euro
Defect	0.50 Euro	0.10 Euro

  - Participants played against a computer program that implemented an extortionate or a generous strategy

- Predictions:

1. *Computer vs Humans:*
   In the treatment with extortion, the computer should get a higher payoff than the participants. In the treatment with generosity, the human participants should get the higher payoff.
2. *Dynamics of cooperation:*
   In all treatments, the best response for humans is to cooperate in every round. Hence we would expect a general trend towards cooperation in all treatments.
3. *Extortion vs generosity:*
   If humans indeed learn to cooperate in all treatments, the extortionate program should get higher payoffs than the generous program.

Direct reciprocity: Extortion in the lab

Extortion subdues human players but is finally punished in the prisoner's dilemma

Christian Hilbe[1,2], Torsten Röhl[1] & Manfred Milinski[3]

Remark 2.16. How do humans react to extortion?

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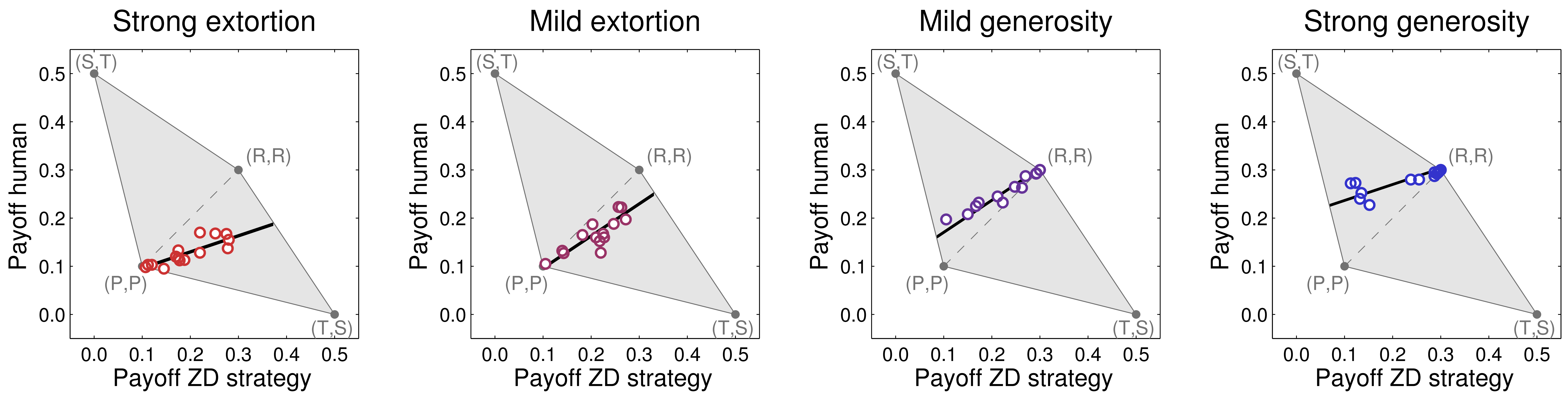

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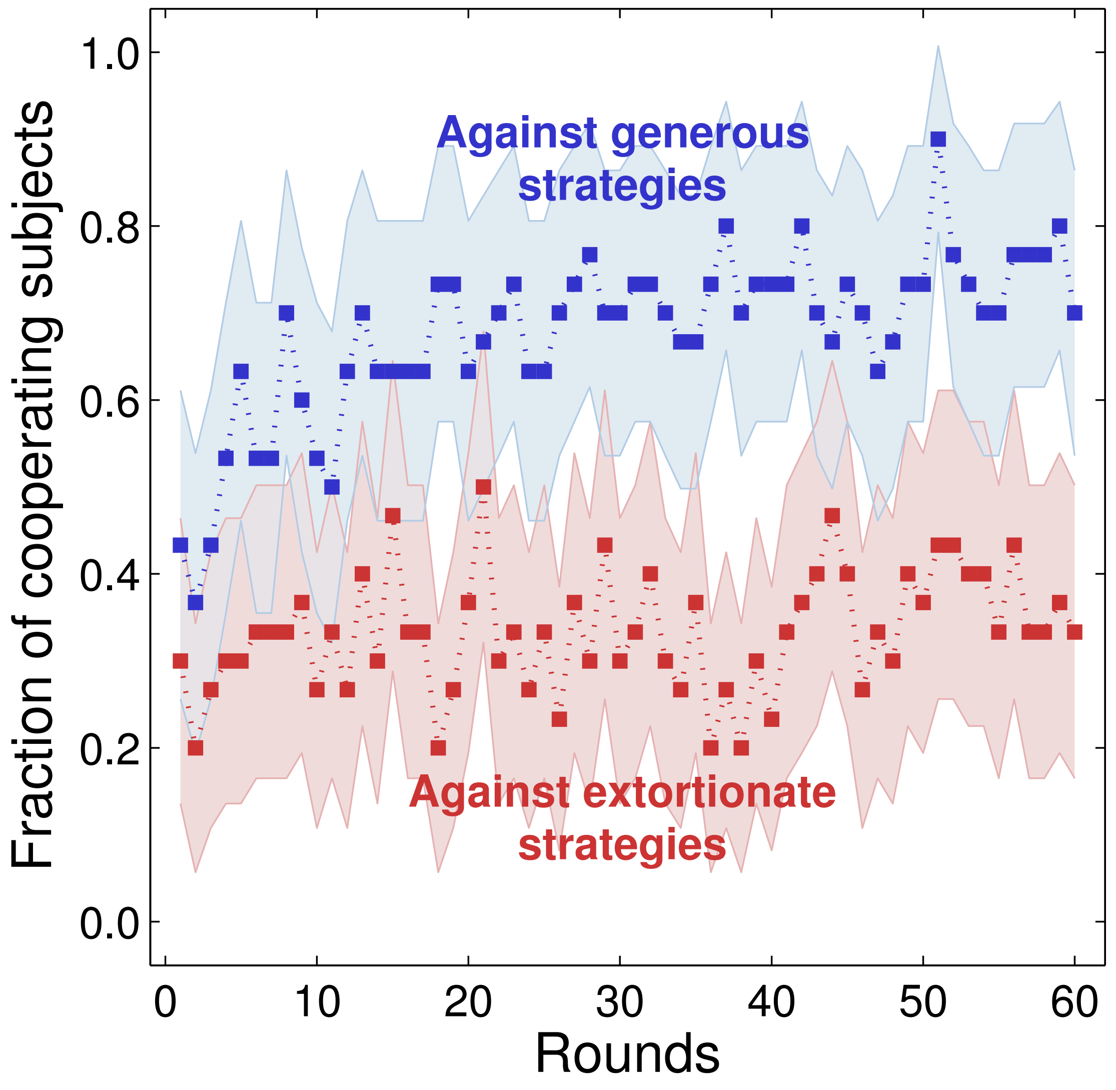

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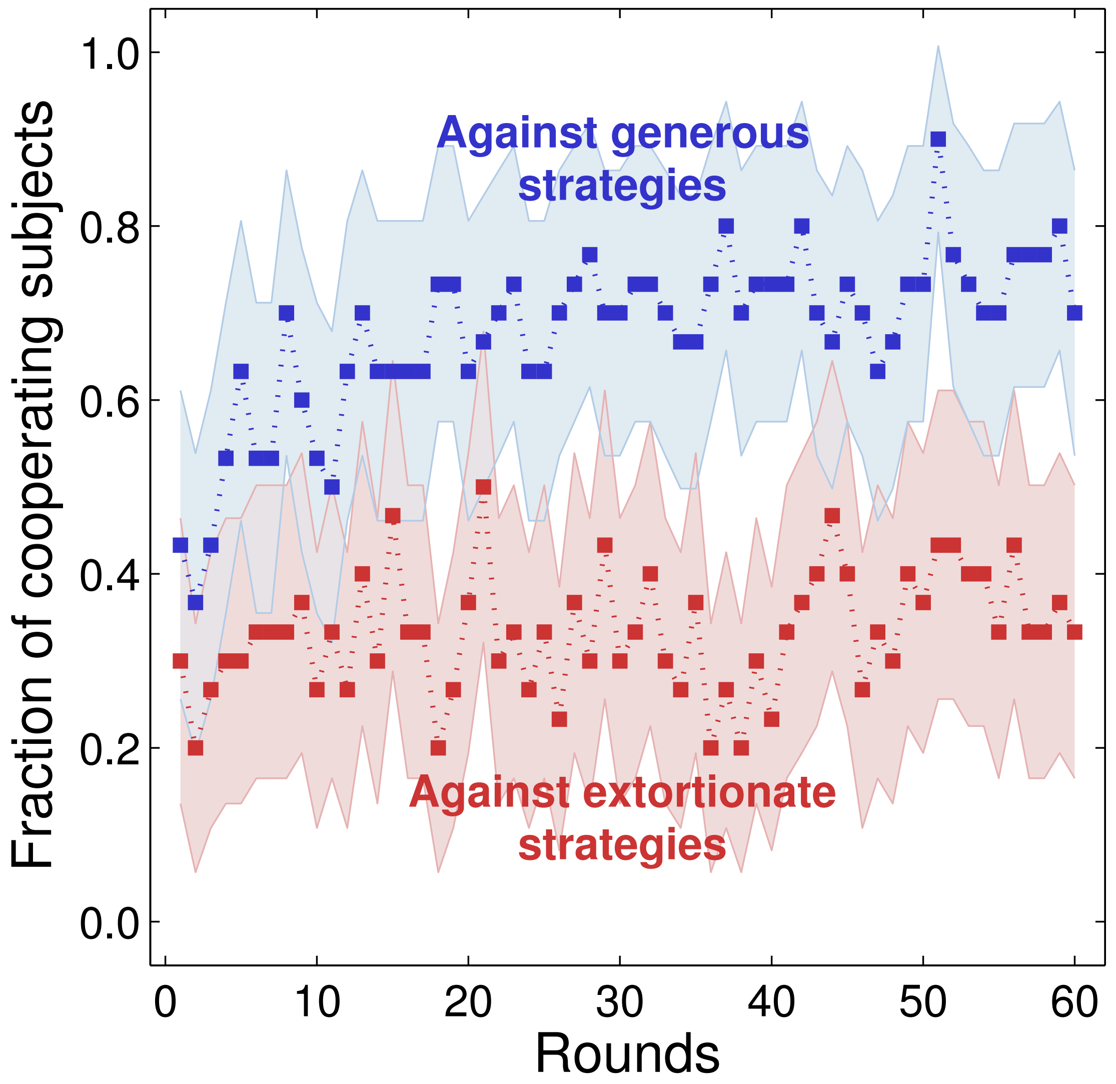

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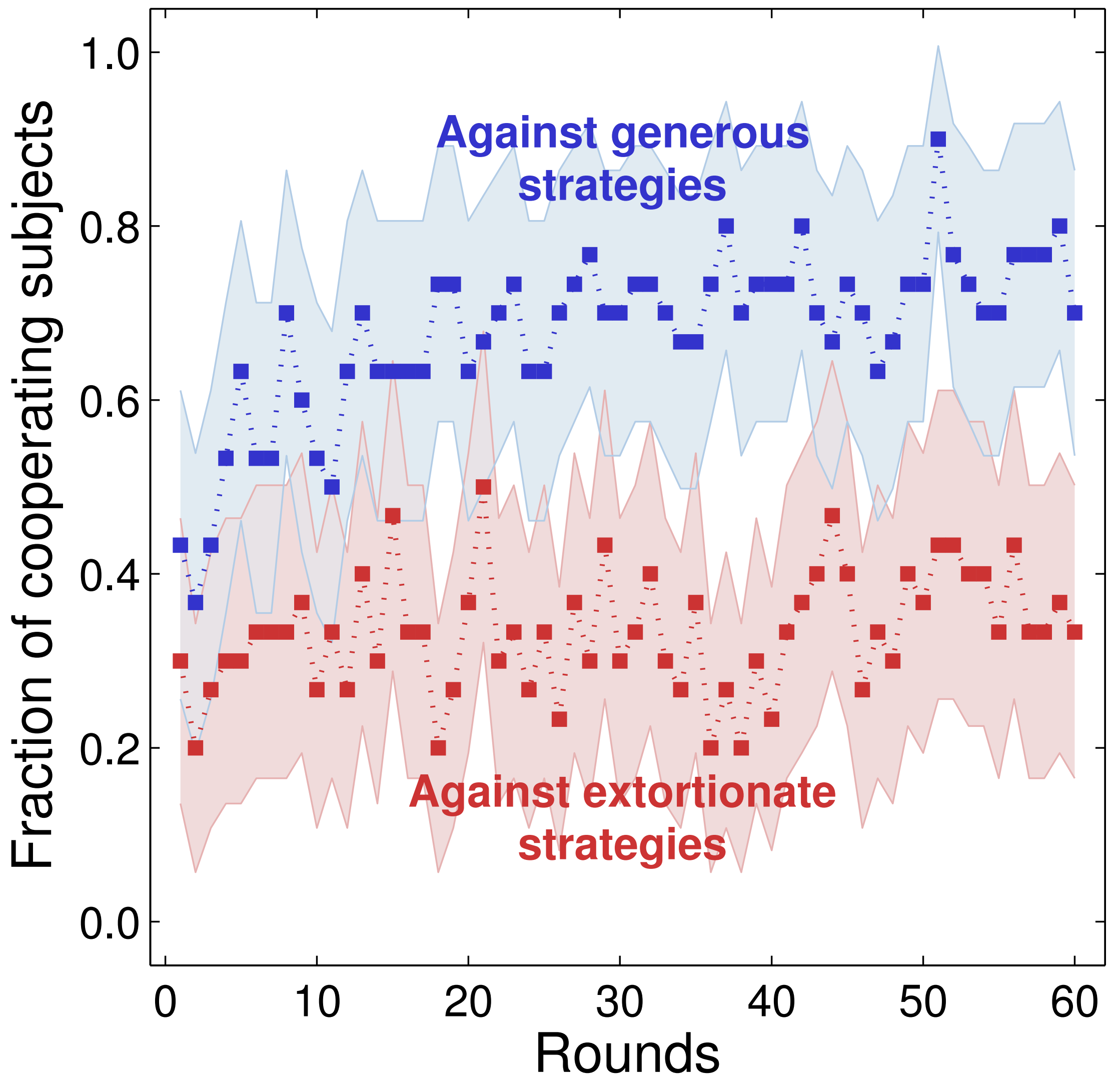

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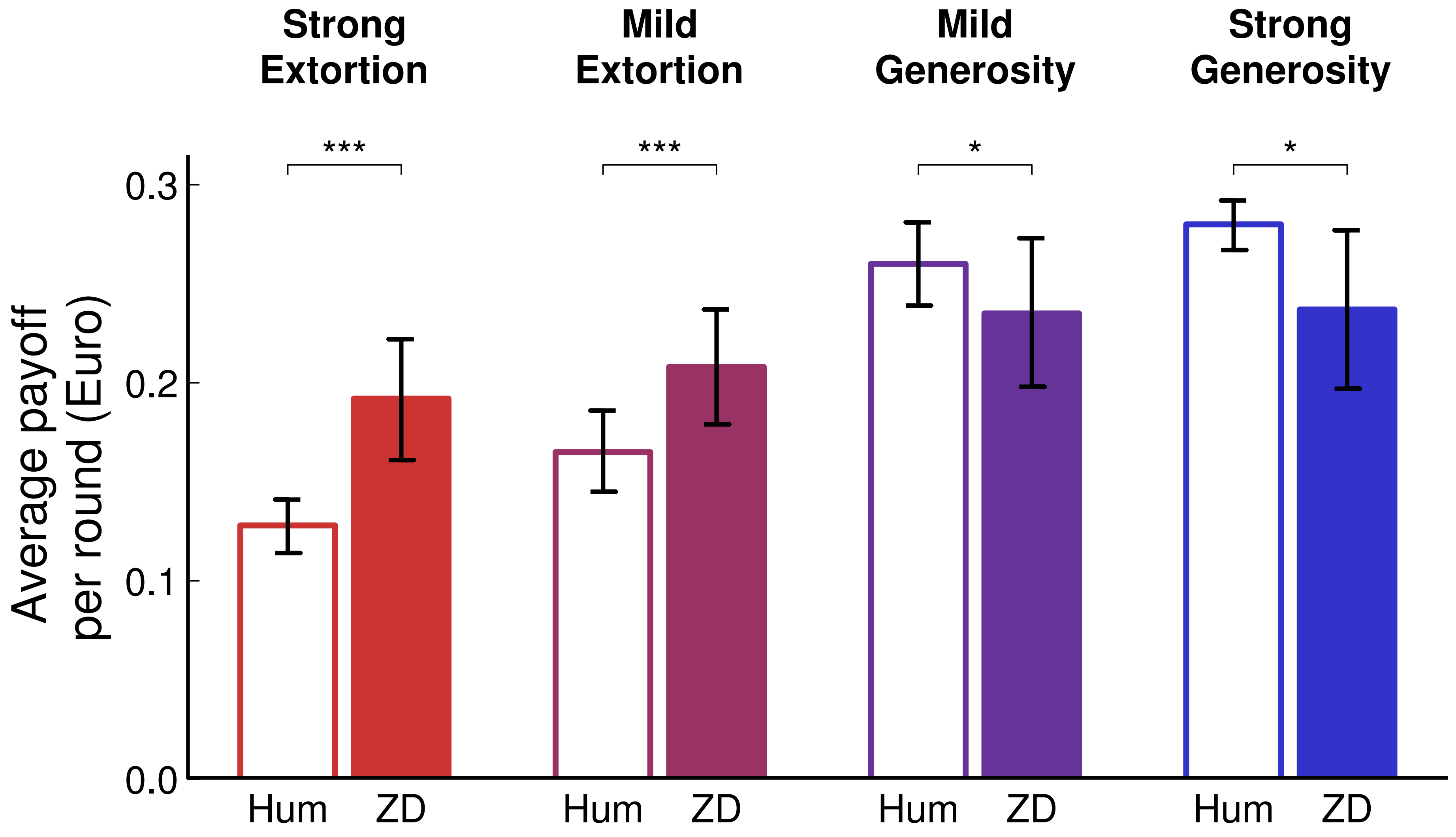

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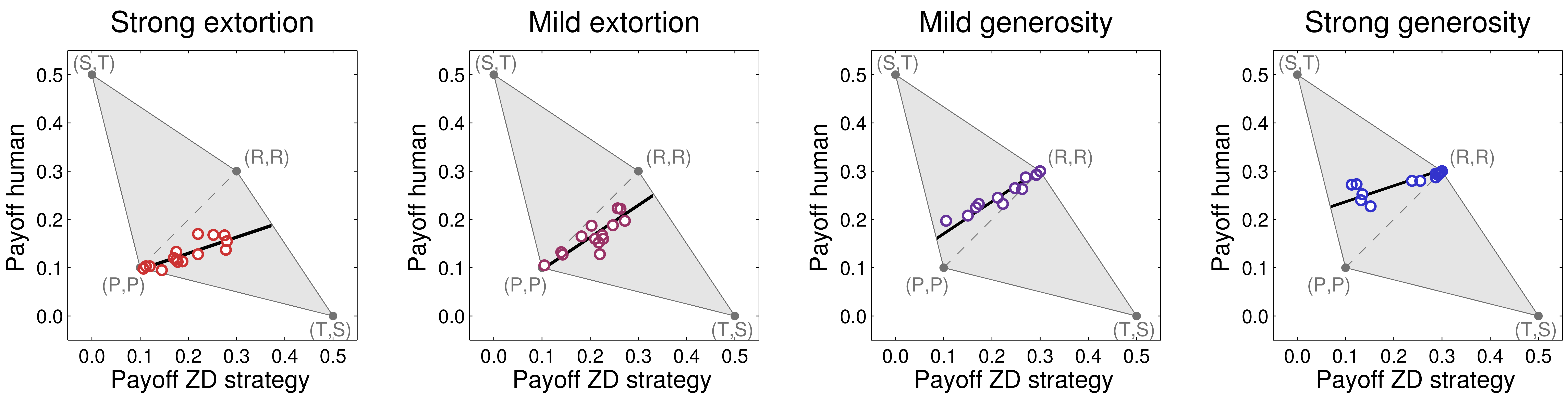

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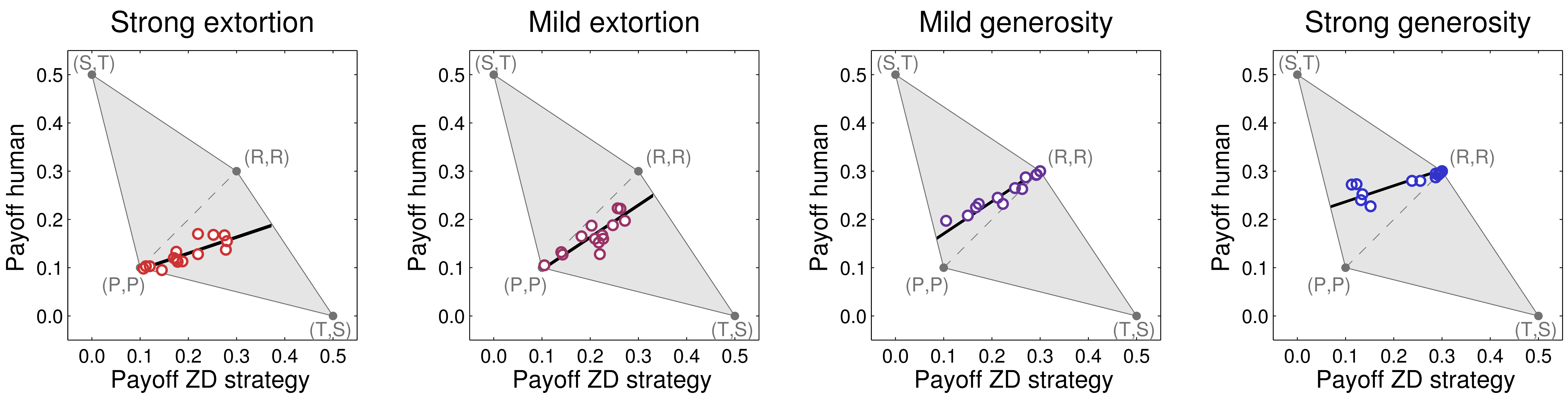

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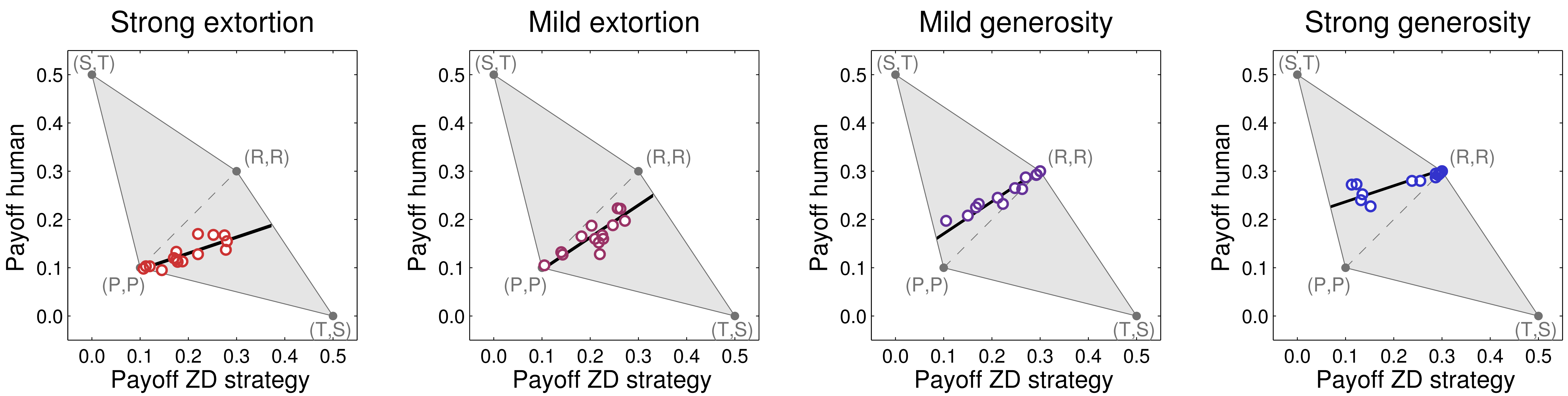

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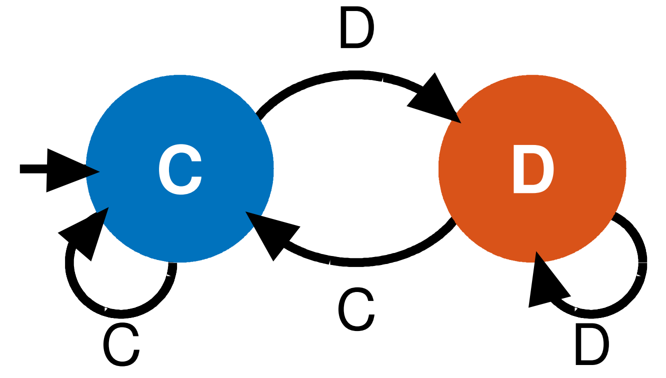
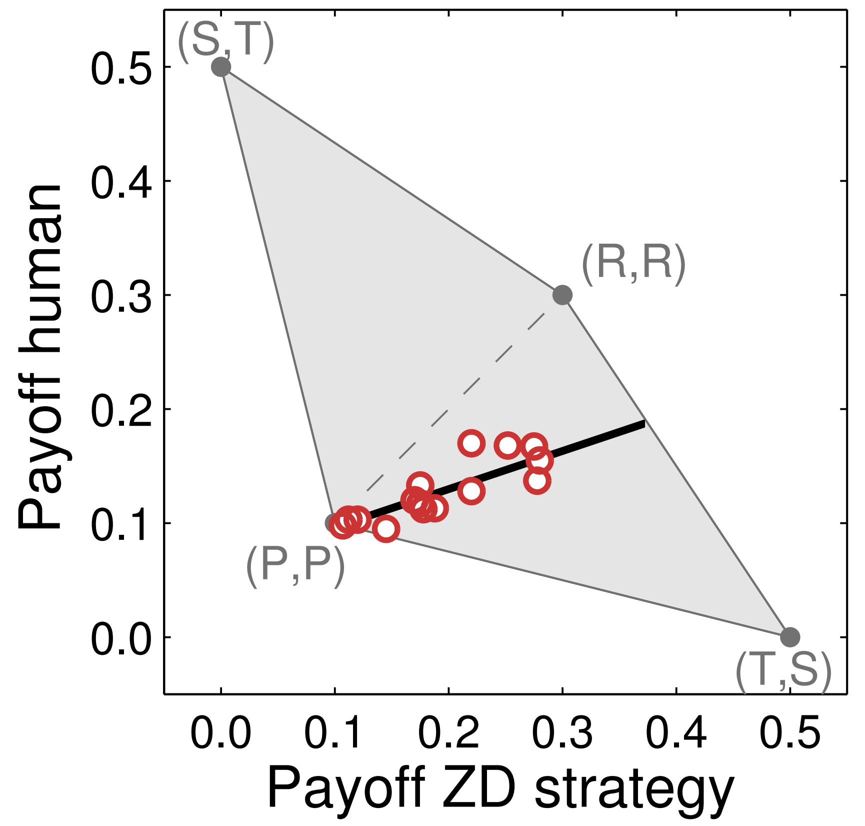
- In line with this interpretation, the effect vanishes if human participants receive information about the nature of their opponent before the experiment.

Summary

Some things you should have learned today:

1. We have used our techniques (Defining a game, characterizing Nash equilibria, evolutionary dynamics) to explore why people may cooperate.





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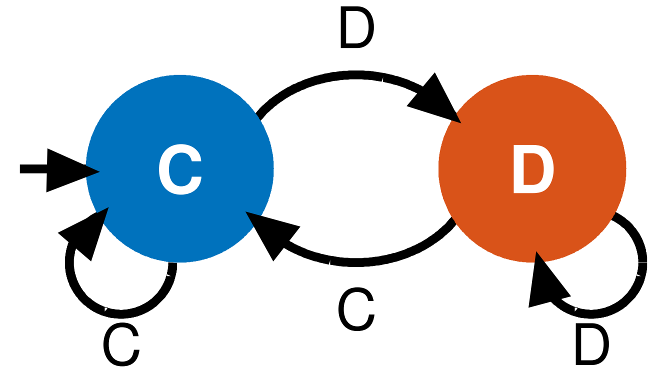

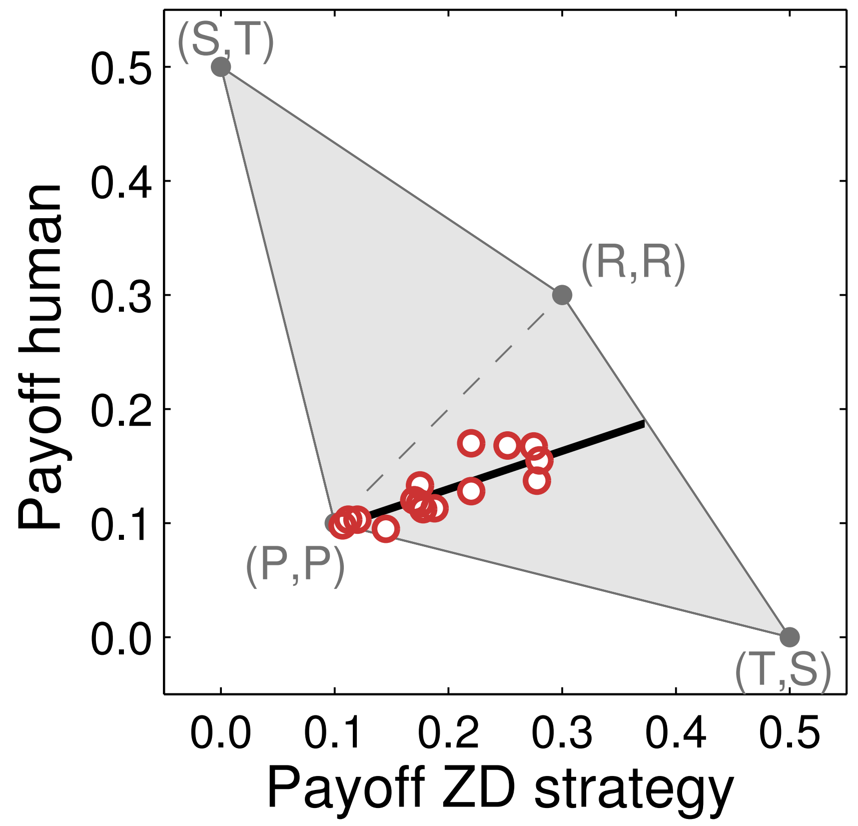

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3. Extortionate (zero-determinant) strategies have interesting mathematical properties, but against humans they don't pay.