

# Exploratory worksheet

January 29th

For each of the statements below, determine whether they are true or false: give a proof if they are true or a counterexample if false. (*Warning: Some statements might be difficult to prove/disprove, in which case you should guess whether you think the result is true or false and provide some computational evidence.*)

1. Every prime number is of the form  $6k + 1$  or  $6k - 1$  for some integer  $k$ .
2. Let  $n > 2$  be an integer.
  - (a) If  $n \mid 2^{n-1} - 1$  then  $n$  is prime.
  - (b) If  $n$  is prime, then  $n \mid 2^{n-1} - 1$ .
3. (a) There are infinitely many primes of the form  $3k + 1$ .  
(b) There are infinitely many primes of the form  $3k + 2$ .
4. There are more primes of the form  $3k + 2$  than of the form  $3k + 1$ . More specifically, if

$$\pi_1(n) = \#\{\text{primes } p \leq n \text{ of the form } 3k + 1\},$$

$$\pi_2(n) = \#\{\text{primes } p \leq n \text{ of the form } 3k + 2\},$$

then  $\pi_1(n) \leq \pi_2(n)$  for any integer  $n \geq 2$ .