Mandatory assignment 2
MEK4300

Christian Pedersen

April 7, 2016

Exercise 1

We are to solve the dimensionless non-linear Falkner-Skan equation

$$f''' + ff'' + \beta\left(1 - f'^2\right) = 0 \tag{1}$$

where $f = f(\eta)$, $\eta = y\sqrt{\frac{U(1+m)}{2\nu x}}$ and $\beta = \frac{2m}{1+m}$. m is is called the Falkner-Skan power-law parameter.

To solve this I introduce a new function $H = f'$. Inserted, this gives me two equations

$$\begin{aligned} H'' + fH + \beta\left(1 - H^2\right) &= 0 \\ H - f' &= 0 \end{aligned} \tag{2}$$

I then introduce two test functions v, q and rewrite the equation set on variational form

$$\begin{aligned} -\int H'v'dx + \int fHvdx + \int \beta vdx - \int \beta H^2 vdx &= 0 \\ \int Hqdx - \int f'qdx &= 0 \end{aligned} \tag{3}$$

This can be implemented directly into FEniCS.

Part a

Velocity profiles and shear-stress profiles for selected β values can be seen in figure 1.

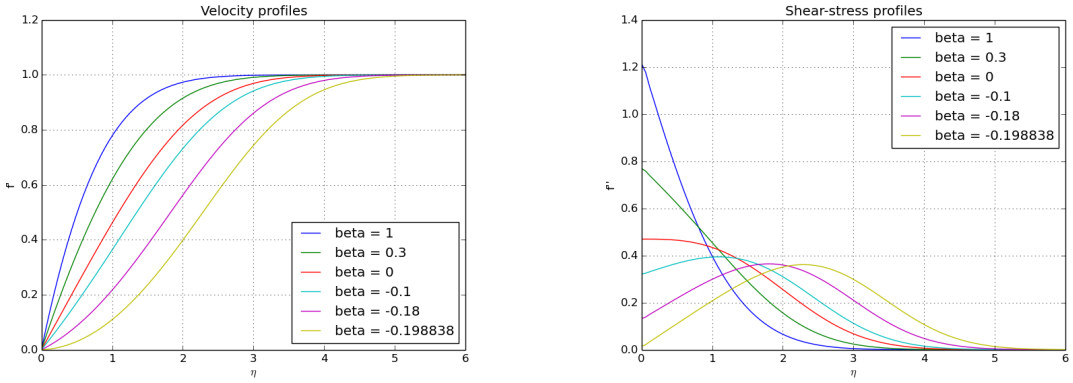

Figure 1: Plots of the velocity profiles and the shear-stress profiles for selected values of β.

Part b

When we have $\beta = -0.198838$ we see from figure 1 that the shear-stress is very close to 0. When $\beta = -0.19884$ we have that the shear-stress is exactly 0. Two different solutions for the shear-stress when $\beta = -0.1$ can be seen in figure 2

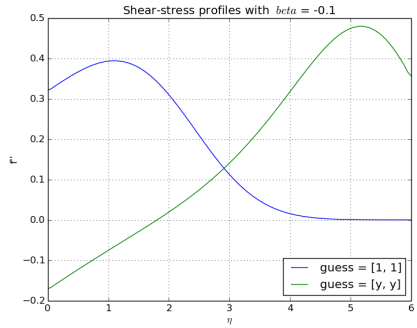

Figure 2: Plots of the shear-stress profile for two different initial guesses.

Exercise 2