COMP 476 Assignment 3 Theory Questions

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**Question #1**

The first thing we do is actually set up our data in hierarchical fashion. This is shown below.

Training Data:

L R R R L R R L L R L R L R L R R L R R L R L R R L L R R L R R

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1-Gram | | | | 2-Gram | | | | 3-Gram | | | |
| Obs. | ..R | ..L | Samp. | Obs. | ..R | ..L | Samp. | Obs. | ..R | ..L | Samp. |
| - | 19/32 | 13/32 | 32 | L | 11/13 | 2/13 | 13 | LL | 2/2 | 0/2 | 2 |
|  | | | | R | 8/18 | 10/18 | 18 | LR | 7/11 | 4/11 | 11 |
|  | | | | RL | 8/10 | 2/10 | 10 |
| RR | 1/7 | 6/7 | 7 |

* Input string is RRR, so the best we can do is look at the last two R’s and use our 3-Gram. So given the fact that we have just seen 2 R’s the predicted next action for the 3-Gram would be L since the probability of this is 6/7. Since we have 7 observations of this, and this exceeds our threshold of 5, we don’t have to go lower in the hierarchy and so we predict that the next action will be L.
* Since we need at least 15 samples for prediction, we cannot use the 3-Gram predictor since we only have 7 samples. So looking at the 2-Gram with the last action as R, we predict that the next action will be L since this has a probability of 10/18 which is greater than R’s probability (8/18) and this 2-Gram has 18 samples which meets our 15 sample requirement for prediction.
* Here, since we need 30 samples for prediction, we have to go all the way down to the 1-Gram to predict since this one has 32 samples to predict with. Here we are simply looking at the total probability of R and L based on no previous knowledge. Since R has the greater probability of occurring overall, we predict the next action to be R in this case.

**Question #2**

The triangle consists of the points (-5, -2, 0), (0, 5, 0) and (4, 0, -1). Since this represents a surface in 3D space, the contact normal is just the normal of the surface. We can find this by taking two vectors conencting these points and doing their cross product.

v1 = (-5, -2, 0) - (0, 5, 0) = (-5, -7, 0)

v2 = (-5, -2, 0) - (4, 0, -1) = (-9, -2, 1)

Now we take the cross product of v1 and v2 to get the contact normal

n = (-5, -7, 0) x (-9, -2, 1) = (-7, 5, -53)

Now this normal could be pointing towards the sphere or away from it.

We need the equation of the plane that the triangle lies in to know which way the normal should point. This is done with the equation:

a(x - x0) + b(y - y0) + c(z - z0) = 0

where (a, b, c) is the normal of the plane, and (x0, y0, z0) is any point on the plane. We can use any vertex of our triangle for this purpose.

We get the plane equation (by using the vertex (0, 5, 0):

-7x + 5(y - 5) - 53z = 0

-7x + 5y - 53z -25 = 0

This vector is in the direction of the line between the sphere's center and the normal of the plane. This allows us to create the parametric line equation:

L={(x, y, z): xS + At, yS + Bt, zS + Ct} where (xS, yS, zS) is the center of our sphere and A, B, C, D are the constants in our plane equation.

L={(x, y, z): 1 + -7t, 1 + 5t, 1 + -53t}

If we substitute this into the equation of our plane we get:

-7(1 + -7t) + 5(1 + 5t) - 53(1 + -53t) - 25 = 0

If we isolate t we get:

-7 + 49t + 5 + 25t - 53 + 2809t - 25 = 0

2883t = 80

t = 80/2883

Now that we have t we get the center of the circle where the sphere intersects with the plane:

xC = 1 + -7\*(80/2883) = 0.805

yC = 1 + 5\*(80/2883) = 1.138

zC = 1 + -53\*(80/2883) = -0.471

To get the penetration depth take the length of the vector from the center of the sphere to the center of the circle of intersection:

l = ((1 - 0.805)^2 + (1 - 1.138)^2 + (1 - -0.471)^2)^(1/2) = 1.489

Now compare this to the radius of the sphere which is 3, meaning that the penetration depth is 3 - 1.489 = 1.511.