COMP 476 Assignment 1 Theory Questions

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Question #1:

a) AI character position: $p_c = [5, 6]$
AI character velocity: $v_c = [3, 1]$
Target position: $p_t = [8, 2]$
Time between updates: $t = 0.4$
Maximum velocity: $v_m = 5$
Maximum acceleration: $a_m = 17$
Next AI character position: p_c'

The first step to computing the next position p_c' is to compute the velocity direction. This is done with:

$$v = p_t - p_c$$

For the first step, the velocity direction is:

$$v = \begin{bmatrix} 8 \\ 2 \end{bmatrix} - \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

Next, we need to normalize the direction of the velocity. We do this by dividing the velocity direction from the next step by its magnitude. This boils down to:

$$v_n = \frac{v}{|v|}$$

Where:

$$|v| = \sqrt{\sum v_i^2}$$

Here, i represents the elements of the vector v.

For the first step the normalized velocity direction is:

$$v_n = \frac{\begin{bmatrix} 3 \\ -4 \end{bmatrix}}{\sqrt{3^2 + (-4)^2}} = \begin{bmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{bmatrix}$$

Now that we have normalized the velocity direction, we can use it to compute the seek velocity using:

$$v_{seek} = v_m * v_n$$

For the first step this gives:

$$v_{seek} = 5 * \begin{bmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

Now that we have the seek velocity we can finally compute the next position with:

$$p_c' = p_c + v_{seek} * t$$

For the first step this gives us the AI character position after the first step:

$$p_c' = \begin{bmatrix} 5 \\ 6 \end{bmatrix} + \begin{bmatrix} 3 \\ -4 \end{bmatrix} * 0.4 = \begin{bmatrix} \mathbf{6.2} \\ \mathbf{4.4} \end{bmatrix}$$

Now we need to do this same procedure for the first 5 steps. The intermediate results are shown in the table below.

STEP	$\boldsymbol{p_c}$	$\boldsymbol{v}$	$\boldsymbol{v_n}$	$\boldsymbol{v_{seek}}$	$\boldsymbol{p_c'}$
1	$[5, 6]$	$[3, -4]$	$[0.6, -0.8]$	$[3, -4]$	$[6.2, 4.4]$
2	$[6.2, 4.4]$	$[1.8, -2.4]$	$[0.6, -0.8]$	$[3, -4]$	$[7.4, 2.8]$
3	$[7.4, 2.8]$	$[0.6, -0.8]$	$[0.6, -0.8]$	$[3, -4]$	$[8.6, 1.2]$
4	$[8.6, 1.2]$	$[-0.6, 0.8]$	$[-0.6, 0.8]$	$[-3, 4]$	$[7.4, 2.8]$
5	$[7.4, 2.8]$	$[0.6, -0.8]$	$[0.6, -0.8]$	$[3, -4]$	$\mathbf{[8.6, 1.2]}$

b) The first step to compute the next position $p_c{}'$ is to compute the acceleration:

$$a = \frac{a_m(p_t - p_c)}{|p_t - p_c|}$$

In the first step this yields:

$$a = 17 * \frac{\left(\begin{bmatrix} 8 \\ 2 \end{bmatrix} - \begin{bmatrix} 5 \\ 6 \end{bmatrix}\right)}{\sqrt{(8-5)^2 - (2-6)^2}} = 17 * \begin{bmatrix} 0.6 \\ -0.8 \end{bmatrix} = \begin{bmatrix} 10.2 \\ -13.6 \end{bmatrix}$$

The next step is to compute the steering seek velocity. This is done using:

$$v = v_c + at$$

For the first step this yields:

$$v = \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 10.2 \\ -13.6 \end{bmatrix} * 0.4 = \begin{bmatrix} 7.08 \\ -4.44 \end{bmatrix}$$

We need to make sure that our velocity is less than the maximum velocity. To do this we take the magnitude of our velocity:

$$|v| = \sqrt{(7.08)^2 + (-4.44)^2} = 8.357$$

Since our velocity is greater than the maximum velocity, we should bound it to the max velocity. To fix this, we need to first normalize the velocity, then multiply it by the maximum velocity. For the first step this yields:

$$v_n = \frac{\begin{bmatrix} 7.08 \\ -4.44 \end{bmatrix}}{8.357} = \begin{bmatrix} 0.847 \\ -0.531 \end{bmatrix}$$

Then take this normalized vector and multiply it by v_m:

$$v = 5 * \begin{bmatrix} 0.847 \\ -0.531 \end{bmatrix} = \begin{bmatrix} 4.235 \\ -2.655 \end{bmatrix}$$

Finally we can compute the new position using:

$$p_c' = p_c + vt$$

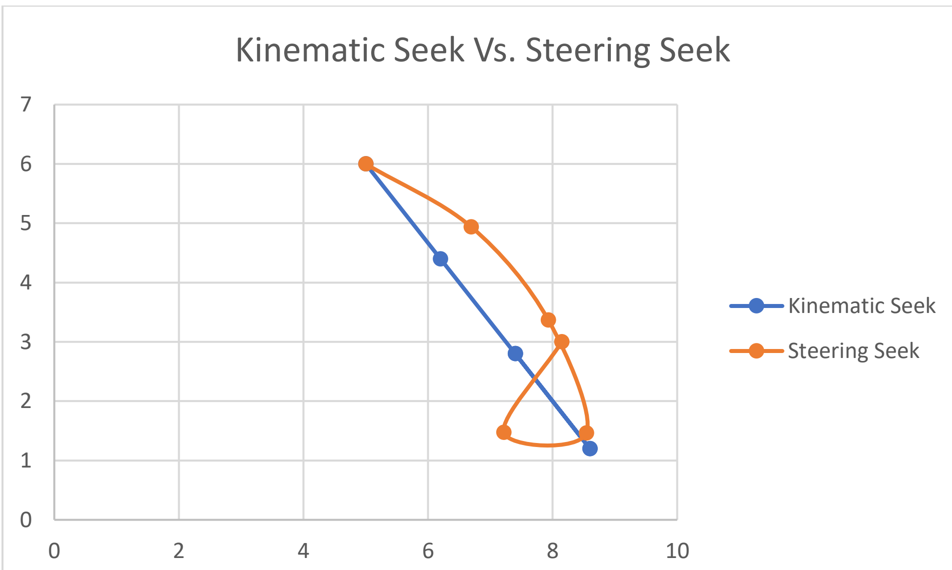
For the first step this yields:

$$p_c' = \begin{bmatrix} 5 \\ 6 \end{bmatrix} + \begin{bmatrix} 4.235 \\ -2.655 \end{bmatrix} * 0.4 = \begin{bmatrix} \mathbf{6.694} \\ \mathbf{4.938} \end{bmatrix}$$

Now we need to do this same procedure for the first 5 steps. The intermediate results are shown in the table below.

STEP	p_c	a	v	$\|v\| < v_m$	v	p_c'
1	$[5, 6]$	$[10.2, -13.6]$	$[7.08, -4.44]$	F	$[4.235, -2.655]$	$[6.694, 4.938]$
2	$[6.694, 4.938]$	$[6.905, -15.535]$	$[6.997, -8.869]$	F	$[3.096, -3.924]$	$[7.932, 3.368]$
3	$[7.932, 3.368]$	$[0.844, -16.987]$	$[3.434, -10.719]$	F	$[1.525, -4.761]$	$[8.542, 1.464]$
4	$[8.542, 1.464]$	$[-12.092, 11.958]$	$[-3.31, 0.022]$	T	$[-3.31, 0.022]$	$[7.218, 1.473]$
5	$[7.218, 1.473]$	$[14.097, 9.501]$	$[2.328, 3.822]$	T	$[2.328, 3.822]$	$\mathbf{[8.149, 3.002]}$

c) Kinematic Seek gives a straight line because we are always moving with maximum velocity. We also ignore acceleration. When we consider acceleration and the starting velocity, the path becomes curved, as seen in the Steering Seek path. This is shown in the plot below.



d) $t2t = 0.55$
$r_{sat}' = 1$
Here we are doing Kinematic Arrive. The first step is to compute the direction of the velocity. This is done by computing:

$$v = p_t - p_c = \begin{bmatrix} 8 \\ 2 \end{bmatrix} - \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

Now that we have the direction of the velocity, we need to normalize it. We do this with:

$$v_n = \frac{v}{|v|} = \frac{\begin{bmatrix} 3 \\ -4 \end{bmatrix}}{\sqrt{3^2 + 4^2}} = \begin{bmatrix} 0.6 \\ -0.8 \end{bmatrix}$$

Next we need to check the magnitude of the velocity to determine if we should slow down. We do this by checking the magnitude of the velocity and setting it to the minimum of the maximum velocity (5 in our case) and the speed based on the time to target. This is done by checking:

$$|v| = \min\left(v_m, \frac{|p_t - p_c|}{t2t}\right) = \min\left(5, \frac{|[3,-4]|}{0.55}\right) = \min(5, 9.09) = 5$$

We do not set the magnitude of the velocity to 0 in this case since $|[3,-4]| = 5 > r_{sat}'$.

Now that we know the magnitude of the velocity, we need to multiply this by the direction of motion to determine how much we should move in this step.

$$v_{move} = 5 * \begin{bmatrix} 0.6 \\ -0.8 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

Now to get the position after the first step we use:

$$p_c' = p_c + vt = \begin{bmatrix} 5 \\ 6 \end{bmatrix} + \begin{bmatrix} 3 \\ -4 \end{bmatrix} * 0.4 = \begin{bmatrix} \mathbf{6.2} \\ \mathbf{4.4} \end{bmatrix}$$

Now we repeat this for five steps. The results are shown in the table below.

STEP	$\boldsymbol{p_c}$	$\boldsymbol{v = p_t - p_c}$	$\boldsymbol{v_n}$	$\boldsymbol{\|v\| = min\left(v_m, \frac{\|p_t - p_c\|}{t2t}\right)}$	$\boldsymbol{v_{move}}$	$\boldsymbol{p_c'}$
1	$[5, 6]$	$[3, -4]$	$[0.6, -0.8]$	5	$[3, -4]$	$[6.2, 4.4]$
2	$[6.2, 4.4]$	$[1.8, -2.4]$	$[0.6, -0.8]$	5	$[3, -4]$	$[7.4, 2.8]$
3	$[7.4, 2.8]$	$[0.6, -0.8]$	$[0.6, -0.8]$	1.81	$[1.086, -1.448]$	$[7.834, 2.221]$
4	$[7.834, 2.221]$	$[0.166, -0.221]$	$[2.173, -2.893]$	0	$[0, 0]$	$[7.834, 2.221]$
5	$[7.834, 2.221]$	$[0.166, -0.221]$	$[2.173, -2.893]$	0	$[0, 0]$	$\mathbf{[7.834, 2.221]}$

e) $r_a = 0.2$
$r_s = 1.5$
$t2t = 0.5$

For the first step, we need to calculate the goal velocity. To do this, we first check if we are outside the slow-down radius. We do this by checking the distance between the character and the target.

$$|p_t - p_c| = \sqrt{(8-5)^2 + (2-6)^2} = 5 > r_s$$

This means that our goal velocity is equal to our maximum velocity.

$$|v_g| = 5$$

We need to transform this to a vector by taking the unit vector direction to the target and multiplying it by our goal velocity magnitude.

$$dir = \frac{p_t - p_c}{|p_t - p_c|} * |v_g| = \frac{\begin{bmatrix} 3 \\ -4 \end{bmatrix}}{5} * 5 = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

Since we are outside the slow down radius at this point, this is essentially a seek problem. Therefore we compute the acceleration with:

$$a = \frac{a_m(p_t - p_c)}{|p_t - p_c|} = 17 * \frac{\left(\begin{bmatrix}8\\2\end{bmatrix} - \begin{bmatrix}5\\6\end{bmatrix}\right)}{\sqrt{(8-5)^2 + (2-6)^2}} = \begin{bmatrix}10.2\\-13.6\end{bmatrix}$$

We can now compute the velocity using:

$$v = v_c + at = \begin{bmatrix}3\\1\end{bmatrix} + \begin{bmatrix}10.2\\-13.6\end{bmatrix} * 0.4 = \begin{bmatrix}7.08\\-4.44\end{bmatrix}$$

Since our velocity is greater than the maximum velocity, we should bound it to the max velocity. To fix this, we need to first normalize the velocity, then multiply it by the maximum velocity. For the first step this yields:

$$v_n = \frac{\begin{bmatrix}7.08\\-4.44\end{bmatrix}}{8.357} = \begin{bmatrix}0.847\\-0.531\end{bmatrix}$$

Then take this normalized vector and multiply it by v_m:

$$v = 5 * \begin{bmatrix}0.847\\-0.531\end{bmatrix} = \begin{bmatrix}4.235\\-2.655\end{bmatrix}$$

Finally we can compute the new position using:

$$p_c' = p_c + vt$$

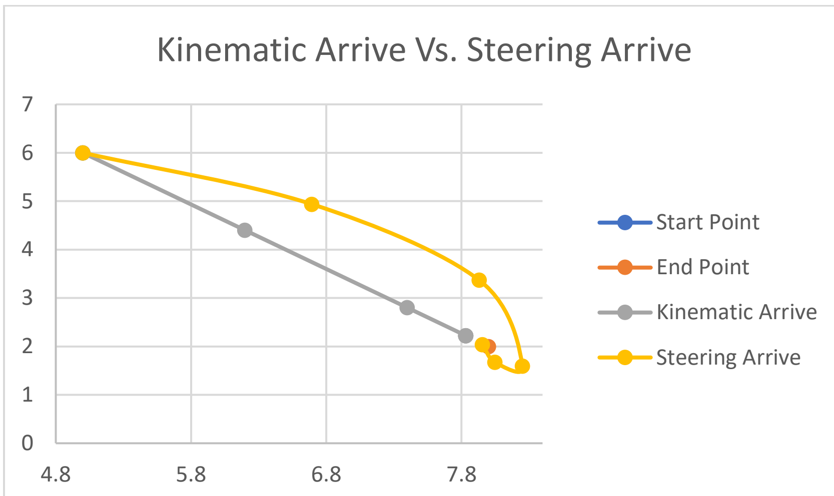
For the first step this yields:

$$p_c' = \begin{bmatrix}5\\6\end{bmatrix} + \begin{bmatrix}4.235\\-2.655\end{bmatrix} * 0.4 = \begin{bmatrix}\mathbf{6.694}\\\mathbf{4.938}\end{bmatrix}$$

STEP	$\boldsymbol{p_c}$	$\boldsymbol{v_c}$	$\lvert\boldsymbol{v_g}\rvert$	$\boldsymbol{a}$	$\boldsymbol{v}$	$\boldsymbol{p_c'}$
1	$[5, 6]$	$[3, 1]$	5	$[10.2, -13.6]$	$[4.235, -2.655]$	$[6.694, 4.938]$
2	$[6.694, 4.938]$	$[4.235, -2.655]$	5	$[6.905, -15.535]$	$[3.096, -3.924]$	$[7.932, 3.368]$
3	$[7.932, 3.368]$	$[3.096, -3.924]$	4.565	$[-5.738, -1.276]$	$[0.801, -4.434]$	$[8.252, 1.594]$
4	$[8.252, 1.594]$	$[0.801, -4.434]$	1.593	$[-3.28, 11.572]$	$[-0.511, 0.195]$	$[8.048, 1.672]$
5	$[8.048, 1.672]$	$[-0.511, 0.195]$	1.103	$[0.704, 1.796]$	$[-0.229, 0.913]$	$[7.956, 2.037]$

f) In the case of the kinematic arrive, the direction of motion never changes, since there is no acceleration. As we get closer, we see that the distance between points goes does, since we are in the slow down radius, until it eventually stops moving when it is within the radius of satisfaction.

In the case of the steering arrive, we actually overshoot the target and we have to turn back. The slow down is clear at the end as we decelerate and the points get closer together. Since we are using acceleration here, the motion is smoother and more realistic since we cannot immediately stop when we get to the target. By reducing time to target we could have stopped more quickly, and maybe avoided overshooting the end point.



Question #2:

a) The center of mass p_c is calculated by taking the average position of the points. This yields:

$$p_c = \begin{bmatrix} \dfrac{21+5+28}{3} \\ \dfrac{16+11+9}{3} \end{bmatrix} = \begin{bmatrix} 18 \\ 12 \end{bmatrix}$$

Now we need the average velocity v_c:

$$v_c = \begin{bmatrix} \dfrac{3+3+6}{3} \\ \dfrac{0+2+4}{3} \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

Using these we can calculate p_{anchor}:

$$p_{anchor} = p_c + k_{offset} v_c = \begin{bmatrix} 18 \\ 12 \end{bmatrix} + 1 \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} \mathbf{22} \\ \mathbf{14} \end{bmatrix}$$

b) First we need to calculate Δp_{s_i} for each character. We do this with the formula:

$$\Delta p_{s_i} = p_{s_i} - p_{anchor}$$

Where p_{s_i} is the assigned slot position of the anchor. For this iteration, because we are computing before the anchor position is updated, p_{anchor} is p_c from part a).

For the 3 characters, this gives:

$$\Delta p_{s1} = \begin{bmatrix} 22 \\ 18 \end{bmatrix} - \begin{bmatrix} 18 \\ 12 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$
$$\Delta p_{s2} = \begin{bmatrix} 6 \\ 13 \end{bmatrix} - \begin{bmatrix} 18 \\ 12 \end{bmatrix} = \begin{bmatrix} -12 \\ 1 \end{bmatrix}$$
$$\Delta p_{s1} = \begin{bmatrix} 29 \\ 12 \end{bmatrix} - \begin{bmatrix} 18 \\ 12 \end{bmatrix} = \begin{bmatrix} 11 \\ 0 \end{bmatrix}$$

Now that we have the deltas, we need to compute the new slot coordinates. The slot coordinates are calculated using:

$$p_{s_i} = \Delta p_{s_i} + p_{anchor}$$

For the three characters, this yields:

$$p_{s_1} = \begin{bmatrix} 4 \\ 6 \end{bmatrix} + \begin{bmatrix} 22 \\ 14 \end{bmatrix} = \begin{bmatrix} \mathbf{26} \\ \mathbf{20} \end{bmatrix}$$
$$p_{s_2} = \begin{bmatrix} -12 \\ 1 \end{bmatrix} + \begin{bmatrix} 22 \\ 14 \end{bmatrix} = \begin{bmatrix} \mathbf{10} \\ \mathbf{15} \end{bmatrix}$$
$$p_{s_3} = \begin{bmatrix} 11 \\ 0 \end{bmatrix} + \begin{bmatrix} 22 \\ 14 \end{bmatrix} = \begin{bmatrix} \mathbf{33} \\ \mathbf{14} \end{bmatrix}$$