BUSI 731 Burke Homework-3

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Worked with:__________________________________

1) **20 Points** A fast-food franchise is considering opening a drive-up window food service operation. Assume that customer arrivals follow a Poisson distribution (= interarrival times follow an exponential distribution, hence CVa = 1), with a mean arrival rate of 24 cars per hour, and that process times follow an exponential probability distribution, (hence CVp = 1). Arriving customers place orders at an intercom station at the back of the parking lot and then drive up to the service window to pay for and receive their order. The following four service alternatives are being considered:

(a) A single channel operation where one employee fills the order and takes the money from the customer. The average process time for this alternative is 2 minutes.

(b) A single-channel operation where two employees **work as a team** to serve each customer. The average process time for this alternative is 1.25 minutes.

(c) A two-channel operation with two service windows and two employees. Newly arriving customers join a queue which feeds both service windows. The employee stationed at each window fills the order and takes the money for customers arriving at that window. The average process time for this alternative is 2 minutes.

(d) There are two service windows and two employees. There is a separate queue for each of the service windows. Newly arriving customers randomly choose one of two queues. The employee stationed at each window fills the order and takes the money for customers arriving at that window. The average process time for this alternative is 2 minutes.

Compute the operating characteristics (T_q, I_q, T, I) for each of the four service alternatives and fill in the following table:

(Note that you should calculate T_q first, then I_q from T_q using Little's Law, then T using the fact that $T = Tq + p$, and finally I using the fact that $I = Iq + Ip$.)

Service Alternative (A)
mu = 1/service time = ½
P = arrival rate/mu = 0.4min/0.5 = 0.8
T_q = 0.8/0.5(1-0.8) = 8min
I_q = T_q x Arrival Rate = 8min x 0.4 = 3.2cars
T = T_q + service time = 8min + 2 = 10min
I = Arrival rate x T = 0.4 x 10 = 4cars

Service Alternative (B)
mu = 1/service time = 1/1.25 = 0.8
P = arrival rate/mu = 0.4min/0.8 = 0.5
T_q = 0.5/0.8(1-0.5) = 1.25min
I_q = Arrival Rate x T_q = 0.4 x 1.25 = 0.5min
T = T_q + service time = 1.25min + 1.25 = 2.5min
I = Arrival rate x T = 0.4 x 2.5 = 1cars

Service Alternative (C)
Two channels and one queue
Mu = 1/service time = ½ = 0.5min
P = arrival rate / n_channels x mu = 0.4/(2 x 0.5) = 0.4
Utilization = arrival rate / n_channels x mu = 0.4/(2 x 0.5) = 0.4
T_q = service time / n_channels x (utilization^(sqrt(2(n_channels+1))-1) / 1 – utilization)
T_q = 0.4^(sqrt(2(2+1))-1) / 1 – 0.4 = 0.4^1.449 / 0.6 = 0.265 / 0.6 = 0.44min
I_q = Arrival Rate x T_q = 0.4 x 0.44min = 0.177cars
T = T_q + service time = 0.44 + 2 = 2.44
I = Arrival Rate x T = 0.4 x 2.44 = 0.976cars

Service Alternative (D)
Two channels with two separate queues
Each queue is equal to the arrival time, 0.4 / n_queues = 0.2
For each queue:
Mu = 1/n_channels = ½ = 0.5
P = arrival rate/mu = 0.2min/0.5 = 0.4
T_q = 0.4/0.5(1-0.4) = 1.33min
I_q = Arrival Rate x T_q = 0.2 x 1.33 = 0.266cars
T = T_q + service time = 1.33min + 2min = 3.33min
I = Arrival rate x T = 0.2 x 3.33 = 0.66cars
For two queues I_q = 2 x 0.266cars = 0.53cars
For two queues I = 2 x 0.666cars = 1.33cars

	Service alternative			
	(A)	(B)	(C)	(D)
Average waiting time (in minutes), T_q	8min	1.25min	0.44min	1.33min
Average number of cars waiting for service, I_q	3.2cars	0.5cars	0.18cars	0.53cars
Average time in the system (in minutes), T	10min	2.5min	2.44min	3.33min
Average number of cars in the system, I	4cars	1cars	0.98cars	1.33cars

(e) Which drive-up window food service operation would you select and why?
I would choose service alternative C. This alternative allows for customers to have the shortest waiting time, smallest number of cars in the system. Alternative B is like C, however – decreasing the waiting time using alternative C would increase customer satisfaction while yielding the very similar outcomes.

2) **10 Points** FestEvents is a concert organizer that is evaluating the installation of portable toilets at an outdoor venue where a concert is going to take place. It is considering an area of the venue that is far away from the existing restrooms. Assume there will be one single queue to access all the portable toilets in this area and that only one person uses a toilet at a time. FestEvents is concerned that if the line is too long, people will find other ways and places to answer nature's call.

FestEvents estimates that there will be 4 arrivals per minute on average to the toilets. The average time a person spends using the toilet is 3.12 minutes.

(a) Suppose FestEvents installs 20 portable toilets. On average, what fraction of the toilets will be occupied during the event?

Service rate = 1/service time = 1 / 3.12 = 0.32
Utilization = arrival rate / n_channels x service rate = 4/(20 x 0.32) = 0.625
62.5% of toilets will be occupied during the event

(b) What is the minimum number of portable toilets that FestEvents must install to ensure that the queue length will not continue to grow without limit (i.e., so that the queue will be stable)? Your answer should be an integer rounded up – they can install 3 portable toilets, but they cannot install 2.3 toilets.

For stability, arrival rate should be < service rate of all toilets

Utilization < 1
Arrival rate / n_channels x service rate < 1 ***solve for the number of channels
Arrival Rate / service rate < n_channels
4/0.32 = 12.5 ~ 13

13 portable toilets should be installed to ensure that the line will not grow substantially.

3) **5 Points** Max Stamp approves study abroad documents for Rice University. Students must wait in line with their forms outside Max's office. One student at a time is allowed in his office and Max takes precisely 25 minutes to evaluate each student's set of documents. On average 2.2 students per hour go to his office and they spend on average 160 minutes trying to get their forms approved (time waiting in queue plus time in Max's office having him evaluate their documents).

(a) On average, how many students are waiting outside of Max's office? (Ignore start-of-the-day and end-of-the-day effects.)

Arrival rate = 2.2 stud/hr
Service time = 25min / 60hr/min = 0.417hrs
Avg time in system = 160min / 60hr/min = 2.67hrs

Using littles law we can find the number of students waiting in the queue:
Avg num students in system (I) = arrival rate x avg time in system = 2.2stud/hr x 2.67hrs = 5.87stud

To find the avg num of students in the queue we can make the assumption that I_p (avg num students being served) is 1, considering that Max can only see one student at a time. Since $I = I_q + I_p$, where I_q is the avg num of students in the queue, and we know I and I_p, we can solve for I_q.

5.87stud = $I_q + 1$
$I_q = 4.87$

This means that, on average, there are 4.87 students in queue outside of Max's office waiting to be seen.