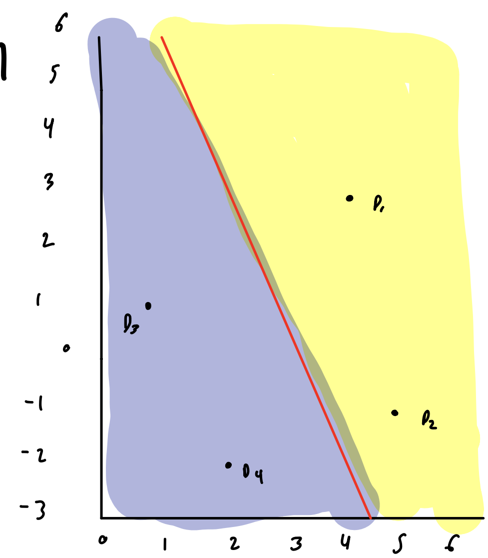
COMP 642 - Christian Ruiz

Q 1.a Perceptron

$L = 0.1$

$$\begin{aligned} D_1 &= (4, 3) \in N = -1 \\ D_2 &= (5, -1) \in N = -1 \\ D_3 &= (1, 1) \in P = 1 \\ D_4 &= (2, -2) \in P = 1 \end{aligned}$$

y^k



$W_k = \underbrace{(1, 0, 0)}_{w_0, w_1, w_2}$

iteration 1: x_0 = bias term

D_1. $h_{w_1}(x^1) = (1 \cdot 1) + (0 \cdot 4) + (0 \cdot 3) = 1 \neq N$

update the weights

$$\left.\begin{aligned} w_0 New &= 1 - 0.1(1 - {}^-1) \cdot 1 = 0.8 \\ w_1 New &= 0 - 0.1(1 - {}^-1) \cdot 4 = -0.8 \\ w_2 New &= 0 - 0.1(1 - {}^-1) \cdot 3 = -0.6 \end{aligned}\right\} (0.8, {}^-0.8, {}^-0.6)$$

D_2. $h_{w_2}(x^2) = (0.8 \cdot 1) + (-0.8 \cdot 5) + (-0.6 \cdot {}^-1) = {}^-2.6 \approx -1 = N$

D_3. $h_{w_3}(x^3) = (0.8 \cdot 1) + (-0.8 \cdot 1) + (-0.6 \cdot 1) = -0.8 \approx -1 \neq P$

update weights

$$\left.\begin{aligned} w_0 New &= 0.8 - 0.1({}^-0.8 - 1) \cdot 1 = 0.98 \\ w_1 New &= {}^-0.8 - 0.1(-0.8 - 1) \cdot 1 = -0.62 \\ w_2 New &= -0.6 - 0.1(-0.8 - 1) \cdot 1 = -0.42 \end{aligned}\right\} (0.98, {}^-0.62, {}^-0.42)$$

D_4. $h_{w_4}(x^4) = (0.98 \cdot 1) + (-0.62 \cdot 2) + (-0.42 \cdot {}^-2) = 0.58 \approx 1 = P$

Iteration 2

$D_1.$ $h_{w1}(x^1) = (0.98 \cdot 1) + (-0.62 \cdot 4) + (-0.42 \cdot 3) = -2.76 = N$

$D_2.$ $h_{w2}(x^2) = (0.98 \cdot 1) + (-0.62 \cdot 5) + (-0.42 \cdot -1) = -1.7 = N$

$D_3.$ $h_{w3}(x^3) = (0.98 \cdot 1) + (-0.62 \cdot 1) + (-0.42 \cdot 1) = -0.06 \neq P$

Update weights

$$\left.\begin{aligned} w_0new &= 0.98 - 0.1(-0.06 - 1) \cdot 1 = 1.086 \\ w_1new &= -0.62 - 0.1(-0.06 - 1) \cdot 1 = -0.514 \\ w_2new &= -0.42 - 0.1(-0.06 - 1) \cdot 1 = -0.314 \end{aligned}\right\}$$

$D_4.$ $h_{w4}(x^4) = (1.086 \cdot 1) + (-0.514 \cdot 2) + (-0.314 \cdot -2) = 0.686 = P$

Iteration 3

$D_1.$ $h_{w1}(x^1) = (1.086 \cdot 1) + (-0.514 \cdot 4) + (-0.314 \cdot 3) = -1.912 = N$

$D_2.$ $h_{w2}(x^2) = (1.086 \cdot 1) + (-0.514 \cdot 5) + (-0.314 \cdot -1) = -1.17 = N$

$D_3.$ $h_{w3}(x^3) = (1.086 \cdot 1) + (-0.514 \cdot 1) + (-0.314 \cdot 1) = 0.258 = P$

D_4 $h_{w4}(x^4) = (1.086 \cdot 1) + (-0.514 \cdot 2) + (-0.314 \cdot -2) = 0.686 = P$

Converged at $(1.086, -0.514, -0.314)$
after 3 iterations

Q 1.b

Dataset:

$D_1: (1,1)^T \in N$

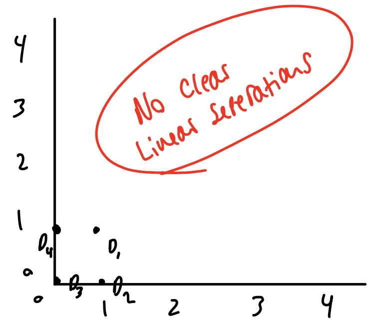
$D_2: (1,0)^T \in P$

$D_3: (0,0)^T \in N$

$D_4: (0,1)^T \in P$

$N = -1 \quad P = 1$

$w = (1, 0, 0)$



Iteration 1:

$D_1 \quad h_{w_1}(x^1) = (1 \cdot 1) + (0 \cdot 1) + (0 \cdot 1) = 1 \neq N$

Update weights:

$$\left.\begin{aligned} w_{new_1} &= 1 - 0.1(1 - {}^-1) \cdot 1 = 0.8 \\ w_{new_2} &= 0 - 0.1(1 - {}^-1) \cdot 1 = {}^-0.2 \\ w_{new_3} &= 0 - 0.1(1 - {}^-1) \cdot 1 = {}^-0.2 \end{aligned}\right\} (0.8, -0.2, {}^-0.2)$$

$D_2 \quad h_{w_2}(x^2) = (0.8 \cdot 1) + ({}^-0.2 \cdot 1) + (-0.2 \cdot 0) = 0.6 = P$

$D_3 \quad h_{w_3}(x^3) = (0.8 \cdot 1) + ({}^-0.2 \cdot 0) + ({}^-0.2 \cdot 0) = 0.8 \neq N$

Update weights:

$w_{new_1} = 0.8 - 0.1(0.8 - {}^-1) \cdot 1 = 0.62$

$w_{new_2} = {}^-0.2 - 0.1(0.8 - {}^-1) \cdot 0 = {}^-0.2$

$w_{new_3} = {}^-0.2 - 0.1(0.8 - {}^-1) \cdot 0 = {}^-0.2$

$D_4 \quad h_{w_4}(x^4) = (0.62 \cdot 1) + ({}^-0.2 \cdot 0) + ({}^-0.2 \cdot 1) = 0.4 = P$

Iteration 2

$D_1. \quad h_{w_1}(x^1) = (0.62 \cdot 1) + ({}^-0.2 \cdot 1) + ({}^-0.2 \cdot 1) = 0.22 \neq N$

Update weights:

$w_1 New = 0.62 - 0.1(0.22 - {}^-1) \cdot 1 = 0.498$

$w_2 New = {}^-0.2 - 0.1(0.22 - {}^-1) \cdot 1 = -0.322$

$w_3 New = {}^-0.2 - 0.1(0.22 - {}^-1) \cdot 1 = {}^-0.322$

$D_2. \quad h_{w_2}(x^2) = (0.498 \cdot 1) + ({}^-0.322 \cdot 1) + ({}^-0.322 \cdot 0) = 0.176 = P$

$D_3.$ $h_{w_3}(x^3) = (0.498 \cdot 1) + (-0.322 \cdot 0) + (-0.322 \cdot 0) = 0.498 \neq N$

- a single perceptron cannot solve the problem; the switching of x_i values from 1's and 0's result in an inseparable non-linear problem.

- We can see a pattern as we adjust the weights; as we attempt to converge to meet the requirements of one data set, updating the weights, we continue to be imbalanced in the others.

- linear problems can be solved by perceptron, however, it cannot solve non-linear problems.