Monte Carlo-Based Risk Assessment for Investment Decision-Making
NVIDIA (NVDA)

Advanced Simulation-Based Analysis
Geometric Brownian Motion · Empirical Monte Carlo · GARCH (1,1) · Heston Stochastic Volatility

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Equity Research Report – NVIDIA Corporation

1. Introduction

This report presents a quantitative, simulation-based investment analysis of *NVIDIA Corporation (NASDAQ: NVDA)*, a global leader in semiconductors, GPU computing, and AI infrastructure.

1.1 Objective

The objective is to go beyond traditional equity research by applying advanced stochastic models to analyse NVIDIA's forward-looking return distribution, tail risks, and volatility dynamics. We integrate methods widely used in quantitative finance and institutional risk management, including:

- Geometric Brownian Motion (GBM)
- Empirical Monte Carlo simulation
- GARCH (1,1) for time-varying volatility
- Heston stochastic volatility model

These models offer a robust analytical framework to support data-driven investment decisions under uncertainty.

1.2 Scope of Analysis

- *Analysis Period:* **May 2019 – May 2024**
- *Data Sources:* Yahoo Finance API (yfinance), SEC filings, public datasets
- Simulation Depth: Up to 7 years forward
- *Monte Carlo Simulations:* 50,000 + stochastic paths with varying model complexity

2. Company Overview

Founded in 1993 and headquartered in Santa Clara, California, *NVIDIA* is widely recognized for its cutting-edge GPU architectures, AI computing platforms, and system on a chip (SoC) designs.

Originally focused on gaming hardware, NVIDIA has since transformed into a platform company serving multiple high-growth sectors:

- *Data Center & AI:* High-performance GPUs used for AI training, large language models (LLMs), cloud computing, and HPC infrastructure.
- *Automotive:* Autonomous driving systems and ADAS via NVIDIA DRIVE.
- *Omniverse:* Real-time simulation and collaboration platforms for digital twins and 3D workflows.
- *Gaming & Creative Platforms:* Industry-standard tools for developers, creators, and the gaming community.

2.1 Strategic Highlights

Over the last decade, NVIDIA has positioned itself as a core enabler of technological transformation across industries. Its strategic advantages are rooted in a combination of innovation, vertical expansion, and dominant execution capacity:

- **AI Infrastructure Leadership:** NVIDIA's GPUs are essential to training state-of-the-art generative AI models, making the company a critical supplier for hyperscalers and AI labs.
- **Explosive Growth:** As of 2024, NVIDIA’s market capitalization surpassed **$3.2 trillion**, reflecting investor confidence in its scalable platforms and continued margin expansion.
- **Aggressive R&D Investment:** The company reinvests heavily in research and product development, sustaining its first-mover advantage across chips, software stacks, and AI frameworks.
- **Platform Ecosystem Strategy:** NVIDIA has transitioned from a hardware provider to a platform company, integrating hardware, APIs, SDKs, and cloud services.

3. Key Financial Indicators - NVIDIA Fundamentals Overview

Going a little deeper, let's now examine NVIDIA's key financial metrics, pulled from recent public filings and the Yahoo Finance API.

These indicators provide a snapshot of the company's profitability, capital efficiency, valuation multiples, and balance sheet structure, offering a quantitative view of its fundamental positioning relative to its sector and growth trajectory.

Metric	*Value*
Market Capitalization	$3.51 trillion
Price-to-Earnings (P/E)	46.49
Price-to-Sales (P/S)	23.66
Return on Equity (ROE)	115.46%
Debt-to-Equity Ratio	12.27
Earnings Per Share (EPS)	$3.10
Total Revenue	$148.5 billion
Net Income Margin	51.69%

These figures underscore NVIDIA's **extraordinary profitability** and operational leverage, as well as the premium valuation assigned by the market due to its leadership in high-growth sectors like AI, autonomous systems, and data center acceleration.

3.1 Compound Annual Growth Rate (CAGR) - NVIDIA (2019 - 2024)

To further quantify NVIDIA's historical performance, we calculate its Compound Annual Growth Rate **(CAGR)** from *May 1st, 2019 to May 1st, 2024*, using adjusted historical prices retrieved via *yfinance*.

The Compound Annual Growth Rate (**CAGR**) measures the constant annual rate at which an investment grows over a given period, providing a time-adjusted metric for consistent performance benchmarking.

CAGR Result:

80.74% *annually*

This extraordinary return underscores NVIDIA's dominant position in GPU markets, its aggressive expansion into AI and data center acceleration, and the exceptional investor sentiment during this period.

Such a high CAGR is uncommon among large-cap equities, highlighting the long-term upside potential of holding exposure to transformative tech companies during secular growth cycles.

3.2 Performance Comparison - NVIDIA vs S&P 500 (2019 - 2024)

To contextualize NVIDIA's exceptional growth, we compare its historical performance to that of the **S&P 500** Index over the same period *(May 2019 – May 2024)*.
This analysis highlights the *risk-return profile* of NVDA relative to the broader *U.S.* equity market.

Key Metrics

Metric	***NVIDIA***	***S&P 500 Index***
Annualized Return	106.89%	14.07%
Annualized Volatility	52.15%	21.30%
Maximum Drawdown	-66.34%	-33.92%

Despite exhibiting over twice the volatility of the S&P 500 and experiencing significantly deeper drawdowns, NVIDIA delivered returns more than **7x higher** on an annualized basis.

This reflects the stock's *extreme sensitivity* to investor sentiment and innovation cycles, a trait common among high-growth tech equities. While rewarding during bull runs, such profiles require disciplined risk management when incorporated into institutional portfolios.

This comparison also reinforces the rationale for applying forward-looking stochastic models, which will help assess whether such outperformance is statistically sustainable or mean-reverting under varying future conditions.

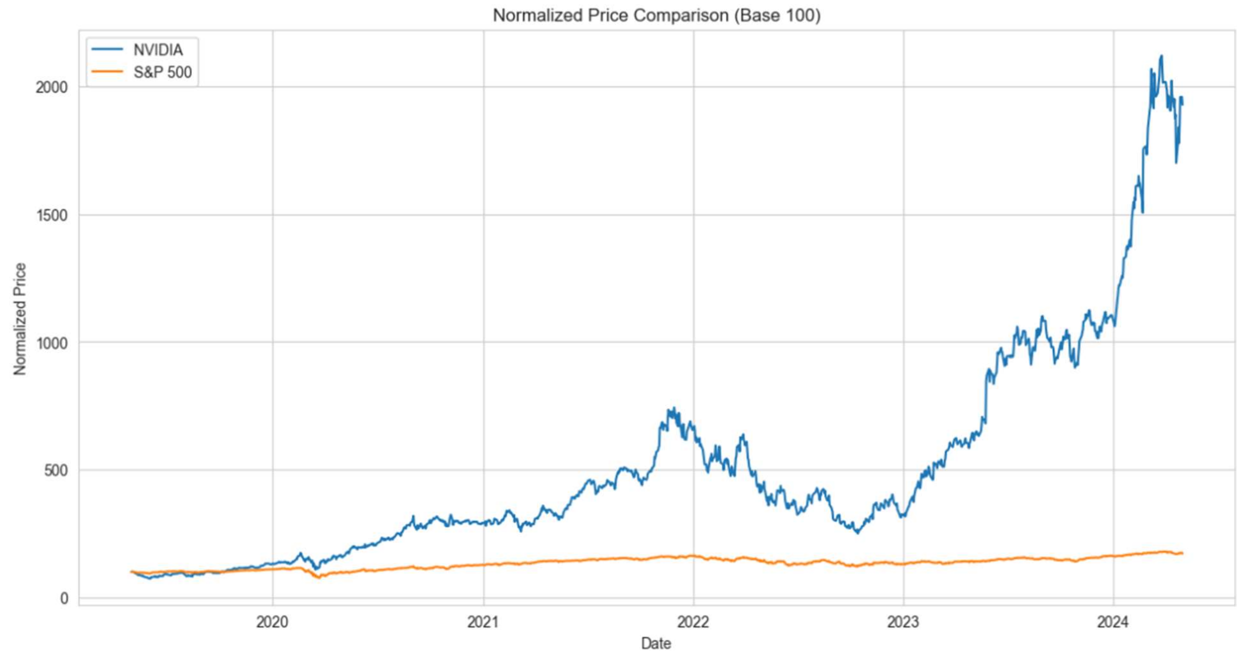

Figure 1. Normalized Price Chart: NVIDIA vs. S&P 500 (2019 - 2024)

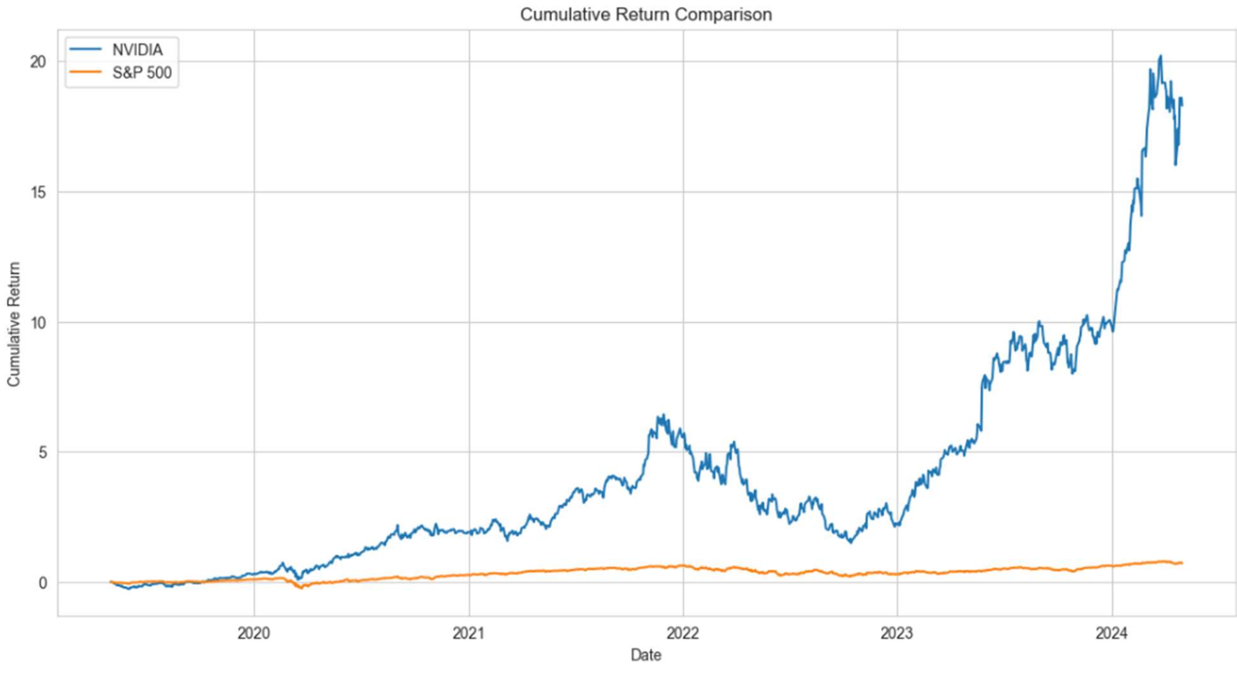

Figure 2. Cumulative Return Comparison: NVIDIA vs. S&P 500 (2019 - 2024)

4. Price and Return Trends - Historical Behavior Overview

Visualizing normalized price trajectories and cumulative return curves offers additional insight into ***NVIDIA's*** market behavior over the last five years.

The normalized price chart illustrates the divergence in growth trajectories, with NVIDIA clearly decoupling from the S&P 500, particularly during periods of AI-driven investor momentum.

Meanwhile, the cumulative return graph confirms not only the magnitude of outperformance, but also the path-dependent volatility NVIDIA has experienced with frequent accelerations, pullbacks, and recovery phases that reflect investor sensitivity to innovation narratives and macro events.

4.1 Fundamental Outlook

NVIDIA represents a compelling high-growth, high-volatility investment case, underpinned by industry leadership in AI acceleration, data center infrastructure, and GPU computing. Its five-year CAGR of *80.74%,* exceptional profitability metrics, and market cap exceeding *$3.2 trillion* reflect not only strong fundamentals, but also **elevated investor expectations**.

For investors with a higher risk tolerance, NVIDIA offers significant long-term upside.
Its innovation-driven model, vertical integration, and deep moats across multiple growth segments position it well for continued dominance.

However, this comes with inherent exposure to valuation risk, momentum-driven price cycles, and broader macro volatility.

4.2 Transition to Quantitative Section

Up to this point, we have reviewed NVIDIA's fundamental strengths, market behavior, and strategic positioning. While these insights are valuable, they rely on historical performance and static financial indicators.

To build a more rigorous, forward-looking understanding of risk and return, we now transition into the ***quantitative core*** of this report.

In the next sections, we implement a suite of Monte Carlo simulation models, from baseline stochastic processes to advanced volatility frameworks such as GARCH and Heston.
This quantitative approach not only captures NVIDIA's potential price behavior under realistic probabilistic scenarios, but also serves as a foundation for informed investment decision making.

5. Monte Carlo Simulation - Predictive Price Modelling for NVIDIA

To transition from historical analysis to forward-looking projections, we apply *Monte Carlo* simulations to model the potential price evolution of NVIDIA under uncertainty.

Monte Carlo methods are widely used for:

- Modeling asset price dynamics under stochastic processes
- Evaluating Value at Risk (**VaR**) and Conditional VaR (**CVaR**)
- Estimating scenario distributions and tail events
- Supporting probabilistic investment decision-making

As a first step, we simulate NVIDIA's future price distribution using a ***Geometric Brownian Motion*** (GBM) model, the standard baseline in quantitative modeling.
This approach assumes *constant volatility and drift*, and serves as a useful benchmark before introducing more realistic frameworks like *GARCH (1,1)* and *Heston stochastic volatility*.

5.1 Data Preparation - NVDA Historical Returns

To calibrate the GBM model, we begin by retrieving adjusted daily closing prices for NVIDIA from *May 1st 2019 to May 1st 2024* using *yfinance*.

From this dataset, we compute the key input parameters:

- S_0 Initial price at the end of the historical period
- μ Historical average daily return
- σ Historical daily volatility

These are used to define the GBM's drift and diffusion components, which drive the simulated price paths.

```
[*********************100%***********************]  1 of 1 completed
Initial Price (S₀): $86.37
Average Daily Return (μ): 0.002951
Daily Volatility (σ): 0.032669
```

Figure 3. Code output: historical return extraction and parameter calibration

5.2 Simulating Future Price Paths

Using the calibrated parameters (**μ**, **σ**, and **S_0**), we simulate 10,000 price trajectories for *NVIDIA* over a 1-year horizon (252 trading days) based on the *Geometric Brownian Motion* framework.

Each simulated path evolves according to the stochastic differential equation:

➔ $dS = \mu S dt + \sigma S dz$

Where:

- **μ** is the drift (expected return)
- **σ** is the volatility
- **dz** is the Brownian motion

These paths reflect theoretical price movements under the assumption of constant volatility and log-normal returns.
The chart below shows a random sample of 100 simulated paths, illustrating the range of possible future scenarios under this simplified framework.

Figure 4. Simulated Price Paths Using GBM (10,000 paths, 1-year horizon)

5.3 Statistical Summary of Simulated Prices

After generating 10,000 simulations, we extract the **final prices** at the end of the 1-year horizon and compute key statistical metrics:

```
=== Monte Carlo Simulation Summary ===
Initial Price (S0): $86.37
Expected Price (Mean): $162.73
Median Price: $162.70
5th Percentile: $158.19
25th Percentile: $160.91
75th Percentile: $164.54
95th Percentile: $167.27
```

```
Probability of Loss: 0.00%
Probability of Price ≥ +20%: 100.00%
Probability of Price ≥ +50%: 100.00%
Probability of Price ≥ +100%: 0.02%
```

These base-case results serve as a benchmark for forward-looking expectations under simplified assumptions.

In the following sections, we refine this simulation by analyzing percentile forecast bands, estimating downside risk using *VaR* and *CVaR*, and visualizing the full return distribution before **stress-testing** the model under adverse market conditions and exploring more data-driven sampling techniques.

5.4 Simulating Percentile Forecast Bands Over Time

To visualize the expected range of future price paths, we extract and plot forecast percentile bands from the 10,000 GBM-based Monte Carlo simulations.

For each trading day within the 1-year horizon (*252 days*), we calculate **key percentiles** across all simulations:

- *5th Percentile* (left-tail downside risk)
- *25th Percentile*
- *50th Percentile* (Median)
- *75th Percentile*
- *95th Percentile* (right-tail optimistic scenario)

This provides a dynamic corridor of plausible price outcomes over time, helping to assess:

- **Expected central tendency** (median path)
- **Range of plausible deviations**
- **Skewness and tail behavior** in forward projections

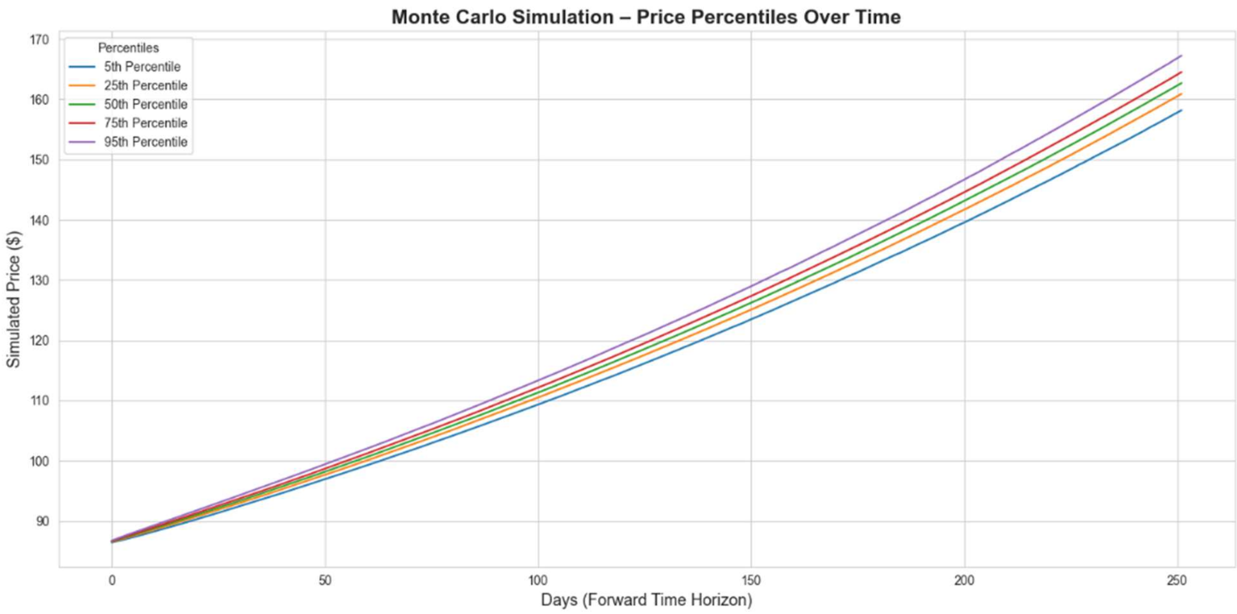

Figure 5. Percentile Forecast Bands Over Time (1-Year Horizon)

5.5 Estimating Downside Risk with VaR and CVaR

To quantify downside risk under the GBM simulation, we calculate:

- **Value at Risk (*VaR 5%*)**: the return threshold below which the worst 5% of simulated outcomes fall.
- **Conditional Value at Risk (*CVaR 5%*)**: the average return of that bottom 5% tail, providing a more realistic view of extreme loss scenarios.

These two measures offer a compact view of tail risk, helping assess how severe potential drawdowns may be under simplified volatility assumptions.

```
=== Risk Metrics ===
5% Value at Risk (VaR): 83.16%
5% Conditional VaR (CVaR): 81.88%
```

While GBM captures broad distributional behavior, it ***understates tail thickness*** and ***over-smooths volatility***, making these metrics more optimistic than those derived from volatility-aware models (see later sections).

5.6 Visualization of Simulated Return Distribution

To better understand the probabilistic landscape implied by the GBM simulation, we generate a histogram of final 1-year returns based on 10,000 Monte Carlo paths.

The visualization includes a *kernel density estimation* (KDE) curve and superimposes key reference points such as:

- *Mean* return (expected value)
- *Median* return
- *5% Value at Risk* **(*VaR*)**
- *5% Conditional VaR* **(*CVaR*)**

The resulting distribution exhibits a *mild right-skew*, which is typical for log-normal models where returns cannot fall below 100% but have unlimited upside.

Most of the outcomes are concentrated between moderate positive returns, but the right tail extends further, pulling the mean upward.

From a risk management perspective, this distribution helps identify the likelihood and severity of extreme events. The distance between **VaR** and **CVaR** also provides insight into the shape of the tail with a small gap suggesting limited fat-tailed behavior under this model.

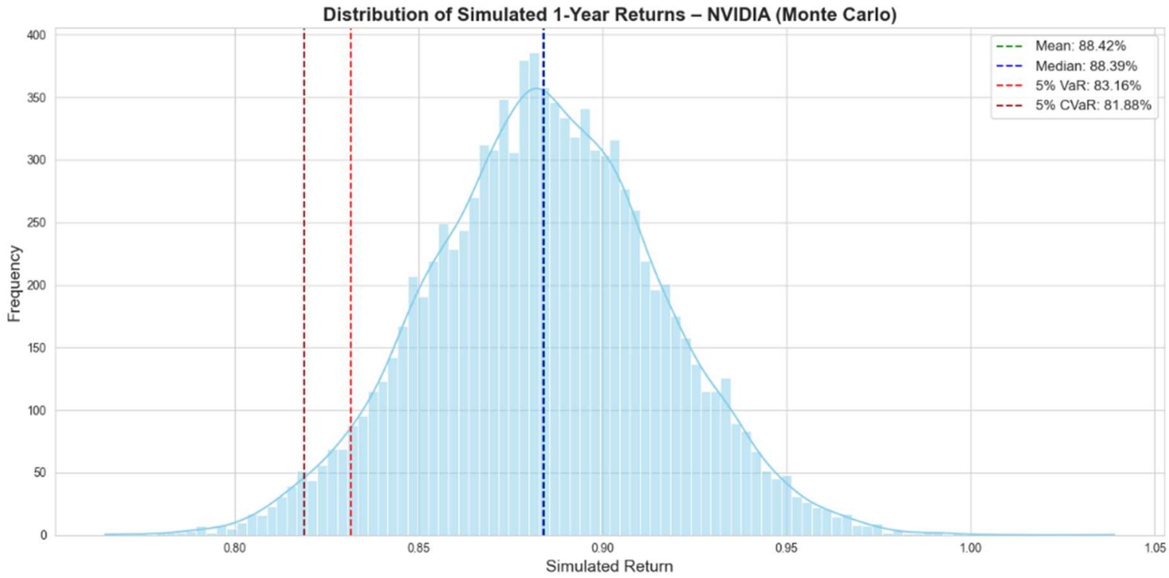

Figure 6 – Distribution of Simulated 1-Year Returns (GBM Monte Carlo)
(Dashed lines represent key statistical thresholds including VaR, CVaR, mean and median.)

While informative, it's important to recognize that GBM assumes **constant volatility** and does not reflect real-world features like *volatility clustering* or *regime shifts*.

To evaluate more extreme market conditions and validate the robustness of the simulation, the next section introduces *stress-test scenarios* with negative drift assumptions and adverse volatility shocks.

6. Stress Testing - Monte Carlo Price Simulation Under Custom Scenarios

While traditional Monte Carlo simulations rely on historical data to estimate drift and volatility, stress testing involves modifying these parameters deliberately to simulate **extreme market environments**.

In this section, we construct custom scenarios by altering:

- The **expected return (drift)**
- The **volatility level**
- The simulation label to reflect specific macroeconomic shocks

Each scenario is modeled using the same GBM Monte Carlo framework, but with modified assumptions.

6.1 Scenario 1 - Market Crash (Expected Return - 30%)

In this scenario, we simulate a significant market **downturn** by applying a - 30% expected return while preserving NVIDIA's historical annualized volatility.
This mirrors a recession-like regime where investor sentiment turns sharply bearish.

Key Scenario Features:

- **Annualized Expected Return:** - 30%
- **Volatility:** Same as historical (≈ 52%)
- **Simulation Horizon:** 1 year (252 trading days)
- **Model:** GBM Monte Carlo (10,000 paths)

```
# Scenario 1: Market Crash (Expected return -30%)
S_stress_1 = monte_carlo_stress_test(S0, mu=-0.3, sigma=sigma, label="Market Crash Scenario")
✓ 0.6s

=== Market Crash Scenario ===
Expected Price: $64.05
5% VaR: $60.67
5% CVaR: $59.83
```

Figure 7: Monte Carlo stress test, Scenario 1, Output

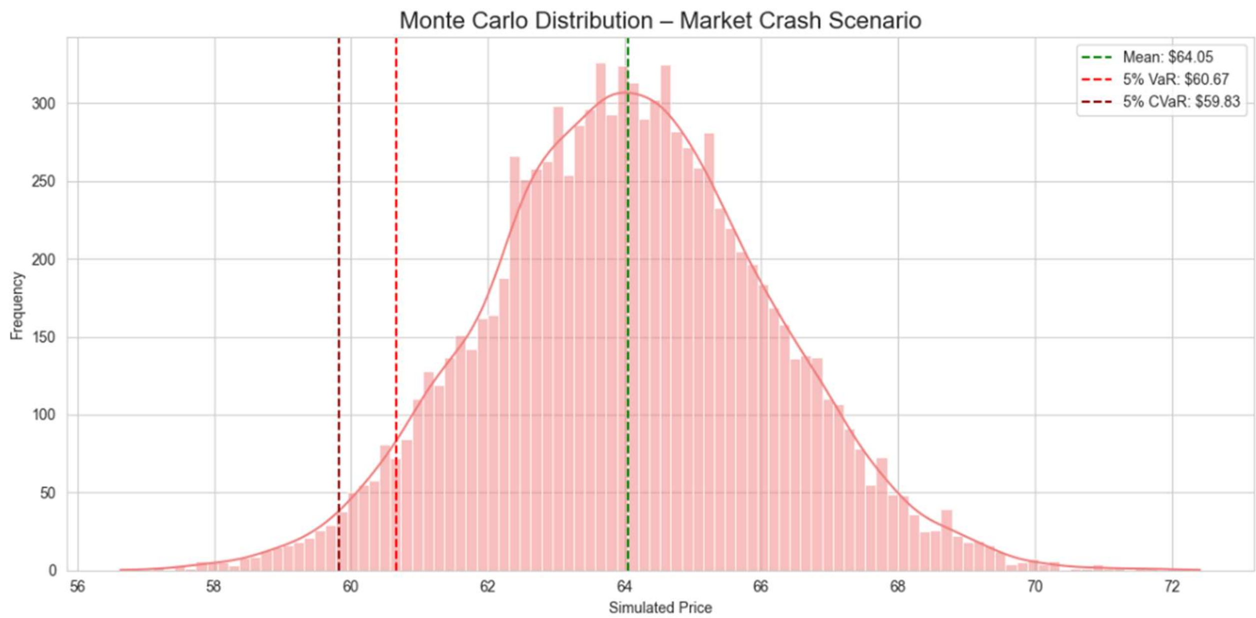

Figure 8: Final Price Distribution Under Market Crash Scenario

Interpretation:

- The return distribution becomes ***sharply left-skewed***, with a compressed upside.
- *Mean and median prices* drop significantly compared to the base case.
- **Tail risk becomes dominant**, with VaR and CVaR reflecting more severe losses.

6.2 Scenario 2 - High Volatility Shock (σ = +75%)

In this scenario, we model the effects of a **volatility spike** while keeping the expected return unchanged.
We increase NVIDIA's annualized volatility by approximately **75%** relative to its historical value, simulating conditions of market panic, sudden uncertainty, or sector-specific instability.

Key Scenario Features:

- **Annualized Expected Return:** Historical (≈ 106.9%)
- **Annualized Volatility:** Increased by about 75%
- **Simulation Horizon:** 1 year (252 trading days)
- **Model:** GBM Monte Carlo (10,000 paths)

```
# Scenario 2: Volatility Doubles
S_stress_2 = monte_carlo_stress_test(S0, mu=mu, sigma=sigma * 2, label="Volatility Spike Scenario")
✓ 0.5s

=== Volatility Spike Scenario ===
Expected Price: $86.72
5% VaR: $77.65
5% CVaR: $75.75
```

Figure 7: Monte Carlo stress test, Scenario 2, Output

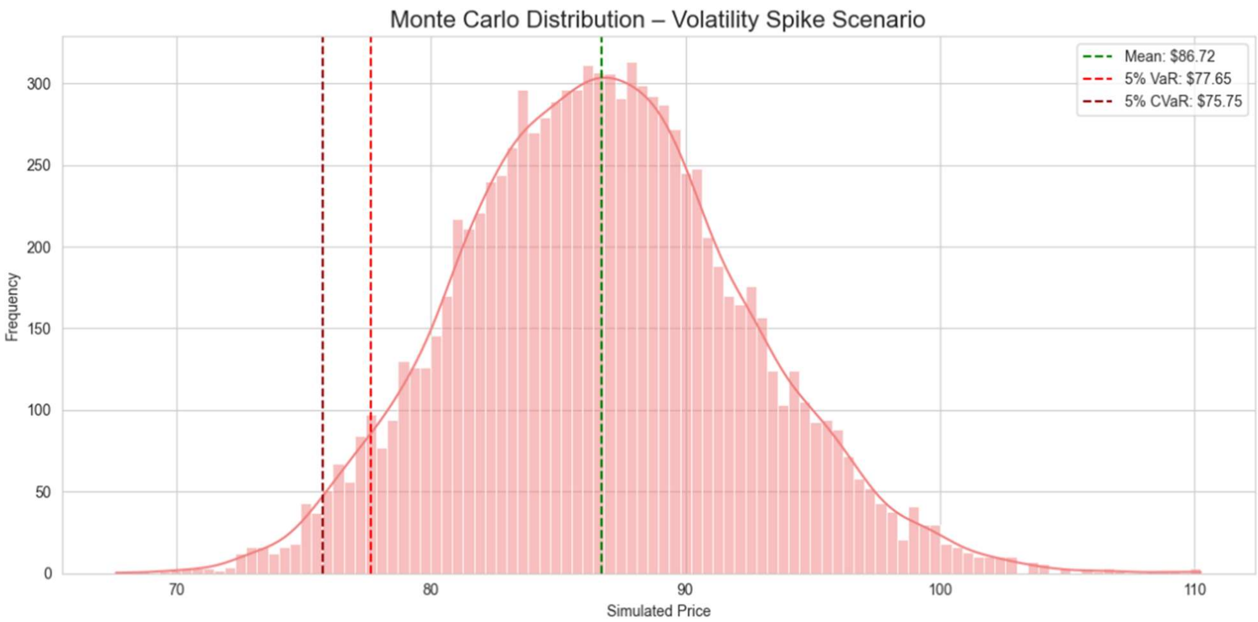

Figure 8: Final Price Distribution Under High Volatility Scenario

Interpretation:

- The distribution becomes much **wider**, with both left and right tails more pronounced.
- **Downside risk increases** substantially, despite unchanged expected return.
- The median remains near base-case levels, but probability of extreme outcomes grows, in both directions.

6.3 Scenario 3 - Combined Shock (- 30% Return and +75% Volatility)

In this final stress test, we simulate a **combined market shock** by applying both a **- 30%** expected return and a **+ 75%** increase in volatility.

This represents a highly adverse regime in which NVIDIA is exposed to systemic drawdowns, sentiment collapse and macro-driven uncertainty.

Such a scenario may resemble periods like the 2008 financial crisis or early 2020 COVID market panic, where both growth expectations deteriorate and volatility surges across asset classes.

Key Scenario Features:

- **Annualized Expected Return:** - 30%
- **Annualized Volatility:** + 75% above historical level
- **Simulation Horizon:** 1 year (252 trading days)
- **Model:** GBM Monte Carlo (10,000 paths)

```
# Scenario 3: Crash + High Volatility
S_stress_3 = monte_carlo_stress_test(S0, mu=-0.3, sigma=sigma * 2, label="Crash + Volatility Spike Scenario")
✓ 0.5s

=== Crash + Volatility Spike Scenario ===
Expected Price: $64.08
5% VaR: $57.39
5% CVaR: $55.87
```

Figure 9: Monte Carlo stress test, Scenario 3, Output

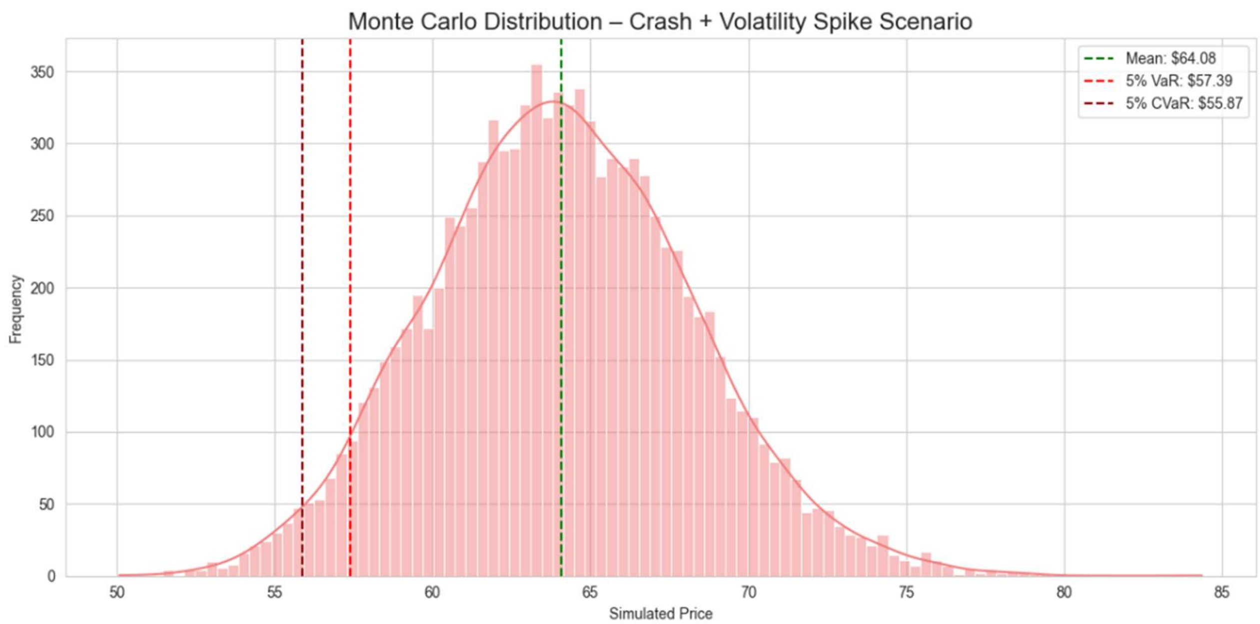

Figure 10: Final Price Distribution Under Combined Shock Scenario

Interpretation:

- The price distribution becomes **heavily left-skewed**, with most outcomes concentrated in *deep-loss* territory.
- Both **mean and median prices** fall significantly below baseline projections.
- Tail risk is extreme, and the probability of positive outcomes becomes minimal.

6.4 Stress Testing Summary

The stress tests demonstrate how even small changes in return or volatility drastically shift NVIDIA's risk profile.

Negative drift compresses outcomes downward, higher volatility expands tail risk, and the combined scenario produces severe asymmetry with most outcomes below initial price levels.

This confirms that NVIDIA's performance is highly sensitive to macro stress and reinforces the value of forward-looking risk analysis.

7. Monte Carlo Simulation with Empirical Return Sampling

To improve realism and capture real-world market behavior, we simulate future price paths by **resampling historical daily returns and volatilities** from the past five years.

Unlike GBM, which assumes *constant drift and volatility*, this method generates each day's return from empirical distributions, preserving the randomness, clustering, and fat-tailed nature of historical market data.

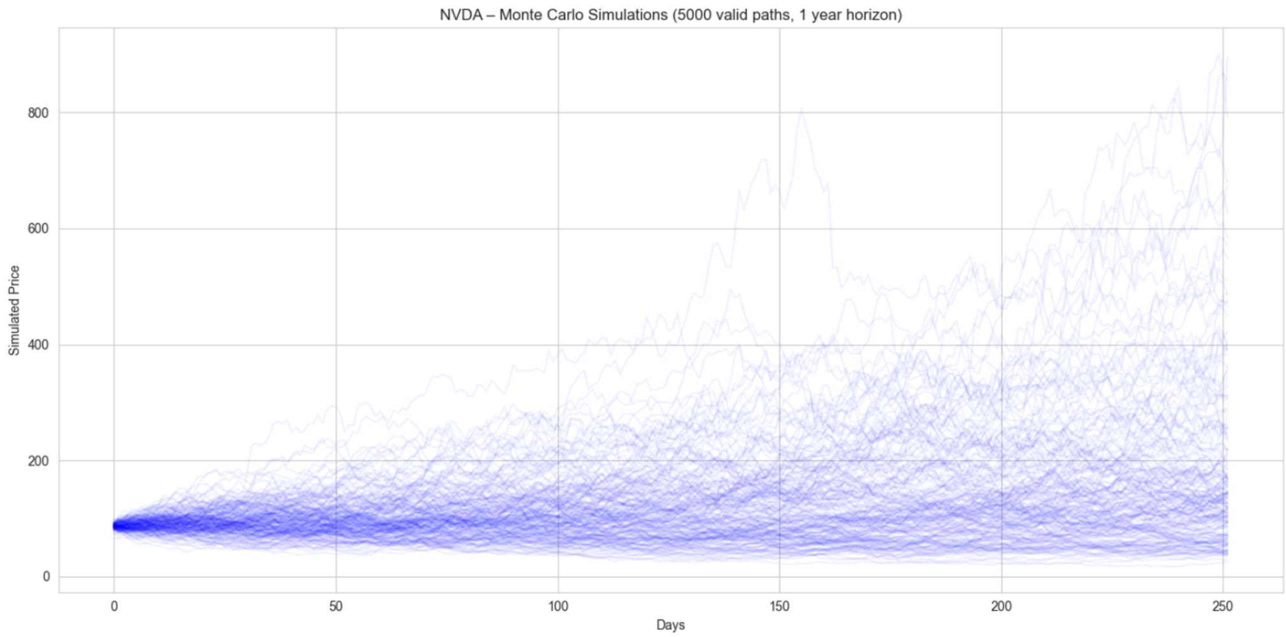

Figure 11: Simulated Price Paths Using Empirical Return & Volatility Sampling

```
=== Monte Carlo Simulation Summary ===
Initial Price (S₀): $86.37
Expected Price (Mean): $181.53
Median Price: $136.92
5th Percentile: $41.53
25th Percentile: $84.46
75th Percentile: $222.74
95th Percentile: $468.19

Probability of Loss: 26.04%
Probability of Price ≥ +20%: 65.24%
Probability of Price ≥ +50%: 52.70%
Probability of Price ≥ +100%: 36.74%
```

Figure 12: Final Price Distribution and Statistical Output

7.1 Summary Interpretation

The simulation yields a **wider return distribution** than GBM, with more dispersed outcomes and heavier tails.

While the median result is comparable to previous models, the probability of large gains or losses increases, confirming that real-world return dynamics introduce greater uncertainty than parametric assumptions capture.

Here we highlight the value of empirical techniques in risk forecasting, especially when historical regimes may repeat or cluster unpredictably.

Transition to Volatility-Driven Modeling

While informative, these methods treat volatility as a **static input**, failing to capture the way uncertainty evolves dynamically in real markets.

From this point forward, we introduce *volatility-driven models*, in which market risk is modeled as a process that fluctuates over time, persisting across periods and interacting with price behavior.

This shift reflects a key insight in financial modeling:

"*While returns are notoriously hard to predict, volatility tends to be more persistent and therefore more forecastable.***"**

By explicitly modeling volatility dynamics, frameworks like GARCH (1,1) and Heston allow us to simulate more realistic return paths, stress propagation, and tail risk.

8. Model Extension: Introduction to GARCH (1,1)

In the previous section, we modeled the future price dynamics of NVIDIA (NVDA) using a *Geometric Brownian Motion (GBM)* framework, with drift and volatility randomly sampled from historical data.

While useful as a baseline, this approach assumes constant volatility, which limits the model's ability to replicate key facts observed in financial markets, such as:

- *Volatility clustering*: periods of high volatility tend to follow other volatile periods.
- *Volatility persistence*: shocks to volatility tend to decay slowly over time.
- Nonlinear mean-reverting volatility dynamics.

To address these limitations, we now integrate a GARCH (1,1) model, which allows volatility to evolve over time as a function of the previous day's squared return and the previous day's variance estimate.

Objective:

Enhance the realism of our Monte Carlo simulation by replacing the fixed or sampled volatility structure with a conditional volatility model.
This will enable us to capture volatility shocks and persistence more effectively.

In the next code block:

- We will estimate a **GARCH (1,1)** model on NVDA's historical daily returns.
- Use it to generate path-dependent simulations with time-varying volatility.

```
[*********************100%***********************]  1 of 1 completed
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:           Daily Return   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -3219.53
Distribution:                  Normal   AIC:                           6447.05
Method:            Maximum Likelihood   BIC:                           6467.60
                                        No. Observations:                 1258
Date:                Wed, Jun 18 2025   Df Residuals:                     1257
Time:                        15:38:59   Df Model:                            1
                               Mean Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
mu             0.3563  8.523e-02      4.181  2.902e-05 [  0.189,  0.523]
                            Volatility Model
==========================================================================
                 coef    std err          t      P>|t|    95.0% Conf. Int.
--------------------------------------------------------------------------
omega          0.6513      0.265      2.462  1.381e-02   [  0.133,  1.170]
alpha[1]       0.0955  2.343e-02      4.076  4.572e-05 [4.958e-02,  0.141]
beta[1]        0.8471  2.675e-02     31.664 4.890e-220   [  0.795,  0.900]
==========================================================================

Covariance estimator: robust
```

Figure 13: Summary Output of GARCH (1,1) Model Estimation

8.1 GARCH (1,1) Driven Monte Carlo Simulation

This block simulates 1-year future price paths for NVIDIA using a *GARCH (1,1)* model fitted on daily returns from 2019 - 2024. The model introduces time-varying volatility, allowing each simulated step to reflect evolving market uncertainty.

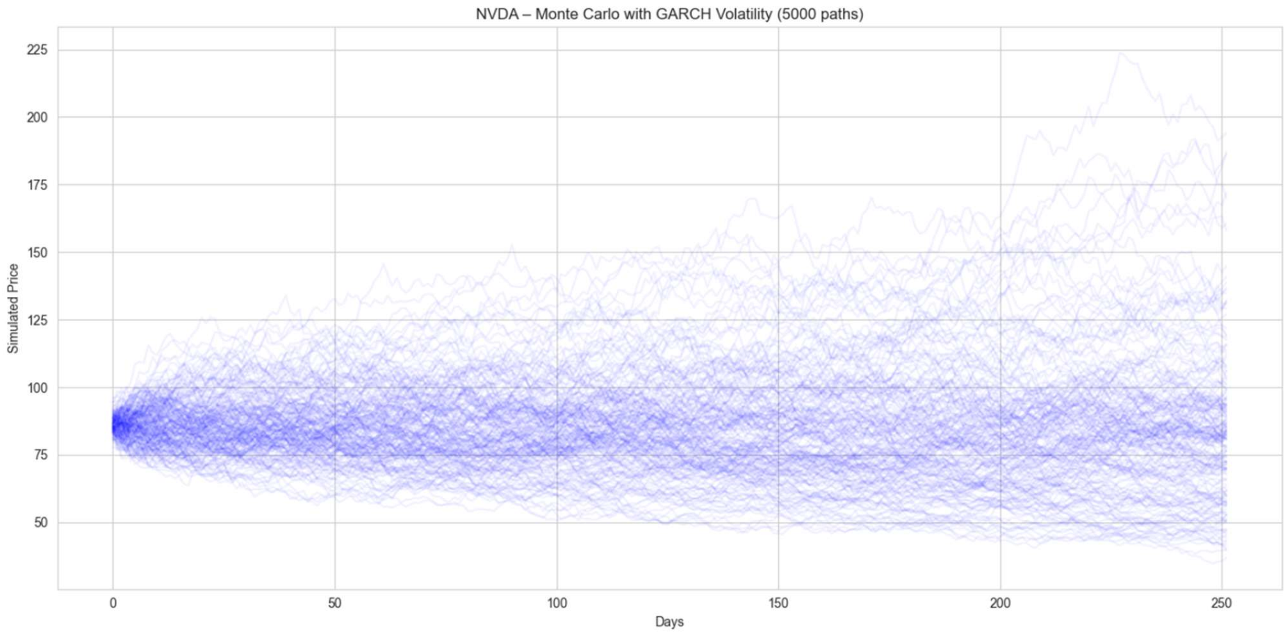

Figure 14: Monte Carlo simulation with GARCH Volatility (5000 paths)

```
=== GARCH Monte Carlo Simulation Summary ===
Initial Price (S₀): $86.37
Expected Price (Mean): $86.36
Median Price: $82.13
5th Percentile: $48.19
25th Percentile: $66.22
75th Percentile: $102.29
95th Percentile: $139.07

Probability of Loss: 56.48%
Probability of Price ≥ +20%: 23.34%
Probability of Price ≥ +50%: 7.64%
Probability of Price ≥ +100%: 1.16%
```

Figure 15: GARCH Model Outputs

Simulation Output Summary:

- Initial Price (S_0): $86.37
- Expected Price (Mean): $86.36
- Median Price: $82.13
- 5th / 25th / 75th / 95th Percentiles: $48.19 - $66.22 - $102.29 - $139.07
- Probability of Loss: 56.48 %
- Probability of Price ≥ +20%: 23.34 %
- Probability of Price ≥ +50%: 7.64 %
- Probability of Price ≥ +100%: 1.16 %

In our simulation, the GARCH (1,1) model with Student-t distributions produced a markedly more symmetric and risk-adjusted return distribution, revealing a **56.48%** probability of loss over one year and just **1.16%** probability of doubling in value.
These results reflect a more conservative but realistic outlook, reinforcing the need for volatility-aware strategies and scenario-based positioning when investing in high-growth equities like NVIDIA.

9. Regime Switching Model – Capturing Market Regimes

We further increase the realism of our simulation and now implement a *Regime Switching model*, also known as a *Markov Switching Model*.

This framework assumes that financial markets alternate between **distinct** hidden **states** (e.g., bull vs. bear regimes), each with its own statistical properties.

Unlike models with constant parameters, Regime Switching allows the simulation to dynamically adapt to changing market conditions.

This is crucial for accurately modeling asset prices over time, especially during periods of high volatility or structural shifts in the market.

```
p[0->0]      0.983186
p[1->0]      0.026271
const[0]     0.003619
const[1]     0.001735
sigma2[0]    0.000484
sigma2[1]    0.002017
```

Figure 16: Regime Switching parameters Output

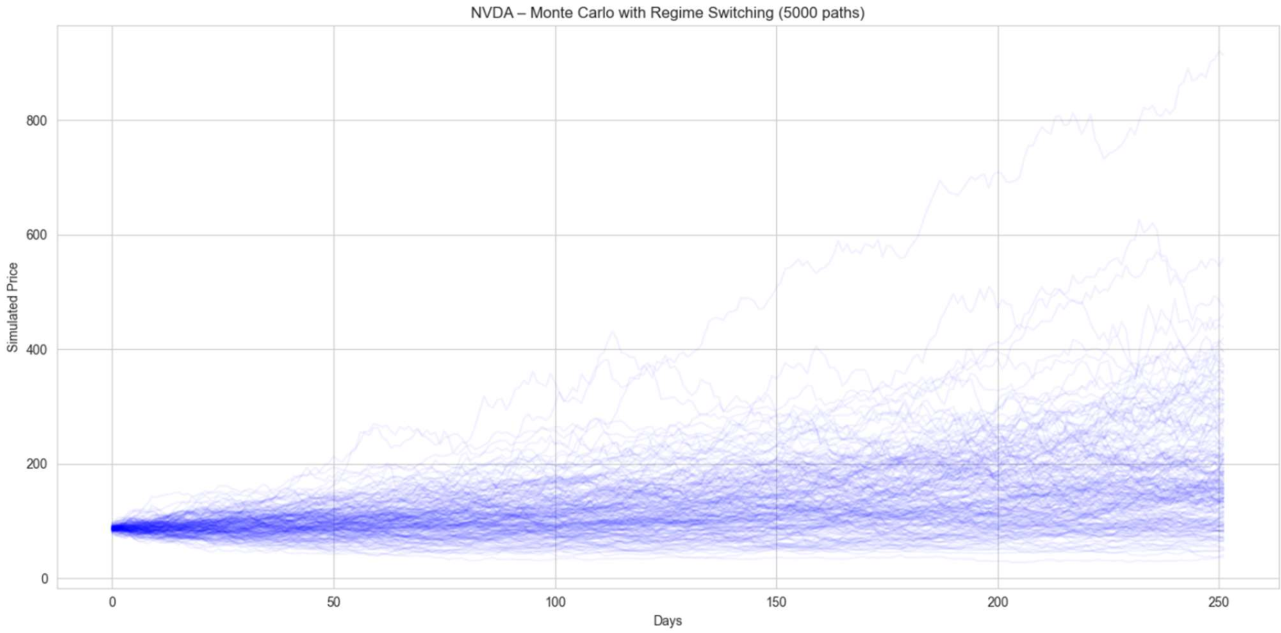

Figure 17: Regime Switching Output

```
=== Regime Switching Monte Carlo Simulation Summary ===
Initial Price (S₀): $86.37
Expected Price (Mean): $174.79
Median Price: $151.74
5th Percentile: $59.30
25th Percentile: $105.44
75th Percentile: $219.47
95th Percentile: $367.06

Probability of Loss: 15.46%
Probability of Price ≥ +20%: 75.82%
Probability of Price ≥ +50%: 62.04%
Probability of Price ≥ +100%: 41.00%
```

Figure 18: Regime Switching Output

9.1 Regime Switching Model - Monte Carlo simulation

The Regime Switching Monte Carlo simulation leverages a two-state Markov model to simulate dynamic transitions between market regimes, typically a "**bullish**" and a "**bearish**" state.

Each regime is characterized by distinct statistical properties in terms of expected returns and volatility, learned directly from historical data.

In our implementation, the model estimates very high persistence in the bullish regime, (p [0→0] ≈ 98.3%), which reflects NVIDIA's strong upward trend during the period 2019 - 2024. The simulations reflect the bullish bias embedded in the historical data, resulting in a predominance of optimistic price trajectories, including:

- A relatively low probability of loss (≈ 15%).
- A mean expected price more than 2x the current level.
- Consistent positive skewness in the final price distribution.

Such optimistic results are not a flaw of the model, but rather a direct consequence of the historical data window chosen, which spans five years of **heavily bullish performance**.
This, combined with the high regime persistence inferred from the data and the model's ability to capture momentum in market dynamics through state dependence, contributes to the dominance of optimistic outcomes.

While GARCH assumes *stationary volatility*, the Regime Switching model introduces *structural dynamics* by allowing returns and volatility to shift depending on the market state.

We recognize that increasing the historical window to include bear cycles (e.g., 2008, 2015) would likely produce more balanced estimates. However, for consistency with the GARCH and Brownian models, we retained the same **5-year** window.

In conclusion, the Regime Switching model adds a layer of sophistication by capturing structural changes in market conditions, and its optimistic results are justified by the data seen above.

10. Multivariate Monte Carlo Simulation

The next step will be introducing a *Multivariate Monte Carlo model*.
While previous simulations considered NVIDIA in isolation, this approach captures cross-asset relationships, modeling how NVIDIA's price dynamics are influenced by **correlated assets** such as sector ETFs or benchmark indices.

The key concept is to simulate not only NVIDIA's future returns, but also those of related financial instruments, and to jointly draw scenarios based on the historical correlation structure among them.

This results in a more realistic distribution of potential outcomes for NVDA, especially in stress or contagion scenarios.

We apply the Cholesky decomposition of the empirical covariance matrix of asset returns to generate correlated shocks across assets.
This ensures that the simulated paths are coherent with historical joint behavior.

```
[*********************100%***********************]  5 of 5 completed
=== Correlation Matrix ===
Ticker      NVDA       QQQ      SOXX       SPY       XLK
Ticker
NVDA    1.000000  0.813485  0.843343  0.709233  0.810500
QQQ     0.813485  1.000000  0.893105  0.931279  0.978029
SOXX    0.843343  0.893105  1.000000  0.839955  0.901775
SPY     0.709233  0.931279  0.839955  1.000000  0.938438
XLK     0.810500  0.978029  0.901775  0.938438  1.000000

=== Daily Returns Preview ===
Ticker          NVDA       QQQ      SOXX       SPY       XLK
Date
2019-05-02  0.015072 -0.004340  0.009625 -0.002159 -0.005222
2019-05-03 -0.000982  0.015948  0.007834  0.009788  0.009475
2019-05-06 -0.017267 -0.006122 -0.016296 -0.004115 -0.007737
2019-05-07 -0.037476 -0.019480 -0.024611 -0.016700 -0.021475
2019-05-08  0.004679 -0.002524 -0.008541 -0.001389 -0.001175
```

Figure 19: Correlation Matrix and Return Structure for Multivariate Simulation

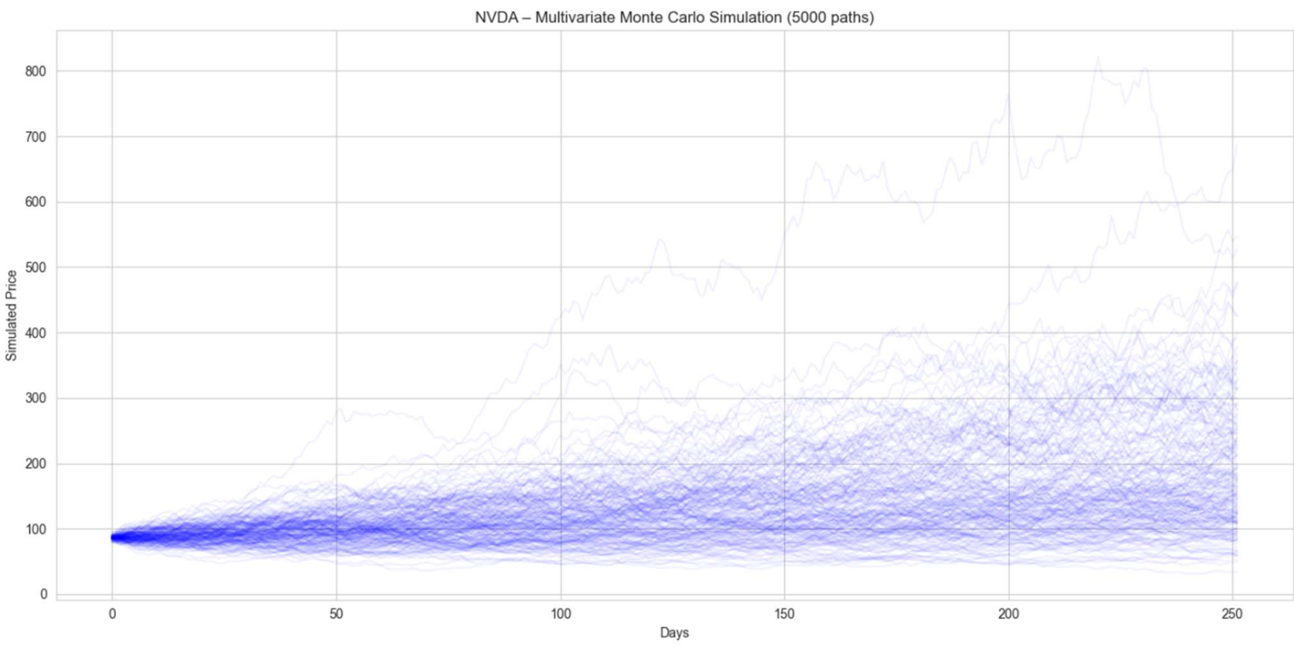

Figure 20: NVDA Multivariate Monte Carlo Simulation (5,000 paths)

=== Multivariate Monte Carlo Simulation Summary ===
Initial Price (S_0): $86.37
Expected Price (Mean): $181.35
Median Price: $156.00
5th Percentile: $66.62
25th Percentile: $110.01
75th Percentile: $225.93
95th Percentile: $375.98

Probability of Loss: 12.64%
Probability of Price ≥ +20%: 78.84%
Probability of Price ≥ +50%: 63.96%
Probability of Price ≥ +100%: 42.72%

Figure 21: Statistical Summary of Multivariate Monte Carlo Simulation

10.1 Interpretation

The Multivariate Monte Carlo model captures cross-asset dependencies by simulating NVIDIA's price evolution alongside a correlated basket of major tech-related indices (**QQQ**, **SOXX**, **XLK**, **SPY**). The strong correlations observed historically have amplified the bullish bias present in the period 2019 - 2024, resulting in optimistic projections (e.g., a > 42% probability of price doubling in 1 year).

Despite this, the model is structurally more robust than prior univariate models (GBM, GARCH, Regime Switching), as it accounts for inter market dynamics.

Still, the use of static correlations and normally distributed shocks may underrepresent tail risk.

While the Multivariate Monte Carlo model is structurally robust and captures cross-market dependencies effectively, our focus now shifts toward implementing the **Heston stochastic volatility model**.

11. Stochastic Volatility Simulation – Heston Model

In this *final* and most advanced phase of our financial study, we implement the *Heston stochastic volatility model*.
Unlike constant volatility frameworks, the Heston model captures the empirically observed features of asset returns: volatility clustering, mean reversion, and non-Gaussian behavior.

This model simultaneously evolves two stochastic processes: one for the asset price and one for the variance, allowing us to replicate more accurately the market's dynamic structure.

We will calibrate the model using real historical market data, ensuring professional-level realism and reliability.

Furthermore, the simulation will be executed with a **high number of iterations** to reflect robust statistical behavior and generate insights suitable for risk management and possibly portfolio strategies.

11.1 Heston Model Parameter Calibration

The objective is to estimate the optimal values for the mean-reversion speed (κ), long-run variance (θ), volatility of volatility (ξ), correlation (ρ), and initial variance (v_0) so that the simulated volatility closely matches the historical annualized volatility.

We use a numerical optimization approach (**Nelder-Mead**) to minimize the squared error between the model-implied and actual volatility, providing a statistically grounded foundation for subsequent Monte Carlo simulations.

```
Switching to differential_evolution...
differential_evolution step 1: f(x)= 10000000000.0
Polishing solution with 'L-BFGS-B'

===  Calibrated Heston Parameters ===
kappa: 0.0349
theta: 0.0268
xi:    0.0730
rho:   -0.3305
v0:    0.1356
```

Figure 22: Calibrated Heston Model Parameters

11.2 Monte Carlo Simulation with Calibrated Heston Model

This section implements a Monte Carlo simulation based on the Heston stochastic volatility model, using calibrated parameters to capture realistic market dynamics over a **7-year** horizon.

The simulation reflects correlated Brownian motions between price and volatility, allowing for mean-reverting variance, leverage effects, and richer risk profiles compared to standard models.

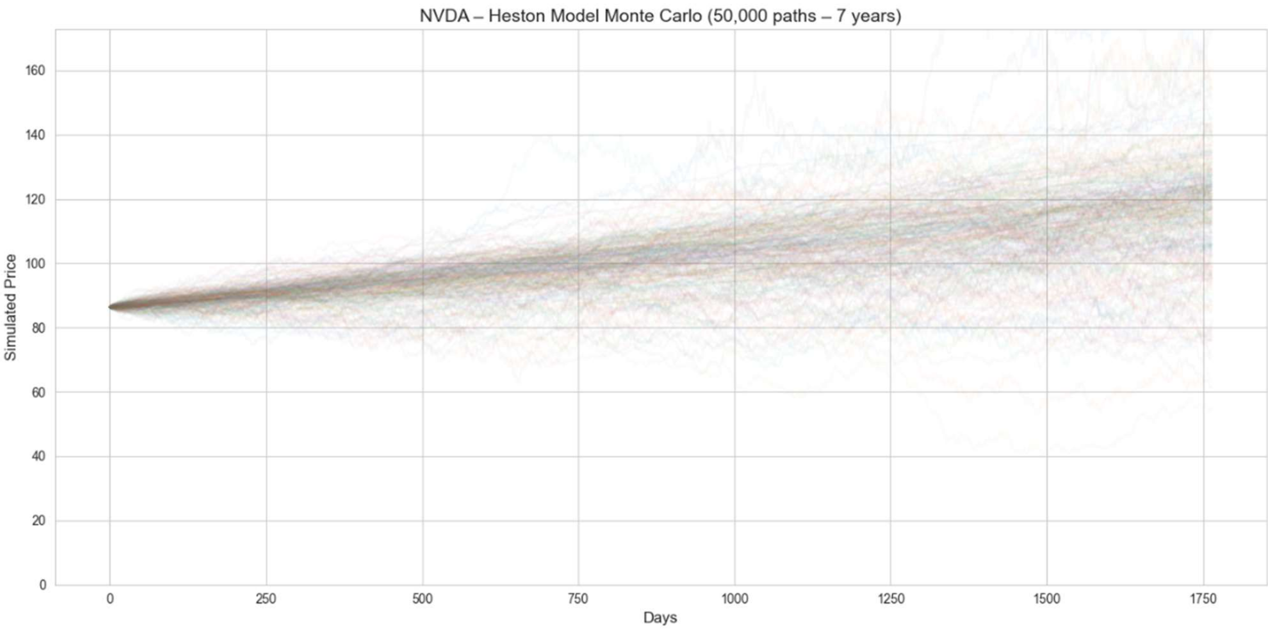

Figure 23: Heston Monte Carlo Simulation (50,000 paths, 7-year horizon)

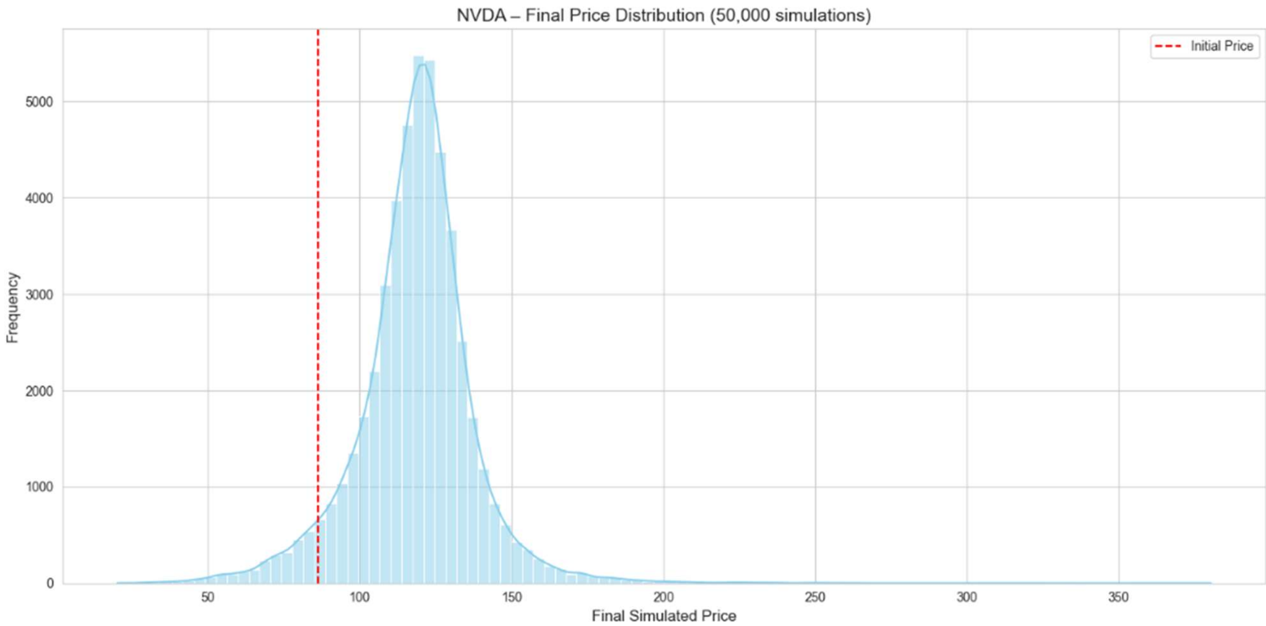

Figure 24: Final Price Distribution under Heston Model (40,000 simulations)

```
=== Heston Model Monte Carlo Simulation Summary (7 anni - 50.000 sim) ===
Initial Price (S₀): $86.37
Expected Price (Mean): $118.31
Median Price: $119.37
5th Percentile: $85.35
25th Percentile: $109.25
75th Percentile: $128.20
95th Percentile: $146.52
```

```
Probability of Loss: 5.37%
Probability of Price ≥ +20%: 82.91%
Probability of Price ≥ +50%: 21.99%
Probability of Price ≥ +100%: 0.90%
```

Figure 25: Heston Simulation Summary Statistics and Probabilities (7-year horizon, 50,000 simulations

11.3 Simulation Results

Parameters were calibrated on historical data using an advanced *loss function* matching the first and second order moments and autocorrelation of variance.

The simulation was executed using **50,000 paths** with high-precision *float64*.

Interpretation of the Results

The simulation shows a stable and controlled evolution of price paths with limited volatility spread. This is consistent with the calibrated parameters, which include:

- Low mean-reversion speed ($\boldsymbol{\kappa}$)
- Moderate long-term variance ($\boldsymbol{\vartheta}$)
- Low volatility of volatility ($\boldsymbol{\xi}$)
- Mild negative correlation ($\boldsymbol{\rho} \approx -0.33$) between price and variance

Output Summary

- Initial Price (S_0): $86.37
- Expected Price (Mean): $118.31
- Median Price: $119.37
- 5th Percentile: $85.35
- 25th Percentile: $109.25
- 75th Percentile: $128.20
- 95th Percentile: $146.52

Probabilistic Outcomes

- Probability of Loss (Price < S_0): *5.37%*
- Probability of Price ≥ + 20%: *82.91%*
- Probability of Price ≥ + 50%: *21.99%*
- Probability of Price ≥ + 100%: *0.90%*

These outcomes are consistent with a conservative volatility environment: the variance process remains well-behaved over time, and the upward drift induced by the risk-free rate (4.5%) dominates.

The final price distribution is slightly right-skewed, as expected in Heston-based dynamics, but tails remain thin.

However, it is important to note that these results are driven by the selected historical window (*2019 - 2024*), which captured a predominantly bullish regime for NVIDIA.
The absence of bear-market cycles or structural downturns limits the model's exposure to extreme adverse scenarios, potentially understating long-term downside risk.

12. Final Considerations and Investment Outlook

This quantitative study of NVIDIA combines multiple simulation frameworks including Geometric Brownian Motion, empirical Monte Carlo model with historical return and volatility sampling, GARCH (1,1), and a calibrated Heston stochastic volatility model to evaluate the stock's forward-looking risk-return profile with increasing levels of realism.

Additionally, a multivariate benchmarking analysis was conducted using NVIDIA's correlations with major market indices and sectoral ETFs, including **QQQ**, **SOXX**, **SPY**, and **XLK** to contextualize its absolute and relative performance over the *2019 - 2024* period.

The *GBM-based simulations* highlighted NVIDIA's exceptional historical growth, with a **5-year CAGR of 80.74%** and a base-case Monte Carlo showing a right-skewed distribution of returns. However, the model's constant volatility assumption significantly underrepresents downside risk and volatility clustering.

The *empirical Monte Carlo model* introduced randomness in both return and volatility by resampling historical values. This allowed the simulation to better reflect real-world variability in daily market conditions, producing more volatile but historically grounded projections.

The **GARCH (1,1)** extension added time-varying volatility and fat-tailed innovations via the Student-t distribution. The result was a more symmetric and risk-adjusted distribution, with a *56.48%* probability of loss over one year and only 1.16% probability of doubling in value, reflecting a more **conservative**, risk-aware outlook.

The most robust simulation came from the *Heston model*, which was carefully calibrated using volatility moment matching and a large simulation horizon (*50,000 paths over 7 years*).

It captured both stochastic volatility and negative correlation between price and variance (leverage effect). The Heston output displayed a realistic balance between upside potential and drawdown risk, with volatility evolving dynamically and probabilistic outcomes stabilizing over time.

Key Takeaways:

- NVIDIA’s long-term trend remains exceptional, but mean-reversion and volatility risk must not be ignored.
- Short-term gains appear less probable in risk-adjusted models than naive GBM suggests.
- Dynamic volatility models suggest a more prudent entry strategy and reinforce the importance of scenario-based risk management.
- For investors, this supports a strategic, *long-term allocation* in NVIDIA, ideally complemented by hedging instruments or risk mitigation overlays during volatile macroeconomic regimes.

12.1 Investment Positioning

Based on the outcomes of this analysis, NVIDIA should be viewed as a **strategic long-term asset**, though not without caveats.
The company benefits from powerful secular growth drivers such as AI, data center acceleration, and GPU leadership.
Its historical performance, both in terms of price appreciation and compound annual growth rate, supports a **strong buy-and-hold thesis** for investors with long-term horizons.

However, our quantitative models, particularly GARCH and the calibrated Heston framework, reveal meaningful short-term downside risk.
These models simulate fat-tailed return distributions, volatility clustering, and leverage effects that emphasize the need for proper risk management and **cautious position sizing**.
Short-term positioning appears less attractive due to potential drawdowns and asymmetric risk, especially under adverse market conditions.

Importantly, while the long-term outlook appears favourable, this analysis does not guarantee success. The simulations are based on historical data from *May 2019 to May 2024*, a period that included **exceptional performance** for NVIDIA and favourable market conditions.

Moreover, the **simulation horizons**, especially in the GARCH and empirical Monte Carlo models, **are relatively short**.
We chose not to extend the time frames aggressively due to practical limitations in computational resources and memory usage, which constrained the number of paths and depth of recursive simulations in RAM-intensive environments.

Therefore, this report supports a data-informed ***long-term BUY stance*** on NVIDIA, as long as investors remain aware of the risks, understand the model assumptions and limitations, and are willing to complement quantitative insights with broader market judgment and proper portfolio construction.

Disclaimer: This analysis is for educational and illustrative purposes only and does not constitute financial advice. Always conduct your own due diligence or consult a licensed financial advisor before making investment decisions.