Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (New) EXAMINATION – WINTER 2018

Subject Code:2130002 Date:17/11/2018

Subject Name:Advanced Engineering Mathematics

Time:10:30 AM TO 01:30 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			MARKS
Q.1	**(a)**	Define: Dirac Delta function, Gamma function, Laplace Transform of a function.	**03**
	(b)	Solve $(D^2-4)y = 1+e^x$; where $D = d/dx$.	**04**
	(c)	Find the Fourier series for $f(x) = \begin{cases} \pi + x \ ; & -\pi < x < 0 \\ \pi - x \ ; & 0 < x < \pi \end{cases}$	**07**
Q.2	**(a)**	Solve $\frac{d^4y}{dx^4} - 2\frac{d^2y}{dx^2} + y = 0$.	**03**
	(b)	Solve $Sinhx\ Cosy\ dx = Coshx\ Siny\ dy$.	**04**
	(c)	Find the Half range Cosine series for $f(x) = (x-1)^2$ in $(0,1)$.	**07**
		OR	
	(c)	Show that current in a circuit containing Resistance R, Inductance L and Constant emf E is given by $i = \frac{E}{R}\left[1 - e^{-\frac{R}{L}t}\right]$.	**07**
Q.3	**(a)**	Solve $x^2y'' + xy' + y = 0$.	**03**
	(b)	Solve by the method of undetermined coefficients. $y'' + 10y' + 25y = e^{-5x}$	**04**
	(c)	Solve using method of variation of parameters. $y'' + 2y' + y = e^{-x}Cosx$	**07**
		OR	
Q.3	**(a)**	State and prove First shifting theorem of Laplace Transform.	**03**
	(b)	Express $f(x) = \begin{cases} Sin\ x\ ; & 0 \ll x \ll \pi \\ 0 \ ; & x > \pi \end{cases}$ as Fourier Sine integral and evaluate $\int_0^\infty \frac{Sin\ \lambda x\ Sin\ \pi\lambda}{1-\lambda^2} d\lambda$	**04**
	(c)	Solve in series the equation $y' = 3x^2y$.	**07**
Q.4	**(a)**	Find $L[t\ Sint]$	**03**
	(b)	Find $L^{-1}\left[\frac{4s+5}{(s-1)^2(s+2)}\right]$	**04**
	(c)	State Convolution theorem and hence find $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$	**07**
		OR	
Q.4	**(a)**	Define unit step function $u(t-a)$. Find $L[t^2u(t-2)]$.	**03**
	(b)	Solve the differential equation. $(D^3 - 2D^2 + 4D - 8)y = 0$; where $D = d/dx$	**04**

(c) Solve differential equation using Laplace transform. **07**

$$y'' + 2y' + y = e^{-t} \quad ; \quad y(0) = -1, y'(0) = 1$$

Q.5 **(a)** Find Radius of convergence of the power series. **03**

$$\sum_{0}^{\infty} \frac{x^n}{n!}$$

(b) Solve the partial differential equation. **04**

$$p + q = z \;\; ; where\ p = \frac{\partial z}{\partial x} \;\; and\ q = \frac{\partial z}{\partial y}$$

(c) Prove that Laplace Equation in polar coordinates is **07**

$$\nabla^2 u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

OR

Q.5 **(a)** Form partial differential equation by eliminating arbitrary functions. **03**

$$f(xy + z^2, x + y + z) = 0$$

(b) Solve partial differential equation. **04**

$\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when $x = 0; z = e^y$ and $\frac{\partial z}{\partial x} = 1$.

(c) Solve the partial differential equation using method of separation of variables. **07**

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y} \quad ; \quad u(0, y) = 8e^{-3y}$$

****