Photon – Theory and Applications

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Preface

The creation of this book has been driven by a deep fascination with light and its fundamental mediator – the photon. In modern physics, the photon plays a central role: as a particle without mass, yet carrying energy and momentum; as the messenger of the electromagnetic interaction; and as a key figure in quantum mechanics. My goal has been to present this multifaceted concept in its historical development, its physical significance, and its technical applications in such a way that both interested laypersons and advanced readers gain a solid understanding – without unnecessary oversimplification, yet always in a clear and accessible manner. In the conception and preparation of this book, modern technology was also employed. The AI system ChatGPT by OpenAI (as of 2025) was used in a supportive role for formulating, structuring, and reflecting on the content. This collaboration made it possible to articulate ideas more quickly, test alternative phrasings, and clarify complex relationships. All content, however, was critically reviewed, revised, and is solely the responsibility of the author. Thus, this book is not only a contribution to the didactics of modern physics – it is also an experiment in how traditional science can gain a new level of clarity and accessibility through modern tools. I hope that the enthusiasm for this subject will resonate with the readers – just as it has accompanied me for many years.

Dipl.-Ing.(FH) Christian Weilharter

[Traunstein, 2025]

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Chapter I

Introduction: Theory and Applications of the Photon

1.1 Motivation and Historical Development

1.1.1 The Central Role of Light in Science and Technology

Light is far more than a mere object of study—it is a universal information carrier, a precise tool, and a fundamental link between theory and measurement. In a sense, light is the "eye of physics": without it, we could neither look into the cosmos nor into the inner structure of matter.

Light as a Means of Knowledge – From Telescope to Particle Accelerator

Astronomy would be unthinkable without light. With Galileo's telescope, the observation of celestial bodies in visible light began. The development of spectroscopy, radio telescopes, and X-ray detectors revealed that every celestial object emits radiation, telling us its temperature, composition, and motion. Light is the only messenger that reliably reaches us across billions of light-years.
At the other extreme, probing the smallest dimensions also requires light. Microscopy—whether with visible light, electrons, or lasers—has opened up an entire hidden world: cells, DNA, atoms. Modern techniques such as scanning tunneling microscopy or optical tweezers rely

on the controlled interaction of light with matter on the smallest scales.

Light in Technology – From Everyday Use to High Precision

Technologically, light has become the central medium. Communication via fiber optics would be impossible without coherent, low-loss light—terabit data streams circle the globe daily as modulated light pulses. In laser technology too, light plays a key role: from CD players to precision material processing and laser surgery, energy and information are controlled by light.
Navigation and timekeeping also rely on light: GPS signals depend on clocks calibrated with lasers, and the most precise clocks in the world—optical lattice clocks—tick with light frequencies.

Light as a Measuring Instrument – Universal and Contact-Free

Light enables non-contact measurement. In spectroscopy, for example, light is used to determine the chemical composition of gases, liquids, and solids—from stellar atmospheres to quality control in industry. Temperature can also be measured via light, using Planck's radiation law. Motion can be determined with the Doppler effect, distances with laser interferometry. Gravitational waves, first detected in 2015, left their trace in the distance between two mirrors—measured with light to a precision of one-thousandth of a proton diameter.

Light as a Theoretical Foundation

In theoretical physics, light is the prime example of fields, quanta, and interactions. Classical electrodynamics describes light as a wave; quantum electrodynamics describes it as an exchange of light quanta. The relativistic structure of spacetime is tightly connected to the constancy of the speed of light—light is not just a phenomenon but a structural element of the laws of nature.

1.1.2 Conclusion

Light is simultaneously a tool, an information carrier, a natural law, and an object of research. In hardly any other area of physics do theory and practice intertwine as deeply as in the study of light. It opens our view into the universe—and at the same time into the heart of matter. In technology, it brings precision, speed, and new possibilities. It rightly stands at the center of modern science.

1.2 Wave-Particle Dualism

A central result of modern physics is the realization that light (and in general, quantum objects) cannot be described solely as a wave or as a particle. Instead, it exhibits a *wave-particle dualism*: depending on the experiment, light appears either as an electromagnetic wave with interference and diffraction patterns (e.g., in the double-slit experiment), or as a stream of light quanta, called photons (e.g., in the photoelectric effect).

This behavior contradicts classical intuition, where waves and particles are strictly separate concepts. In quantum physics, however, they are two complementary aspects of the same physical phenomenon. The mathematical description of this dualism requires a fundamental reformulation of physics: the classical trajectory of a particle is replaced by the wave function, whose squared modulus gives the probability distribution. This marks the beginning of quantum mechanics.

Wave-particle dualism is not a lack of knowledge but a deeper property of nature, confirmed experimentally and generalized mathematically by quantum field theory.

Voices of Great Physicists on Wave-Particle Dualism

Albert Einstein (1909)[2]

"It seems as though we are forced to attribute to the electromagnetic field certain quantum-like properties in order to explain the observed phenomena."

Niels Bohr (1933)[1]

"The opposite of a correct statement is a false statement. But the opposite of a profound truth may well be another profound truth."

Richard P. Feynman (1965)[5]

"I think I can safely say that nobody understands quantum mechanics."

1.3 The Photon Concept in the New Worldview of Quantum Mechanics

1.3.1 From the Classical Concept of Light to the Quantum Revolution

In classical physics, light was understood either as particles (Newton) or as waves (Huygens, later Maxwell). With Maxwell's equations, one had an elegant theory of electromagnetic waves that fully explained light as a wave phenomenon. A particle character seemed unnecessary.

But at the end of the 19th century, this worldview began to crumble. The attempt to explain the radiation spectrum of black bodies with classical physics led to the so-called *ultraviolet catastrophe*. The Rayleigh–Jeans law predicted an unphysical divergence of energy at high frequencies.

1.3.2 Planck's Quantization Idea

Max Planck (1900) found a way out by postulating that energy is not emitted continuously but only in discrete units:

$$E = nhf, \quad n \in \mathbb{N} \tag{I.1}$$

Here h is Planck's constant and f the frequency of the radiation.

1.3.3 Einstein's Light Quantum Hypothesis

In 1905, Albert Einstein interpreted Planck's assumption more radically: light consists of *discrete energy packets*, called **light quanta** or **photons**, each carrying an energy of

$$E_\gamma = hf \tag{I.2}$$

He thereby explained the *photoelectric effect*, in which electrons are emitted from a metal only if the frequency of the light exceeds a certain threshold—independent of light intensity.
This insight challenged the classical wave picture and laid the foundation for a new understanding of light.

1.3.4 Wave-Particle Dualism of the Photon

Modern experiments, such as the double-slit experiment with single photons, clearly show: photons behave both like particles and like waves. They produce localized momentum transfer in individual detections, but interference patterns collectively.
A photon has:

- energy $E = hf$
- momentum $p = \dfrac{h}{\lambda}$
- no rest mass
- constant speed c in vacuum

(A detailed derivation of the photon's energy and momentum relations can be found in Appendix A, Section A.1.)

1.3.5 Photons in Quantum Electrodynamics (QED)

In quantized field theory, the photon is understood as an *excitation of the electromagnetic field*. It is the interaction particle of the electromagnetic force, a so-called *exchange particle* in the framework of quantum field theory.
The photon:

- is massless but has spin 1
- has no well-defined position in the classical sense
- exists only as a *detection event* in the measurement process

(The mathematical description of the electromagnetic field, including the field strength tensor and Lagrangian density, is given in Appendix A; see Section A.2.)

1.3.6 Worldview Shift Through the Photon

The photon concept transcends the limits of classical ideas and exemplifies the central paradigms of quantum mechanics:

1. Physical quantities are often not continuous but quantized.
2. Measurement alters the system and brings out certain properties.
3. Wave and particle are not opposites but complementary descriptions.

Quantum Object Instead of Light Ball

"The photon is not a light ball but a quantum object—defined not by being, but by happening."[a]
This quote captures the change in worldview: the photon is not a classical object but an event that becomes concrete only through measurement. It embodies interaction rather than substance—one of the deepest insights of quantum mechanics.

[a] Paraphrased, inspired by modern interpretations such as Zeilinger [15].

1.3.7 Outlook and Applications

Photons play a central role today in numerous fields:

- *Photonics:* light as information and energy carrier in technology
- *Laser physics:* stimulated emission of coherent photons
- *Quantum optics:* single-photon sources, entanglement, teleportation
- *Quantum cryptography:* secure communication through photon states

1.3.8 Philosophical Reflections on the Photon Concept

The photon concept has changed not only our physical but also our philosophical worldview. Several great thinkers of quantum physics have expressed this shift concisely:

What Is Reality in Quantum Physics?

"There is no quantum world. There is only an abstract quantum theory." – **Niels Bohr** [**bohr1934**]
"Quantum theory has taught us that we cannot ascribe properties to nature without considering the act of observation." – **Werner Heisenberg** [6]
"The photon is pure information—it exists only when it interacts with the world." – **Anton Zeilinger** [14]

These statements underline: the photon is not a material particle in the classical sense, but a quantum event—something that becomes concrete only through measurement. The classical notion of a well-defined object is replaced by a probabilistic description in space, time, and interaction.

1.4 Methodology and Experiments: Making the Invisible Visible

The idea that light consists of tiny portions of energy—so-called photons—may seem obvious today. But how can such a thing be measured? How can we prove something that is so small and fleeting that it has no fixed form?
Indeed, the path from theory to measurement in the case of the photon is particularly fascinating. Quantum physics shows us that light is not simply “seen”—it only becomes evident in its interaction with matter.

1.4.1 The Single Photon: When Light Clicks

Imagine darkening a room completely and firing just one single light particle onto a highly sensitive surface. There—if everything works—**it clicks**: an electrical pulse occurs, triggered by exactly this one photon.
Such detectors really exist. They are called *single-photon detectors* and can count individual photons. Particularly sensitive devices—so-called *avalanche detectors*—trigger a small electron avalanche when hit by a photon, turning a tiny event into a measurable signal.

The Single Photon – When It Clicks

A photon can be detected individually—by detectors so sensitive that they react to a single quantum of light.
When a photon strikes the active surface, it produces a measurable electrical signal. This “click” is the direct evidence of a single photon—and thus proof of its particle character.
Particularly sensitive devices, such as avalanche photodiodes or superconducting detectors, count photons individually. This technology underpins modern quantum optics.

1.4.2 Double-Slit: The Magic of Many Singles

A famous experiment demonstrates the dual nature of the photon most impressively: the double-slit experiment. Photons are sent one by one through a wall with two slits. Each individual photon strikes the screen seemingly at random. But after many hits, an interference pattern emerges—as if all photons had gone through both slits simultaneously and interfered with each other.

How is this possible? Quantum physics says: each photon *behaves like a wave* as long as it is not observed—and like a particle when detected. It is both—or, more accurately, something entirely new beyond classical categories.

1.4.3 When Two Photons Know More Than One

Things become even stranger when two photons are produced at the same time—so-called *entangled photon pairs*. Measuring one immediately fixes the state of the other—even if it is far away.

Such experiments show that the microscopic world does not work as our everyday intuition suggests. Cause and effect, space and time—all acquire new meaning. Physics speaks here of *entanglement* and *non-classicality*. For us, it means: nature is deeper, more interconnected, and more surprising than we ever imagined.

What Does Entanglement Mean?

Two entangled photons form a joint quantum system—their properties cannot be defined independently.

- Measuring one photon immediately determines the state of the other—regardless of distance.
- There is no hidden classical information—the correlation arises only upon measurement.
- The “knowledge” is not in a single photon but in the entirety of the entangled system.

This quantum connection contradicts the classical notion of local forces—and has been confirmed in many experiments.

How Are Entangled Photons Produced?

In quantum optics, entangled photons are usually produced using nonlinear crystals—for example, by *spontaneous parametric down-conversion* (SPDC).
A single high-energy photon (pump laser) strikes a special crystal. With small probability, a pair of lower-energy photons is created, conserving momentum and energy.

- These two photons are entangled—for example, in polarization state.
- The result is not a "decay" in the classical sense, but a quantum coupling of two states.
- The photons move in opposite directions, allowing them to be analyzed separately.

This technique underpins modern experiments on quantum entanglement, Bell tests, and quantum communication.

(A formal description of entangled states using Dirac notation can be found in Appendix A, Section A.3.)

1.4.4 The Meaning of the Experiments

What the Experiments Teach Us

These experiments did more than confirm the photon concept—they revolutionized our worldview.
Photons show that nature cannot always be forced into clear categories like "wave" or "particle." They show that measurement and observation play a deeper role than previously thought. And they open the door to technologies such as quantum computers, quantum cryptography, and ultrafast light switches.

1.4.5 Conclusion

What we know about light today is owed to a combination of theory and experimental skill. Without modern detectors, precise timekeeping, and clever setups, we could only talk about photons—but not know that they truly exist. Experiments are the proof: the photon is no fantasy, but a building block of reality—and at the same time a window into a world full of mysteries yet to be uncovered.

What We Take Away from Chapter I

The photon is not a theoretical construct—it is real, measurable, and confirmed by experiments.

- It has no mass but does carry energy and momentum.
- It shows wave and particle properties—depending on the experiment.
- It is indivisible, but not a classical "little ball."
- It stands at the beginning of many modern technologies (lasers, quantum cryptography, light detection).

The photon is the prototype of a quantum object—it forces us to rethink reality, information, and measurement.

1.5 Structure of This Book

This book is divided into eight thematically coordinated chapters. It takes the reader from the physical foundations of light, through the quantization of the electromagnetic field, to modern applications and open questions in research. Each chapter highlights the role of the photon from a particular perspective—historical, theoretical, experimental, or technological.

- **Chapter II** describes the path from classical light theory to the introduction of the light quantum, covering central experiments such as the photoelectric effect and blackbody radiation that led to quantum theory.
- **Chapter III** addresses the physical properties of the photon, including energy, momentum, spin, polarization, and masslessness.
- **Chapter IV** explores the experimental confirmation of the photon—from Millikan's measurements of the photoelectric effect to modern single-photon experiments and quantum erasers.

- **Chapter V** introduces quantum electrodynamics (QED) and shows how the photon is understood as the gauge boson of the electromagnetic interaction.
- **Chapter VI** describes practical applications of the photon, such as in laser technology, medical imaging, and communication technology.
- **Chapter VII** highlights current developments and future potential, particularly in quantum information, photonics, and fundamental research.
- **Chapter VIII** places the photon in the Standard Model of particle physics, showing how its masslessness arises from the symmetries of the theory and what open questions remain.

Numerous illustrations, quotations, and reflections accompany the scientific content, also shedding light on philosophical, historical, and epistemological dimensions. Mathematical tools are introduced carefully, so that the work remains accessible to interested readers with a general scientific background.

> *With an understanding of the historical, conceptual, and experimental foundations of the photon, we now step into the terrain of quantization—the transition that carries light from the classical world into quantum physics.*

How to Read This Book

To clearly distinguish different aspects of the presentation, colored boxes are used throughout the book. They provide orientation and highlight central content:

- **Physics boxes** (blue) provide physical explanations and background information.
- **Math boxes** (green) contain derivations, formulas, and mathematical details.
- **Didactics boxes** (yellow) point out conceptual pitfalls or offer alternative explanations.
- **Note boxes** (gray) provide structural hints or references to other chapters and appendices.

- **Warning boxes** (red) highlight particularly critical aspects or misunderstandings.
- **Hypothesis boxes** (orange) discuss hypothetical scenarios or "what-if" questions.

This way, readers can decide whether to focus on the main text or to explore the additional perspectives in the boxes.

Chapter II

The Path to the Light Quantum

2.1 Classical Theories of Light and Their Limitations

Over the centuries, classical physics developed two fundamental models to describe light: the particle model and the wave model. Both models offered convincing explanations for various phenomena, yet they faced fundamental limitations that ultimately led to the development of quantum theory.

2.1.1 Newton's Corpuscular Theory of Light

Isaac Newton assumed that light consisted of tiny particles – so-called *corpuscles* – propagating in straight lines. This model could explain reflection and rectilinear propagation quite well, but it failed when it came to phenomena such as diffraction or interference.

Isaac Newton (1704) – Corpuscular Theory *[12]*

"Are not the rays of light very small bodies emitted from shining substances?"

Comment: Newton formulated the first systematic corpuscular theory of light. Although his model could explain many phenomena, it failed with diffraction and interference – which later paved the way for the development of the wave theory.

2.1.2 Huygens' Wave Model

Christiaan Huygens opposed Newton's idea with a wave model, in which light propagates as advancing wavefronts in a hypothetical medium called the *ether*. This model successfully described phenomena such as interference and diffraction, especially after Young's double-slit experiment (1801).

Huygens (1960) on the Propagation of Light [7]

"Light is produced by a very small agitation of the ether and spreads successively."

Comment: Huygens developed a consistent wave model of light, in which light propagates as a mechanical oscillation in the ether – much like sound in air. This theory was groundbreaking for explaining refraction and interference, even though the existence of the ether was later disproved.

2.1.3 Maxwell's Electrodynamics

A milestone was James Clerk Maxwell's theory of electrodynamics (1865). It unified electricity and magnetism into a single theory and showed that electromagnetic waves propagate at the speed of light – a breakthrough that identified light as an electromagnetic wave. With this, the wave nature of light was firmly established in classical physics.

Maxwell (1873) on Light and Electromagnetic Waves [9]

"The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws."

Comment: Maxwell was the first to connect light with the electromagnetic field. In his equations, light no longer appeared as a mechanical motion of an ether but as an independent electromagnetic wave propagating through space with a finite speed. This laid the foundation for a new understanding of light propagation without a material medium.

2.1.4 Limits of the Classical Theory

Despite its successes, classical light theory encountered serious limitations by the end of the 19th century:

- **Blackbody Radiation:** Classical theory predicted an infinite energy emission at high frequencies (the *ultraviolet catastrophe*).
- **Photoelectric Effect:** Observations (e.g., the frequency-dependent emission of electrons) contradicted classical predictions.
- **Compton Effect:** The scattering of X-rays by electrons could only be explained using particle momentum.

These contradictions marked a fundamental crisis of classical physics and ushered in the paradigm shift toward quantum theory, in which light exhibits both particle and wave properties.

2.2 Blackbody Radiation

2.2.1 Introduction

Blackbody radiation was one of the phenomena that shook the classical worldview of physics. Although the radiation spectrum emitted by heated bodies could be measured with great precision, all classical theories failed to explain it. Especially in the short-wavelength range, the equations led to unphysical predictions: an infinite energy density – known as the *ultraviolet catastrophe*.

This fundamental contradiction between theory and experiment forced physics to a radical rethinking. Only Max Planck's assumption that energy is emitted in discrete portions – so-called quanta – led to a formula that could correctly describe the observed spectrum. This laid the foundation for quantum theory.

2.2.2 What Is a Blackbody?

A *blackbody* is an idealized physical object that completely absorbs all incident electromagnetic radiation – regardless of wavelength or direction. In thermal equilibrium, such a body emits radiation that depends only on its temperature and has a characteristic spectrum: the so-called *blackbody radiation*.

A perfect blackbody reflects no light at all – which is why at room temperature it appears perfectly black. In practice, such bodies can only be approximated, for example by a hollow sphere with a small opening: radiation entering the cavity is reflected many times and almost completely absorbed.

The thermal radiation of a blackbody is universal: it depends neither on the material nor on the shape, but only on temperature. It therefore serves as a central reference model in thermodynamics and radiation physics.

What Is a Blackbody?

A blackbody is an ideal physical model:

- It absorbs all incident electromagnetic radiation – independent of frequency or angle of incidence.
- The radiation it emits depends solely on its temperature – not on its composition.
- The spectral distribution follows Planck's radiation law and shows a maximum that shifts with temperature (Wien's displacement law).

Such models are used to describe the thermal radiation of stars, filament lamps, or cavity radiators.

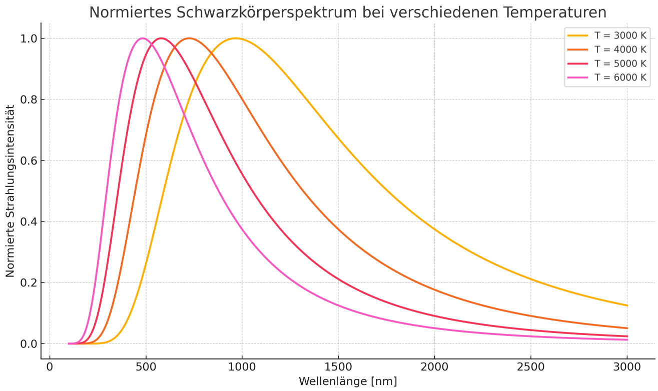


Figure II.1: Normalized blackbody spectrum for different temperatures. The maximum shifts to shorter wavelengths as the temperature increases (Wien's displacement law).

2.2.3 Procedure of the Cavity Experiment

To experimentally investigate thermal radiation, so-called *cavity radiators* are used. They serve as nearly ideal realizations of a blackbody. The typical setup consists of a solid metal block with an inner cavity and a small opening to the outside world.

Radiation entering through this opening is reflected many times at the walls and is almost completely absorbed. Conversely, a tiny portion of the internal equilibrium radiation exits through the opening – this corresponds almost exactly to the theoretical blackbody radiation at the block's temperature T.
The radiation emerging from the opening is studied with a spectrometer. The resulting spectrum shows the typical blackbody curve: an intensity profile with a maximum that shifts with temperature (Wien's displacement law), and a characteristic drop in the ultraviolet region. These results fundamentally contradicted classical physics and ultimately led to the development of quantum theory.

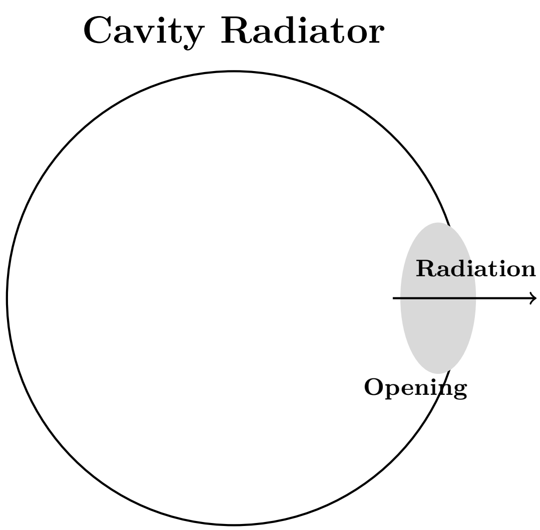


Figure II.2: Model of a cavity radiator producing nearly ideal blackbody radiation.

2.3 Failure of the Classical Theory – the Rayleigh–Jeans Law

In the 19th century, Lord Rayleigh and Sir James Jeans attempted to describe blackbody radiation using classical electrodynamics and statistical mechanics. They treated standing electromagnetic waves in a cavity as harmonic oscillators and calculated the energy distribution by applying the equipartition principle of classical thermodynamics.

The result was the so-called *Rayleigh–Jeans Law*, which gives the spectral energy density as a function of wavelength λ and temperature T:

$$u(\lambda, T) = \frac{8\pi kT}{\lambda^4}$$

Here:

- $u(\lambda, T)$: radiation energy per wavelength interval and volume,
- k: Boltzmann constant,
- T: absolute temperature.

For long wavelengths the formula yields reasonable results and agrees well with experimental data. But at short wavelengths the expression diverges to infinity – leading to an unphysical, infinite energy density:

$$\lim_{\lambda \to 0} u(\lambda, T) \to \infty$$

This problem became known as the *ultraviolet catastrophe.* It stood in fundamental contradiction to experimental observations (see Fig. II.1), where radiation in the UV region does not increase but instead falls off sharply.
This catastrophe revealed the limitations of classical physics. A new idea was required to explain the observations – and it came from Max Planck. (The derivation via mode counting in a cavity and the classical equipartition principle is presented in Appendix A, Section A.4.)

2.4 Failure of the Classical Theory – Wien's Radiation Law

Even before Planck, Wilhelm Wien in 1896 provided an approximation for the radiation spectrum of blackbodies in the short-wavelength range. He derived the so-called *Wien's Radiation Law* from thermodynamic arguments and dimensional analysis – but without the quantum concept we know today.

The spectral energy density in wavelength form is given by:

$$u(\lambda, T) = \frac{c_1}{\lambda^5} \exp\left(-\frac{c_2}{\lambda T}\right)$$

with:

- $u(\lambda, T)$: energy per wavelength interval and volume,
- λ: wavelength,
- T: absolute temperature,
- $c_1 = 2\pi hc^2$, $c_2 = \frac{hc}{k}$: constants from Planck units (fully understood only later).

Wien's law describes the radiation distribution in the high-frequency range ($\lambda \to 0$) very well and predicts an exponential decrease in intensity. In the long-wavelength range ($\lambda \gg 1\ \mu$m), however, it deviates significantly from the measurements.
Wilhelm Wien received the Nobel Prize in 1911 for his contributions to radiation theory. His law was an important step toward Planck's complete theory. (The mathematical derivation of Wien's radiation law is given in Appendix A; see Section A.5.)

Why Does the Classical Theory Fail?

The Rayleigh–Jeans law correctly describes thermal radiation at low frequencies – but at high frequencies it gives absurd results:

- Intensity rises without bound – the higher the frequency, the greater the radiation.
- This "ultraviolet catastrophe" contradicts all observations.
- The cause: classical physics assumes that every mode in the cavity can absorb unlimited energy.

This assumption fails in reality. Only the quantization of energy – as introduced by Planck – explains why high frequencies are suppressed.

2.5 Emergence of Planck's Radiation Law

At the end of the 19th century, describing the radiation of a blackbody posed a fundamental problem for theoretical physics. Empirically, the spectral energy density $I(\lambda, T)$ was well known, but no theory could correctly explain the entire spectrum.

2.5.1 The Crisis of Classical Physics

Classical physics could only explain limiting cases:

- For large wavelengths ($\lambda \to \infty$), the **Rayleigh–Jeans law** was valid:

  $$I(\lambda, T) = \frac{8\pi c}{\lambda^4} \cdot kT$$

  This law, however, led to an unphysical divergence at short wavelengths – the so-called **ultraviolet catastrophe**.

- For short wavelengths ($\lambda \to 0$), the **Wien's law** was known:

  $$I(\lambda, T) = c_1 \lambda^{-5} \cdot e^{-c_2/(\lambda T)}$$

  Yet it failed in the long-wavelength region.

2.5.2 Planck's Interpolation Approach

Max Planck was familiar with both limiting laws and searched for a mathematical function that could interpolate across the entire observed curve. In October 1900, this led him to his famous formula:

$$I(\lambda, T) = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{e^{hc/(\lambda kT)} - 1} \tag{II.1}$$

with:

- h: Planck's constant
- c: speed of light
- k: Boltzmann constant
- T: absolute temperature

(The detailed derivation of Planck's radiation law using energy quantization is presented in Appendix A, Section A.6.)

2.5.3 Remark on the Meaning of h

Planck initially regarded his formula as a **mathematical interpolation**, not as the expression of a fundamental natural constant. He introduced h to fit the curve – without realizing the revolutionary consequences. Only Albert Einstein recognized its deeper meaning in 1905 within the framework of his light quantum hypothesis.

Max Planck – Scientific Autobiography [Planck1948]

“I had the feeling that I had introduced something monstrous against my will.”

Planck's Radiation Law: A Mathematical Interpolation [Hoffmann2008]

At the end of 1900, Max Planck was searching for a function to correctly describe the observed blackbody radiation curves. He combined known limiting laws (Wien for high, Rayleigh for low frequencies) and developed an interpolation formula that later became known as Planck's radiation law:

$$I(\nu, T) = \frac{8\pi\nu^2}{c^3} \cdot \frac{h\nu}{e^{h\nu/kT} - 1} \tag{II.2}$$

At the time, he himself was unaware of the deeper meaning of the constant h – for him it was a mathematical tool. Only later did Einstein, with his light quantum hypothesis of 1905, interpret this formula as evidence of a fundamental quantization of nature.

2.5.4 Important Historical Insight

Important Historical Insight [Hoffmann2008]

On December 14, 1900, Max Planck presented his famous radiation formula to the Physical Society. Today this day is regarded as the *"birthday of quantum physics"*. Yet neither Planck nor his audience recognized the significance of the discovery at the time.

Quote:

> "Although none of the scientists present – including Planck himself – were aware of the meaning and scope of the formula or the constant. Planck's result was initially seen merely as a formula that correctly represented the radiation data."

As Hoffmann points out, the revolution only became clear through Einstein's light quantum hypothesis (1905) and the critical analysis of Planck's law by Einstein and Ehrenfest. Planck himself spoke of a fundamental upheaval only years later.

2.5.5 Graphical Comparison of the Three Radiation Laws

The following figure shows the spectral energy density of blackbodies as a function of wavelength. It compares the classical Rayleigh–Jeans law, Wien's radiation law, and the full Planck law. The diagram clearly illustrates:

- The Rayleigh–Jeans law diverges in the UV region.
- Wien's law is valid only in the short-wavelength region.
- Only Planck's law correctly describes the entire spectrum.

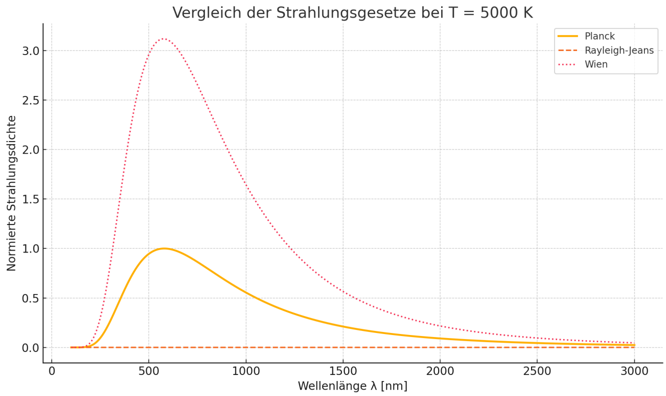


Figure II.3: Comparison of the three radiation laws. Only Planck's law agrees with experimental data across the entire spectrum.

2.5.6 Conclusion

- The error of the classical theory is not only large but catastrophic.
- The deviation at short wavelengths amounts to several orders of magnitude.
- Only Planck's law explains the entire spectrum correctly – by quantizing energy.

2.6 The Photoelectric Effect and Einstein's Light Quantum

In 1905, Einstein went a step further: not only the emission of energy, but light itself was quantized. He postulated that a light quantum (photon) carries the energy $E = h\nu$. Only if this energy exceeds the work function A can an electron be emitted:

$$E_{\text{kin}} = h\nu - A$$

This explained experimental observations that were incompatible with the wave theory – in particular the frequency dependence of the electron energies.

Einstein (1905)[3]

"The generation of light does not occur everywhere along the wavefront uniformly, but only at certain places, at individual points."
Comment: Here Einstein already describes the quantization of light – the starting point of photon theory.

(A detailed derivation of the photoelectric equation can be found in Appendix A, Section A.12.)

2.7 First Reactions and Significance

The idea that light possesses not only wave properties but also particle properties was difficult for many physicists at the beginning of the 20th century to accept. The wave theory of light had seemed fully confirmed by interference and diffraction experiments as well as by Maxwell's electrodynamics. Albert Einstein's light quantum theory of 1905 – the radical thesis that light consists of discrete energy packets (later called *photons*) – contradicted this established view.
Einstein himself was aware of the explosive nature of his hypothesis and described it as a "heuristic point of view." Yet the concept was more than a mere computational trick – it fully explained the photoelectric effect and led to precise, experimentally testable predictions.
A striking example of the initial skepticism was Robert A. Millikan. Although in 1916 he himself confirmed the energy formula $E = h\nu$ with high precision in a series of experiments, he long remained critical of the light quantum concept:

Only decades later did the light quantum hypothesis become accepted as a fundamental part of quantum physics – especially after phenomena such as *Compton scattering* (1923) confirmed the momentum nature of the photon.

The introduction of the photon revolutionized not only optics but was also crucial for:

- the development of *quantum mechanics*,
- the understanding of *atoms and molecules*,
- the theory of *quantum electrodynamics (QED)*,
- as well as numerous *technological applications* (lasers, photodetectors, solar cells, quantum computers).

Today the photon is not merely a theoretical construct, but a real object, demonstrated in countless experiments and used in technology – a *quantum of the electromagnetic interaction*.

Robert A. Millikan on Einstein (1916) [10]

"Einstein's photoelectric equation ... cannot in my judgment be looked upon at present as resting upon a satisfactory theoretical foundation."

Comment: Despite his experimental confirmation of Einstein's formula, Millikan initially rejected the light quantum concept – a striking example of how deeply rooted theoretical convictions cannot be immediately overturned, even by clear experimental evidence.

2.8 Conclusion

What If There Were No Quantization?

Without the assumption that energy can only be absorbed or emitted in discrete portions ($E = nhf$):

- Fundamental experiments such as blackbody radiation and the photoelectric effect would have remained unexplained.
- Classical physics would have collapsed under the contradictions with reality.
- No modern understanding of the interaction between light and matter would have developed.

Quantization was the first step toward a new physical worldview. It is not optional – it is essential for understanding the photon.

Chapter III

Properties of the Photon

3.1 Photons as Quanta of Energy

The idea that energy does not exist continuously but in discrete portions – so-called quanta – was revolutionary at the beginning of the 20th century. The starting point, however, was not a deliberate physical revolution but a mathematical device introduced by Max Planck in 1900.

3.1.1 Planck's Formula as a Makeshift Solution

Planck attempted to correctly describe the radiation spectrum of a black body. To achieve this, he formally introduced a quantization of energy. He assumed that the energy elements absorbed or emitted by hypothetical oscillators must be integer multiples of a smallest unit of energy of the form

$$\varepsilon = h\nu$$

with:

- ε: energy of an oscillator,
- ν: its oscillation frequency,
- h: the Planck constant, $h \approx 6.626 \cdot 10^{-34}$ Js.

Planck himself did not regard this assumption as a statement about reality, but as a mathematical–statistical device for deriving the radiation law [13]. In his *Scientific Autobiography* he wrote in retrospect:

Max Planck (1905)[13]

"I had resolved to derive the radiation formula at any cost, and I accomplished this, no matter what it took. [...] The introduction of an elementary quantum of action was a purely formal assumption – I did not at all intend to introduce a physical quantum theory."

3.1.2 Einstein Turns Them into Light Quanta

It was **Albert Einstein** who realized in 1905, in his work on the photoelectric effect, that quantization might not merely be a calculational trick, but a real property of light. He postulated that light consists of discrete packets of energy – later called *photons*. The energy of such a photon is given by:

$$E = h\nu$$

This laid the foundation of quantum theory [4].

3.1.3 Examples from Different Frequency Ranges

The energy of a photon depends only on its frequency ν (or equivalently its wavelength λ) – independent of light intensity. This is impressively demonstrated by typical examples from nature and technology:

Green Light

Given the frequency of typical green light:

$$\nu = 6.0 \cdot 10^{14} \text{ Hz}$$

The wavelength follows from:

$$\lambda = \frac{c}{\nu}$$

with the speed of light $c = 3.0 \cdot 10^8$ m/s. Thus:

$$\lambda = \frac{3.0 \cdot 10^8 \text{ m/s}}{6.0 \cdot 10^{14} \text{ Hz}} = 5.0 \cdot 10^{-7} \text{ m} = 500 \text{ nm}$$

Now we compute the energy of a single photon:

$$E = h \cdot \nu = 6.626 \cdot 10^{-34} \text{ Js} \cdot 6.0 \cdot 10^{14} \text{ Hz} = 3.976 \cdot 10^{-19} \text{ J}$$

This tiny amount of energy is sufficient to trigger a chemical reaction in the retina of the eye – the beginning of vision.

Green Light

A photon of green light with $\nu = 6 \cdot 10^{14}$ Hz has a wavelength of $\lambda = 500$ nm and an energy of about $4 \cdot 10^{-19}$ J.

X-Rays

X-radiation has an extremely high frequency and correspondingly short wavelength (in the range of tenths of a nanometer). A single photon carries much more energy than visible light – enough to eject electrons from inner atomic shells. This is the physical basis of medical X-ray imaging – but also the reason why such radiation is biologically effective and potentially harmful.

X-Rays

An X-ray photon with $\nu = 3 \cdot 10^{18}$ Hz has a wavelength of about $\lambda = 0.1$ nm and an energy of about $2 \cdot 10^{-15}$ J $= 12.4$ keV.

Microwaves

Microwaves have very low frequencies and correspondingly long wavelengths – typically several centimeters. A single photon carries very little energy. It is insufficient to remove electrons or directly trigger chemical reactions, but it is just right to excite water molecules in food into motion. This generates heat – the physical principle behind microwave ovens.

Microwaves

A photon of typical microwave radiation with $\nu = 2.45 \cdot 10^{9}$ Hz has a wavelength of about $\lambda = 1.22 \cdot 10^{8}$ nm and an energy of about $1.62 \cdot 10^{-24}$ J.

3.1.4 Summary

Photons as Quanta of Energy

The equation $E = h\nu$ was originally a mathematical device for describing blackbody radiation [13]. Only in 1905 did Einstein realize that it describes the real nature of light: light consists of discrete packets of energy – photons [4]. This marked the beginning of quantum theory.

3.2 Momentum of the Photon

An essential feature of the photon is its momentum – even though it has no rest mass. In classical mechanics, the momentum of a particle is defined as the product of mass and velocity:

$$p = m \cdot v$$

Because photons are massless ($m_0 = 0$), this relation seems inapplicable at first sight. Nevertheless, photons carry momentum – a fact spectacularly confirmed by experiments such as Compton scattering. The explanation is provided by quantum physics in conjunction with special relativity.

3.2.1 Momentum from the Quantization of Light

The energy of a photon is, according to Planck and Einstein,

$$E = h \cdot \nu$$

At the same time, special relativity gives for massless particles like the photon:

$$E = p \cdot c$$

Equating both expressions yields the photon momentum:

$$p = \frac{E}{c} = \frac{h \cdot \nu}{c}$$

Since frequency and wavelength are related by $\nu = \frac{c}{\lambda}$, we obtain

$$p = \frac{h}{\lambda} \quad \text{(III.3)}$$

(A formal derivation via the relativistic energy–momentum relation is given in Appendix A; see Sec. A.8.)

This expression shows: The momentum of a photon is inversely proportional to its wavelength – the shorter the wavelength, the larger the momentum.

Photon Momentum

A photon has momentum

$$p = \frac{h}{\lambda}$$

despite having no rest mass. The momentum increases with increasing frequency or decreasing wavelength.

3.2.2 Momentum Transfer in Practice

That light carries momentum is evident, for example, from the radiation pressure exerted on matter – an effect already suspected by Kepler and later confirmed experimentally.
In modern technology, photon momentum plays a role in, for example:

- **Solar sails:** using radiation pressure to propel spacecraft.

  Real-World Image Material

  A real photo of the deployed solar sail of the **IKAROS** spacecraft was taken in 2010 by the mini camera **DCAM2**. You can find it online at The Planetary Society: https://www.planetary.org/space-images/ikaros-spacecraft-from-dcam2

- **Optical tweezers:** manipulation of small particles with focused light beams.

  Image Material for Optical Tweezers

  A real image and schematic of optical tweezers (laser trap) is publicly available on Wikipedia: https://en.wikipedia.org/wiki/Optical_tweezers

3.2.3 Connection to the de Broglie Wavelength

The expression $p = \frac{h}{\lambda}$ is not limited to photons; it also holds for matter waves (the de Broglie relation). For massive particles:

$$\lambda = \frac{h}{p}$$

This links wave character and momentum – another confirmation of wave–particle duality also for light.

3.3 Health Aspects of Electromagnetic Radiation

3.3.1 Ionizing Radiation: UV Light and Beyond

Ultraviolet radiation, especially below 280 nm, has photons with energies of several eV. These are sufficient to remove electrons from atoms or to break chemical bonds – this is **ionizing radiation**.

$$E = h\nu = \frac{hc}{\lambda}$$

For UV light with $\lambda = 200\,\mathrm{nm}$,

$$E = \frac{6.626 \cdot 10^{-34}\,\mathrm{J\,s} \cdot 3.0 \cdot 10^{8}\,\mathrm{m\,s^{-1}}}{200 \cdot 10^{-9}\,\mathrm{m}} = 9.939 \times 10^{-19}\,\mathrm{J} \approx 6.2\,\mathrm{eV}$$

This energy suffices to damage DNA molecules – a physical mechanism known to cause **skin cancer**.

3.3.2 Non-Ionizing Radiation: Mobile Networks, WLAN, Microwaves

Radiation in the radio, WLAN, or microwave range has much lower photon energies:

- Mobile networks: $\nu \approx 900\,\mathrm{MHz} \Rightarrow E \approx 3.7 \times 10^{-6}\,\mathrm{eV}$
- WLAN: $\nu \approx 2.4\,\mathrm{GHz} \Rightarrow E \approx 1.0 \times 10^{-5}\,\mathrm{eV}$
- Microwave: $\nu \approx 2.45\,\mathrm{GHz} \Rightarrow E \approx 1.01 \times 10^{-5}\,\mathrm{eV}$

These energies are **not sufficient** to cause ionization. Nevertheless, there is discussion about thermal or biological effects through **cumulative energy absorption**.

3.3.3 Summary – Ionizing vs. Non-Ionizing Radiation

Ionizing and Non-Ionizing Radiation

- **Non-ionizing:** photons have too little energy to remove electrons from atoms. Examples:
  - Radio waves, microwaves
  - Infrared, visible light
  - Mobile networks, WLAN, Bluetooth
- **Ionizing:** photons have enough energy to ionize atoms, potentially causing cell or DNA damage. Examples:
  - Ultraviolet (especially UV-B, UV-C)
  - X-rays
  - Gamma rays

3.3.4 Limits and Protective Measures

In practice, safety limits are set to avoid excessive exposure. The key metric is the **Specific Absorption Rate (SAR)** measured in $\mathrm{W\,kg^{-1}}$, describing how much electromagnetic energy per kilogram of body tissue is absorbed.

- EU limit for mobile devices: $2\,\mathrm{W\,kg^{-1}}$
- WLAN routers: typically well below this

3.3.5 Case Study: UV-C Radiation for Disinfection

Ultraviolet radiation in the range 200 nm–280 nm – so-called **UV-C** – has photon energies up to 6.2 eV. This is sufficient to **directly damage DNA and RNA of microorganisms**, e.g., by forming pyrimidine dimers, thereby inhibiting their replication.

Applications:

- Hospitals: UV-C lamps for disinfection of surfaces, air, and equipment.
- Drinking water treatment: UV-C without chemical additives.

Physical basis:

$$E = \frac{hc}{\lambda} = \frac{6.626 \cdot 10^{-34}\,\mathrm{J\,s} \cdot 3.0 \cdot 10^{8}\,\mathrm{m\,s^{-1}}}{254 \cdot 10^{-9}\,\mathrm{m}} \approx 7.83 \times 10^{-19}\,\mathrm{J} \approx 4.9\,\mathrm{eV}$$

This energy suffices to break covalent bonds in DNA – a clear physical mechanism of disinfection.

Note on Hazard

UV-C affects not only microorganisms but can also damage human skin and eyes. Direct exposure must be strictly avoided.

3.4 The Electromagnetic Spectrum

3.4.1 Overview

The electromagnetic spectrum encompasses all forms of electromagnetic radiation – from long-wavelength radio waves to short-wavelength gamma rays. One typically distinguishes by frequency ν, wavelength λ, or photon energy $E = h\nu = \frac{hc}{\lambda}$.

3.4.2 Representation

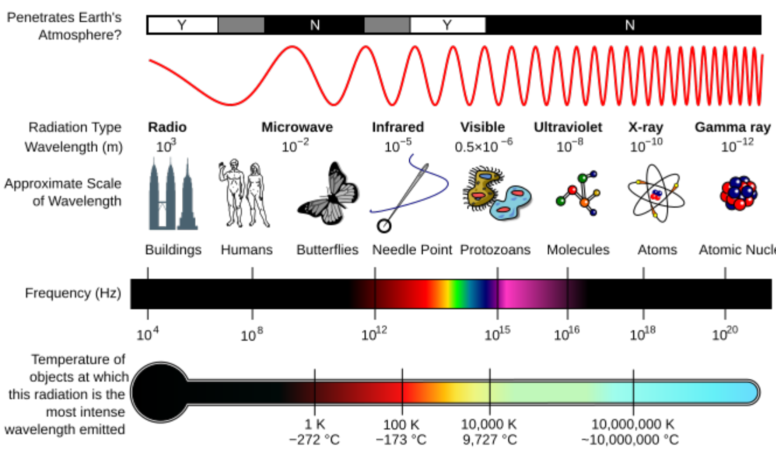


Figure III.1: The electromagnetic spectrum with typical applications. Source: Wikimedia Commons

3.4.3 Frequency, Wavelength, and Energy Ranges

Region	Frequency	Wavelength	Photon energy
Radio waves	1×10^3–1×10^8 Hz	km – m	$\sim 1 \times 10^{-9}$ eV
Microwaves	1×10^8–1×10^{11} Hz	m – mm	$\sim 1 \times 10^{-6}$ eV
Infrared	1×10^{11}–4×10^{14} Hz	mm – 750 nm	1×10^{-3} eV – 1 eV
Visible light	4×10^{14}–8×10^{14} Hz	750 nm – 380 nm	1.6 eV – 3.3 eV
Ultraviolet	8×10^{14}–3×10^{16} Hz	380 nm – 10 nm	up to 100 eV
X-rays	3×10^{16}–3×10^{19} Hz	10 nm – 0.01 nm	100 eV – 100 keV
Gamma rays	$>3 \times 10^{19}$ Hz	$<$ 0.01 nm	$>$ 100 keV

Table III.1: Typical regions of the electromagnetic spectrum with frequency, wavelength, and energy

3.4.4 Applications Across the Spectrum

Typical applications in different spectral regions:

- **Radio waves:** broadcasting, radio communication, MRI
- **Microwaves:** mobile networks, WLAN, microwave oven
- **Infrared:** remote controls, thermography
- **Visible:** optics, photography, biological vision
- **UV:** sunburn, disinfection (UV-C)
- **X-ray:** medical imaging, materials testing
- **Gamma:** radiation therapy, astrophysics

Takeaways on the Electromagnetic Spectrum

Physical takeaways:

- The electromagnetic spectrum spans waves of all frequencies – from radio to gamma.
- Photon energy increases with increasing frequency (or decreasing wavelength) according to $E = h\nu = \frac{hc}{\lambda}$.
- The transition to **ionizing radiation** typically begins in the UV, at photon energies around $10\,\text{eV}$.

Didactic takeaways:

- Placing typical applications – e.g., mobile networks, microwaves, UV disinfection, X-ray diagnostics – in the spectrum clarifies their physical and biological effects.
- A visual link between frequency, wavelength, energy, and biological relevance is especially helpful.
- **Rule of thumb:** the shorter the wavelength, the higher the photon energy – and the greater the potential for biological damage.

3.5 Masslessness and Propagation at the Speed of Light

3.5.1 No Rest Mass – Yet Energy and Momentum

The photon has no rest mass, i.e., $m_0 = 0$. Nevertheless, it carries energy and momentum. The photon energy (Planck–Einstein) is

$$E = h\nu$$

and the momentum follows as

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

Thus the photon does not contradict relativity – on the contrary: these relations are direct consequences of special relativity for massless particles.

3.5.2 What If the Photon Had a Tiny Mass?

Relativistic energy: If the photon were not exactly massless, one would have to use the general relativistic energy formula

$$E = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and the momentum

$$p = \gamma m_0 v$$

Then the simple relation $E = pc$ would no longer be exact; instead

$$E^2 = (pc)^2 + (m_0 c^2)^2$$

(A detailed mathematical discussion for massless and hypothetically massive photons is given in Appendix A, Sec. A.9.)

Physical Consequences:

- The speed of light would no longer be the same for all photons.
- The speed would depend on energy (frequency) $\Rightarrow$ violation of Lorentz invariance.
- Long-wavelength light would travel more slowly than short-wavelength light.
- Coulomb's law would have to be modified $\Rightarrow$ the range of the electric force would be finite.

Experimental Bounds: Precise measurements show: if the photon has a mass, it must be extremely small:

$$m_\gamma < 10^{-54}\ \text{kg} \approx 10^{-18}\ \text{eV}/c^2$$

This number 10^{-54} kg **is not a law of nature nor a theoretical prediction**, but an upper bound inferred from many experiments and observations – under the hypothesis that the photon might have a mass. No such tiny deviations have been observed.

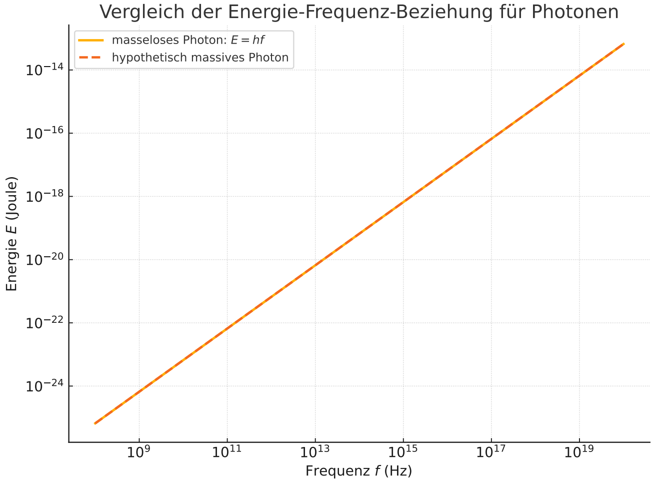


Figure III.2: Didactic comparison of the energy–frequency relation $E(f)$ for a massless photon (blue) and a hypothetical massive photon with exaggerated mass (red, dashed).

Why No Difference Is Visible in the Plot

The $E(f)$ relation is shown for massless and (exaggerated) massive photons. Because the experimental upper bound on the photon mass is so tiny ($m_\gamma < 10^{-54}$ kg), the two curves are practically indistinguishable even on log–log scales.

This lack of deviation strongly supports the photon's masslessness.

What if the Photon Had a Mass?

If the photon had even a tiny mass ($m_\gamma > 0$), then:

- It would travel more slowly than the speed of light ($v < c$),
- The energy–frequency relation would not be $E = hf$, but
$$E = \sqrt{(hf)^2 + (m_\gamma c^2)^2}\,,$$
- The speed of light would not be universal – e.g., long-wavelength light would be slower than short-wavelength light,
- Coulomb's law would be *screened* – electric fields would have finite range.

Since none of this is observed, the photon is, with overwhelming evidence, truly massless.

3.5.3 Why Would $E = hf$ Fail for a Massive Photon?

The formula $E = hf$ holds exactly only for massless particles. If the photon had mass, one would have

$$E = \sqrt{(hf)^2 + (m_\gamma c^2)^2} \; > \; hf\,,$$

so the energy would not depend on frequency alone – something that would be experimentally detectable.

Confirmed Experiments:

- Photoelectric effect
- Compton effect
- Spectral lines and laser spectroscopy
- Quantum optics and precision QED experiments

All confirm, to high precision, the relation $E = hf$. No systematic deviation has ever been found.

Conclusion: Why the Photon Is Massless

- The photon has no rest mass ($m_0 = 0$) but does have energy and momentum.
- It necessarily propagates at the speed of light: $v = c$.
- Only under this condition do $E = pc$ and $E = hf$ hold.
- If $m_\gamma > 0$, numerous observations and special relativity would be contradicted.

3.6 Spin and Polarization

3.6.1 Spin of the Photon

In quantum mechanics, **spin** is a fundamental property of elementary particles – comparable to an intrinsic angular momentum. Unlike classical rotation, spin does not refer to literal spinning in space; it is a purely quantum property with discrete values.

Photons have spin 1 and thus belong to the class of *bosons*. While half-integer spin particles (like electrons with spin 1/2) are fermions and obey the Pauli principle, bosons can occupy the same quantum state – a principle with fundamental consequences for light, e.g., laser amplification.

Properties of Photon Spin

- Photons have **spin 1** but no rest mass.
- Only two measurable spin states exist: **helicity** $+1$ **and** -1.
- The **spin direction** is always along the photon's direction of motion.

The restriction to only two spin states is a direct consequence of the photon's **masslessness**. In contrast to massive spin-1 particles (e.g., the Z boson), there is no rest frame for the photon; thus the longitudinal (0) spin state is absent. Only the two transverse helicity modes ± 1 remain.
(A formal derivation of the allowed photon helicities is given in Appendix A, Sec. A.10.)

Helicity is the projection of spin onto the direction of motion:

- **Helicity $+1$:** right-circularly polarized light.
- **Helicity -1:** left-circularly polarized light.

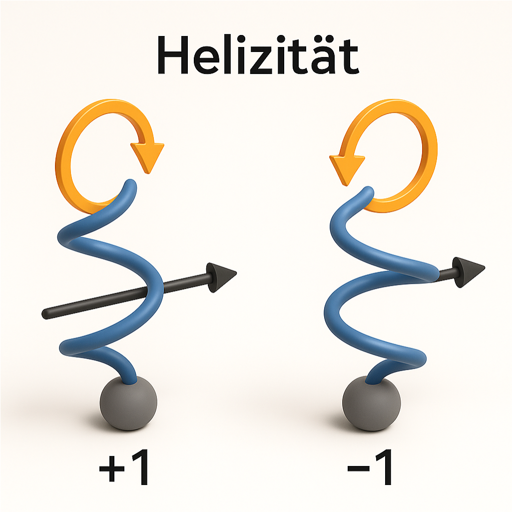


Figure III.3: Visualization of photon helicity. Left: helicity $+1$ (right-circular), right: helicity -1 (left-circular). Orange arrows indicate spin (field rotation), black arrows indicate propagation direction.

Comment on the Illustration

The spiral depicts how the electric field vector rotates for circular polarization. The photon travels straight along the propagation direction (momentum arrow). The spiral shows rotation of the field – not the photon's path. This distinction is crucial for understanding helicity.

These two states underpin *circular polarization*, discussed next.

For comparison:

- An **electron** has spin 1/2 with two states (up/down).
- A **Z boson** (massive, spin 1) exhibits three states: -1, 0, $+1$.
- The **photon** (massless, spin 1) exhibits only -1 and $+1$ – the 0 state is absent.

Didactic Rule of Thumb

Massless spin-1 particles such as photons possess only two helicity states: ± 1. A longitudinal (0) state does not exist because no rest frame can be defined.

Conclusion

The photon's spin fundamentally differs from that of massive particles. As a massless spin-1 particle, the photon has only two helicity states: $+1$ and -1. A longitudinal helicity-0 state is excluded because no rest frame exists.

Spin can thus only be considered along the direction of motion – it manifests as helicity:

- Helicity $+1$: spin aligned with motion (right-circular)
- Helicity -1: spin opposite to motion (left-circular)

A classical picture of literal rotation is inapplicable. Spin is an intrinsic quantum property that appears, for photons, as circular polarization.

3.6.2 Polarization as a Macroscopic Manifestation of Spin

Polarization is an observable phenomenon directly linked to the quantum spin of photons. While spin is an intrinsic property of each photon, polarization describes the collective state of a light field – i.e., many photons together.

Classically, polarization is the orientation of the electric field vector of an electromagnetic wave. This vector oscillates transverse to the direction of propagation and can take different orientations and rotations:

- **Linearly polarized light:** the electric field oscillates in a fixed direction (e.g., vertical or horizontal).

- **Circularly polarized light:** the field vector rotates at constant amplitude – clockwise (right-circular) or counterclockwise (left-circular).

- **Elliptically polarized light:** the most general case – the field vector traces an ellipse.

In quantum optics, polarization corresponds to the collective helicity content of many photons. A fully right-circularly polarized field consists of photons with helicity $+1$, a left-circular field of photons with -1. Linear polarization arises from a quantum superposition of both helicity states:

$$|\text{linear}\rangle = \frac{1}{\sqrt{2}}\big(|+1\rangle + |-1\rangle\big)$$

(A more detailed treatment using Dirac notation and Jones vectors is given in Appendix A, Sec. A.11.)

Superposition and Polarization

A linearly polarized photon is a quantum superposition of the two helicity states $|+1\rangle$ and $|-1\rangle$. This superposition leads to the fixed oscillation direction of the electric field.

Thus polarization is a directly observable consequence of photon spin. In many experiments – e.g., with polarizers, polarization cameras, or double-slit setups using polarized light – polarization appears as a macroscopic manifestation of quantum states.

Superposition and Polarization

A linearly polarized photon is in a quantum superposition of the two helicity states $|+1\rangle$ (right-circular) and $|-1\rangle$ (left-circular):

$$|\text{linear}\rangle = \frac{1}{\sqrt{2}}\left(|+1\rangle + |-1\rangle\right).$$

This implies:

- The photon has no definite helicity.
- It is simultaneously in $|+1\rangle$ and $|-1\rangle$ with equal amplitude and fixed phase.
- The fixed oscillation direction (linear polarization) arises from this coherent superposition.
- Measuring helicity destroys the superposition: the photon is found with helicity $+1$ or -1 – never both at once.

Polarization is therefore more than a field direction – it is the expression of a quantum state that is fully captured only via superposition.

3.6.3 Measuring and Detecting Polarization

Polarization is a macroscopic observable consequence of the photon's quantum spin structure. Various optical methods are available – from simple polarizers to precise single-photon detectors in quantum optics.

Linear polarization can be tested classically with two polarizers: a linearly polarized beam passes a second polarizer (the "analyzer"). The transmitted intensity varies with the relative angle according to **Malus's law**:

$$I = I_0 \cos^2 \theta$$

where θ is the angle between the incident polarization and the analyzer axis.

Circular polarization is revealed by combining a linear polarizer with a quarter-wave plate, which converts circular to linear polarization for analysis.

Single-photon experiments in quantum optics directly probe quantized polarization:

- A photon impinges on a polarizer and is either transmitted or absorbed.
- Repeated measurements on identically prepared photons yield polarization statistics.
- These experiments show: polarization is a single-photon phenomenon, not merely a classical wave effect.

Technical applications abound:

- LCD displays, polarized sunglasses, remote sensing
- Communications using polarization multiplexing (e.g., in fiber optics)
- Quantum communication with polarized single photons (QKD)

What Polarization Reveals About Photons

Polarization is not only a property of electromagnetic waves but a directly measurable result of photon spin. Polarization measurements allow fundamental statements about the states of single light quanta – up to and including entanglement in quantum physics.

3.7 Conclusion

Chapter III has shown how light can be understood, from the quantum-mechanical viewpoint, not only as an electromagnetic wave but also as a particle – the photon. This understanding unites classical concepts such as frequency and polarization with quantized properties such as energy, momentum, and spin.

Key insights:

- A photon's energy is directly proportional to its frequency: $E = h\nu$.

- A photon's momentum is determined by its wavelength: $p = \frac{h}{\lambda}$.
- Photons have spin 1 but, due to masslessness, only the helicity states $+1$ and -1 occur.
- The polarization of macroscopic light fields is a collective phenomenon rooted in the quantum states of many photons.

These quantized properties of the photon will play a central role in Chapter IV, where we explore the wave nature of particles and the particle nature of waves in the form of wave–particle duality.

Chapter IV

Experimental Confirmation of the Photon

4.1 The Photoelectric Effect

4.1.1 Introduction and Classical Expectation

The so-called photoelectric effect – the emission of electrons from a metal surface when irradiated with light – was already known in the 19th century. Heinrich Hertz discovered the phenomenon in 1887 by accident during his investigations of electromagnetic radiation. Philipp Lenard later studied it systematically and found: electrons are released from the metal when exposed to certain kinds of light.

Classical electrodynamics explained this behavior using the wave model of light. According to that model, the energy of light should be transferred continuously to the electron via the electric field strength. From this, three seemingly plausible expectations followed:

- **Intensity decides:** The higher the light intensity, the more energetic the emitted electrons should be.
- **Delay:** With weak light, a measurable time should pass until an electron has absorbed enough energy to be emitted.
- **Any wavelength possible:** In principle, any wavelength of light – even long-wave red light – should trigger electron emission if it is only intense enough.

These expectations seemed logical within the classical theory. But reality drastically contradicted them – a turning point in the history of physics.

Philipp Lenard (1902) *[8]*

"It turned out that light seemed to act with greater energy on electrons the shorter its wavelength was – regardless of how bright it was."

4.1.2 Experimental Observations

The systematic investigation of the photoelectric effect by Philipp Lenard, Robert Millikan, and others led to observations that could not be reconciled with the classical wave theory. The central findings were:

- **Immediate emission:** Electrons are released without measurable delay – even at extremely low light intensities.
- **Frequency dependence:** There exists a *threshold frequency* ν_{min}, below which no electrons are emitted – regardless of intensity.
- **Energy independent of intensity:** The kinetic energy of the emitted electrons depends solely on the frequency of the light – not on its intensity.
- **Linear relationship:** The electron energy increases linearly with the light frequency:

$$E_{\mathrm{kin}} = h\nu - A$$

(A more detailed derivation of Einstein's equation for the photoelectric effect can be found in Appendix A, Section A.12.)
These results fundamentally contradicted the classical expectation of continuous energy absorption from the electromagnetic field.

Albert Einstein (1905) *[4]*

"The generation of light does not occur uniformly across the wavefront, but only at specific places, at individual points."

Robert A. Millikan (1916) *[11]*

"Although I confirmed Einstein's equation through years of experiments, I long resisted the idea that light consists of particles."

Conclusion: The observations could only be explained by assuming that light consists of discrete quanta – *photons* – transferring their energy to an electron in a single collision. The classical wave picture had to be abandoned or profoundly modified.

4.1.3 Einstein's Explanation (1905)

In 1905 Albert Einstein published his groundbreaking paper entitled *"On a Heuristic Point of View Concerning the Production and Transformation of Light"* [4]. In it he fundamentally questioned the classical idea of light as a continuous wave.
Einstein proposed that light consists of individual energy portions – so-called **light quanta**. These quanta carry a definite amount of energy:

$$E = h\nu$$

(A formal representation of the Planck–Einstein relation can be found in Appendix A, Section A.13.)
A photon with frequency ν thus possesses a discrete energy proportional to frequency. This idea was revolutionary, as it assigned a *particle character* to light – contrary to the well-established Maxwell theory.
Einstein further postulated that such a light quantum could transfer its energy completely to an electron in a collision.
Only if the energy of the photon is greater than the so-called **work function** A, an electron is released from the metal:

$$E_{\text{kin}} = h\nu - A$$

This equation explains directly:

- Why only light of sufficiently high frequency can release electrons,
- Why emission occurs immediately (a single photon suffices),

- Why the kinetic energy of electrons grows linearly with frequency.

Einstein's explanation was radical – and at first highly controversial. Even Max Planck, the founder of quantum theory, considered the light quantum hypothesis too speculative.

Albert Einstein (1905) *[4]*

"The phenomena of thermal radiation demand a viewpoint according to which light consists of discrete quanta of energy in the direction of propagation."

Assessment: Einstein turned Planck's mathematical trick into physical reality. With this he laid the foundation for the modern concept of the photon and the quantized nature of the electromagnetic field.

4.1.4 Millikan's Measurements (1916)

Although Einstein's light quantum hypothesis provided a convincing explanation for the photoelectric effect, it was initially heavily disputed. Many physicists – including Max Planck – considered it unimaginable that light should consist of real particles. One of the most prominent skeptics was **Robert A. Millikan**.
In a series of experiments (1909–1916), Millikan developed a particularly precise setup to systematically test Einstein's equation. Using a vacuum photo cell, finely adjustable counter voltages, and monochromatic light sources, he was able to directly measure the maximum kinetic energy of the electrons at different light frequencies.
The result: Despite all his doubts, Millikan found a crystal-clear confirmation of Einstein's prediction:

$$eU = h\nu - A$$

(A detailed derivation of the stopping voltage equation and its experimental significance can be found in Appendix A, Section A.14.)
Here U is the counter voltage required to fully suppress the electron current. The slope of the resulting straight line (electron energy vs. frequency) yielded with high accuracy Planck's constant h.

Robert A. Millikan (1916) [11]

"Einstein's equation matches the data with astonishing accuracy – and yet I cannot bring myself to regard it as theoretically satisfying."

Remarkable: With utmost precision, Millikan disproved the classical theory – and confirmed Einstein's light quantum – but was reluctant to accept the concept. Only years later did he recognize the quantized nature of light as physical reality.
Significance: Millikan's measurements are considered one of the most important experimental proofs of the photon concept. They strengthened the acceptance of Einstein's theory – although it was only fully appreciated with the development of quantum mechanics.

4.1.5 Mathematical Description

The central formula for describing the photoelectric effect is based on Einstein's assumption that each photon possesses a discrete energy $E = h\nu$. When such a photon hits an electron in the metal, its energy is transferred to the electron. The energy balance is:

$$E_{\text{Photon}} = E_{\text{Work Function}} + E_{\text{kin}} \quad \Rightarrow \quad h\nu = A + E_{\text{kin}}$$

Where:
- h is Planck's constant - ν is the frequency of the incident light - A is the material-dependent **work function** - E_{kin} is the kinetic energy of the electron
If a counter voltage U is applied to suppress the electron current, then the energy eU corresponds to the maximum kinetic energy of the electrons:

$$eU = h\nu - A$$

This is the experimentally measurable form of Einstein's equation.

What is the Work Function A?

The work function A is the minimum energy required to release an electron from the metal. It depends on the material and typically lies between 2 eV (e.g. cesium) and 5 eV (e.g. platinum). Only if $h\nu > A$, an electron is emitted.

Graphical representation:

The equation describes a *linear dependence* of electron energy (or stopping voltage U) on frequency ν. The graph is a straight line:

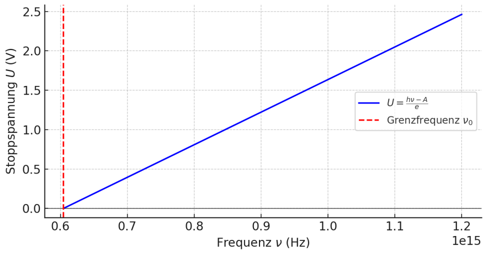


Figure IV.1: Linear dependence of electron energy on frequency.

- The **slope** of the line corresponds to h/e
- The **x-intercept** ν_0 is the threshold frequency: $h\nu_0 = A$
- For $\nu < \nu_0$ no emission occurs – regardless of intensity

This linear relationship was experimentally confirmed with high accuracy by Millikan.

Consequence:

> The energy of a photon depends solely on its frequency – not on light intensity.

4.1.6 Comparison: Wave Model vs. Photon Model

Explaining the photoelectric effect marked a profound paradigm shift in physics: from the classical wave picture of light to a quantized particle model. To illustrate the significance of this change, it is useful to directly compare both models.

Classical Wave Model	**Photon Model (Einstein)**
Light is a continuous wave in the electromagnetic field	Light consists of individual quanta (photons) with energy $E = h\nu$
Energy is transferred continuously via field strength	Energy is transferred abruptly in discrete portions
Intensity determines the energy transfer	Intensity only determines the *number* of photons, not their energy
Any frequency can release electrons if intensity is high enough	Only photons with $h\nu > A$ release electrons
Energy is accumulated slowly; time delay possible	Immediate emission when a photon strikes

Table IV.1: Comparison between the classical wave model and Einstein's photon model for the photoelectric effect

Didactic Clarification

In the wave model, light intensity determines the strength of energy transfer – in the photon model, however, it only determines the number of incident quanta.
The energy of an individual photon depends exclusively on frequency.

Typical Misunderstanding: "Bright light must release more energetic electrons." → *False*, because with light below the threshold frequency nothing happens – regardless of brightness.
Assessment: The photoelectric effect was the first direct proof that electromagnetic radiation has not only wave character, but also particle properties. This duality was later generalized in the concept of wave–particle duality.

4.1.7 Significance for Physics

The photoelectric effect was the first phenomenon that could only be explained by assuming a quantized nature of light. Einstein's light quantum hypothesis of 1905 contradicted classical electrodynamics and was initially dismissed as speculative. But Millikan's precise confirmation in 1916 forced the scientific community to fundamentally rethink the concept of light.

Physical Consequences:

- Energy transfer of light does not occur continuously, but in discrete quanta – the photons.
- The energy of a photon is proportional to frequency: $E = h\nu$.
- The photoelectric effect provided the first direct experimental proof of this quantization.

Einstein himself received the Nobel Prize in 1921 – explicitly not for relativity, but:

> "for his services to theoretical physics, and especially for his discovery of the law of the photoelectric effect."

Millikan was also awarded the Nobel Prize in 1923 – for his determination of the elementary charge and his work on the photoelectric effect.

Conclusion

The photoelectric effect marks the beginning of the photon concept – and thus the dawn of quantum physics.
It shows: light not only has wave properties, but under certain conditions behaves like a particle.

Outlook: In the next section we will consider another key experiment: **Compton scattering**. It demonstrates not only the energy transfer, but also the *momentum transfer* of photons – a decisive proof of the particle character of light.

4.2 Compton Scattering

4.2.1 The Compton Experiment (1923)

In 1923 the American physicist **Arthur H. Compton** published the results of a scattering experiment with X-rays that would revolutionize physics. He directed high-energy photons onto nearly free electrons – for example in graphite – and analyzed the scattered radiation as a function of angle.
The central observation was striking: the scattered light had a longer wavelength (lower energy) than the incident light, and the wavelength shift depended systematically on the scattering angle.

This change could not be explained by classical scattering (such as Thomson scattering) or interference. Compton interpreted the result as an **elastic collision** between a photon and an electron – fully in line with a particle model of light. Thus the photon was not only a carrier of energy, but also of momentum.

Physical principle:

- A photon with wavelength λ strikes a stationary electron.
- During the collision the photon is deflected (scattering angle θ) and transfers momentum and energy to the electron.
- The scattered photon has a new wavelength $\lambda' > \lambda$.

Key equation:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

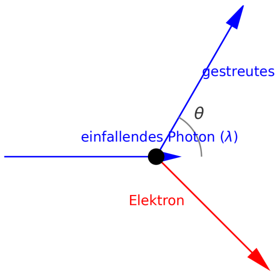


Figure IV.2: Schematic representation of Compton scattering: a photon transfers momentum to an electron.

Significance: Compton scattering was the first experimental proof that photons possess momentum – a decisive step toward accepting the photon as a real particle. Arthur Compton received the Nobel Prize in Physics in 1927 for this.

4.2.2 Compact Derivation of the Compton Formula

The central observation in the Compton effect is: the scattered photon has a longer wavelength than the incident photon. This shift can only be explained if the photon is assigned not only energy $E = h\nu$, but also momentum $p = h/\lambda$.

Basic idea of the derivation: A photon strikes a stationary electron. During the collision, energy and momentum are transferred to the electron. The situation resembles an elastic collision of two particles – with the difference that one of them is massless.

Core assumptions:

- **Energy conservation:** The sum of photon energy and electron rest energy is conserved.
- **Momentum conservation:** The momentum vectors before and after the collision must balance – in both x- and y-directions.
- **Photon momentum:** For the photon $p = \frac{h}{\lambda}$, for the electron the relativistic energy–momentum relation applies.

Applying these conservation laws and simplifying yields the **Compton formula**:

Compton Formula

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

(A more detailed derivation of the Compton formula can be found in Appendix A, Section A.16.)

Physical interpretation:

- The wavelength of the scattered photon increases with the scattering angle θ.
- The factor $\frac{h}{m_e c} \approx 2.43 \cdot 10^{-12}\,\mathrm{m}$ is known as the *Compton wavelength* of the electron.
- The effect shows that the photon transfers *momentum* – a clear proof of its particle character.

4.3 Double-Slit Experiment with Single Photons

A central argument for the wave nature of light since the 19th century was the observation of interference patterns – in particular in the famous double-slit experiment. But modern experiments show: even single photons, sent one after another through the setup, produce an interference pattern on the screen. This is only explainable if the photon also possesses wave character.

Experimental setup:

- A weak light source emits single photons – so weak that never more than one is in the setup at the same time.
- The photons pass through two closely spaced slits (double slit).
- Behind them is a light-sensitive screen or detector.

Observation:

- Each individual photon is registered as a point – like a particle.
- Over time, however, an interference pattern emerges – like a wave.

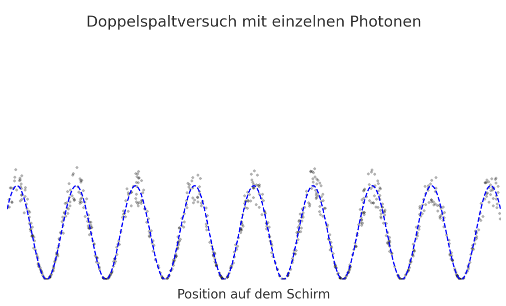


Figure IV.3: Double-slit experiment with single photons: particle detections with a wave pattern.

What the Photon Graphic is Meant to Show

- The dots represent individual photon detections – the particle aspect of light.
- Over time a pattern emerges that is typical of wave interference – as with water waves.
- The dashed line shows only the statistical frequency of detection locations – it is not a light ray and not a real wave.

(A mathematical formalization of the double-slit experiment with single photons and the superposition representation can be found in Appendix A, Section A.17.)

Interpretation: The photon apparently interferes with itself – it "passes through both slits simultaneously." Only upon detection on the screen does the state collapse into a single event. This behavior can only be understood within quantum mechanics: the photon is neither a classical wave nor a classical particle – it exhibits both, depending on the experiment.

Key Idea

The double-slit experiment with single photons shows: the quantum nature of light encompasses both wave and particle properties. The interference pattern emerges even though never two photons are in the system at the same time.

Wave as well as Particle Properties

The interference pattern disappears immediately if one tries to determine the photon's path through one of the slits. The possibility of interference is tied to the *unknowability of the path* – a basic principle of quantum physics.

Summary

Conclusion: An Apparently Paradoxical Behavior

Even when photons are sent one by one – successively – through the double slit, an interference pattern emerges over time.
This raises a profound question: How does a single photon "know" that it is part of a pattern?
In classical terms there would have to be some kind of communication between photons – but that is not the case. Each photon seems to interfere with itself. Quantum mechanics explains this by the **superposition of all possible paths**: the photon has not gone through one or the other slit – but through both, as long as the path is not measured.
This notion contradicts our everyday experience – but is experimentally beyond doubt. The double-slit experiment is therefore a key result of quantum physics.
The double-slit experiment with single photons challenges our classical thinking: How can a single photon contribute to building an interference pattern, although no second photon is present in the setup at the same time?
Apparently each photon interferes with itself. In the language of quantum mechanics this means: as long as no measuring device determines the path, all possible paths are superposed – including "through both slits at once."
This superposition collapses only upon detection into a single point. The interference pattern arises not from interaction between photons, but from the *statistics of many single measurements* – and from the quantum-mechanical structure of state space.
What looks like "communication" is in fact an expression of the nonclassical nature of the quantum world.

4.4 Antibunching: The Proof of Single Photons

A particularly convincing experimental proof for the existence of single photons is provided by the phenomenon of **antibunching**. Here a light source is used that emits only one photon at a time – for example, a fluorescing atom or a quantum dot.

In a setup with a beam splitter and two detectors, it turns out: it *never happens* that both detectors register a signal at the same time. This means: there is no such thing as "half a photon" – but always exactly one, which arrives either here or there.

Why does this contradict the classical wave picture?
In classical theory, light is a continuous electromagnetic wave. When such a wave strikes a beam splitter, it is *divided*: one part goes left, the other right. Thus – at least with strong intensity – both detectors should sometimes register a signal simultaneously.

In contrast, antibunching shows: *never* are both detectors triggered at once. This means that the light does not arrive split, but in **indivisible energy packets** – single photons. This directly contradicts the classical wave picture.

What Antibunching Shows

Light cannot be split into two directions simultaneously if it consists of single photons.
This contradicts every classical wave picture – but precisely matches the behavior of indivisible light quanta.

(A mathematical description of the second-order correlation function $g^{(2)}(0)$ can be found in Appendix A, Section A.18.)

4.5 Hong–Ou–Mandel Effect: Interference of Two Photons

Another striking experiment is the **Hong–Ou–Mandel (HOM) effect**. Two identical photons are sent from opposite directions onto a half-transparent mirror (beam splitter).

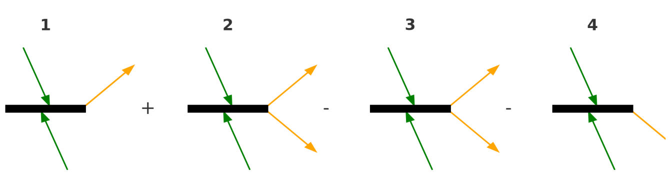


Figure IV.4: Four possible paths of two photons at a beam splitter. The middle cases would lead to coincidence – but cancel due to interference.

What This Diagram Shows

The diagram shows the four possible paths two identical photons can take when hitting a beam splitter.
In cases 1 and 4 both photons exit together through the same output – exactly as observed experimentally.
Paths 2 and 3 would lead to the photons arriving at different detectors (coincidence). But these cases cancel out due to destructive interference – because of the indistinguishability of the photons and the quantum-mechanical superposition of their amplitudes.
The result: with perfect overlap both detectors are never triggered simultaneously – a clear sign of the quantum nature of light.

According to classical expectation, each photon should be reflected or transmitted with 50
This phenomenon arises from the interference of the *probability amplitudes* of the two processes. It shows: photons are indistinguishable and can interfere at the quantum level – not as fields, but as particles with wave nature.

What the HOM Effect Shows

Two photons can "prevent" themselves from taking different paths – through the interference of their quantum states.
This can only be explained if photons are considered indistinguishable quantum particles.

4.6 Conclusion

What the Experiments Show About Light

The four experiments discussed in this chapter – the photoelectric effect, Compton scattering, the double slit with single photons, and modern quantum experiments such as antibunching and the Hong–Ou–Mandel effect – demonstrate impressively that light cannot be explained by classical models alone.

- The **photoelectric effect** shows: light transfers *energy* in discrete portions (photons).
- **Compton scattering** shows: photons also carry *momentum* – like particles.
- The **double-slit experiment** shows: photons display *wave patterns* in their distribution – even when occurring one at a time.
- **Antibunching** and the **Hong–Ou–Mandel effect** show: photons are *indivisible*, *non-classically indistinguishable*, and subject to the rules of quantum mechanics.

(A more detailed derivation of quantum interference at the beam splitter in the HOM effect can be found in Appendix A, Section A.19.)
Together these experiments prove: light is not an either–or of wave and particle – it is both at once, depending on the experiment. The photon concept is therefore not just a calculational tool – but physically real.

Chapter V

The Photon in Quantum Electrodynamics (QED)

5.1 From the Photon to Quantum Electrodynamics

The history of the photon begins with a paradox: Light, which since the 19th century was understood as an electromagnetic wave, showed behavior in certain experiments that could only be explained with a particle picture—particularly in the photoelectric effect and in Compton scattering. These experiments led Einstein in 1905 to introduce the concept of the *light quantum*. Although the photon model proved successful experimentally, it remained controversial for decades.

A complete theoretical understanding of light–matter interaction was achieved only with the development of **Quantum Electrodynamics** (QED). It unites Maxwell's classical field theory with the principles of quantum mechanics and special relativity. In QED, the photon is the exchange particle of the electromagnetic interaction—a massless, spin-1 gauge boson that can appear not only in real but also in virtual states. (A compact introduction to the QED field formalism and the potential A^μ can be found in Appendix A, Section A.20.)

What is Quantum Electrodynamics?

Quantum Electrodynamics describes how charged particles (e.g., electrons) exchange photons and thereby interact electromagnetically.
The theory is based on quantized fields, Feynman diagrams, and a local gauge principle. It is the most precise physical theory ever tested experimentally.

The transition from classical to quantum field–theoretic understanding was anything but straightforward. At first, photons were treated as discrete packets of classical wave energy—a compromise between particle and wave pictures. Only in the 1940s, through the work of Dirac, Feynman, Schwinger, and Tomonaga, did a consistent theory emerge that mathematically describes the photon as the quantum of the electromagnetic field.
What QED changes:

- Photons are no longer seen as light rays but as excitations of a quantized field.
- Interaction occurs not continuously but in discrete processes (vertices).
- Virtual photons—not directly observable but mathematically necessary—play a central role.

(For the transition from the classical field to quantization and to Feynman diagrams, see Appendix A, Section A.21.)
This chapter introduces the properties of the photon within QED, beginning with its vector character, followed by its role as an exchange particle in Feynman diagrams, and ending with experimental confirmations of utmost precision.

5.2 The Vector Character of the Photon in QED

The transverse polarization of the photon and its property as a spin-1 particle were already described in Section 3.6. In Quantum Electrodynamics, however, this structure arises not only from experimental observation or classical equations, but from the underlying *field formalism* and the *gauge symmetry* of the theory. (A formal derivation of photon helicity is given in Appendix A, Section A.10; for polarization in Jones/Dirac notation, see A.11.)

What is the Field Formalism?

In QED, photons are not "particles with trajectories," but quantized excitations of a field: the electromagnetic potential $A^\mu(x)$. Just as a water wave is a local displacement of the water surface, a photon is a discrete oscillation of the field—described simultaneously by quantum mechanics and relativity.

(A short derivation of the Lorentz-covariant form of the four-potential A^μ is given in Appendix A, Section A.20.)
The central mathematical starting point is the so-called *Lagrangian density* $\mathcal{L}$, from which the equations of motion of the field are derived. For the electromagnetic field it reads:

$$\mathcal{L}_{\text{EM}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad \text{with} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

(Derivations of the field strength tensor and EM Lagrangian density: Appendix A, Sections A.22 and A.23; for the energy–momentum relation see A.1.)
This Lagrangian density leads—via the principle of least action—to Maxwell's equations in their relativistic form. It is invariant under so-called **gauge transformations**:

$$A^\mu(x) \to A^\mu(x) + \partial^\mu \Lambda(x)$$

(For local $U(1)$ gauge symmetry, gauge fixing, and the Lorenz condition see Appendix A, Section A.24.)

What Does Gauge Symmetry Mean?

Gauge symmetry means that the electromagnetic potential $A^\mu(x)$ can be changed locally without altering measurable quantities such as the electric field. This freedom is not just a mathematical trick but a structural principle: it determines which terms are allowed—and which are not.

Why is the photon massless? A mass term of the form $\frac{1}{2}m^2 A_\mu A^\mu$ would *not* be invariant under this transformation. The requirement of gauge symmetry forbids it—the photon must be massless. (Formal argument via the Proca Lagrangian and broken gauge invariance: Appendix A, Section A.25.)

Why is the photon transverse? A vector field has four components, but not all are physically independent. Gauge freedom and Lorentz invariance allow the elimination of non-physical modes. In the end, only two permissible polarization states remain: the transverse ones with helicity +1 and −1. (Reduction of degrees of freedom, Lorenz condition, and trace/projector method: Appendix A, Section A.26; see also A.10.)

Consequences of Gauge Symmetry

The gauge symmetry of QED explains two fundamental properties of the photon:

- **Masslessness:** A mass term would not be gauge invariant—therefore the photon is necessarily massless.
- **Transversality:** The symmetry allows only two polarization modes—both perpendicular to the direction of propagation.

Virtual photons—an exception While real photons possess only transverse states, so-called *virtual photons*—appearing in the internal lines of Feynman diagrams—can also contain longitudinal or scalar components. However, these are not observable and cancel completely in physical predictions (e.g., in the scattering amplitude). (Longitudinal/scalar contributions in propagators and their cancellation in observables: Appendix A, Section A.27.)

Conclusion: In QED, the structure of the photon does not result from extra assumptions but from the symmetric construction of the theory itself. Gauge symmetry replaces intuition—and creates order.

5.3 Virtual Photons and Feynman Diagrams

5.3.1 What Are Virtual Particles?

Virtual particles—especially virtual photons—are central computational elements in quantum field theory. They differ fundamentally from real particles, which can be detected in a detector. Their existence is **mathematical in nature**—and yet their effects show up in real experiments.

Difference: Real or Virtual Particles

Property	Real Photons	Virtual Photons
Observable in a detector	Yes (e.g., light quantum in the photoelectric effect)	No
Energy–momentum relation	$E = pc$ (on-shell)	$E^2 \neq p^2c^2$ (off-shell)
Lifetime	Arbitrarily long (for stable particles)	Extremely short (due to uncertainty relation)
Physical existence	Yes	Only as a computational element in diagrams

On-Shell Condition

A particle is **on-shell** if it fulfills the relativistic energy–momentum relation:

$$E^2 = p^2c^2 + m^2c^4$$

For massless particles (e.g., photons) this reduces to:

$$E = pc$$

Off-shell states occur for virtual particles, which appear only in intermediate steps of calculations.

Time–Energy Uncertainty

According to Heisenberg's uncertainty relation:

$$\Delta E \cdot \Delta t \gtrsim \hbar$$

For very short times Δt, it becomes possible to "violate" energy conservation temporarily—for example, by the appearance of a virtual photon with seemingly "wrong" energy. This effect is not a rule violation but a consequence of quantum mechanics.

Interpretation

Virtual particles occur in **intermediate states**—for instance, when an electron briefly emits a photon that is immediately reabsorbed. These processes are represented in **Feynman diagrams**. The virtual particles correspond to the **internal lines** of the diagram.

Virtual Particles in the Quantum Vacuum

Virtual particles arise as intermediate states in quantum-mechanical interactions. They do not fulfill the classical energy–momentum relation (they are *off-shell*) and cannot be observed directly.
Nevertheless, their effects appear in the most precise experiments—for example, in the **Lamb shift** in the hydrogen spectrum or in the **anomaly of the electron's g-factor**.

Indirect Evidence for Virtual Photons

- **Lamb shift:** In high-precision spectra of the hydrogen atom, a fine shift of the energy levels is observed, which can only be explained with quantum field–theoretic corrections—particularly the influence of virtual photons on the electron.
- **g-factor anomaly:** The electron possesses a magnetic moment that deviates slightly from the classical value $g = 2$. This deviation arises from loop processes in Feynman diagrams—with virtual photons as mediators.

Does Virtual Mean Less Real?

The term "virtual" can easily be misleading. Virtual particles are **not simply "unreal" particles**—they are precisely defined mathematical objects in quantum field theory.
They follow their own rules and contribute decisively to the correct result—precisely because they *do not* fulfill the conditions of real particles.

Transition

In the next section, we consider how exactly these virtual photons mediate electromagnetic interaction in quantum field theory—from the classical Coulomb force to scattering processes.

5.3.2 Virtual Photons as Exchange Particles

In classical physics, the electromagnetic force is described as a field produced by charges and acting on other charges—as in Coulomb's law or in Maxwell's equations. In quantum field theory, by contrast, the force is mediated through the exchange of virtual photons.
These virtual photons appear in interaction processes between charged particles, without ever being observed as real light quanta. They are not detectable—yet they are the central element through which forces can be explained quantum mechanically.

Exchange Picture vs. Force Picture

Instead of imagining a "force" in the classical sense, QED describes particles as sending each other *virtual photons* that carry momentum. This creates the impression of an interaction.

Virtual Photons as Force Mediators

Virtual photons transfer momentum between charged particles and thereby mediate the electromagnetic interaction. This process is the quantum field–theoretic explanation of forces such as the Coulomb force or magnetism.
No real photon "flies" between the particles—the effect arises exclusively from mathematically described intermediate states in the Feynman diagram.

Example: Electron–Electron Scattering

A particularly illustrative example is so-called *Møller scattering*, in which two electrons scatter elastically. In classical physics, one would speak of a repulsive Coulomb force—in QED, however, the process is described as the exchange of a virtual photon between the electrons.

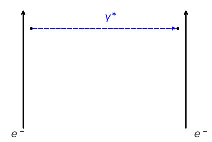


Figure V.1: The virtual photon is shown here as the internal line of the Feynman diagram. It transfers momentum—although it never exists as a real particle.

What Does This Diagram Show?

The Feynman diagram represents a typical **Møller scattering**—the elastic scattering of two electrons via the exchange of a virtual photon. The time axis runs from bottom to top.

- **Two electrons** enter the diagram from below (left and right sides). They approach each other and interact.
- The **virtual photon** (depicted by the dashed blue line and the symbol γ^*) is exchanged between the electrons. It is *not real*, but an intermediate state in the quantum field–theoretic calculation.
- The photon transfers momentum and energy—causing the electrons to change direction and leave the interaction region scattered upward.
- The two **vertices** (connection points) show where the interaction is localized. Mathematically, the momentum transfer takes place at these points.

What Does the Feynman Diagram Really Show?

The Møller scattering diagram does not show a real photon "flying" between electrons. Rather, it describes a quantum-mechanical intermediate state:
A **virtual photon** mediates the interaction—mathematically within perturbation theory. This photon does not satisfy the classical relation $E = pc$, it exists only as an *off-shell* intermediate state and transfers momentum.
Real effects such as scattering angles and energy distributions can be calculated from it—without a photon ever being detected.

No Direct Observability—but Measurable Effect

Virtual photons cannot be detected. Yet they influence measurable quantities: cross sections, scattering angles, and energy distributions can be described with high accuracy through the underlying exchange mechanism.

Distinction from Real Photon Production

An important difference: In the *Compton effect*, a **real** photon is scattered—with detection in the experiment. In **virtual exchange**, by contrast, no photon participates as a real particle. This is the fundamental distinction between **exchange diagrams** and **photon processes**.

No Need for an "Invisible Force"

In quantum field theory, there is no longer a need for a classical force field acting between two charges. Instead, interactions arise through the exchange of virtual particles—in this case, virtual photons.
The classical picture of action at a distance is replaced by a local, quantized exchange principle.

5.3.3 Feynman Diagrams as Visual Aids for Calculation

The so-called **Feynman diagrams** are a central tool of quantum field theory—especially in Quantum Electrodynamics (QED). They serve as a *graphical notation* for mathematical terms in perturbation theory and help structure complex processes in an illustrative way. A diagram does not represent a real "sequence in space and time," but encodes a probability amplitude. Nevertheless, measurable physical quantities such as scattering angles, cross sections, or lifetimes can be calculated from it.

Elements of a Feynman Diagram

- **Fermion lines:** solid lines with arrows—e.g., electrons or positrons
- **Photon lines:** wavy or dashed lines—depending on convention
- **Vertices:** points where particles "interact," i.e., momentum is transferred
- **Time axis:** usually bottom to top, or left to right

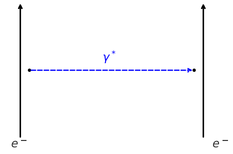


Figure V.2: Feynman diagram

What a Feynman Diagram Really Shows

A Feynman diagram is not a classical motion picture, but a symbolic representation of a mathematical integral over all possible paths of a process. It encodes the structure of the probability amplitude, not the exact trajectory of individual particles.

Example: Single-Photon Exchange

In elastic electron–electron scattering, the corresponding Feynman diagram consists of two incoming and two outgoing electron lines, plus one virtual photon line in between. This diagram symbolically represents a formula summing over all possible momentum transfers and times—it does not replace reality but reflects it probabilistically.

More than Just Pictures—Diagram Rules

Every Feynman diagram corresponds to a mathematical expression. The so-called *Feynman rules* assign, for example, a fraction term (propagator) to each line, a coupling constant (e.g., e) to each vertex, and an integral over momenta to the whole diagram. The sum of all possible diagrams up to a certain order yields the physically measurable quantity.

A Diagram Is Not Reality

A common misunderstanding is that a Feynman diagram shows a real process—for example, a photon "flying." In fact, it describes a *superposition of all possible intermediate states*, summarized in a quantum-mechanical amplitude. It is therefore a computational tool, not a film of what happens.

5.3.4 Illustrative Diagrams

To better understand the role of virtual photons compared with real photons, it is worth looking at typical processes in Quantum Electrodynamics. Feynman diagrams clearly indicate whether a photon is real (detectable) or only virtual (mediating interaction).

1. Virtual Exchange: Møller Scattering (Elastic e^-e^- Collision)

In this process, two electrons scatter elastically through the exchange of a virtual photon. The photon is not real—it only transfers momentum.

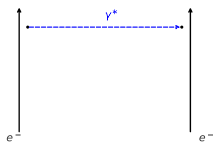


Figure V.3: Virtual photon

Virtual Photon

The exchanged photon is virtual—it does not exist as a real particle. It transfers momentum and energy but does not fulfill $E = pc$.

2. Real Photon: Compton Scattering

In the Compton effect, a **real** photon is scattered by an electron. Both the incoming and outgoing photons are real, detectable particles. The Feynman diagram shows two vertices with an internal electron propagator.

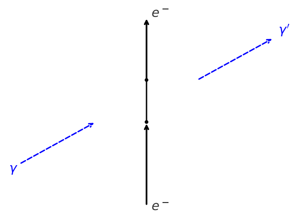


Figure V.4: Real photon

Real Photons

In Compton scattering, photons appear as real particles. They are measurable and fulfill the on-shell condition $E = pc$.

3. Loop Diagrams: Self-Interaction and g-Factor

In higher orders of perturbation theory, closed loops appear in Feynman diagrams—for example, when an electron interacts with itself via a virtual photon. Such diagrams explain, among other things, the anomaly of the g-factor.

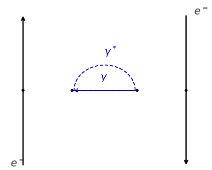


Figure V.5: Loop diagram

Loop Diagrams and Precision Effects

Loops with virtual photons provide corrections to simple processes—for example, for the exact value of the electron g-factor or the Lamb shift. These effects have been confirmed experimentally with high precision.

Didactic Comparison

- **Virtual photons** appear as internal lines in diagrams. They are not directly observable but are physically effective.
- **Real photons** are external lines. They can be measured and are on-shell.

Real or Virtual Photons in the Diagram

Whether a photon is real or virtual is seen from whether it is shown as an incoming or outgoing particle. Only real photons are observable. Virtual photons are internal lines—they “exist” only within the calculation.

5.3.5 Significance for QED

Quantum Electrodynamics (QED) is the most precisely confirmed theory in physics. Its enormous accuracy would be unthinkable without the concept of virtual photons and Feynman diagrams.

Central Role of Virtual Photons

Virtual photons enable the calculation of electromagnetic interaction between charged particles on the quantum level. They appear in all orders of perturbation theory—from simple exchange processes to complex loop corrections.

No QED Without Virtual Photons

Virtual photons are the foundation of the quantum field–theoretic description of electromagnetic processes. Without them, QED could neither mediate forces nor make quantitative predictions.

Feynman Diagrams as Methodological Backbone

Feynman diagrams provide not only an illustrative representation but also a precise computational prescription. For every possible interaction, all allowed diagrams can be drawn and translated into a mathematical expression. The sum of all diagrams yields the observable amplitude.

Example: Calculation of the g-Factor

The deviation of the electron's gyromagnetic factor from the classical value $g = 2$ is one of the most accurate experiments in modern physics. The correction arises from a loop process with a virtual photon—that is, from a second-order Feynman diagram.

Accuracy of the Theory

The theoretically calculated values for quantities such as the Lamb shift, the electron g-factor, or scattering angles in electron collisions agree with measurements to many decimal places.

Quantum Electrodynamics as a Success Story

QED is not only an elegant theory—it delivers, with the help of virtual particles and Feynman diagrams, quantitative predictions with an accuracy of up to 12 decimal places. The role of the photon—even as a virtual particle—is central.

5.3.6 Limits and Misconceptions

Despite their illustrative representation, Feynman diagrams must not be misunderstood as pictorial descriptions of real processes. Virtual photons do not exist in the classical sense—they are mathematical objects within an approximation method.

Virtual Photons Do Not Fly

It is misleading to say that a virtual photon "flies back and forth between two electrons." Virtual particles are not measurable, not localized, and do not satisfy a classical equation of motion. Their role consists exclusively in mediating interaction within a quantum-mechanical computational model.

No Particle Trajectory in the Diagram

A Feynman diagram does not describe motion in space or a classical flight path. It represents a symbolic structure for calculating an amplitude. The "lines" in the diagram are not trajectories but terms in a formula.

Off-Shell Means: No Energy–Momentum Relation

Virtual photons do not satisfy the familiar relation $E = pc$. They are *off-shell*, i.e., they temporarily violate the energy–momentum balance—within the framework of Heisenberg's uncertainty relation. This is not a weakness of the theory but one of its central features.

Diagrams Are Not Exact Pictures

Another misconception is to assume that Feynman diagrams show "what really happens." In fact, they contain no statement about the sequence of events, real locations, or classical times. The diagrams represent probability amplitudes—not observations.

Warning: Do Not Take Feynman Diagrams Literally!

Feynman diagrams are a powerful computational tool—but not sketches of reality. Interpreting them as "sequences of particle flights" misses the essence of quantum field theory.

Limits of Validity

QED—and with it the concept of virtual photons—is a perturbation theory. It works excellently for small coupling constants (e.g., in electrodynamics), but fails for strong interactions. There, other methods (e.g., lattice theory) are required.

Conclusion of This Section

Virtual photons and Feynman diagrams are central concepts in QED—but they must be interpreted correctly: not as "flying particles" but as building blocks of a mathematical theory with enormous predictive power.

5.3.7 Summary

Virtual photons do not appear in detectors—but they determine what is measured there. They are the invisible carriers of electromagnetic interaction in the quantum world. Feynman diagrams help capture their effect mathematically, without attributing to them a classical reality.

QED would not be possible without these concepts—and it is precisely these seemingly abstract building blocks that make it the most precise theory in physics.

5.4 The QED Formalism

Quantum Electrodynamics—QED for short—is the field theory of the electromagnetic interaction. It describes how electrons, positrons, and photons behave quantum mechanically and interact with each other.

In the previous chapter we saw how Feynman diagrams are used to represent processes such as electron scattering, the Compton effect, or self-interactions. But these diagrams are not a substitute for the underlying theory—they are *derived tools*.

In this chapter, we go one step deeper: we look at the **mathematical structure** of QED. At the center is the so-called **Lagrangian density**, from which all predictions of the theory can be derived—from the structure of the interaction to the Feynman rules.

It turns out: The whole of Quantum Electrodynamics can be built from just a few principles—especially the requirement of **gauge invariance** and of **relativity**. These principles determine not only the form of the equations but also how the photon fits into the theory as a massless interaction particle.

The goal of this chapter is therefore to understand QED from its foundations—not as a collection of computational rules, but as an elegantly structured theory with enormous predictive power.

Note for Readers

This chapter introduces the mathematical structure of Quantum Electrodynamics (QED). It is aimed at readers interested in the theoretical derivation of photon interaction.

Readers primarily interested in experimental confirmation and applications may skip directly to Chapter 5.5—without losing the main thread of the book.

5.4.1 Basic Idea of QED as a Field Theory

Quantum Electrodynamics is a **field theory**. This means: it does not describe individual particles but fields that propagate through space and interact with each other.

An electron is not treated as a point particle but as an excitation of a *Dirac field*. Light—or more precisely: the photon—is likewise neither a wave nor a classical particle, but the excitation of an *electromagnetic field*, which itself is quantized.

Particles as Fields

Instead of describing the exchange of particles, QED works with fields such as:

- **Electron field** $\psi(x)$ (Dirac field)
- **Photon field** $A^\mu(x)$ (Four-potential)

The state of an electron (e.g., position, momentum, spin) is described by a wave function within the Dirac field. A photon, on the other hand, is an excitation of the quantized field A^μ—more precisely: a **Fock state** with one photon.

Interaction Through Coupling of Fields

The central idea of QED is that these two fields are *coupled* to each other. The interaction does not take place directly between particles, but through terms in the so-called **Lagrangian density**:

$$\mathcal{L}_{\text{int}} = -e\,\bar{\psi}\gamma^\mu\psi\,A_\mu$$

This expression describes that the electron field ψ couples to the electromagnetic field A_μ—in other words, it "feels" the presence of a photon. The coupling term contains exactly the structure that later appears in Feynman diagrams as a vertex.

Field Theory Instead of Particle Mechanics

In QED, electrons and photons are not treated as classical particles but as quantized fields. Their interaction arises from a coupling term in the Lagrangian density—not from a classical force.

Why This Approach Is Necessary

Classical electrodynamics (Maxwell's equations) cannot explain many phenomena:

- No quantized energy transfer
- No concept of the photon
- No way to describe scattering at the quantum level

Only a quantized field theory makes it possible to describe processes such as the photoelectric effect, Compton scattering, or electron–positron annihilation correctly.

Why Not Simply Classical?

Quantum field theory extends quantum mechanics to systems with variable particle number. Only in this way can creation and annihilation of photons or electrons be described rigorously. Without this approach, one could not derive Feynman diagrams—nor achieve the precision results of QED.

5.4.2 The QED Lagrangian Density

At the heart of QED is the so-called **Lagrangian density**, from which all equations of the theory can be derived. It contains both the free fields (electron, photon) and their interaction.
The full QED Lagrangian density reads:

$$\mathcal{L}_{\text{QED}} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e\bar{\psi}\gamma^\mu \psi A_\mu$$

Structure of the QED Lagrangian Density

- $\bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$: Dirac term for the free electron field
- $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$: Kinetic term for the photon (Maxwell part)
- $-e\bar{\psi}\gamma^\mu \psi A_\mu$: Coupling between electron field and electromagnetic field

Meaning of the Terms

Each part of the Lagrangian density corresponds to a component of the theory:
- The first term describes the **free Dirac field**—that is, an electron or positron without external influence. - The second term is the **free electromagnetic field strength**—the quantum version of the Maxwell field. - The third term describes the **interaction** between electron and photon—the core of QED.

Field Strength Tensor

The expression $F_{\mu\nu}$ is the so-called **field strength tensor**, defined as:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

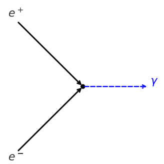
It contains both the electric and the magnetic field—packaged into a single relativistic object. The term $F_{\mu\nu}F^{\mu\nu}$ is invariant under Lorentz transformations.

Photon Field from the Lagrangian Density

The photon's quantum nature arises in QED from the quantization of the field A^μ within the Maxwell structure. The field strength tensor $F_{\mu\nu}$ contains the dynamics of the photon as a quantized gauge field.

Interaction as Coupling Terms

The interaction term
$-e\bar{\psi}\gamma^\mu\psi A_\mu$ describes the coupling of electrons to the photon. From this term follows directly the structure of the QED vertex in Feynman diagrams:



$$\Rightarrow \quad -ie\gamma^\mu$$

Figure V.6: Interaction term

From Lagrangian Term to Feynman Vertex

The coupling in the Lagrangian density generates the QED vertex: one electron, one positron, one photon meet at a point. This structure forms the basis of all diagrams in QED.

Gauge Invariance as a Principle

The form of the QED Lagrangian density is not chosen arbitrarily. It results from the requirement that the theory be **locally gauge invariant**—that is, symmetric under transformations of the form:

$$\psi(x) \to e^{i\alpha(x)}\psi(x), \qquad A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu \alpha(x)$$

Only with this structure does the theory remain consistent, relativistically invariant, and quantizable.

5.4.3 Coupling Between Electrons and Photons

In QED, the interaction between electron and photon does not arise "from the outside," but follows from a fundamental principle: **local gauge invariance**. This symmetry forces us to introduce the photon as the coupling partner of the electron field.

Global and Local Phase Transformations

A Dirac field ψ is invariant under a **global** phase transformation:

$$\psi(x) \to e^{i\alpha}\psi(x)$$

The Lagrangian density remains unchanged—this is a symmetry.
If, however, one demands that α depends on position, i.e.

$$\psi(x) \to e^{i\alpha(x)}\psi(x)$$

one speaks of a **local gauge transformation**. This destroys the invariance of the Lagrangian density—unless one introduces an additional field $A_\mu(x)$ that co-transforms:

$$A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu \alpha(x)$$

Covariant Derivative and Minimal Substitution

To keep the theory locally gauge invariant, one replaces the ordinary derivative in the Dirac Lagrangian by the **covariant derivative**:

$$\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu$$

This automatically generates an interaction term in the Lagrangian density:

$$\mathcal{L}_{\text{int}} = -e\,\bar{\psi}\gamma^\mu \psi A_\mu$$

Coupling From Principle

The coupling between electron and photon is not an added term, but follows necessarily from the demand for local gauge invariance. It uniquely determines the form of the interaction.

Physical Meaning

The expression $\bar{\psi}\gamma^\mu\psi$ is the **four-current density** of the electron field. The term A_μ couples to this current—exactly as a classical electromagnetic field couples to an electric current. The difference is that here both quantities are quantized.

Simple Picture: "Field Feels Field"

One can picture the process as follows: The electron field "feels" the photon field—because its derivative is modified. Where formerly only the gradient appeared, there is now a term that includes the photon as well.

The Force Arises From the Derivative

In QED, the electromagnetic interaction arises because the electron field responds to the *covariant* derivative. This derivative contains the photon—and with it, the coupling is generated.

5.4.4 Feynman Rules From the Lagrangian Density

Feynman diagrams are not mere sketches—they follow from the structure of the Lagrangian density. Each term in the QED Lagrangian density has a clear correspondence to a building block of the diagrams.

Basic Principle: Perturbative Expansion

Since the interaction in QED is weak (the fine-structure constant $\alpha \approx 1/137$ is small), physical quantities such as transition probabilities or scattering amplitudes can be expanded as a **series in the coupling constant**. Each term in this series corresponds to a Feynman diagram.

Elements and Their Meaning

From the Lagrangian density, the following correspondences emerge:

Feynman Rules of QED (Simplified)

- **Electron line:** $\dfrac{i(\not{p}+m)}{p^2-m^2+i\varepsilon}$ (Dirac propagator)
- **Photon line:** $\dfrac{-ig^{\mu\nu}}{q^2+i\varepsilon}$ (Photon propagator)
- **Vertex:** $-ie\gamma^\mu$
- **Each internal line:** Integration over four-momentum d^4p
- **Complete diagram:** Product of all factors, integration over loop momenta

Example: Single-Photon Exchange

The simplest application is Møller scattering (electron–electron scattering via a virtual photon). From the vertex $-ie\gamma^\mu$ and the propagators, the formula for the scattering amplitude results. The calculation follows directly from the Feynman rules—and yields measurable quantities such as cross sections and scattering angles.

Order of the Diagrams

The more vertices a diagram contains, the higher its order in e or in $\alpha = e^2/(4\pi\hbar c)$. Lower orders yield the main contributions, higher orders the corrections—in the form of loops (loop corrections).

Why the Rules Work

The Feynman rules arise systematically from the QED Lagrangian density by applying perturbation theory. They are therefore not a collection of arbitrary instructions—but the direct result of the structure of the theory.

Connection to Measurement

Every observable quantity—scattering angle, energy distribution, g-factor correction—can be calculated using the Feynman rules. The agreement with experiment makes QED the most precise theory in physics.

5.4.5 Quantization of the Electromagnetic Field

In classical electrodynamics, the electromagnetic field is described by the four-potential $A^\mu(x)$. But as long as this field only satisfies classical equations (e.g., Maxwell's equations), there are no photons. Only through **quantization** does the field become a quantum system—and the photon appears as an *excitation* of this field.

From Wave to Photon

The electromagnetic field consists of infinitely many oscillating degrees of freedom—similar to a guitar string with infinitely many possible overtones. Each mode can be quantized. In this way, a **Fock space** arises, describing states such as

$$|0\rangle, \quad |1_{\vec{k}}\rangle, \quad |2_{\vec{k}}\rangle, \quad \ldots$$

—that is, states with zero, one, or several photons of a given wave number $\vec{k}$.

The Photon as Excitation

A photon is nothing more than a state with a single excitation of the quantized field $A^\mu(x)$ in a particular mode. This perspective is fundamentally different from the classical view of a wave or particle.

What Is a Photon in QED?

A photon is the simplest excitation of the quantized electromagnetic field. It has no rest mass, but a definite energy $E = \hbar\omega$ and momentum $\vec{p} = \hbar\vec{k}$.

Polarization and Transversality

Because the photon is massless, it has only two physically observable polarization states—not three, as a massive spin-1 particle would have.

This is a direct consequence of gauge invariance and the transversality condition:

$$\vec{k} \cdot \vec{\epsilon} = 0$$

QED handles this correctly via the **Gupta–Bleuler formalism** or through Feynman gauges, in which unphysical degrees of freedom are carried along at first and later canceled.

Gauge Symmetry and Degrees of Freedom

Gauge symmetry makes it possible to eliminate certain components of the field A^μ through transformations. Only the transverse components remain physically relevant—and these are precisely the two polarization directions of the photon.

Why Doesn't the Photon Have a Spin-3 State?

A massive spin-1 particle would have three polarization states. But because the photon is massless and the electromagnetic field is gauge invariant, only two states remain. These correspond to linear or circular polarization.

5.4.6 Summary

Quantum Electrodynamics (QED) is the quantized field theory of electromagnetic interaction. Its mathematical structure rests on the following key elements:

- **Field description:** Electrons and positrons are described by the Dirac field $\psi(x)$, the photon by the four-potential $A^\mu(x)$.
- **Lagrangian density:** The QED Lagrangian density contains three components:
  - the free Dirac term for the electron field,
  - the free Maxwell term for the photon field (field strength tensor $F_{\mu\nu}$),
  - and the coupling term $-e\bar{\psi}\gamma^\mu\psi A_\mu$.
- **Coupling via gauge symmetry:** The form of the interaction term follows from the requirement of local gauge invariance. It requires the introduction of a covariant derivative.

- **Feynman rules:** From the Lagrangian density, the building blocks of Feynman diagrams can be derived: propagators, vertices, and integration rules.

- **Quantization of the photon field:** By quantizing the electromagnetic field, photons emerge as excitations—with two physical polarization states.

QED is internally consistent, relativistically invariant, and agrees with experimental reality in all tested domains. The explicit calculation of physical quantities proceeds via perturbation series and Feynman diagrams—this is the subject of the next chapter.

5.5 Precision Experiments

This section highlights the role of the photon in modern high-precision experiments. Such experiments provide not only impressive confirmations of Quantum Electrodynamics (QED), but also of special relativity and fundamental symmetries of nature. The photon is not only an object of observation but often also the central tool for probing the finest physical effects.

5.5.1 The Anomalous Magnetic Moment of the Electron

One of the most accurately tested results of QED is the so-called *anomalous magnetic moment* of the electron, i.e. the tiny deviation of the g-factor from exactly 2:

$$g_e \approx 2.00231930436...$$

This deviation results from quantum field–theoretic corrections, in particular from the exchange of virtual photons:

Physical Meaning

The difference between $g_e = 2$ and the experimentally measured value is explained by loop processes in Feynman diagrams, in which virtual photons mediate between the electron and itself. The agreement between theory and experiment is a triumph of QED.

5.5.2 Lamb Shift

Another spectacular example of experimental confirmation of QED is the *Lamb shift* of the energy levels in the hydrogen atom. The classical Dirac formalism predicts the same energy for the 2s and 2p levels. Experimentally, however, a tiny energy shift was found:

$$\Delta E_{\text{Lamb}} \approx 1057\,\text{MHz}$$

Why Is This Important?

The Lamb shift shows that empty space—the quantum vacuum—is not "empty." Virtual photons cause fluctuations that influence the energy levels.

5.5.3 Spectroscopy and Constant Measurements

Photon-based high-precision spectroscopy also provides the most accurate measurements of fundamental constants such as:

- the fine-structure constant α
- Planck's constant h
- the speed of light c (now fixed by definition)

These measurements rely on laser technology, frequency combs, and atomic clocks—all photon-based techniques.

5.5.4 Tests of Fundamental Symmetries

Photon experiments also contribute to testing fundamental symmetries:

- **CPT invariance:** Precision measurements on antihydrogen compare spectral lines with ordinary hydrogen.
- **Lorentz invariance:** Directional dependence of the speed of light is tested with resonator-based laser systems.

What If Light Were Not Isotropic?

If a directional dependence of the speed of light were measured, it would indicate a fundamental violation of Lorentz symmetry—with drastic consequences for relativity and our physical worldview.

5.5.5 Summary

Precision experiments are among the strongest pillars for testing our physical theories. They show how deeply anchored and reliable the concept of the photon is in modern physics. In particular, Quantum Electrodynamics demonstrates here its unparalleled accuracy—with the photon as both exchange particle

5.5.6 Summary

Precision experiments are among the strongest pillars for testing our physical theories. They show how deeply anchored and reliable the concept of the photon is in modern physics. In particular, Quantum Electrodynamics demonstrates its unparalleled accuracy—with the photon as both *exchange particle* and *information carrier*.

5.6 Conclusion

In Chapter V we have encountered the photon as the central mediator of the electromagnetic interaction within Quantum Electrodynamics (QED). Beginning with the transition from the classical light quantum to the quantized field, it became clear how deeply the structure of QED is intertwined with the photon.
We have seen:

- how the photon can be described mathematically as a vector field with gauge freedom,
- how virtual photons in Feynman diagrams mediate the interaction between charged particles,
- how the QED formalism is built from gauge symmetry, Lagrangian density, and perturbation theory,
- and how precision experiments—from the g-factor to the Lamb shift—confirm the theory with unprecedented accuracy.

QED ranks among the most successful theories in physics. It not only delivers precise predictions, but also a deep understanding of the role of the photon—as a massless yet powerful quantum object.

Outlook to Chapter VI

In the next chapter we explore applications of the photon in practice and research—from laser technology to quantum sensors and the role of photons in modern communication.

Chapter VI

Applications of the Photon

6.1 Introduction

Photons are not only fundamental carriers of quantum-physical properties – they also form the basis of countless applications in technology, research, and everyday life. This chapter presents selected fields of use where the control, detection, and utilization of individual photons play a central role. The range extends from laser technology and quantum sensing to medical imaging, optical communication, and astronomical observation. The goal is to make the physical principles behind these applications understandable and to show their significance for science and society.

6.2 Photons in Laser Technology

6.2.1 Operating Principle of Lasers

The term “laser” stands for **Light Amplification by Stimulated Emission of Radiation**. A laser does not produce light through incandescence or chemical reactions, but by the controlled amplification of photons in an active medium – based on the principle of *stimulated emission.*
The three central processes, theoretically described by Einstein already in 1916, are:

- **Spontaneous emission**: An excited atom decays to a lower state without external influence and emits a photon.

- **Absorption**: A photon excites an atom in the ground state to a higher energy state.
- **Stimulated emission**: A photon interacts with an already excited atom – which then emits a second photon, phase-coherent with the first.

For effective light amplification, stimulated emission must dominate. This requires a so-called *population inversion* – i.e., more atoms in the excited than in the ground state, which under normal conditions is not the case.

Stimulated Emission as the Basis of Lasers

If a photon of the right energy strikes an excited atom, it can force the atom to emit a second photon. Both photons are:

- **phase-coherent** (same wave phase),
- **frequency-identical** (same energy), and
- **propagating in the same direction**.

This property enables the controlled amplification of light – the operating principle of the laser.

6.2.2 Structure and Types

A laser essentially consists of three functional components:

- **Active medium:** A material whose atoms or molecules can be brought into excited states by energy supply (pumping). This is where stimulated emission occurs.
- **Pump unit**: Provides the necessary energy to achieve a population inversion. Possible methods include optical, electrical, or chemical pumping.
- **Resonator**: Two mirrors that reflect light back and forth. One mirror is partially transparent, allowing part of the amplified light to exit as the laser beam.

Variety of Laser Types – An Overview

Lasers are mainly distinguished by their active medium:

- **Solid-state lasers** (e.g., Nd:YAG): High power, used in industry and medicine.
- **Gas lasers** (e.g., helium–neon, CO_2): Stable and precise, e.g., for surveying.
- **Semiconductor lasers** (e.g., laser diode): Compact, efficient, e.g., in CD/DVD players or laser pointers.
- **Fiber lasers**: Amplify light in an optical fiber – high beam quality with robust design.
- **Dye lasers**: Particularly flexible in wavelength, using organic molecules as medium.

6.2.3 Applications in Technology and Research

Applications of Lasers — Technology (Overview)

- **Materials processing (cutting, welding, hardening):** High power density, precise focus, narrow heat-affected zone, easily automated.
- **Additive manufacturing (SLS/SLM, PBF):** Point-by-point melting for complex, high-strength parts with minimal material use.
- **Measurement technology & metrology (interferometry, laser tracker):** Precision length/angle measurements down to the nanometer scale due to coherence and stability.
- **LIDAR & distance measurement:** Fast 3D mapping and robust ranging (autonomous driving, drones, geodesy).
- **Fiber-optic communication:** Very high data rates over long distances (DWDM, coherent transmission).
- **Surface structuring & lithography:** Direct micro-/nanopatterning ("laser direct write"), prototyping, micromechanics.
- **Process analytics (Raman, LIBS):** Contactless, fast material sensing inline without sample prep.
- **Holography, displays & projection:** High contrast, wide color gamut, real holograms and AR optics.
- **Barcode scanning & sensors:** Fast, high-contrast scanning in retail, logistics, and automation.
- **Alignment & adjustment:** The laser beam as a precise reference line in construction and mechanical engineering.

Applications of Lasers — Research (Overview)

- **Spectroscopy (absorption, fluorescence, Raman, CARS):** Narrowband/tunable with high selectivity for structural and molecular information.
- **Optical tweezers & micromanipulation:** Contactless control of particles or cells using strongly focused beams.
- **Atomic physics (laser cooling, MOT, BEC):** Resonant cooling down to nK and precise control of neutral atoms.
- **Precision metrology (optical clocks, frequency combs):** Extremely stable frequencies and new time/length standards.
- **Nonlinear optics & attosecond physics:** Frequency conversion, high harmonics, and ultrafast dynamics.
- **Quantum optics & quantum communication (QKD):** Single photons, entanglement, secure key distribution.
- **Laser–plasma, accelerators, fusion:** Extreme intensities for dense plasmas, compact accelerators, and inertial confinement fusion.
- **Atmosphere & astronomy (lidar, laser guide stars):** Probing aerosols/winds; adaptive optics for large telescopes.
- **Biomedical imaging (two-photon, STED):** Deep, gentle, and super-resolution microscopy.
- **Femtochemistry & pump–probe:** Making reaction dynamics visible in real time.

6.3 Photon Detectors and Sensing

The detection of individual photons is a key technology in modern physics and engineering. Whether in astrophysics, quantum optics, medical imaging, or industrial quality control – sensitive and precise sensors determine the reliability of an experiment or the success of an application. This section introduces central types of photon detectors, their operating principles, and typical applications.

6.3.1 Photomultipliers and Semiconductor Detectors

Photomultiplier tubes (PMTs) exploit the photoelectric effect: An incoming photon knocks an electron out of a photocathode. This electron is multiplied in a cascade of dynodes and detected as a macroscopically measurable electrical signal. PMTs offer:

- high gain (up to 10^6–10^8),
- extremely high sensitivity in the UV to visible range,
- fast response times in the nanosecond range.

Disadvantages include sensitivity to magnetic fields, the requirement of high voltage, and the mechanical fragility of the glass tubes.

Semiconductor detectors, in particular avalanche photodiodes (APDs) and single-photon avalanche diodes (SPADs), also work via the photoelectric effect, but within a semiconductor material. Advantages:

- compact design, robust, and easily integrable,
- good quantum efficiency (often exceeding 50%),
- possibility of array integration (pixel detectors, SiPMs).

They are the foundation of modern photonic sensor arrays in research and industry.

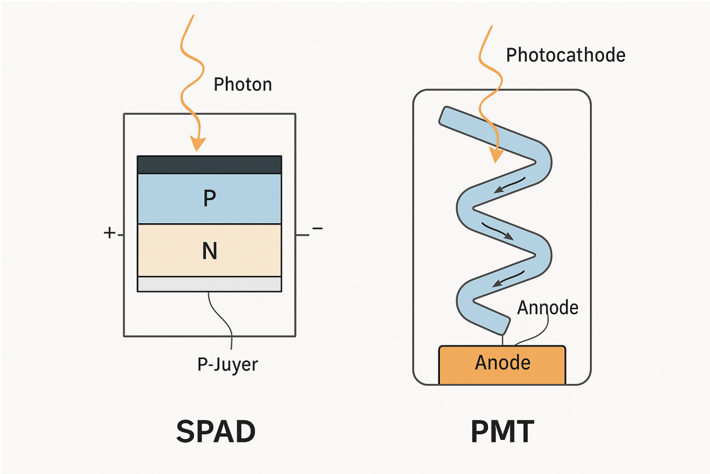


Figure VI.1: Schematic comparison of a SPAD (Single-Photon Avalanche Diode, left) and a PMT (Photomultiplier Tube, right).

Didactic Comparison: SPAD vs. PMT

Single-Photon Avalanche Diode (SPAD)	**Photomultiplier Tube (PMT)**
Semiconductor device with a special PN junction.	Vacuum tube with photocathode and amplification stages.
A photon generates an electron–hole pair in the depletion region.	A photon knocks an electron out of the photocathode.
The electron triggers an avalanche breakdown – producing an electrical pulse.	The electron is accelerated and multiplied across multiple dynodes.
Very high time resolution, ideal for compact, integrated quantum sensors.	High gain, ideal for weak light sources (e.g., scintillators).
Operates at room temperature, easily integrated into chips.	Requires high voltage, sensitive to magnetic fields.
Well suited for arrays and imaging systems (SiPM).	Traditionally used in nuclear physics, astronomy, PET, etc.

Concept: Avalanche Breakdown

In an **avalanche breakdown**, a free electron in the strong electric field of a semiconductor generates additional electrons through impact ionization. This process multiplies like an avalanche and produces a measurable current pulse – enabling the detection of a single photon.

Concept: Scintillator

A **scintillator** is a material that emits light when struck by ionizing particles. This so-called *scintillation light* can be registered by photon detectors such as PMTs or SiPMs, enabling the indirect measurement of radiation.

6.3.2 Single-Photon Counting and Quantum Efficiency

The ability to **detect individual photons** is a milestone in quantum technology. Here, not only "counting" is important, but also the **quantum efficiency**, i.e., the fraction of incident photons that actually produce a measurable signal.

Physical Terms

- **Single-photon counting:** Detection methods in which each registered signal can be assigned to a single photon.
- **Quantum efficiency (QE):** Ratio of registered to incident photons, typically wavelength-dependent.

Technically, SPADs or superconducting nanodetectors are most commonly used. The latter achieve quantum efficiencies close to 100%, but must be operated cryogenically.

How Does Single-Photon Counting Work?

In single-photon counting, each individual photon pulse is registered as a discrete event. The detector (e.g., a SPAD) generates an electrical pulse when a photon arrives – typically a short voltage or current spike. These pulses are electronically processed:

- Each pulse corresponds to one detected photon.
- A counter sums these pulses over defined time intervals.
- The result is a photon count per interval – e.g., "125 photons in 1 ms."

For this to work reliably, the pulses must:

- exceed a clear **threshold** (threshold detection),
- be temporally well separated (observe dead time),
- and not be confused with **thermal noise** or dark current.

What Counts as a Photon?

Not every registered pulse originates from a photon. Detectors also produce *dark counts* (false signals without incoming light). High-quality single-photon detectors, however, achieve dark count rates below 100 pulses per second and quantum efficiencies of 50–90%.

6.3.3 Applications in Industry and Fundamental Physics

The applications of photonic sensing are diverse:

- **Fundamental physics:** Detection of individual photons in quantum optics experiments (e.g., antibunching, HOM effect), particle detection in high-energy physics, astronomical observations (e.g., Hubble, JWST).
- **Industrial applications:** Quality control, laser scanners, light curtains, position sensors, photon spectroscopy.
- **Medicine and life sciences:** Fluorescence microscopy, PET scanners, optical tomography.

Note on the Importance of Detection Technology

The continuous improvement of photon detection is a prerequisite for progress in quantum communication, medical imaging, and fundamental research. Many developments in quantum technology directly depend on the ability to detect photons precisely and with minimal loss.

6.4 Photons in Medical Imaging

Photons play a central role in many imaging techniques of modern medicine. Their range of use extends from high-energy X-rays to gentle visible laser light in optical diagnostics. Different physical processes – absorption, scattering, fluorescence, or emission – are employed to make contrast in tissue visible or to identify pathological structures.

6.4.1 X-ray, CT, and PET

X-rays are based on high-energy photons, which generate an image through absorption and attenuation when passing through the body. Denser tissue (e.g., bone) absorbs more photons and appears brighter on the detector.

Computed tomography (CT) extends this technique by reconstructing a 3D image from many X-ray projections. Rotating X-ray sources and detector arrays are used for this purpose.

Positron emission tomography (PET) uses a different mechanism: Radiopharmaceuticals emit positrons, which annihilate with electrons – producing two photons with an energy of 511 keV emitted in opposite directions. These are registered by ring detectors, and from the coincidence of detection, the origin is reconstructed.

Photons in PET

In PET, two gamma photons of 511 keV are created in the annihilation of an electron and a positron. These photons leave the body almost unhindered and allow for precise localization of metabolism – e.g., in tumors.

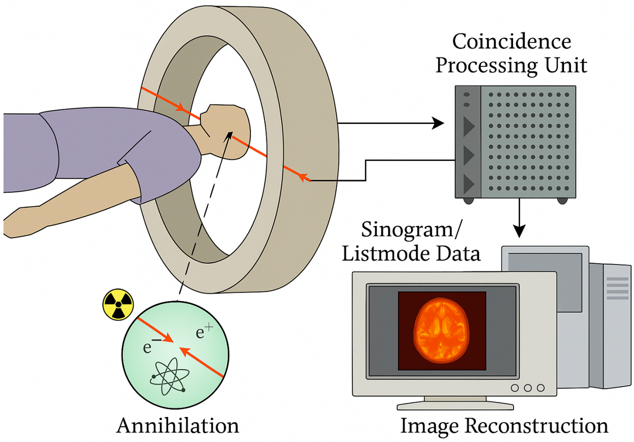


Figure VI.2: Principle of positron emission tomography (PET): A radiopharmaceutical administered into the body emits a positron, which annihilates with an electron. Two gamma photons of 511 keV each are emitted in opposite directions and registered by detectors in the PET ring. From the coincidence of signals, the point of origin is reconstructed.

Concept: Annihilation

In **annihilation**, a particle and its antiparticle collide and annihilate each other. Their mass is fully converted into energy – usually in the form of photons. In PET, annihilation of an electron with a positron produces a photon pair of 511 keV each.

Concept: Radiopharmaceutical

A **radiopharmaceutical** is a radioactively labeled compound administered specifically into the body. In PET, for example, [^{18}F]FDG is used – a sugar-like substance that accumulates in metabolically active tissues (such as tumors). The positrons produced there provide the image signal through annihilation.

6.4.2 Optical Tomography and Fluorescence Imaging

In contrast to ionizing radiation, these techniques rely on visible or near-infrared light.

Diffuse optical tomography (DOT) uses light sources and detectors on the skin surface. Photons penetrate the tissue, are scattered and absorbed. From the light distribution, the optical properties inside can be reconstructed.
Fluorescence imaging employs substances (fluorophores) that are excited by light and re-emit at a different wavelength. This method allows, for example, the visualization of molecular processes or tumor markers.

Advantage of Optical Methods

Optical methods are non-invasive, free of ionizing radiation, and are particularly suitable for surface and functional imaging – e.g., in infants, neurodiagnostics, or molecular imaging.

6.4.3 Lasers in Surgery and Diagnostics

Lasers are used both for **imaging** and for **tissue interaction**:

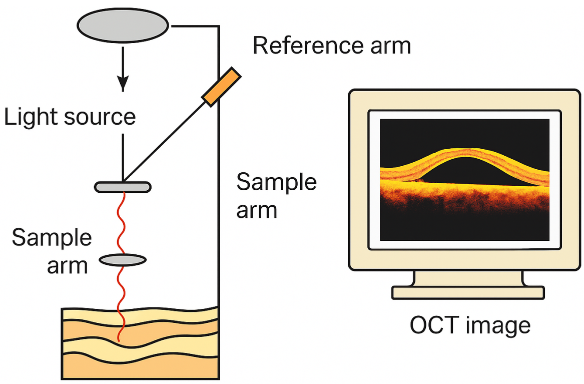
- **Diagnostic:** Confocal laser scanning microscopy, optical coherence tomography (OCT), spectroscopy.
- **Surgical:** Tissue cutting, coagulation, ablation – depending on wavelength, pulse duration, and power.

Laser surgery exploits the precise focusing of photons for targeted tissue treatment without mechanical stress. Typical applications are:

- Ophthalmology (e.g., LASIK),
- Dermatology (e.g., tattoo removal),
- Tumor surgery (precise cuts with minimal blood loss).

Laser Parameters

The effect of a laser strongly depends on its wavelength, pulse duration, energy, and focusing. Short pulses with high power allow precise cuts without damaging the surrounding tissue.



OCT Principle

Figure VI.3: Schematic representation of optical coherence tomography (OCT): Light from a broadband source is split by a beam splitter. One part hits a movable reference mirror (reference arm), the other the tissue to be examined (sample). The reflected light portions interfere, and from the interference patterns the depth profile is reconstructed. The result is a high-resolution image, e.g., of the retina.

Didactic Explanation: Interference in OCT

Interference occurs when two light waves overlap – depending on the phase, they either reinforce or cancel each other. In OCT, this property is used to determine the position of reflecting structures in tissue with high precision.
Only when the optical path lengths in the *sample arm* and the *reference arm* are nearly equal does an interference signal occur. From the measured intensity as a function of the reference arm position, an **A-scan** – a depth profile along a line in the tissue – is obtained.
By laterally shifting the measurement beam, many A-scans next to each other are recorded, which together form a **B-scan** (2D cross-section). This produces a detailed image of the examined tissue – e.g., of the retina.

6.5 Photons in Communication

Photons form the backbone of modern data transmission – both in classical fiber optic networks and in future quantum-based systems. The low-loss propagation of light in optical media, combined with high bandwidth and low latency, makes photonic communication technology a decisive factor in global information infrastructure.

6.5.1 Optical Fiber and Optical Networks

Optical fibers guide light by total internal reflection in a thin quartz core. Photons can thus be transmitted over distances of several hundred kilometers with minimal attenuation.
Advantages:

- extremely high data rates (terabits/s),
- immunity to electromagnetic interference,
- low attenuation (e.g., 0.2 dB/km at 1550 nm).

Optical networks use wavelength division multiplexing (DWDM), amplifiers (EDFAs), and modulated laser systems to transmit large amounts of data in parallel and efficiently – from backbone networks to fiber-to-the-home connections.

Total Internal Reflection in Optical Fibers

Light guidance in an optical fiber is based on total internal reflection at the interface between the higher-refractive-index core and the cladding. The critical angle determines whether the light remains "trapped" inside the core.

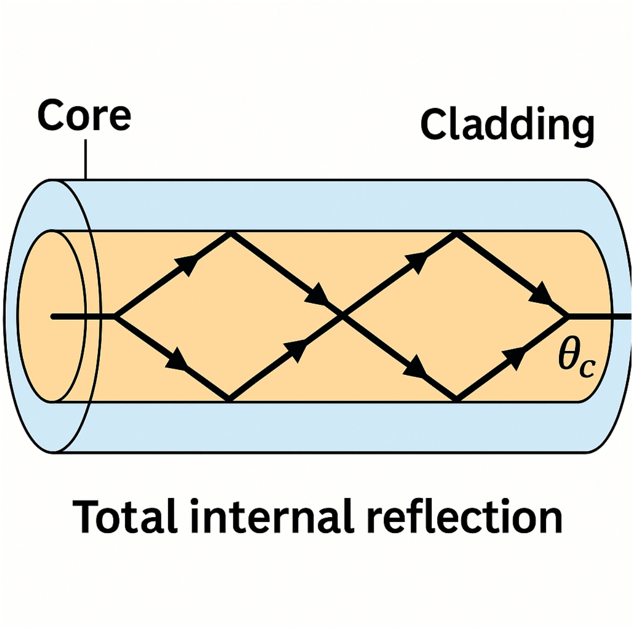


Figure VI.4: Schematic illustration of total internal reflection in an optical fiber: Light (photons) is guided in the higher-index core and fully reflected at the interface with the cladding. This keeps the light “trapped” in the core even over long distances.

6.5.2 Quantum Communication and QKD

Quantum communication exploits quantum states of photons – such as polarization or entanglement – to transmit information in a way that is absolutely secure against eavesdropping.
QKD (Quantum Key Distribution) is a practical application: Two parties (Alice and Bob) exchange a secret key. Any eavesdropping attempt (Eve) inevitably alters the quantum state of the photons and can thus be detected.

What Makes QKD Secure?

The security relies on two quantum-physical principles:

- **No-cloning theorem:** Unknown quantum states cannot be perfectly copied.
- **Measurement disturbance:** Any measurement affects the state – an eavesdropping attempt measurably disturbs the system.

BB84 is the first and best-known QKD protocol. It uses random polarizations and basis changes to generate a key, which is then verified over a classical channel.

How Does QKD Work (e.g., BB84)?

In the BB84 protocol, Alice sends single photons whose polarization is randomly chosen – e.g., horizontal (→), vertical (↑), diagonal (↘), or anti-diagonal (↖). Bob also measures in random bases. Only when sender and receiver bases match does a valid bit result. After the exchange, Alice and Bob (publicly) compare the bases used and keep only the matching ones. From these bits, the key is generated.
If a photon is intercepted, its polarization changes – allowing Alice and Bob to detect disturbances in the key and reveal eavesdropping attempts.

6.5.3 Photonic Chips and Future Systems

With the advent of **photonic chips**, light signals are no longer guided solely through optical fibers but directly through integrated optical circuits. These enable:

- compact, energy-efficient data processing,
- light-based logic and modulation systems,
- new concepts for neural networks and AI accelerators.

Photonic chips combine lasers, modulators, waveguides, and detectors on a single substrate – often silicon photonics.

Future of Photonic Communication

Photon-based communication is not only faster than classical electronics – it will become a prerequisite for secure quantum networks, light-based processors, and global, tap-proof communication.

6.6 Photons in Astronomical Observation

The observation of photons from space is the foundation of modern astronomy. Since photons – unlike, for example, gravitational waves or neutrinos – are comparatively easy to detect, they provide most of our information about the universe. Whether visible light, radio waves, or high-energy gamma rays – every photon reaching Earth carries a message from space and time.

6.6.1 Detection of Photons from Space

Telescopes and detectors on Earth and in space measure photons across various wavelength ranges:

- **Optical range:** CCDs (charge-coupled devices), CMOS sensors
- **Infrared and radio:** Bolometers, radio telescopes
- **X-rays and gamma rays:** Space telescopes with scintillators and semiconductor detectors

The analysis of these photons provides information about temperature, motion (Doppler shift), composition, and distance of cosmic objects.

Why Don't All Photons Reach Earth?

The Earth's atmosphere is opaque to many wavelength ranges – especially UV, X-rays, and gamma rays. Therefore, corresponding telescopes must be positioned outside the atmosphere – e.g., in Earth orbit.

6.6.2 Adaptive Optics, Spectroscopy, and Telescopes

Telescopes collect photons and focus them onto detectors. Modern large telescopes (e.g., the VLT or ELT) use adaptive optics to improve image quality:
- **Adaptive optics** compensates for atmospheric distortions in real time using deformable mirrors. - **Spectroscopy** disperses light into its wavelengths and allows the analysis of chemical composition, temperature, and motion of celestial bodies. - **Interferometry** combines several telescopes into a virtual giant telescope with extremely high angular resolution.

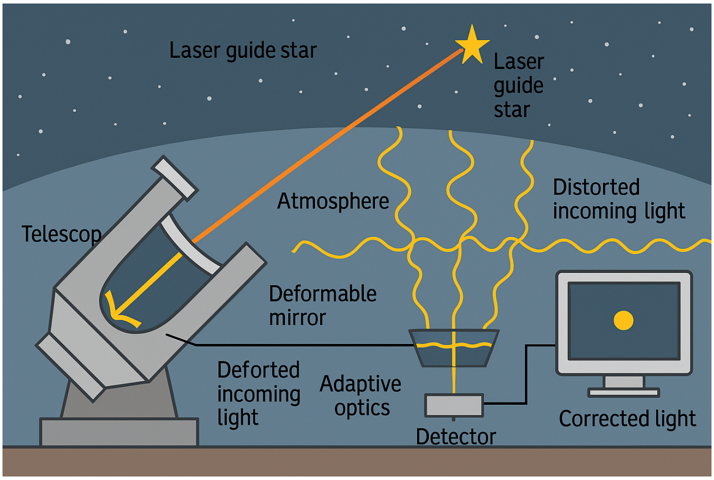


Figure VI.5: Schematic illustration of a telescope with adaptive optics. A wavefront sensor analyzes distortions caused by the atmosphere. A deformable mirror surface corrects these in real time, producing a sharp image.

What Does a Spectrum Show?

A **spectrum** is the distribution of photon intensity as a function of wavelength. Typical features include:

- **Emission lines** $\rightarrow$ hot, radiating gases,
- **Absorption lines** $\rightarrow$ cold gases in front of hot sources,
- **Redshift** $\rightarrow$ motion of the object away from the observer.

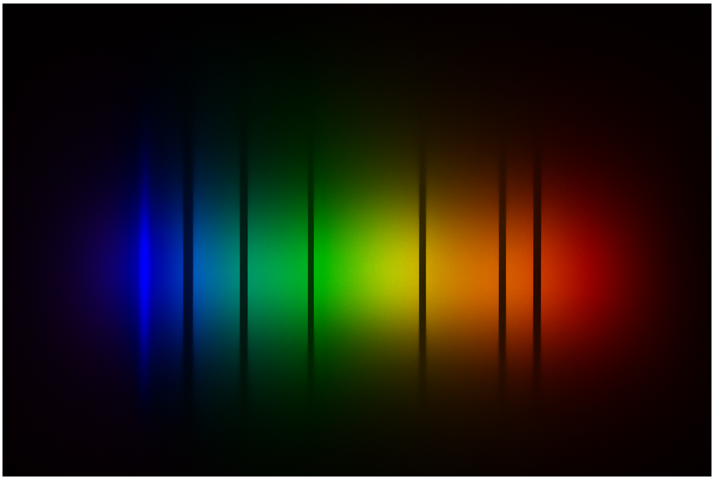


Figure VI.6: Emission spectrum of glowing hydrogen gas.

Spectral Lines and Redshift

- **Emission lines** arise when atoms emit specific photons. These lines are bright and characteristic of the respective element.
- **Absorption lines** occur when photons of certain frequencies are absorbed by a gas through which the light passes – they then appear missing in the spectrum.
- **Redshift:** When a light source moves away, all lines shift to longer wavelengths – toward red. This is used to measure cosmic motion.

6.6.3 Photons in Gravitational Wave Astronomy

Gravitational waves are fundamental distortions of spacetime – triggered by accelerated masses, such as the merger of black holes. They are not electromagnetic waves and are not made of photons. Nevertheless, it is precisely photons that enable their detection: as precise probes in highly sensitive interferometers.

LIGO, Virgo, and other gravitational wave detectors use kilometer-scale laser interferometers to measure minute changes in spacetime. Laser beams (consisting of coherent photons) are sent into two perpendicular arms, reflected by mirrors, and recombined at the output. A passing gravitational wave slightly changes the arm lengths – and thus the interference pattern.

Photons as a Tool for Measuring Spacetime Curvature

Laser interferometers such as LIGO use photons to detect differences in arm length as small as 1×10^{-19} m – about a thousand times smaller than a proton. The time difference a photon needs to traverse the two arms changes measurably due to a passing gravitational wave.

This technique relies on the interference of coherent light waves. When the two beams are recombined after traveling through the arms, constructive or destructive interference occurs depending on the phase shift. Even the smallest changes in optical path length affect this pattern. In this way, even minute curvatures of spacetime can be detected.

Thanks to this photon-based precision measurement, events such as black hole mergers, neutron star collisions, or traces of the cosmic gravitational background have been experimentally confirmed. Photons thus make visible what would otherwise lie beyond direct observation – further evidence of their central role in modern physics.

How Does an Interferometer Work?

An interferometer uses the principle of superposition (interference) of light waves to make the smallest length differences visible.

- A laser beam is split into two partial beams using a beam splitter.
- These travel along two perpendicular arms to mirrors, are reflected, and recombined.
- If the optical path lengths of the two arms are exactly equal, the waves cancel each other partly or completely – the interference pattern is constant.
- If one arm length changes slightly (e.g., due to a gravitational wave), the phase of one wave shifts, and the interference pattern measurably changes.

Thus, even length changes smaller than an atom's diameter can be detected by analyzing the light intensity at the detector. Light serves as a precise "measuring stick" in space.

Why Are Photons So Precisely Measurable?

Photons are ideal measurement tools – for several reasons:

- **High coherence:** Laser light consists of coherent photons – i.e., waves with exactly the same frequency and stable phase. This enables extremely sensitive interference effects.
- **Low interaction:** Photons hardly interact with matter. This allows long propagation paths without disturbance or deflection.
- **Quantum nature:** Single photons are discrete quantum objects. Their detection produces clear, countable signals – ideal for precise time or position measurements.
- **Speed of light as a constant:** The velocity of photons in a vacuum is constant. This makes them natural "standards" for time and length.

These properties make photons indispensable in modern metrology – from interferometers to quantum sensors to optical atomic clocks.

6.6.4 Conclusion

Photons are not only central objects of modern physics but also indispensable tools in science, engineering, medicine, and communication. This chapter has shown how diverse and precise photons can be controlled, generated, and detected:

- In **laser technology**, stimulated emission and coherent light amplification enable applications ranging from materials processing to quantum optics.
- **Photon detectors** such as SPADs or PMTs allow single-photon counting with high quantum efficiency – a foundation for quantum technologies and precise imaging.
- In **medical diagnostics**, photons are used for imaging with high spatial resolution and minimal exposure – from X-rays to fluorescence microscopy.
- **Optical communication** uses photons for fast, low-loss, and secure data transmission – in fibers as well as in quantum communication systems.
- In **astronomical observation**, photons provide crucial information about the structure, motion, and composition of the universe – up to the detection of gravitational waves through interferometric measurement.

The ability to deliberately generate, guide, and measure single photons marks a technological turning point: From classical applications to quantum information technology, the photon opens new horizons – in research, industry, and society.

Chapter VII

Photons and the Future of Physics

7.1 Introduction

Photons not only shape today's technology but also open doors to entirely new areas of research. While they are already indispensable in communication, metrology, and data processing, we are also at the beginning of an era in which photons take on central roles in photonic computers, quantum communication networks, and the most precise experiments in fundamental physics. This chapter spans the arc from current developments in photonics to forward-looking applications and the great open questions of physics — from the search for the hypothetical graviton, to the puzzle of dark energy, to possible extensions of the Standard Model. The topics presented show how the smallest quantum of light could become the key to the technologies and discoveries of tomorrow.

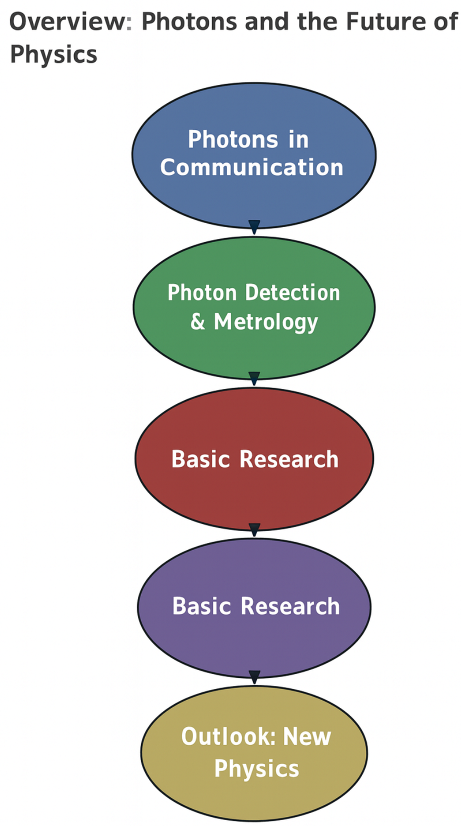


Figure VII.1: Overview: Photons and the Future of Physics. The graphic shows the main themes of this chapter in a vertical arrangement, from photonic logic to open questions of new physics.

7.2 Single Photons and Quantum Information

The ability to generate, control, and detect individual photons has taken on a key role in quantum information technology in recent years. Single photons serve as ideal carriers of quantum information: they are robust against disturbances, travel at the speed of light, and can be prepared in well-defined quantum states.

In classical information technology, bits are stored and processed in the form of electrical voltages or magnetic states. In quantum information, by contrast, the smallest unit — the *qubit* — is based on the coherent superposition of states. For photons, these states are often realized through polarization directions, time bins, or path information.

What Makes a Photon an Information Carrier?

Photons can carry different physical degrees of freedom that are usable as qubits:

- **Polarization:** Horizontal ($|H\rangle$) and vertical ($|V\rangle$) as basis states.
- **Time bins:** Early and late arrival times as logical states.
- **Path:** Two different optical paths in an interferometer.
- **Frequency:** Different spectral modes for encoded information.

These degrees of freedom can be precisely controlled and transmitted over long distances without destroying the quantum information.

7.2.1 Generation of Single Photons

Single-photon sources are central building blocks of quantum optics. Common methods include:

- *Spontaneous parametric down-conversion* (SPDC) in nonlinear crystals.
- *Single atoms* or ions in optical traps.
- *Quantum dots* in semiconductors.

The goal is to generate photons with high purity (low multi-photon probability) and high indistinguishability.

What Does "Indistinguishability" Mean?

In quantum optics, *indistinguishability* means that two photons are identical in all physical properties:

- same frequency (energy)
- same polarization
- identical spatial mode (same optical path)
- identical arrival time within the coherence time

Only if photons are perfectly indistinguishable can they exhibit quantum interference effects such as the *Hong–Ou–Mandel dip*.

The Hong–Ou–Mandel Dip Phenomenon

The *Hong–Ou–Mandel dip* experiment is a central method for verifying the *indistinguishability* of two photons. When two indistinguishable photons simultaneously strike a 50:50 beam splitter, they always leave it together through the same output — a purely quantum interference effect. The depth of the measured "dip" in the coincidence rate is a direct measure of the indistinguishability of the photons. A detailed description and figure can be found in **Chapter IV.5**.

7.2.2 Decoherence and Sources of Error

Quantum information is sensitive to disturbances. For photons, the main causes of decoherence are:

- Interaction with the transmission medium (e.g., fiber attenuation, scattering).
- Phase noise due to temperature fluctuations and vibrations.

Error correction and quantum repeaters are necessary to preserve quantum information over long distances.

7.2.3 Applications

Single photons form the basis for:

- Quantum cryptography.
- Quantum communication via satellites.
- Hybrid systems in which photons serve as interfaces between matter qubits.

These applications mark the beginning of a new era of information processing, in which photons are not only carriers but also mediators between different quantum systems.

7.3 Quantum Cryptography and Quantum Communication

The security of classical communication systems is based on mathematical methods whose safety relies on the practical impossibility of certain computations — such as factoring large numbers in RSA. This security may be threatened by future quantum computers. Quantum cryptography, on the other hand, uses fundamental physical laws to ensure security — independent of an attacker's computational power.

Core Principle of Quantum Cryptography

Quantum cryptography rests on two central features of quantum mechanics:

1. **Measurement disturbance:** Any attempt to measure a quantum state changes it (Heisenberg uncertainty principle).
2. **No cloning:** Unknown quantum states cannot be copied without error (no-cloning theorem).

Thus: eavesdropping inevitably leaves traces that can be detected by the legitimate communication partners.

7.3.1 Quantum Key Distribution (QKD)

The best-known protocol is **BB84** (Bennett & Brassard, 1984). Here, single photons are transmitted in random polarization states. The process in short:

- Sender (*Alice*) randomly chooses one of two possible bases (e.g., horizontal/vertical or diagonal).
- Receiver (*Bob*) measures in randomly chosen bases.
- After the transmission, both compare their basis choices publicly and discard measurements where the bases don't match.
- From the remaining data, a shared key is extracted.

An eavesdropper (*Eve*) introduces extra errors that can be detected statistically.

7.3.2 Quantum Communication over Large Distances

The range of direct quantum communication is limited by losses in optical fibers and atmospheric disturbances. Possible solutions include:

- **Quantum repeaters:** Nodes that generate entangled photon pairs and distribute them over large distances.
- **Satellite-based quantum communication:** Avoiding fiber losses by free-space transmission (e.g., the Chinese quantum communication satellite *Micius*).

7.3.3 Applications and Outlook

Quantum cryptography is currently being tested for highly secure government and financial communications. In combination with classical networks, hybrid systems are emerging that will enable long-term secure communication even in the age of quantum computers. Remarkably, quantum physics plays a double role here: On one hand, quantum computers threaten the security of today's encryption methods through their computational power. On the other, the very same physics provides with quantum cryptography an entirely new approach that allows in principle eavesdrop-proof communication. This interplay between challenge and solution makes quantum communication one of the most exciting research fields in modern physics.

7.4 Photonics as a Future Technology

Photons are not only fundamental carriers of information in quantum physics but also the basis of numerous modern technologies. *Photonics* encompasses all technologies based on the generation, control, and detection of light — from lasers in medicine to fiber optics in global communication networks.

What Does "Photonics" Mean?

The term *photonics* describes the engineering and technological use of photons — analogous to electronics, which deals with electrons. Photonics includes:

- Light sources (lasers, LEDs, quantum light sources)
- Light guidance (fiber optics, photonic chips)
- Light detection (cameras, photon detectors)
- Light manipulation (modulators, filters, nonlinear crystals)

7.4.1 Photonics in Communication

In telecommunications, photonics is increasingly replacing electronics to handle the growing volumes of data. Fiber networks transmit information at the speed of light with minimal energy loss. Photonic switches and routers on microchips promise ultra-fast signal processing directly with photons.

7.4.2 Photonics in Medicine

Photon-based methods such as laser surgery, optical imaging (OCT), and fluorescence diagnostics have revolutionized medicine. Future developments include minimally invasive operations with ultrashort laser pulses and photonic biosensors for real-time diagnostics.

7.4.3 Photonics in Sensing and Metrology

Photonic sensors enable high-precision measurements in industry, geoscience, and space research. Examples include LIDAR systems for autonomous driving and interferometric gravitational-wave detectors.

Photonic Circuits vs. Electronic Circuits

Photonic circuits offer several decisive advantages over electronic approaches:

- **Higher speed:** Light moves through a medium much faster than electrons through conductors.
- **Lower losses:** No ohmic heating from electrical resistance.
- **Greater bandwidth:** A photon signal can carry many wavelengths simultaneously (multiplexing).
- **Low crosstalk:** Hardly any electromagnetic interference between adjacent lines.

These features make photonic circuits a key factor for future high-speed and high-bandwidth technologies.

7.4.4 Outlook

Photonics is considered a key technology of the 21st century. Its combination with quantum physics — in quantum communication, quantum computers, or quantum metrology — promises entirely new applications. The development toward integrated photonic circuits could trigger a transformation in information processing similar to that of microelectronics in the 20th century.

7.5 Optical Logic and Photonic Computers

The miniaturization of electronic circuits is increasingly reaching physical limits: transistors are becoming so small that quantum and thermal effects impair their function. At the same time, the energy demand of modern data centers is rising rapidly. A promising alternative is to use photons instead of electrons for information processing.

Why Photons Are Attractive for Logic Circuits

- **High speed:** Light moves nearly at the speed of light — optical signals can be processed extremely fast.
- **No ohmic losses:** Unlike electric currents, optical lines hardly heat up.
- **Parallel processing:** Multiple wavelengths can be used simultaneously through multiplexing.
- **Direct coupling to fiber communication:** No conversion between electron and photon signals needed.

7.5.1 Basic Principle of Optical Logic Gates

Optical logic circuits work with components that redirect, attenuate, or amplify light beams depending on input conditions. Examples include nonlinear crystals, optical modulators, or photonic crystal structures. Logic gates such as AND, OR, and NOT can be realized via interference, absorption, or polarization changes.

From Electronics to Photonics

In electronics, logic gates are based on transistors that block or allow current flow. In photonics, components such as Mach–Zehnder interferometers or microresonators take on this role — but for light.

7.5.2 Photonic Computers

A photonic computer uses optical circuits for central computational operations. This technology is particularly suitable for:

- **Artificial intelligence:** Matrix multiplications can be carried out extremely fast and energy-efficiently in optical networks.
- **Signal processing:** Broadband processing without electrical bottlenecks.
- **Quantum information processing:** Combination of photonic logic and quantum bits (qubits).

What If Optical Computers Replaced Electronics?

If photonic computers could fully replace electronics, the energy consumption of large data centers could be drastically reduced. At the same time, clock rates in the terahertz range might be achieved — far beyond today's processors.

Optical logic gates can also be realized with a Mach–Zehnder interferometer (MZI). A beam splitter divides the laser beam into two paths, each containing a phase modulator. Only if both modulators apply a certain phase shift do the beams interfere at the second beam splitter in such a way that light appears at the desired output. Choosing the phases so that this happens only when both inputs are active makes the MZI work like a classical AND gate — but in a purely optical way.

Photonic AND Gate in a Mach–Zehnder Interferometer

A Mach–Zehnder interferometer can be configured to work as an AND gate. The inputs A and B control phase modulators in the two arms of the interferometer. Only if both apply a phase shift of π, the phases add up to 2π, resulting in constructive interference at the "1" output.

A	B	Phase A	Phase B	Output "1"	Output "0"
0	0	0	0	1	0
0	1	0	π	0	1
1	0	π	0	0	1
1	1	π	π	1	0

Only for $A = 1$ and $B = 1$ is the total phase 2π, so the upper output becomes bright. In all other cases, the light is directed into the "0" output.

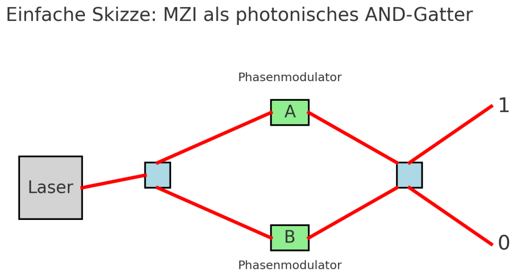


Figure VII.2: Simple schematic of a Mach–Zehnder interferometer with two phase modulators (A and B) acting as a photonic AND gate. Only if both modulators apply a phase shift of π, the phases add to 2π and light appears at the "1" output.

7.5.3 Challenges

Despite the advantages, several open problems remain:

- Efficient generation and control of single photons on a chip.
- Miniaturization of optical components to the nanometer scale.
- Integration with existing electronics.

7.5.4 Summary

Optical logic and photonic computers offer a fascinating possibility to increase computing power and reduce energy demand. Whether they will completely replace classical electronics or dominate only in special applications depends on solving the technical challenges.

7.6 Photons in Fundamental Research

Photons play a central role not only in technology but also in modern fundamental research. Their properties as massless, bosonic quantum objects make them ideal tools for studying fundamental questions of physics — from the smallest scales of quantum mechanics to cosmological distances.

- **Testing quantum mechanics:** Experiments with single photons — such as double-slit experiments, Bell tests, or quantum tomography — test the limits and predictions of quantum mechanics with the highest precision.
- **Astrophysics and cosmology:** Photons from distant galaxies and the cosmic microwave background provide information on the origin and evolution of the universe.
- **Precision measurements:** Laser interferometers such as LIGO or Virgo detect tiny changes in length caused by gravitational waves — based on coherent photon beams.
- **Tests of fundamental symmetries:** Polarization, frequency, and flight time of photons are used to probe Lorentz invariance, CPT symmetry, and other fundamental principles.

Photons as Messengers of Natural Laws

Photons interact only weakly with their environment, move at the speed of light, and carry information about their source across billions of years and light-years. This makes them unique messengers that provide insights into processes neither directly accessible nor reproducible — from the first moments after the Big Bang to the subtlest effects in quantum field theory.

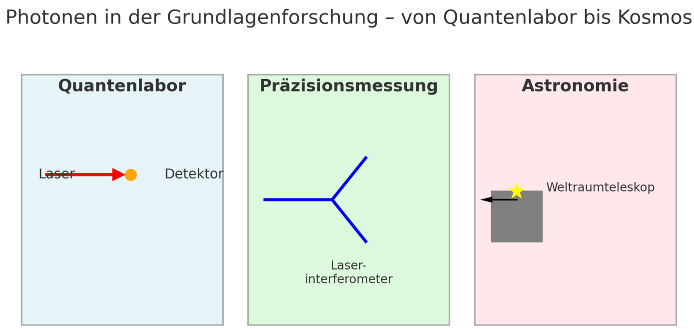


Figure VII.3: Photons in fundamental research: from quantum labs and precision measurements to astronomy. The illustration shows central fields of application: experiments with single photons in the lab, laser interferometry for gravitational-wave detection, and space-based observations of stars and galaxies.

7.6.1 Outlook

Fundamental research with photons is far from complete. New detection methods, improved sources for single and entangled photons, and space-based experiments promise even deeper insights into the structure of natural laws.

7.7 Outlook: Graviton, Dark Energy, New Physics?

Even though the photon as the quantum of light is well understood in modern physics, many fundamental questions remain — and photons often play a key role in answering them.

- **The graviton:** The hypothetical exchange particle of gravitation has not yet been observed. Precise measurements with photons — via gravitational lenses or interferometry — could provide indirect evidence.
- **Dark energy:** The accelerated expansion of the universe points to an as-yet unknown form of energy. Photometry and spectroscopy of distant supernovae and galaxies use photons as the only information source to study this mysterious component.

- **New physics beyond the Standard Model:** High-precision experiments with photons could reveal deviations from established theories, such as tiny violations of Lorentz invariance or hints of additional spatial dimensions.

What If the Photon Were Not the Only Massless Boson?

The existence of additional massless exchange particles — such as the graviton — would fundamentally change our understanding of the fundamental forces. Photon experiments could, through subtle effects such as deviations in light propagation or polarization patterns, provide the first hints of such new physics.

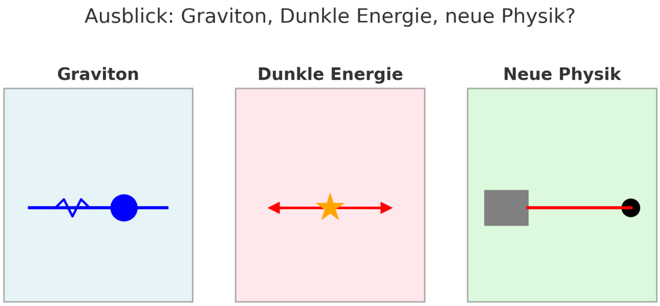


Figure VII.4: Symbolic outlook on open questions in physics: **left:** the hypothetical graviton as a possible candidate for another massless exchange particle; **center:** dark energy, recognizable in the accelerated expansion of the universe; **right:** laboratory experiments with photons searching for hints of new physics.

7.7.1 Outlook

The future of photon research lies not only in technical applications but also in the photon's role as a precise messenger of new natural laws. Space telescopes, gravitational-wave observatories, and laboratory experiments with unprecedented sensitivity could bring us closer to answering these fundamental questions.

7.8 Conclusion

Photons are far more than just carriers of light — they are tools, data transmitters, measuring instruments, and messengers of the fundamental laws of nature. From optical communication and precision metrology to photonic computers and the study of cosmic phenomena, their versatility is evident. The applications presented here make clear that photons not only form the foundation of modern technologies but are also crucial for exploring open questions of physics — from the nature of gravitation and dark energy to possible extensions of the Standard Model. The future of photon research therefore lies both in the further development of technical systems and in the search for new physics.

The photon is not just a particle of light — it is a key to today's technology and to the physics of tomorrow.

What If We Could Fully Control Photons?

A thought experiment:

- Photonic chips could replace classical electronics — with nearly light-speed computations.
- Global quantum communication networks would be absolutely secure against eavesdropping.
- Single-photon labs could revolutionize medical diagnostics at the molecular level.
- Photons as "probes" could provide direct insights into dark matter or the structure of spacetime.

Such visionary scenarios go beyond today's technology — they form the bridge to the next chapter on *The Photon in the Standard Model and Visionary Applications*.

Chapter VIII

The Photon in the Standard Model of Particle Physics

8.1 The Standard Model: Overview

The **Standard Model of Particle Physics** is a highly successful theory that describes the known fundamental particles and their interactions—with the exception of **gravitation**. It combines **quantum electrodynamics (QED)**, the **quantum theory of the weak interaction**, and **quantum chromodynamics (QCD)** into a consistent framework.

The fundamental building blocks are **fermions**, which form matter, and **bosons**, which act as exchange particles for the fundamental forces. Bosons are particles with integer spin that mediate the fundamental interactions. The gauge bosons include the **photon** (carrier of the electromagnetic interaction), the $W^\pm$ **and** Z^0 **bosons** (carriers of the weak interaction), and the **gluons** (carriers of the strong interaction). The **Higgs boson** plays a special role: it gives elementary particles their mass through the **Higgs mechanism**.

The interactions are described by **gauge symmetries**, formulated in the mathematical language of **Lie groups**. The Standard Model is based on the symmetry group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. Each factor corresponds to a fundamental interaction: SU(3) for the strong interaction, $\mathrm{SU}(2) \times \mathrm{U}(1)$ for the **electroweak theory**.

Despite its success, the Standard Model is incomplete: It explains neither gravitation nor the nature of **dark matter** or **dark energy**.

8.2 U(1) Gauge Symmetry and the Photon

The electromagnetic interaction can be elegantly formulated as a **U(1) gauge symmetry**. The group U(1) consists of all complex numbers of absolute value 1, expressed as $e^{i\theta}$. It is an **abelian group**, i.e. the group operation (here: multiplication) is commutative. In mathematics, U(1) is a **Lie group**, a continuous symmetry group parametrized by continuous parameters.

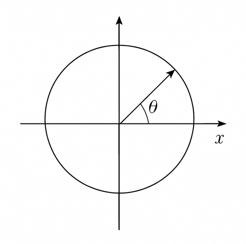
In quantum mechanics, a **global U(1) symmetry** describes the invariance of the wave function under a phase change $\psi \to e^{i\alpha}\psi$. According to **Noether's theorem**, this symmetry is directly linked to **charge conservation**.

If the symmetry is made *local*—i.e. the phase angle α may depend on space and time—one speaks of a **local U(1) gauge symmetry**. To preserve this invariance, a new field must be introduced: the electromagnetic potential A_μ. This field compensates the phase changes and leads to the **gauge coupling** between charged particles and the field.

Quantizing this field yields the **photon** as the massless **gauge boson** of the electromagnetic interaction. Its properties—especially masslessness and spin 1—follow directly from the structure of the U(1) symmetry.

The U(1) gauge theory is a special case of an abelian gauge theory and forms the electromagnetic sector of the **electroweak theory**. Its mathematical simplicity makes it an ideal starting point for understanding more complex theories such as SU(2) or SU(3).

U(1) explained intuitively



The group U(1) can be pictured as all possible rotations on a circle. Each point on the circle is defined by an angle θ. In quantum mechanics this corresponds to a phase change of the wave function – the distance to the origin always stays the same, only the direction changes.

8.3 Electroweak Unification

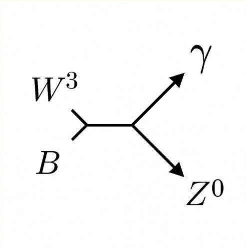
The **electroweak theory** unifies the **electromagnetic interaction** and the **weak interaction** in a single theoretical framework. It was developed in the late 1960s by **Sheldon Glashow**, **Abdus Salam**, and **Steven Weinberg** and forms a central part of the **Standard Model of Particle Physics**. For this achievement they received the **Nobel Prize in Physics** in 1979.

Mathematically, the electroweak theory is based on the symmetry group SU(2) × U(1). The SU(2) symmetry describes the weak interaction with its three gauge bosons W^1, W^2, W^3, while the U(1) symmetry represents the electromagnetic part. Through a mixing (*Weinberg angle*) of the fields W^3 and B (the U(1) boson), the massless **photon** and the neutral Z^0 **boson** emerge.

The electroweak symmetry is spontaneously broken by the **Higgs mechanism**. This gives the $W^\pm$ and Z^0 bosons mass, while the photon remains massless. This property is a direct manifestation of the remaining unbroken U(1) symmetry of electrodynamics.

The experimental confirmation of the electroweak theory came in the early 1980s at **CERN** through the direct detection of the W and Z bosons. This discovery is regarded as one of the greatest successes of modern particle physics.

From W^3 and B to the Photon



In electroweak theory, the **photon** γ and the neutral $\mathbf{Z^0}$ boson arise through a mixing of the fields W^3 and B – described by the **Weinberg angle** θ_W.

Mathematically:

$$\begin{aligned} \gamma &= \quad \cos\theta_W\, B + \sin\theta_W\, W^3 \\ Z^0 &= -\sin\theta_W\, B + \cos\theta_W\, W^3 \end{aligned}$$

This rotation of fields explains why the photon remains massless, while the Z^0 boson acquires mass.

8.4 Why Is the Photon Massless?

Within the **Standard Model of Particle Physics**, the **photon** is a massless **gauge boson** of the electromagnetic interaction. Its masslessness is a direct consequence of the unbroken **U(1) gauge symmetry** in **quantum electrodynamics (QED)**.

In the **Higgs mechanism**, which gives the $W^{\pm}$ and Z^0 bosons mass, exactly one gauge symmetry remains unbroken: the U(1) symmetry of electrodynamics. The associated gauge field is identified with the photon—and since unbroken gauge symmetries always lead to massless exchange particles, the photon has no rest mass.

Mathematically this appears in the Lagrangian density of the electromagnetic field: no mass term of the form $\frac{1}{2}m^2 A_\mu A^\mu$ occurs. Such a term would violate gauge invariance and is therefore forbidden.

Experimental bounds on a possible photon mass are extremely strict: Measurements set an upper limit of less than $10^{-18}\,\mathrm{eV}/c^2$. Practically the photon is perfectly massless—an essential property for the infinite range of the electromagnetic force.

Masslessness and range

The range of a fundamental interaction is directly linked to the mass of its exchange particle.

- **Massless exchange particles** (e.g. photons) mediate forces with unlimited range.
- **Massive exchange particles** (e.g. $W^{\pm}$ and Z^0 bosons) mediate forces with finite range, determined by the *Compton wavelength* $\lambda_C = \hbar/(mc)$.

For the photon, its masslessness implies the infinite range of the electromagnetic interaction.

Massless photon and spin correlations over long distances

A massless photon has no range limitation in transmitting its quantized properties. This has two essential consequences:

- **Infinite interaction range:** The electromagnetic force acts in principle over arbitrary distances.
- **Conservation of spin direction (polarization):** Since the photon is massless and travels at the speed of light, its spin or polarization orientation is preserved even across cosmological distances.

In entangled quantum states this means that the polarization correlations of two photons remain measurable even when they are far apart—a result of the combination of masslessness and quantum entanglement.

8.5 Comparison with Other Gauge Bosons

In the **Standard Model of Particle Physics** there are several **gauge bosons** that mediate fundamental interactions. The **photon** differs from them in essential properties:

- **Photon (QED):** Massless, spin 1, mediates the **electromagnetic interaction**. Range: infinite.
- $\mathbf{W}^{\pm}$ and $\mathbf{Z^0}$ **bosons** (electroweak theory): Spin 1, heavy masses ($\approx 80{-}91\,\mathrm{GeV}/c^2$), range: about $10^{-18}\,\mathrm{m}$. Mediators of the **weak interaction**.
- **Gluons** (QCD): Spin 1, formally massless, mediate the **strong interaction**. Effective range is very limited due to **confinement**—gluons never appear as free particles.
- **Graviton** (hypothetical): Spin 2, massless, would mediate gravitation. Not experimentally confirmed.

The decisive difference: Only the photon appears as a freely measurable, massless exchange particle with unlimited range. W and Z bosons are massive and thus short-range, gluons are massless but confined in bound states.

Comparison of gauge bosons

Particle	Spin	Mass	Range	Interaction
Photon	1	0	infinite	electromagnetic
$W^{\pm}$	1	$\approx 80\,\mathrm{GeV}/c^2$	$10^{-18}\,\mathrm{m}$	weak
Z^0	1	$\approx 91\,\mathrm{GeV}/c^2$	$10^{-18}\,\mathrm{m}$	weak
Gluon	1	0	*effectively short*	strong (confinement)
Graviton (hyp.)	2	0	infinite	gravitation

8.6 Open Questions and Extensions

Despite the success of the **Standard Model of Particle Physics**, fundamental questions remain unanswered that also involve the **photon** or could extend its theoretical framework:

- **Photon mass:** Experimentally the photon is considered massless, but an extremely tiny, so far unmeasurable mass cannot be excluded. Future precision measurements may further constrain this limit—or surprisingly reveal a finite mass.
- **Interaction with dark matter:** Whether photons couple in any way to dark matter is unknown. Direct and indirect searches may provide hints in the future.
- **Unified theories:** In models beyond the Standard Model—such as **Grand Unified Theories (GUT)** or **string theories**—the photon is part of a larger symmetry structure. These models may predict new properties or partner particles.
- **Quantum gravity:** A consistent theory uniting gravitation with the quantum fields of particle physics is still missing. The interaction of the photon with hypothetical **gravitons** or with the structure of spacetime on the smallest scales remains largely unexplored.
- **New symmetries or particles:** Extensions of the Standard Model could involve additional gauge bosons or symmetries that influence the role of the photon.

Answering these questions will require a combination of precise experiments, new observational technologies, and theoretical breakthroughs. Thus, the photon remains not only a central tool of physics but also a key to possible new physical worlds.

8.7 Conclusion

The Photon – Particle, Wave, and Window to the Future

The **photon** has played a unique role in the history of physics: It was the key to the birth of quantum theory, the starting point for the development of **quantum electrodynamics (QED)**, and remains an indispensable tool of both experimental and theoretical research. From the **photoelectric effect** to **Compton scattering** to modern applications such as **quantum communication** and **photonics**, the photon has not only shaped our physical models but also enabled technologies that transform daily life.

Within the **Standard Model of Particle Physics**, the photon embodies an unbroken **U(1) gauge symmetry**—a mathematical elegance expressed in its masslessness and infinite range. At the same time, the open questions show that we are still far from a complete understanding.

The photon unites fundamental properties of nature:

- **Wave and particle** in a single quantum description.
- **Massless messenger** with infinite range.
- **Precise probe** in astronomy, particle physics, and quantum optics.
- **Key actor** in future technologies such as quantum computers and photonic circuits.

Its dual role as theoretical foundation and practical tool makes the photon one of the most fascinating objects in physics. It is not only a component of our physical worldview but also a gateway to yet unknown aspects of the universe—a gateway that we will continue to push open in the decades ahead.

Even though the photon appears in a clear and consistent role in the Standard Model, it simultaneously symbolizes the limits of this theory. To highlight this dual function—as foundation and as bridge to new physics— we conclude with a didactic summary and then take a speculative look into the future.

Didactic conclusion: The photon in the Standard Model

The photon is not only a tool of quantum optics and modern technology, but also a key figure in the theoretical foundation of physics.

Core message: Its role as a massless gauge boson of the unbroken U(1) symmetry combines mathematical elegance, experimental precision, and cosmic range. Thus the photon exemplifies both the strength—and the limits—of the Standard Model.

What if the Standard Model were only a stepping stone?

A thought experiment:

- If photons could interact with dark matter, our understanding of cosmology would be revolutionized.
- Detecting a tiny photon mass would fundamentally change the structure of electrodynamics.
- New symmetries or hidden partner particles of the photon could appear in an extended theory beyond the Standard Model.
- Perhaps the photon is even the key to linking quantum field theory and gravitation.

Such speculative perspectives go beyond the Standard Model—and open the door to a future chapter on *new physics*.

Appendix A

Mathematical Background and Derivations

This appendix provides a formal and mathematical deepening of the physical concepts discussed in the main text. The aim is to preserve the didactic readability of the chapters while at the same time giving interested readers access to the complete derivations.
The sections are thematically structured according to the central properties of the photon, including the energy–momentum relation, the mass hypothesis, helicity, and polarization. In this way, the appendix builds a bridge between the intuitive explanations in the main text and the mathematical rigor of quantum field theory.

A.1 Energy–Momentum Relation of the Photon

This section formally derives why a photon has the energy

$$E = hf$$

and the momentum

$$p = \frac{h}{\lambda}$$

Starting from Maxwell's equations and their wave equation, it is shown via the Poynting vector and the energy density of the electromagnetic field that quantization of the fields leads to discrete energy portions. This derivation supplements the intuitive presentation in the main text (Chapter I).

A.2 The Electromagnetic Field in the Relativistic Formulation

Here we introduce the formal description of the electromagnetic field:

- Four-potential $A^\mu = (\phi, \vec{A})$
- Field strength tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$
- Antisymmetry property $F^{\mu\nu} = -F^{\nu\mu}$
- Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- Connection to quantization: photon as the gauge boson of $U(1)$ electromagnetism

This provides the mathematical foundation for the statement in the main text that photons are "excitations of the electromagnetic field."

A.3 Formal Description of Entangled Photons

The experiments on the generation of entangled photon pairs (SPDC) presented in the main text (Chapter I) can be formally described in Dirac notation. A typical entangled polarization state pair is given by:

$$|\psi\rangle = \frac{1}{\sqrt{2}} \big(|H\rangle_A \otimes |V\rangle_B + |V\rangle_A \otimes |H\rangle_B \big)$$

- $|H\rangle$: horizontally polarized state
- $|V\rangle$: vertically polarized state
- Indices A, B: the two photons

This formalism makes the correlations observed in the experiment transparent and shows why classical hidden-variable models are not sufficient.

A.4 Derivation of the Rayleigh–Jeans Law

The Rayleigh–Jeans law arises when the electromagnetic modes in a cavity are treated as harmonic oscillators:

1. Count the number of standing waves in the cubic volume V.
2. Each mode has two polarization directions.
3. According to the equipartition principle of classical thermodynamics, each degree of freedom in equilibrium contributes the mean energy kT.

The number of modes between the frequencies ν and $\nu + d\nu$ is

$$g(\nu)\,d\nu = \frac{8\pi V \nu^2}{c^3}\,d\nu.$$

Multiplied by kT, this yields the spectral energy density

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3}\,kT,$$

which, in wavelength form, becomes $u(\lambda, T) = \frac{8\pi kT}{\lambda^4}$. The law is accurate at long wavelengths but diverges for $\lambda \to 0$ – the so-called *ultraviolet catastrophe*.

A.5 Wien's Radiation Law

In 1896 Wilhelm Wien derived an approximation for blackbody radiation. His reasoning was based on:

- Thermodynamic considerations: adiabatic compression of a cavity shifts the radiation spectrum.
- Dimensional analysis: the intensity depends on T and λ and must have the correct units.

The result was

$$u(\lambda, T) = \frac{c_1}{\lambda^5}\,\exp\Bigl(-\frac{c_2}{\lambda T}\Bigr),$$

with constants c_1, c_2, which were only fully understood through Planck's approach. Wien's law is valid in the UV region but fails at long wavelengths.

A.6 Derivation of Planck's Radiation Law

Planck combined the limiting cases of Rayleigh–Jeans and Wien and introduced the quantization of energy:

- An oscillator can only take on energies $E_n = nh\nu$.
- The occupation probability follows the Boltzmann distribution.

The mean energy per oscillator is

$$\langle E \rangle = \frac{h\nu}{e^{h\nu/kT} - 1}.$$

Multiplied by the mode number $g(\nu)\,d\nu = \frac{8\pi V\nu^2}{c^3}\,d\nu$ this gives the energy density

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3}\,\frac{h\nu}{e^{h\nu/kT} - 1}.$$

This is **Planck's radiation law**, which agrees with experiment at all frequencies.

A.7 Mathematical Description of the Photoelectric Effect

In the photoelectric effect, an electron in a metal absorbs a photon with energy $E_\gamma = h\nu$. To release the electron from the metal, the *work function* A must be overcome. The excess energy goes into the electron's kinetic energy:

$$E_{\text{kin}} = h\nu - A.$$

This leads to a *threshold frequency*

$$\nu_{\min} = \frac{A}{h},$$

below which no electrons are emitted – independent of light intensity. This linear relation between electron energy and light frequency was confirmed precisely in Millikan's experiments (1916).

A.8 Photon Momentum

The momentum of a photon can be derived from the relativistic energy–momentum relation. For arbitrary particles,

$$E^2 = (pc)^2 + (m_0c^2)^2,$$

where E is the energy, p the momentum, c the speed of light, and m_0 the rest mass.

- For massless particles ($m_0 = 0$) this equation reduces to

$$E = pc.$$

- For the photon, the quantization condition holds simultaneously:

$$E = hf = \frac{hc}{\lambda}.$$

- Equating both expressions for E immediately yields

$$p = \frac{E}{c} = \frac{h}{\lambda}.$$

Thus, the momentum of a photon is directly linked to its wavelength. This relation is one of the central bridges between the wave and particle description of light.

A.9 Energy–Momentum Relation of the Photon

For relativistic particles, the general energy–momentum relation is

$$E^2 = (pc)^2 + (mc^2)^2.$$

- **Massless photon:** Setting $m = 0$ immediately gives

$$E = pc.$$

  This is consistent with the relations $E = hf$ and $p = h/\lambda$.

- **Hypothetically massive photon:** If the photon had a rest mass $m_\gamma \neq 0$, then

$$E^2 = (pc)^2 + (m_\gamma c^2)^2.$$

Such a photon would always move slower than c, and the speed of light would no longer be universally constant. Even minimal deviations from $m = 0$ would be revealed in precision experiments.

Experimentally, only an upper bound for the photon mass has been established so far. Current limits are

$$m_\gamma < 10^{-18}\,\mathrm{eV}/c^2,$$

which effectively means that the photon is considered massless.

A.10 Helicity of the Photon

The photon has spin $s = 1$, but due to its masslessness not all three spin projections ($m_s = -1, 0, +1$) are physically realizable.

- **General spin-1 state:** For massive spin-1 particles, three polarization states are possible, corresponding to the projections $m_s = -1, 0, +1$ onto the direction of motion.
- **Massless photon:** Since the photon has no rest mass, there is no rest frame in which the spin orientation can be defined independently of the momentum vector. Mathematically, the gauge invariance of Maxwell's equations (or of the QED formalism) enforces the vanishing of the longitudinal component ($m_s = 0$).
- **Helicity states:** Two possible states remain:

$$\begin{aligned} \lambda &= +1 \quad \text{(right-handed, right circular polarization)} \\ \lambda &= -1 \quad \text{(left-handed, left circular polarization)} \end{aligned}$$

  These are called the two helicity states of the photon.

Thus, the photon is a two-state massless boson whose degrees of freedom are fully described by the two possible helicities.

A.11 Polarization of the Photon

Polarization describes the transverse oscillation direction of a photon's electric field. Formally, this degree of freedom can be represented in two ways:

- **Dirac notation:** In quantum mechanics, polarization states are written as basis vectors in a two-dimensional Hilbert space:

$$|H\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |V\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

  where $|H\rangle$ stands for horizontal and $|V\rangle$ for vertical polarization. Arbitrary polarization states can be expressed as linear combinations:

$$|\psi\rangle = \alpha|H\rangle + \beta|V\rangle, \quad |\alpha|^2 + |\beta|^2 = 1.$$

- **Jones vectors:** In classical optics, the same state is described by *Jones vectors*:

$$\vec{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix},$$

  where E_x and E_y are the complex amplitudes of the electric field components in the x- and y-directions. Here too, normalizing the intensity corresponds to normalizing the state vector in Dirac notation.

Both descriptions are equivalent — Dirac notation emphasizes the quantum mechanical state space, while Jones vectors reflect the classical electromagnetic wave optics. Linking these representations is a central tool in quantum optics.

A.12 Derivation of Einstein's Photoelectric Equation

Einstein's equation

$$E_{\text{kin}} = h\nu - A$$

follows from a simple energy balance between a photon and an electron.

1. A photon carries the energy

$$E_{\text{photon}} = h\nu,$$

   where h is Planck's constant and ν is the frequency of the incident light.

2. To liberate an electron from a metal, the **work function** A must be overcome. This corresponds to the minimal binding energy of electrons in the solid.

3. If energy remains after overcoming A, it appears as the electron's kinetic energy:
$$E_{\text{kin}} = E_{\text{photon}} - A.$$

4. Hence, directly:
$$E_{\text{kin}} = h\nu - A.$$

Remark. If a retarding (stopping) voltage U is applied, then

$$eU = h\nu - A,$$

where e is the elementary charge. This form allows a direct experimental determination of h by measuring the stopping potential as a function of the frequency.

A.13 Planck–Einstein Relation $E = h\nu$

Goal. Justify why a single light quantum (photon) carries the energy

$$E = h\nu = \hbar\omega.$$

Route 1: Quantizing the normal modes of the electromagnetic field.

The free electromagnetic field in a volume V can be decomposed into plane normal modes with angular frequencies $\omega_{\mathbf{k}}$. Each mode is a harmonic oscillator with Hamiltonian

$$\hat{H}_{\mathbf{k}} = \hbar\omega_{\mathbf{k}}\left(\hat{a}^{\dagger}_{\mathbf{k}}\hat{a}_{\mathbf{k}} + \tfrac{1}{2}\right).$$

The eigenvalues are $(n + \frac{1}{2})\hbar\omega_{\mathbf{k}}$, $n \in \mathbb{N}_0$. An excitation $\Delta n = 1$ raises the energy by $\Delta E = \hbar\omega$. We identify this increase with *one photon* in that mode:

$$E_{\text{photon}} = \hbar\omega = h\nu.$$

Route 2: From Planck's quantum hypothesis to Einstein's light quantum.

Planck postulated in 1900 discrete energies $E_n = nh\nu$ for matter oscillators. Einstein in 1905 applied quantization to the *radiation field* itself: the energy of radiation behaves as if bundled into spatially localized *energy packets* of size $h\nu$. Only then can, among other things, the entropy properties behind Wien's law and the photoelectric effect be explained consistently. Thus for a single light quantum,

$$E_{\text{photon}} = h\nu.$$

Consequences.

(i) Photon energy depends *only* on frequency (not on intensity). (ii) Together with $E_{\text{kin}} = h\nu - A$ this explains the threshold frequency in the photoelectric effect. (iii) With field quantization this leads to the number operator $\hat{N} = \hat{a}^\dagger \hat{a}$ and a clear assignment of energy per photon.

A.14 Derivation of the Stopping-Potential Equation

Goal. Connect photon energy, work function, and the measurable retarding voltage U.

Starting point.

The energy balance in the photoelectric effect is

$$E_{\text{photon}} = h\nu = A + E_{\text{kin,max}}.$$

Experimental principle.

In a photocell, a *retarding (stopping) voltage* U is applied between cathode and anode. Electrons with kinetic energy E_{kin} must do work eU to reach the anode. At the **stopping potential** U_0 the energy is just exhausted:

$$E_{\text{kin,max}} = eU_0.$$

Derivation.

Inserting into the balance,

$$h\nu = A + eU_0 \;\Rightarrow\; eU_0 = h\nu - A.$$

Experimental significance.

– The graph $U_0(\nu)$ is a straight line with slope h/e. – The intercept yields the material-dependent work function A. – Millikan (1916) determined Planck's constant with high precision this way, confirming Einstein's hypothesis.

Consequence.

The stopping potential enables a direct measurement of fundamental constants, independent of light intensity or photon number.

A.15 The Work Function

The **work function** A is the minimum energy required to free an electron from a metal. It depends on the material and the electronic structure of the surface.
Formal definition:

$$A = E_{\text{Fermi}} + E_{\text{binding}} - E_{\text{vacuum}},$$

where E_{vacuum} is the energy level of an electron in vacuum.
Typical categories:

- Alkali metals (e.g., cesium, potassium)
- Transition metals (e.g., iron, copper)
- Noble metals (e.g., platinum)

The work function explains why only photons above a *threshold frequency* $\nu_0 = A/h$ can liberate electrons. It is material-specific and can vary with surface condition, temperature, or coatings.

A.16 Derivation of the Compton Formula

The derivation of the **Compton formula** is based on energy and momentum conservation in the collision of a photon with a stationary electron.
Setup:

- A photon with wavelength λ hits an electron at rest.
- After the collision the photon has wavelength λ' and is scattered by an angle θ.
- The electron acquires a recoil momentum $\vec{p}_e$.

Conservation laws:

$$\begin{aligned} E_\gamma + m_e c^2 &= E'_\gamma + E_e, \\ \vec{p}_\gamma &= \vec{p}'_\gamma + \vec{p}_e, \end{aligned}$$

with

$$E_\gamma = \frac{hc}{\lambda}, \quad E'_\gamma = \frac{hc}{\lambda'}, \quad p_\gamma = \frac{h}{\lambda}, \quad E_e^2 = (p_e c)^2 + (m_e c^2)^2.$$

Result:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta).$$

This shift is independent of photon energy and depends only on the scattering angle. The factor

$$\lambda_C = \frac{h}{m_e c} \approx 2.43 \times 10^{-12}\,\mathrm{m}$$

is the **Compton wavelength** of the electron.

A.17 The Double-Slit in the Quantum-Mechanical Formalism

The double-slit experiment with single photons can only be understood using quantum mechanics. Unlike classical wave theory or classical particle mechanics, one considers the photon's **wavefunction** and its superposition.
Superposition principle. Given two possible paths W_1 and W_2, the total amplitude is

$$\Psi_{\text{total}} = \Psi_1 + \Psi_2.$$

Dirac-notation representation. Let $|1\rangle$ be "photon goes through slit 1" and $|2\rangle$ be "photon goes through slit 2." Without which-path measurement:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + |2\rangle\right).$$

A.18 Antibunching and the Second-Order Correlation Function

The phenomenon of **antibunching** shows that photons are emitted *one by one*. Mathematically, this is described by the second-order correlation function.

Definition:

$$g^{(2)}(\tau) = \frac{\langle I(t)\, I(t+\tau)\rangle}{\langle I(t)\rangle^2},$$

where $I(t)$ is the intensity (or count rate) at the detector and τ the time delay between two measurements.

Antibunching. For an ideal single-photon source,

$$g^{(2)}(0) = 0.$$

Physical consequences. – Antibunching contradicts any classical wave picture. – It shows the **indivisibility of the photon**: it is detected here or there — but never simultaneously at two places. – Thus, antibunching is a direct proof of the quantum nature of light.

A.19 Hong–Ou–Mandel Interference at a Beam Splitter

The **Hong–Ou–Mandel (HOM) effect** describes two-photon interference of identical photons at a 50:50 beam splitter. For perfect indistinguishability the coincidences at the two outputs vanish ("HOM dip").

Beam-splitter transformation (Heisenberg picture).

For input modes $\hat{a}, \hat{b}$ and output modes $\hat{c}, \hat{d}$ of a lossless 50:50 beam splitter we choose the unitary map

$$\begin{pmatrix}\hat{c}\\ \hat{d}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}\begin{pmatrix}\hat{a}\\ \hat{b}\end{pmatrix}, \qquad \begin{pmatrix}\hat{a}\\ \hat{b}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -i\\ -i & 1\end{pmatrix}\begin{pmatrix}\hat{c}\\ \hat{d}\end{pmatrix}.$$

For the creation operators,

$$\hat{a}^\dagger = \frac{\hat{c}^\dagger - i\hat{d}^\dagger}{\sqrt{2}}, \qquad \hat{b}^\dagger = \frac{-i\,\hat{c}^\dagger + \hat{d}^\dagger}{\sqrt{2}}.$$

Input and output states.

Two single photons, one per input mode, $|\psi_{\rm in}\rangle = \hat{a}^\dagger\hat{b}^\dagger|0\rangle$, lead to

$$|\psi_{\rm out}\rangle = \hat{a}^\dagger\hat{b}^\dagger|0\rangle = \frac{1}{2}\,(\hat{c}^\dagger - i\hat{d}^\dagger)(-i\,\hat{c}^\dagger + \hat{d}^\dagger)\,|0\rangle = -\frac{i}{2}\Big(\hat{c}^{\dagger 2} + \hat{d}^{\dagger 2}\Big)\,|0\rangle.$$

Since the $\hat{c}^\dagger\hat{d}^\dagger$ terms cancel exactly, the output state contains *no* $|1_c, 1_d\rangle$ component (no coincidences). After normalization one may write equivalently

$$|\psi_{\rm out}\rangle \propto |2_c, 0_d\rangle \,\pm\, |0_c, 2_d\rangle,$$

where the relative sign depends only on the beam-splitter phase convention; the physics (vanishing coincidences) is unchanged.

Imperfect overlap and the HOM dip.

Real photons are finite wave packets in time/spectrum/polarization. Let $\Lambda(\tau) = \int dt\, f_a(t)\, f_b^*(t+\tau)$ be the (complex) temporal overlap integral (delay τ). Then the coincidence probability at the outputs is

$$P_{\rm coinc}(\tau) = \frac{1}{2}\Big(1 - |\Lambda(\tau)|^2\Big).$$

For perfectly overlapping, indistinguishable photons, $|\Lambda(0)| = 1 \Rightarrow P_{\rm coinc}(0) = 0$. For two Gaussian wave packets with coherence time τ_c, $|\Lambda(\tau)|^2 = \exp\big[-(\tau/\tau_c)^2\big]$, so

$$P_{\rm coinc}(\tau) = \tfrac{1}{2}\Big(1 - e^{-(\tau/\tau_c)^2}\Big)$$

shows the characteristic *HOM dip*.

Role of indistinguishability.

Any distinguishability (polarization angle $\Delta\phi$, spectral or spatial mode mismatch) reduces the visibility $V \in [0,1]$:

$$P_{\rm coinc}(\tau) = \frac{1}{2}\Big(1 - V\,|\Lambda(\tau)|^2\Big), \qquad V = |\langle\xi_a|\xi_b\rangle|^2,$$

where $|\xi_{a,b}\rangle$ collect all *internal* degrees of freedom (e.g., polarization).

Remark.

The disappearance of coincidences is not a classical field-interference effect but a *two-photon interference* of probability amplitudes, proving indistinguishability and the bosonic nature of photons.

A.20 Field Formalism and Four-Potential

In quantum electrodynamics (QED) the photon is not described as a classical particle, but rather as an excitation of the *electromagnetic field*. This field is represented by the **four-potential** $A^\mu(x)$, which in the relativistic formulation comprises four components:

$$A^\mu(x) = \big(\Phi(x), \vec{A}(x)\big),$$

where $\Phi(x)$ is the electric potential and $\vec{A}(x)$ is the magnetic vector potential. The temporal and spatial components combine into a Lorentz vector.

Field strength tensor.

From the four-potential one obtains the **field strength tensor**

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

which contains the physical fields:

$$\vec{E} = -\nabla\Phi - \frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \nabla \times \vec{A}.$$

Lagrangian density.

The dynamics of the electromagnetic field are derived from the **Lagrangian density**

$$\mathcal{L}_{\mathrm{EM}} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}.$$

Through the principle of least action, this formulation leads to Maxwell's equations in their relativistic form.

Gauge symmetry.

The potential A^μ is not uniquely determined:

$$A^\mu(x) \quad \to \quad A^\mu(x) + \partial^\mu \Lambda(x).$$

This freedom is called **gauge symmetry** and guarantees that only the physically measurable quantities $\vec{E}$ and $\vec{B}$ are independent of the choice of potential.

Thus, the four-potential is the central mathematical structure from which both classical electrodynamics and the quantized form of QED can be systematically developed.

A.21 From the Classical Field to QED

Classical electrodynamics in the sense of Maxwell describes electric and magnetic fields as continuous waves propagating through space. In this view, the electromagnetic field is a *deterministic solution* of Maxwell's equations.

Limits of the classical model.

Phenomena such as the photoelectric effect and Compton scattering demonstrate that light does not interact with matter in arbitrarily divisible amounts, but rather in discrete energy packets $h\nu$. This points to an underlying quantum nature of the field.

Quantization of the field.

Quantum electrodynamics (QED) goes beyond the classical theory by *quantizing* the electromagnetic field itself. Specifically:

- The potential $A^\mu(x)$ becomes an operator field.
- Its Fourier modes are identified with creation and annihilation operators for photons.
- Field states are described in Fock space, with the possibility of creating arbitrarily many photons in specified modes.

New perspective.

The photon thus appears as the **quantum of the electromagnetic field**, no longer as a classical particle or wave packet. Interactions such as the scattering of two electrons can be understood as *photon exchange*, represented mathematically by **Feynman diagrams**.

In this way, QED bridges classical field theory, quantum mechanics, and special relativity. It provides a consistent theoretical foundation in which the photon is described as a fundamental exchange particle—a vector boson.

A.22 Field Strength Tensor $F_{\mu\nu}$

The foundation of the relativistic formulation of electrodynamics is the **field strength tensor** $F_{\mu\nu}$. It combines the electric and magnetic fields into a covariant form and is directly derived from the four-potential $A^\mu(x)$:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

Properties.

- $F_{\mu\nu}$ is *antisymmetric*, i.e. $F_{\mu\nu} = -F_{\nu\mu}$.
- It contains exactly six independent components, which correspond to the three components of the electric field $\vec{E}$ and the three components of the magnetic field $\vec{B}$.

Matrix representation.

In 3+1 notation one obtains:

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}.$$

Lorentz covariance.

By its very definition, $F_{\mu\nu}$ is a rank-two tensor and transforms consistently under Lorentz transformations. This guarantees that electric and magnetic fields are not independent entities, but instead transform into one another depending on the observer.

Physical significance.

- The field strength tensor is the central quantity in the Lagrangian formulation of electrodynamics.
- It allows for a compact representation of Maxwell's equations.
- In quantum electrodynamics (QED) it provides the basis for defining photon fields and their interactions.

Thus, $F_{\mu\nu}$ unifies the classical fields $\vec{E}$ and $\vec{B}$ into a single relativistically invariant structure.

A.23 EM Lagrangian Density and Equations of Motion

The dynamics of the electromagnetic field can be elegantly expressed through a **Lagrangian density**. The starting point is the field strength tensor $F_{\mu\nu}$ (see Section A.22).

Lagrangian density.

The canonical form is

$$\mathcal{L}_{\text{EM}} = -\tfrac{1}{4}\, F_{\mu\nu}F^{\mu\nu}.$$

- The prefactor $-\frac{1}{4}$ is necessary to obtain the correct normalization under variation.
- The contracted form $F_{\mu\nu}F^{\mu\nu}$ is a Lorentz scalar, i.e. invariant under Lorentz transformations.

Coupling to matter.

For the field to interact with charged particles, an interaction term is added:

$$\mathcal{L}_{\text{int}} = -j_\mu A^\mu,$$

where j_μ is the four-current.

Variation and field equations.

Applying the **principle of least action** to

$$\mathcal{L} \; = \; -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - j_\mu A^\mu$$

and varying with respect to the potential A^μ, one obtains:

$$\partial_\nu F^{\mu\nu} \; = \; j^\mu.$$

These are Maxwell's equations in compact, covariant form. The inhomogeneous equations ($\nabla \cdot \vec{E} = \rho$, $\nabla \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{j}$) are included within them.

Homogeneous equations.

The remaining two Maxwell equations ($\nabla \cdot \vec{B} = 0$, $\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$) follow from the definition of the field strength tensor and the identity

$$\partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} = 0.$$

This shows that the entirety of classical electrodynamics can be derived from a compact Lagrangian formulation — an elegant starting point for quantization within the framework of QED.

A.24 Gauge Symmetry, Gauge Fixing, and the Lorenz Condition

A central structural principle of electrodynamics is **gauge symmetry**. It states that the four-potential $A^\mu(x)$ is not uniquely determined, but only up to a *gauge transformation*:

$$A^\mu(x) \;\; \to \;\; A^\mu(x) + \partial^\mu \Lambda(x),$$

where $\Lambda(x)$ is an arbitrary scalar function.

Physical consequences.

- The observable fields $\vec{E}$ and $\vec{B}$ remain unchanged under this transformation.
- Only gauge-invariant quantities are physically measurable.
- A mass term $\frac{1}{2} m^2 A_\mu A^\mu$ is not gauge invariant and is therefore excluded in QED.

Gauge freedom and degrees of freedom.

A vector field A^μ has four components. Gauge symmetry allows one to remove redundant degrees of freedom:

- The gauge transformation removes one component.
- The equations of motion (Lorentz invariance) remove another.
- Exactly two independent degrees of freedom remain — the two transverse polarization states of the photon.

Gauge fixing.

For practical calculations one chooses a specific *gauge*:

- **Lorenz gauge:** $\partial_\mu A^\mu = 0$. It is Lorentz covariant and well-suited to relativistic formulations.
- **Coulomb gauge:** $\nabla \cdot \vec{A} = 0$. Used frequently in quantum optics.

Lorenz condition.

In Lorenz gauge the equations of motion reduce to a wave (d'Alembert) equation:

$$\Box A^\mu(x) = j^\mu(x),$$

with the d'Alembert operator $\Box = \partial_\mu \partial^\mu$. This makes the wave nature of the electromagnetic field explicit.

Gauge symmetry is thus not merely a mathematical convenience, but the reason the photon is **massless** and has exactly two transverse polarization states.

A.25 Why the Photon Is Massless (Proca Argument)

In classical field theory one could formally add a mass term for a vector field A^μ. The corresponding Lagrangian density (Proca theory) is

$$\mathcal{L}_{\text{Proca}} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m^2 A_\mu A^\mu.$$

Consequences of the mass term.

- The equations of motion are modified and yield a massive wave equation.
- A massive spin-1 field has **three** independent polarization states (not two).
- The propagation speed would be less than the speed of light.

Gauge symmetry is violated.

The mass term $\frac{1}{2}m^2 A_\mu A^\mu$ is not invariant under the gauge transformation

$$A^\mu \;\to\; A^\mu + \partial^\mu \Lambda(x),$$

thus breaking the fundamental $U(1)$ gauge symmetry of electrodynamics.

Experimental evidence.

All observations show that:

- electromagnetic waves always propagate at the speed of light c,
- the photon has only two transverse polarization states,
- and no deviation from perfect masslessness has been detected.

Hence gauge symmetry holds exactly, and the photon has exactly zero rest mass.

The **Proca argument** thus shows: Gauge symmetry in QED forbids a mass term and forces the photon to be massless.

A.26 Transversality and Helicity ±1

The photon is a massless spin-1 particle. Its physically allowed polarization states follow from **gauge symmetry** and **Lorentz invariance**.

Reduction of degrees of freedom.

A vector field A^μ starts with four components.

- Gauge freedom removes one component (by a gauge choice).
- The equations of motion (e.g., the Lorenz condition $\partial_\mu A^\mu = 0$) remove another.
- Exactly **two independent degrees of freedom** remain.

Transversality.

The two remaining polarization modes are orthogonal to the direction of propagation:

$$\vec{k} \cdot \vec{\epsilon}_\lambda = 0,$$

where $\vec{k}$ is the wave vector and $\vec{\epsilon}_\lambda$ the polarization vector.

Helicity.

For a massless particle the **helicity** — the projection of spin onto the direction of motion —

$$h = \frac{\vec{S} \cdot \vec{p}}{|\vec{p}|}$$

is a well-defined, Lorentz-invariant quantity. The photon has exactly two helicity states:

$$h = +1 \quad \text{and} \quad h = -1.$$

Physical interpretation.

- Helicity $+1$: right-circularly polarized photons.
- Helicity -1: left-circularly polarized photons.

Thus the photon is **transversely polarized** with only two possible helicities — a direct consequence of gauge symmetry and masslessness (see Section A.25).

A.27 Virtual Photons and Unphysical Modes

Whereas real photons have only two transverse helicity states ($h = \pm 1$), **virtual photons** in QED can involve additional modes. They appear on internal lines of Feynman diagrams and are not observable particles, but mathematical auxiliaries of the formalism.

Unphysical components.

Photon propagators can contain, besides transverse parts, longitudinal or even scalar components. These arise from gauge freedom and are unavoidable when one allows propagating solutions in all components.

Consistency of the theory.

Although such unphysical modes appear in intermediate steps, they cancel out in all *physical observables*. This occurs due to:

- the gauge invariance of the theory,
- coupling only to the conserved electric current j^μ,
- and the Ward identities, which ensure the cancellation of non-transverse contributions.

Example: photon propagator.

In Feynman gauge the photon propagator is

$$D_{\mu\nu}(k) = \frac{-ig_{\mu\nu}}{k^2 + i\epsilon},$$

which formally includes longitudinal and time-like parts. In physical amplitudes these couple in such a way that they cancel exactly.

Conclusion: Virtual photons are a computational device in QED. They may carry seemingly unphysical modes, but gauge symmetry and current conservation guarantee that only the two transverse helicity states of the photon are physically realized.

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Appendix C

AI in Science – Tool, Not Truth

Motivation

This book was born from the desire to present complex physical concepts —particularly the photon and its role in modern physics—in a clear and well-founded way. A new tool was used in the process, one that is becoming increasingly relevant in scientific work: **artificial intelligence**, specifically the language model ChatGPT by OpenAI. But how can AI be meaningfully used in science without compromising understanding, precision, or responsibility? And how can this use be disclosed without undermining the scientific integrity of the work itself? This appendix offers a transparent look into how this book was developed and advocates for a responsible use of AI—as a tool, not a source of truth.

What AI Can—and Cannot—Do

AI-based language models like ChatGPT are powerful aids for writing and structuring. They can:

- assist in drafting initial versions of text,
- smooth out complex explanations,
- offer inspiration or propose outlines,
- suggest alternative formulations.

However, what they **cannot** do:

- **understand** scientific content in the proper sense,
- **verify** whether a formula is derived correctly,
- **grasp** the meaning of physical concepts,
- **critically assess or classify** scientific sources.

Therefore: AI can be a valuable *support*, but it **cannot and must not replace the scientific process of understanding**. Anyone using AI must still think for themselves—and critically review all results.

How This Book Was Created

The contents of this book—including its structure, physical explanations, and mathematical derivations—were conceived, researched, and authored by the writer. ChatGPT was used in the following supporting roles:

- for **formulating individual passages**, such as introductions, summaries, or didactic sections,
- for **stylistic review** of technical passages,
- for **developing outlines** in early stages of work,
- for reflecting on **clarity and reader guidance**.

What is essential: **All scientific statements, formulas, and interpretations were reviewed, questioned, revised, or discarded by the author.** No AI was involved in the development of physical arguments or core content.

Ethical Questions and Scientific Responsibility

The use of AI in scientific work raises important and justified questions:

- How much automation is acceptable without blurring authorship?
- How should potential errors be handled?
- How transparently must AI usage be disclosed?

The answer lies in a fundamental principle of scientific integrity: **responsibility**. Anyone using AI remains responsible for the result—regardless of whether certain formulations were proposed by a model.
In this sense, AI is not an author but a tool. It can accelerate processes but cannot replace what science is fundamentally about: **critical thinking, careful examination, and methodological work**.

Recommendations for Use in Research

For researchers, educators, and students, the following principles can guide a constructive use of AI:

- Use AI **consciously and selectively**—for linguistic support, not for argumentation or proof.
- **Verify all content independently**—especially when it involves complex material.
- **Disclose AI usage clearly** when relevant—e.g., in prefaces, appendices, or submission statements.
- Do not use AI for **deception or window dressing**, but as a tool to better express your own ideas.

Conclusion: AI as a Tool—But Human Responsibility Remains

Artificial intelligence is neither a substitute for nor an opponent of human insight. It is a **tool** that can assist in scientific communication—**if used consciously, thoughtfully, and responsibly**.
In this sense, this book is also a contribution to a new, enlightened way of working with technology in science. Not because technology can do everything—but because we have learned how to use it wisely.

Guiding Principle

AI is only as powerful as the human using it.
It can help structure, formulate, and vary text—but without critical thinking, subject knowledge, and human responsibility, it remains a tool without purpose.

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