Exhibit 1 – Dimensions and units of measurement

The following table (Table E1.1) shows the dimensions of a variety of physical quantities in terms of the basic units *mass, length, time* (M,L,T) or *force, length, time* (F,L,T). The table also shows the preferred units for those quantities in both the International System (S.I.) and the English System (E.S.) of units. Additional units commonly used for the quantities listed are shown in the last column of the table.

Table E1.1 – Dimensions and units of measurement

	Dimensions		Preferred units		Other
Quantity	(M,L,T)	(F,L,T)	S.I.	E.S.	**units**
Length (L)	L	L	m	ft	in, mi
Time (T)	T	T	s	s	h, d
Mass (M)	M	FT^2L^{-1}	kg	slug	
Area (A)	L^2	L^2	m^2	ft^2	Ac
Volume (Vol)	L^3	L^3	m^3	ft^3	Ac-ft
Velocity (V)	LT^{-1}	LT^{-1}	m/s	ft/s or fps	--
Acceleration (a)	LT^{-2}	LT^{-2}	m/s^2	ft/s^2	--
Discharge (Q)	L^3T^{-1}	L^3T^{-1}	m^3/s	ft^3/s or cfs	--
Kinematic viscosity (ν)	L^2T^{-1}	L^2T^{-1}	m^2/s	ft^2/s	St
Force (F)	MLT^{-2}	F	N	lb	--
Pressure (p)	$ML^{-1}T^{-2}$	FL^{-2}	Pa	lb/ft^2	psi, atm
Shear stress (τ)	$ML^{-1}T^{-2}$	FL^{-2}	Pa	lb/ft^2	psi
Density (ρ)	ML^{-3}	FT^2L^{-4}	kg/m^3	$slug/ft^3$	--
Specific weight (ω)	$ML^{-2}T^{-2}$	FL^{-3}	N/m^3	lb/ft^3	--
Energy/Work/Heat (E)	ML^2T^{-2}	FL	J	lb ft	--
Power (P)	ML^2T^{-3}	FLT^{-1}	W	lb ft/s	hp
Dynamic viscosity (μ)	$ML^{-1}T^{-1}$	FTL^{-2}	N s/m^2	lb s/ft^2	P

The symbols for the units used in Table E1-1 are listed next:

Ac	: acre, a unit of area
Ac-ft	: acre × feet
atm	: atmosphere
cfs	: cubic feet per second
fps	: feet per second
ft	: foot or feet
hp	: horse power
in	: inch
J	: joule
kg	: kilogram
cfs	: cubic feet per second
lb	: pound
m	: meter
N	: newton
mi	: mile
P	: poise
Pa	: pascal
psi	: pounds per square inch
s	: second
St	: stokes
W	: watt

PHY 115: Mechanics and Properties of Matter	Hamattan Semester 2014

Lecture 6: November-December, 2014

Lecturer: Ibrahim T. T. *Department: Physics* *Institution: Unilorin*

Disclaimer: *This Lecture note introduces some concepts of Mechamics. However it has not been subjected to the usual scrutiny reserved for formal publications and may be distributed outside this class only with the permission of the Instructor.*

6.1 Particle Kinematics

Particles may be described as point-like objects with mass but no size i.e. they exhibit negligible size and internal structure. Kinematics is the mathematical description of motion of an object without referring to the cause of the motion. Here we shall be concerned with the study of one-dimensional straight line motion of a particle and its generalization to two- and three-dimensional motion.

6.1.1 One-Dimensional Rectilinear Motion

A rectilinear motion is the motion of a particle along a straight line. It may, for example, be the motion of a ball rolling across the floor, the motion of a car on a straight road, the motion of a ball tossed straight up or that of an object dropped from a height.

Consider a particle moving from an initial position X_i to a final position X_f along a straight line, the change in position $\Delta X = X_f - X_i$, is equal to the DISTANCE covered by the particle. If the direction of motion is specified along with the magnitude ΔX, then the change in position is a vector quantity referred to as the DISPLACEMENT. If the corresponding initial time t_i at the start of the motion and the final time t_f are noted, then the time interval for the duration of the motion is given by $\Delta t = t_f - t_i$. The ratio $\Delta X/\Delta t$ therefore defines the SPEED of the particle. If the direction is specified then the ratio is referred to as the VELOCITY. In summary, Speed is the rate of change of distance covered with time, and Velocity is the rate of change of displacement with time.

When the rate of change of distance covered (or displacement) with time is *constant* the Speed (Velocity) is said to be UNIFORM. This means that the particle travels equal distance in equal time interval.

Similarly the rate of change of the velocity of a particle, say from an initial value U to a final value V over a time interval Δt, defines the ACCELERATION of the particle i.e. $\vec{a} = \Delta V/\Delta t$.

6.1.1.1 Average Speed and Instantaneous Speed

Average speed is defined as the ratio of the total distance covered with time. Instantaneous Speed on the other hand is the speed of the particle at an instant or a particular time t.

6.1.1.2 1-D Motion plus Uniform Acceleration

Uniform acceleration is therefore a constant rate of change of velocity with time. Positive acceleration is associated with increasing velocity, while negative acceleration or DECELERATION refers to a decreasing velocity.

Consider a uniformly accelerated particle whose velocity changes from an initial velocity U to a final velocity V over a time interval t, the acceleration of the particle is given by

$$a = \frac{V - U}{t}. \tag{6.1}$$

For the motion, the area on the velocity-time $(V - t)$ corresponds to the total distance covered by the particle and the slope or the gradient gives the acceleration a. For a uniformly accelerated body, the $(V - t)$ graph is a straight line graph whose area is given by the area of the trapezium formed by the line and the two axes. Thus the total distance is given by either,

$$S = \int V dt \tag{6.2}$$

$$= Ut + \frac{1}{2}at^2. \tag{6.3}$$

or

$$S = \int V dt \tag{6.4}$$

$$= \frac{(U + V)t}{2} \tag{6.5}$$

A quick look at equations 6.1, 6.3 and 6.5 show that each contains four of the five parameters a, V, U, S, and t, describing the particle motion. Eliminating the time t from equation 6.3 or 6.5 using equation 6.1 gives the fourth equation

$$V^2 = U^2 + 2aS. \tag{6.6}$$

The four equations are together referred to as the *Newtons's equations of motion*.

Note, objects falling freely on the earth's surface experiences the force of attraction towards the earth's surface. The associated acceleration is a constant called the *acceleration due to gravity g=9.8 m/s^2*. The valus of g is a function of the radius of the earth but is independent on the mass m of the falling body. For example, the values of g at the pole is greater than that at the equator.

Xamples

Q1.) A car starts from rest and travels 0.25 Km in 25 s at constant acceleration.
a.)Calculate the acceleration of the car, b.) Determine the final speed of the car.
Soln:
Rest implies the initial velocity $U = 0$ Distance covered $S = 0.25$ Km $= 250$ m, time interval $t = 25$ s.
Require to calculate the velocity $V =?$, and the acceleration $a =?$

$$S = Ut + 1/2at^2, \text{ Use an equation that does not contain the unknown } V. \tag{6.7}$$

$$a = 2(S - Ut)/t^2 \tag{6.8}$$

$$a = 2 * (250 - 0)/625 = 0.8m/s^2 \tag{6.9}$$

$$S = (U + V)t/2, \tag{6.10}$$

$$V = 2S/t - U = 2 * 250/25 - 0 = 20m/s. \tag{6.11}$$

Q2.) A man sitting on the ground throws a ball vertically upward with an initial speed 30 m/s.
a.) How long does it take the ball to return to the ground?
b.)What is its velocity on striking the ground?
Soln:
$U = 30$ m/s, deceleration $a = g = -9.8$ m/s^2, t=? V=?
If the time required to travel up to the maximum height is t_1, at this height the particle is momentarily at rest and as such the final velocity V=0, then

$$V = U - gt_1 \tag{6.12}$$
$$t_1 = (V - U)/ - g = (0 - 30)/ - 9.8 = 3.06\ s \tag{6.13}$$

therefore the actual time to return to the ground $t = 2 * t_1 = 6.12$ s
For final velocity on striking the ground V=?

$$V = U - gt_1 \tag{6.14}$$
$$V = 30 - 9.8 * 6.12 = -29.98 \text{ m/s} \tag{6.15}$$

The negative sign indicates the direction of motion.

6.2 Center of Mass (CM)

The center of mass concept is central to the study of the motion of many-particle systems. Here we shall not be concerned with the study of the motion of such systems but with their mass disribution. Recall that particles are bodies with NEGLIGIBLE spatial extent and internal structure. Real objects on the other hand are many-body system consisting of very large number of particles. At the extreme, these objects may either be a RIGID body or a FLUID.

The Center of mass of a many-body system is defined as the point in the system which moves as if the total mass is concentrated at that point. For an isolated n-body system with no external force, the total momentum $\vec{P}$ is given by

$$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + m_4\vec{v}_4 + m_5\vec{v}_5 + \cdots + m_n\vec{v}_n \tag{6.16}$$
$$= m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + \cdots m_n\frac{d\vec{r}_n}{dt} \tag{6.17}$$
$$= \frac{d}{dt}(m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n) \tag{6.18}$$

where $r_i(i = 1, 2, ..., n)$ are the position vectors of each particle in the system. Note that the masses m_i are time independent (ie. $\frac{dm}{dt} = 0$). The centre of mass of the system is then defined by the position vector

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots + m_n\vec{r}_n}{m_1 + m_2 + m_3 + \cdots + m_n} \tag{6.19}$$
$$= \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{\sum m_i} \tag{6.20}$$
$$= \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{M} \tag{6.21}$$

where $M = \sum m_i$ is the total mass of the system. The corresponding cartesian coordinate of the position vector $\vec{R}$ is given by

$$X = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{m_1 + m_2 + m_3 + \cdots + m_n} \tag{6.22}$$
$$Y = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots + m_ny_n}{m_1 + m_2 + m_3 + \cdots + m_n} \tag{6.23}$$
$$Z = \frac{m_1z_1 + m_2z_2 + m_3z_3 + \cdots + m_nz_n}{m_1 + m_2 + m_3 + \cdots + m_n} \tag{6.24}$$

6.2.1 Velocity and Acceleration of CM

Since the velocity $\vec{V}$ is define as the rate of change of displacement with time then

$$\vec{V} = \frac{d\vec{R}}{dt} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n}{m_1 + m_2 + m_3 + \cdots + m_n} \tag{6.25}$$
$$= \frac{\sum \vec{p}_i}{\sum m_i} = \frac{\vec{P}_{tot}}{M} \qquad \text{(Statement of First Law)} \tag{6.26}$$

The acceleration $\vec{A}$ is the rate of change of velocity with time

$$\vec{A} = \frac{d\vec{V}}{dt} = \frac{d^2\vec{R}}{dt^2} \tag{6.27}$$
$$M\vec{A} = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} + \frac{d\vec{p}_3}{dt} + \cdots + \frac{d\vec{p}_n}{dt} \tag{6.28}$$
$$= \frac{d\vec{P}_{tot}}{dt} = \vec{F}_{ext}. \tag{6.29}$$

We see from above that the CM moves like a single point mass i.e. it moves with constant velocity in the absence of the external force and accelerates when the resultant force is non-zero. The CM therefore exhibit a simple motion irrespective of the motion of the constituent bodies.
Xamples
Q 1.)Two masses are placed on a 1.5 m long massless rod. The masses are arranged such that a 1.6 kg mass is placed at the left end of the rod and a 1.8 kg mass is placed at 1.2 m from the left end.
a.) What is the location of the CM ?
b.) By moving the 1.8 kg mass, can you arrange to have the CM in the middle of the rod?
Soln:
a.) Arrange the masses on a 1-D coordinate such that the left most mass is placed at the origin

$$m_1 = 1.6\ kg, \qquad x_1 = 0, \qquad m_2 = 1.8\ kg, \qquad x_2 = 1.2\ kg, \tag{6.30}$$
$$X = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}, \tag{6.31}$$
$$X = \frac{1.6 \times 0 + 1.8 \times 1.2}{1.6 + 1.8} = 0.64\ m. \tag{6.32}$$

b.) Let the 1.8 kg mass be at x_2 and the mid-point be $x_m = 0.75$ m. If the CM is located at x_m, then

$X = 0.75$ m.

$$m_1 = 1.6\ kg, \qquad x_1 = 0, \qquad m_2 = 1.8\ kg, \qquad x_2 = ? \tag{6.33}$$
$$X = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}, \tag{6.34}$$
$$x_2 = \frac{(m_1 + m_2)X - m_1x_1}{m_2} \tag{6.35}$$
$$x_2 = \frac{0.75 \times 3.4 - 1.6 \times 0}{1.8} = 1.4\ m. \qquad \text{It is possible!} \tag{6.36}$$

Q 2.)In the question above, determine the CM of the new system if a third mass of 2.3 kg is placed at a distance of 1.1 m vertically above the 1.6 kg such that a straight line connects the CM of the 1.6 kg and that of 2.3 kg.
Soln:
This is now a 2-D configuration, and the x,y coordinate system will be considered.

$$m_1 = 1.6\ kg, x_1 = 0,\ y_1 = 0 \qquad m_2 = 1.8\ kg, x_2 = 1.2m,\ y_2 = 0 \qquad m_3 = 2.3kg, x_3 = 0, y_3 = 1.1m. \tag{6.37}$$
$$X = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3},\ Y = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} \tag{6.38}$$
$$X = \frac{1.6 \times 0 + 1.8 \times 1.2 + 2.3 \times 0}{1.6 + 1.8 + 2.3} = 0.38\ m. \tag{6.39}$$
$$Y = \frac{1.6 \times 0 + 1.8 \times 0 + 2.3 \times 1.1}{1.6 + 1.8 + 2.3} = 0.44\ m. \tag{6.40}$$
$$\vec{R} = 0.38\hat{\imath} + 0.44\hat{\jmath}, |R| = \sqrt{0.38^2 + 0.44^2} = 0.58m, \theta = tan^{-1}(0.44/0.38)\text{From the origin} \tag{6.41}$$

Xercise
1.)Two masses $m_1 = 15$ kg amd $m_2 = 25$ kg are joined by connecting a rod of length 0.8 m. Determine the distance of the CM of the system from the m_1 if a.) the connecting rod is massless, and b.) the connecting rod is a uniform rod of mass 15 kg.
2.)Find the location of the CM of a system of three particles arranged such that two of the particles with mass m are separated by distance l along the y-axis. The third mass $2m$ lies on the x-axis and is separated by a distance l from one of the masses.

6.2.2 Continous Mass Distribution

A solid object is a many-particle system consisting of tiny particles that are not visible to our naked eyes. Such systems are considered as CONTINUUM OF MATTER. The CM of a continous distribution of mass may be defined by considering a rod of length L (with ends a and b) divided into equal length ΔX but not necessarily of equal mass unless the mass distribution is uniform. If the i-th segment has a mass Δm_i, the local mass density is given by $\frac{\Delta m_i}{\Delta x}$. The total mass of the rod M may then be expressed as

$$M = \Delta m_1 + \Delta m_2 + \Delta m_3 + \cdots \Delta m_n = \left(\frac{\Delta m_1}{\Delta x} + \frac{\Delta m_2}{\Delta x} + \frac{\Delta m_3}{\Delta x} + \cdots + \frac{\Delta m_n}{\Delta x}\right)\Delta x. \tag{6.42}$$
$$= \sum_{i=1}^{n} \frac{\Delta m_i}{\Delta x}\Delta x \tag{6.43}$$
$$= \int_a^b \left(\frac{dm}{dx}\right)dx \qquad \text{For infinitesimal } \Delta x \tag{6.44}$$

The corresponding CM of the linear mass distribution may then be written as

$$X = \frac{\Delta m_1 x_1 + \Delta m_2 x_2 + \Delta m_3 x_3 \cdots \Delta m_n x_n}{M}. \tag{6.45}$$

$$X = \frac{1}{M}\left[\frac{\Delta m_1}{\Delta x}x_1 + \frac{\Delta m_2}{\Delta x}x_2 + \frac{\Delta m_3}{\Delta x}x_3 \cdots \frac{\Delta m_n}{\Delta x}x_n\right]\Delta x. \tag{6.46}$$

$$= \frac{1}{M}\int_a^b \left(\frac{dm}{dx}\right) x dx \qquad \text{For infinitesimal } \Delta x. \tag{6.47}$$

$$= \frac{1}{M}\int_a^b \lambda x dx. \tag{6.48}$$

where $\lambda = \frac{dm}{dx}$ is the mass density.

Xample

Q.1) Consider a rod of length L, whose mass density is given by $\lambda = C(1+ax^2)$, where x is the distance from the light end and C is a constant with a dimension of mass per unit length. Calculate the CM of the rod.

soln:

Require to determine the CM X given eqns 4.29 and 4.33.

$$\lambda = C(1+ax^2), X = ? \tag{6.49}$$

$$M = \int_0^L \left(\frac{dm}{dx}\right) dx \tag{6.50}$$

$$= C\left|x + ax^3/3\right|_0^L = C(L + aL^3/3) \tag{6.51}$$

$$X = \frac{1}{M}\int_0^L \lambda x dx = \frac{C}{M}\int_0^L x(1+ax^2)dx \tag{6.52}$$

$$= \frac{C}{M}\left|x^2/2 + ax^4/4\right|_0^L = \frac{\left[L^2/2 + aL^4/4\right]}{\left[L + aL^3/3\right]} = \left(\frac{L}{2}\right)\frac{1+aL^2/2}{1+aL^2/3} \tag{6.53}$$

PHY 115: Mechanics and Properties of Matter I

(Newton’s laws of Motion, Friction, Work and Energy)

Newton’s Laws of Motion

Newton’s laws deal with force and motion / acceleration.

Newton’s First law of Motion

The law states that every object will continue in its state of rest, or of uniform motion in a straight line, unless an external force acts on it. But it is possible to apply a force to an object (i.e. an external force acting on an object) without producing motion. A good example of this is when a car is to be pushed to start the engine by gradually increasing the applied force. It will get to a particular value of the applied force that the car will start to move, that is any values of the applied force below this value is not strong enough to cause the car to move. Therefore it can be said that though an external force is applied, there is no movement or change of state of the car.

$$a = 0, \text{ then } F_{applied} = F_{friction}$$

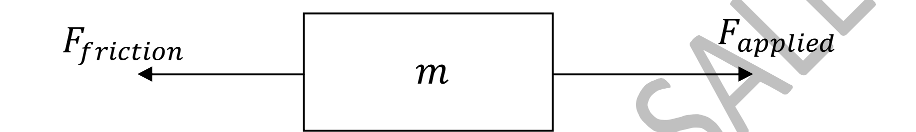


Fig. 1: Applied force is equal to the frictional force between the box of mass m and the surface, only when acceleration $a = 0$.

Our basic assumption here, for now, is that there is no friction between the surfaces (i.e. the object and the horizontal plane) in contact. The explanation above is to introduce a new concept called Net Force, F_{Net}, to replace the external force $F_{applied}$. Therefore we can state the Newton’s first law of motion as follow:

Every object will continue in its state of rest, or of uniform motion in a straight line, unless a net force acts on it. This tendency of an object to resist change of state of motion is called as inertia.

Inertia: the resistance an object has to a change in its state of motion.

If $\boldsymbol{F_{Net} = 0}$, then $\boldsymbol{a = 0}$ statement of the 1st law.

Consider an object of mass m under the influence of three forces as shown in Fig, 2 below, the net force F_{Net} is the vector sum of all the three forces.

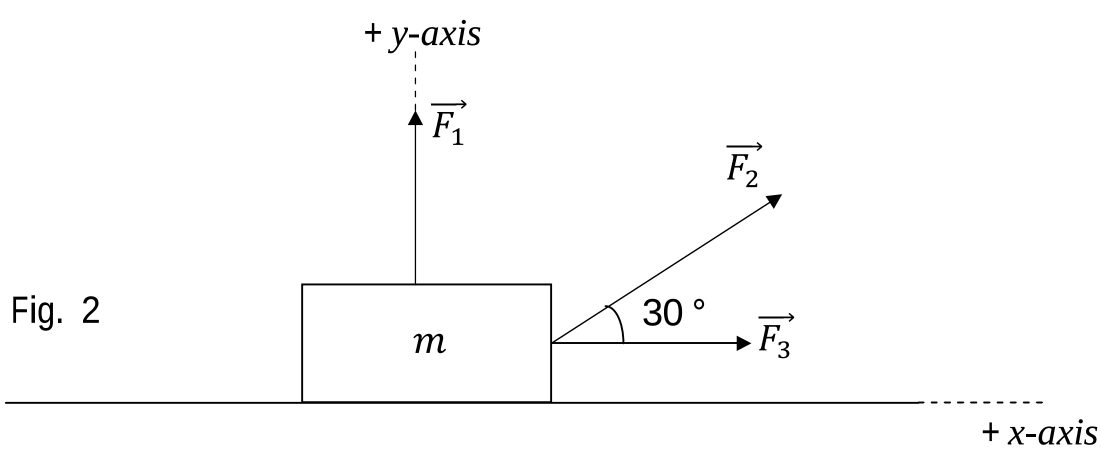


Fig. 2

$$\vec{F}_{Net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

Remember that a vector, $\vec{V}$, can expressed as $\vec{V} = Vcos\theta\ i + Vsin\theta\ j = V_x\ i + V_y\ j$

where V= magnitude of the vector and θ = angle the direction of the vector makes with the positive x-axis.

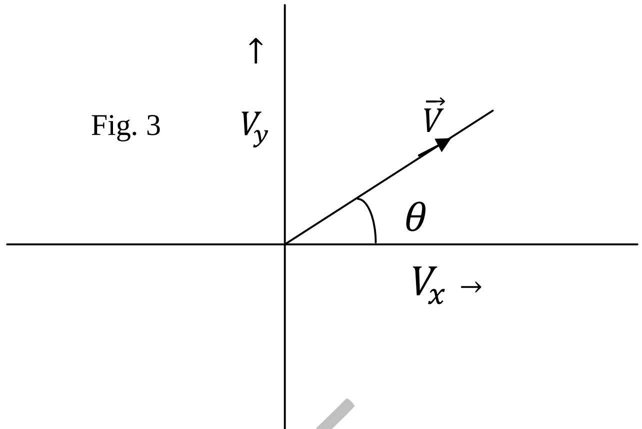
Y-component of vector $\vec{V}$,

$V_y = Vsin\theta$

X-component of vector $\vec{V}$,

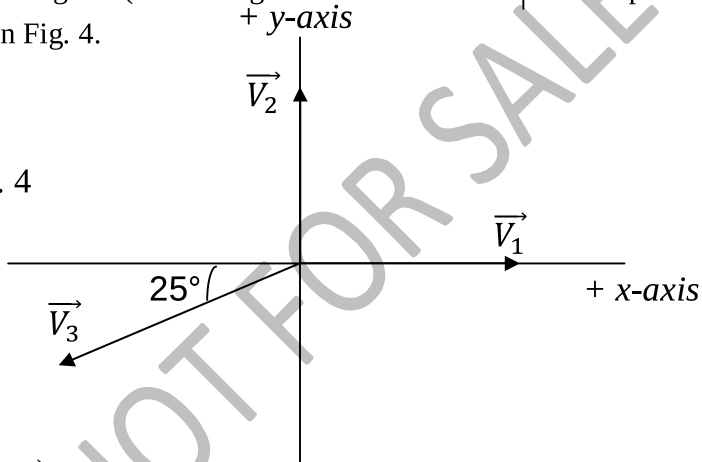
$V_x = Vcos\theta$

Fig. 3



Example 1: Determine angle θ (i.e. the angle each vector makes with the positive x-axis) for vectors $\vec{V_1}, \vec{V_2}\ and\ \vec{V_3}$ shown in Fig. 4.

Fig. 4



Solution:

The direction of vector $\vec{V_1}$, as indicated the diagram, is along the positive x-axis

Therefore, the angle $\vec{V_1}$ makes with the positive x-$axis$ $\theta_1 = 0\,°$.

The direction of vector $\vec{V_2}$, as indicated in the diagram, is along the positive y-$axis$, and the angle between the positive y-$axis$ and positive x-$axis$ is $90\,°$.

Therefore, the angle $\vec{V_2}$ makes with the positive x-$axis$ $\theta_2 = 90\,°$.

For vector $\vec{V_3}$, the angle θ_3 that $\vec{V_3}$ makes with positive x-$axis$ is $180\,° + 25\,°$,

therefore $\theta_3 = 205\,°$.

Example 2: Using fig. 4, if the magnitude of vectors $\vec{V_1}, \vec{V_2}\ and\ \vec{V_3}$ is 10 N, 15 N and 20 N respectively, calculate the y- and x-components of each of the vectors.

Solution:

Vector $\vec{V_1}$, magnitude of the vector $V_1 = 10\,N\ and\ \theta_1 = 0\,°$.

x-$component$ $V_{1,x} = 10\cos 0\,° = 10\,N$, and *y-component* $V_{1,y} = 10 \sin 0° = 0\ N$

For $\overrightarrow{V_2}$, magnitude V_2 = 15N, $\theta_2 = 90^0$

$V_{2,x}$ = 15 Cos90^0 = 0N x – component

$V_{2,y}$ = 15 Sin90^0 = 15N y - component

For $\overrightarrow{V_3}$, V_3 = 20N, $\theta_3 = 205^0$

$V_{3,x}$ = 20 Cos205^0 = 20 x (-0.9063) = - 18.13N x – component

$V_{3,y}$ = 20 Sin205^0 = 20 x (-0.4226) = - 8.45N y – component

Note: If the direction of a vector is along *x-axis* or *y-axis*, the vector will have only one component e.g. $\overrightarrow{V_2}$ is along *y-axis*, $V_{2,y}$ = 15N while $V_{2,x}$ = 0N.

Now to $\overrightarrow{F_{Net}}$ in Fig. 2, we can write each of the forces in a vector equation and the net force will be the vector sum of all the three forces.

$$\overrightarrow{F_{Net}} = F_1 Cos 90^0 i + F_1 Sin 90^0 j$$
$$+ F_2 Cos 30^0 i + F_2 Sin 30^0 j$$
$$+ F_3 Cos 0^0 i + F_3 Sin 0^0 j$$

$$\overrightarrow{F_{Net}} = (0 + F_2 Cos 30^0 + F_3) i + (F_1 + F_2 Sin 30^0 + 0) j$$

NEWTON'S SECOND LAW OF MOTION

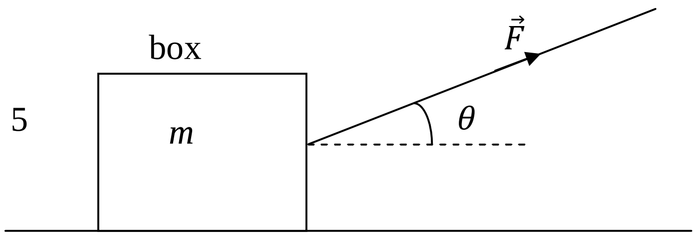
The net force $\overrightarrow{F_{Net}}$ on a body is equal to the product of the body's mass (m) and its acceleration.

$$\overrightarrow{F_{Net}} = m\vec{a}$$

$\overrightarrow{F_{Net}}$ can also be written in form of its components; $F_{net,x} = ma_x$ & $F_{net,y} = ma_y$

The implication of these two equations above is that x-component of the resultant or net force is responsible for acceleration along *x-axis* while y-component is responsible for acceleration along *y-axis*. A good illustration of this is shown in Fig. 5 below.

Fig. 5



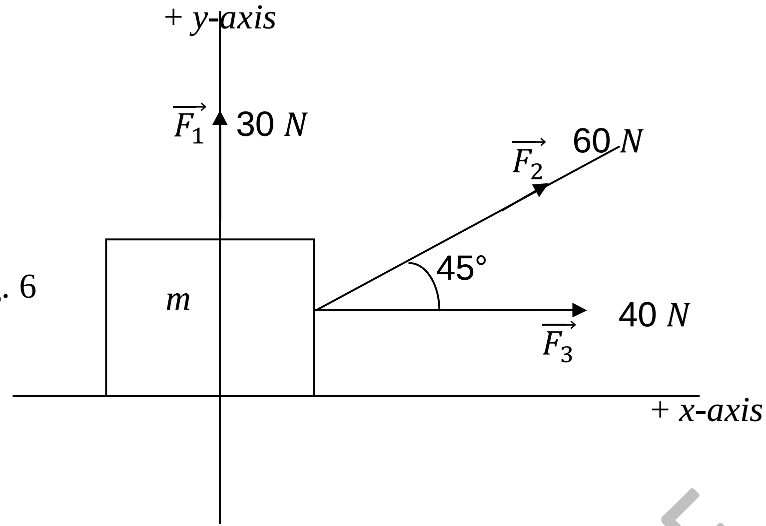
If the value of the applied or net force, in this case F, is gradually increased from zero upward, it will get to a particular value of $\vec{F}$ at which the box will be lifted (or just about to be lifted) off the surface. At this value of $\vec{F}$, in the direction shown, the y-component (i.e. F_y) is equal to the weight of the box. To keep the box moving along the horizontal surface, the applied force needs to be reduced.

Therefore, from this illustration, the x-component of the net-force $(F_{net,x})$ will drag the box along the horizontal surface while the y-component $(F_{net,y})$ will try to lift the box off the surface.

We can also state the Newton's second law of motion as: "The effect of a net force is to change the state of motion of the object on which it acts".

Example 3: Find the magnitude and direction of the net force acting on the object shown in Fig. 6 below.

Fig. 6



Solution:

$\vec{F_1}$, $F_1 = 30N$, $\theta_1 = 90^0$

$\vec{F_2}$, $F_2 = 60N$, $\theta_2 = 45^0$

$\vec{F_3}$, $F_3 = 40N$, $\theta_3 = 0^0$

The net force $\overrightarrow{F_{Net}} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

$= 30Cos90^0 i + 30Sin90^0 j$

$+ 60Cos45^0 i + 60Sin45^0 j$

$+ 40Cos0^0 i + 40Sin0^0 j$

$\Rightarrow$

$\overrightarrow{F_{Net}} = 0i + 30j$

$+42.4i + 42.42j$

$+40.0i + 0j$

$= (0 + 42.4 + 40.0)i + (30 + 42.4 + 0)j$

$\overrightarrow{F_{Net}} = 82.42i + 72.42j$

The magnitude of $\overrightarrow{F_{Net}}$, $F_{Net} = \sqrt{82.42^2 + 72.42^2} = 109.72\ N$

Angle θ that $\overrightarrow{F_{Net}}$ makes with positive x-$axis$,

$$\theta = \tan^{-1}(\frac{72.42}{82.42}) =$$

The magnitude $F_{net} = 109.72N$ and θ which indicates direction =

Example 4: If the mass of the object in Fig. 6 is 15Kg, find the acceleration (in magnitude and direction).

Solution:

$\overrightarrow{F_{Net}} = m\vec{a}$, From example 3, $\overrightarrow{F_{Net}} = 82.42i + 72.42j$

Therefore, $\vec{a} = \frac{F_{Net}}{m} = \frac{82.42i+72.42j}{15}$

$\vec{a} = 5.495i + 4.828j$

Magnitude of $\vec{a}$, $a = \sqrt{5.495^2 + 4.828^2} = 7.31m/s^2$

The angle θ, $\vec{a}$ makes with positive x-axis, $\theta = \tan^{-1}\frac{4.828}{5.495}$

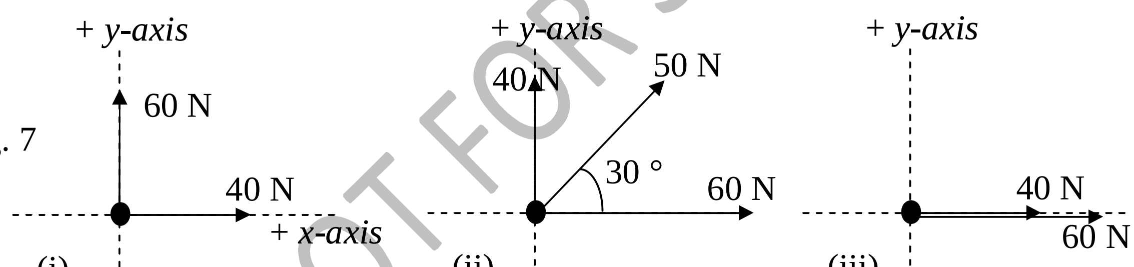
The magnitude of acceleration $a = 7.31\, m/s^2$, and $\theta =$

Alternative Solution: From $\overrightarrow{F_{Net}} = m\vec{a}$, the direction of the net force $\overrightarrow{F_{Net}}$ is the direction of the resulting acceleration. For the magnitude of acceleration, one can divide the magnitude of net force F_{net} by the mass i.e. $a = \frac{F_{net}}{m} = \frac{109.72}{15} = 7.31 m/s^2$

Problems

1. A 1580Kg car is travelling at a speed of 15.0m/s. What is the magnitude of the horizontal net force that is required to bring it to a halt in a distance of 50.0m? Ans = 355N.
2. A 900Kg car travelling at 25m/s brakes hard and comes to rest in 5s. What is the average breaking force? (Ignore drag) Ans = 4500N.
3. A net force acts on mass m_1 and creates an acceleration. A mass m_2 is added to mass m_1. The same net force acting on the two masses together creates one-third the acceleration. Determine the ratio $\frac{m_2}{m_1}$. Ans = 2:1 or 2.
4. An empty plane whose mass is 30, 400Kg has a maximum takeoff acceleration of 1.20m/s^2. What is its maximum acceleration when it is carrying a load of 8200Kg? Ignore friction. Ans = 0.945m/s^2.
5. Find the magnitude and direction of the net force acting on each of the three objects shown in Fig. 7 below.

Fig. 7



Ans: 72.11 N, 56.31 ^{0}, Ans: 122.05 N, 32.18 ^{0}, Ans: 100 N, 0 ^{0}

6. If the masses of the objects in Fig. 7 above are 7.5Kg, find their accelerations (both magnitude and direction). Ans: 9.61 m/s^2, 56.31 ^{0}; 16.27 m/s^2, 32.18 ^{0}; 13.33 m/s^2, 0 ^{0}

Example 4

In fig. 8 below, If M = 3.3Kg and m=2.1K and assuming that the cord and pulley have negligible mass, find

a. The acceleration of the sliding block
b. The acceleration of the hanging block
c. The tension in the cord

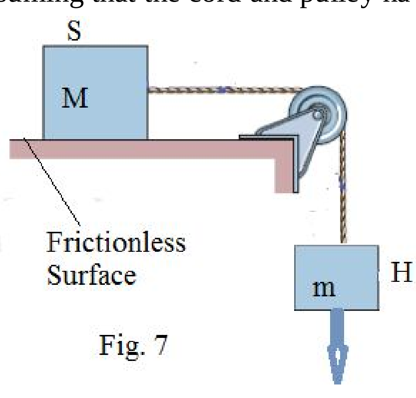


Fig. 7

Solution:

The forces and their direction acting on block S and block H are shown. Since block S is on a frictionless surface, the net force on it is T. The forces on block H are T and F_g (Please, take note of the direction of T and F_g).

Applying second law of motion, $\overrightarrow{F_{Net}} = m\vec{a}$

On block S, T = ma (1)

On block H, mg – T = ma (2)

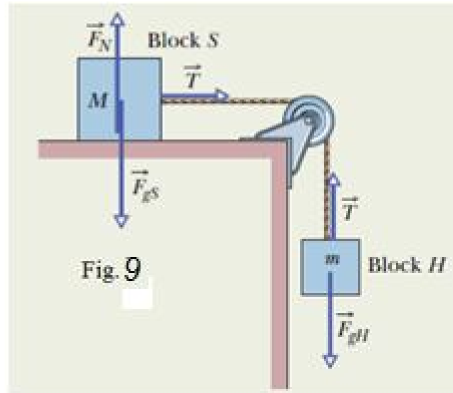


Fig. 9

Using eqn (1), T = ma, replace T in (2) with Ma

Mg – Ma = ma

Mg = Ma + ma = (M + m)a

a = $\frac{M}{M+m}g = \frac{2.1}{3.3+2.1}x10 = 3.89m/s^2$

Since the cord is inextensible, bloth blocks (S and H) will accelerate at a= 3.89m/s^2.

(c) Using equation1, T = 3.3 x 3.90 = 12.83N

SECOND LAW OF MOTION (MOMENTUM)

Force is equal to the rate of change of momentum,

$F = \frac{dp}{t}$ = force acting on a body in motion is equal to the rate of change of momentum of the body.

Where dp is change in momentum and t, the time for the change. Suppose an external force F acts on an object of mass m moving with initial velocity v_1 to change its velocity to v_2 at time interval t.

Then F = $F = \frac{dp}{t} = \frac{mv_2-mv_1}{t} = \frac{m(v_2-v_1)}{t}$

But $F = ma = \frac{m(v_2-v_1)}{t}$,

$a = \frac{(v_2-v_1)}{t}$ - definition of acceleration.

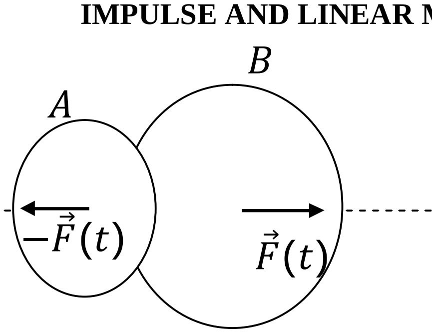
This is another statement of Newton's second law of motion

COLLISION

Collision is an isolated event in which two or more bodies exert relatively strong forces on each other for a relatively short time.

IMPULSE AND LINEAR MOMENTUM

$Fig. 10$



The diagram in Fig. 10 above shows the collision between two objects A and B of different masses. If the collision is head-on, the direction of their final velocity will be along x-axis. The two forces $\overrightarrow{F(t)}$

and $-\overrightarrow{F(t)}$ will change the linear momentum of both bodies. The amount of change dp is

$d\vec{p} = \overrightarrow{F(t)}dt$ according to Newton's second law of motion

Integrating over the interval Δt – from initial time t_i to final time t_f

$$\int_{t_i}^{t_f} d\vec{p} = \overrightarrow{F(t)}dt$$

$\int_{t_i}^{t_f} d\vec{p} = \vec{p}_f - \vec{p}_i$, the change in linear momentum of the body.

$\int_{t_i}^{t_f} \overrightarrow{F(t)}dt = \vec{j}$ i.e. the impulse

For each body, change in the linear momentum is equal to the impulse that acts on the body.

MOMENTUM AND KINETIC ENERGY IN COLLISIONS

Elastic Collision: Both momentum and Kinetic energy are conserved i.e. change in momentum dp = constant and $k_i = k_f$.

Inelastic Collision: Momentum is conserved by Kinetic energy decreases.

i.e. dp = constant but $k_i \neq k_f$.

LAW OF CONSERVATION OF LINEAR MOMENTUM

In a closed, isolated system containing a collision, the linear momentum of each colliding body may change but the total linear momentum $\vec{p}$ of the system cannot change, whether the collision is elastic or inelastic.

Collision in one dimension: For objects moving along x-axis or y-axis, velocity has a single component. Under **in-elastic collision**, the two bodies join together after collision and move with common velocity v. Therefore, we can write;

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad \text{i.e. dp = constant}$$

$$\frac{1}{2}m_1{u_1}^2 + \frac{1}{2}m_2{u_2}^2 = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 \qquad k_i = k_f$$

Elastic collision in one dimension:

The two bodies of masses m_1, m_2 and moving with initial velocities u_1, u_2 respectively, if after collision the velocities change to V_1, V_2 respectively, then we can write

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2 \quad \text{i.e. } dP = Constant$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 \quad \text{i.e. } k_i = k_f$$

Collision in Two Dimensions

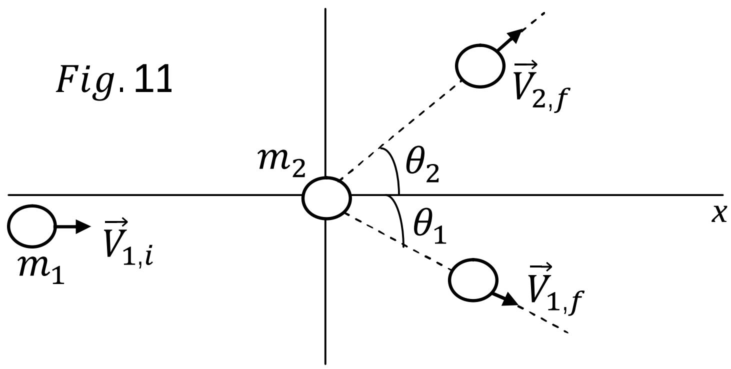
When collision between two bodies is not head-on, the direction of their final velocities is not along their initial axis. For this type of collision, linear momentum is still conserved.

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

If collision is elastic, total kinetic energy is conserved.

$$k_{1i} + k_{2i} = k_{1f} + k_{2f}$$

Fig. 11



From Fig. 11 above, after collision, the impulse between the bodies sends the bodies off at angles θ_1 and θ_2 to the *x axis*.
Momentum is a vector quantity, so the components along x-axis is

$$m_1 V_{1i} = m_1 V_{1f} \cos\theta_1 + m_2 V_{2f} \cos\theta_2$$

And along y-axis is

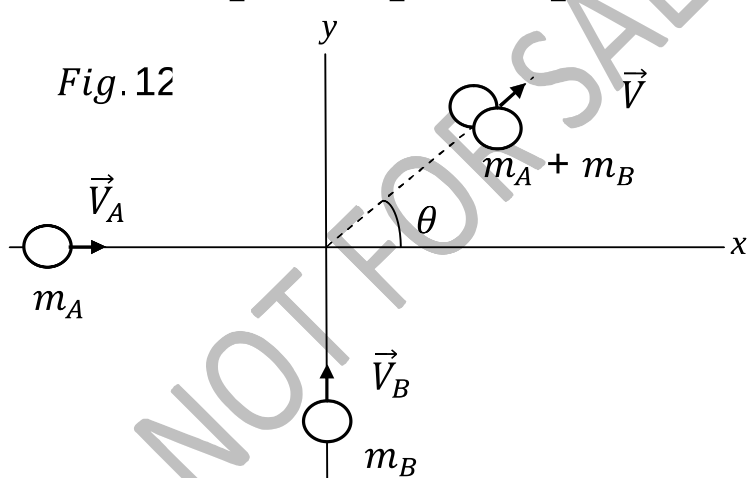
$$0 = -m_1 V_{1f} \mathrm{Sin}\theta_1 + m_2 V_{2f} \mathrm{Sin}\theta_2$$

Also, kinetic energy is conserved.

$$\frac{1}{2} m_1 {V_{1i}}^2 = \frac{1}{2} m_1 {V_{1f}}^2 + \frac{1}{2} m_2 {V_{2f}}^2$$

Example

Fig. 12



Two objects A and B collide and embrace, in a completely inelastic collision. Thus, they stick together after impact as shown in the diagram above, where the origin is placed at the point of collision. A is of mass 83 Kg originally moving east with speed 6.2 Km/h and B is of mass 55 Kg and is originally moving north with speed 7.8 Km/h. What is the common velocity $\vec{v}$.

Questions

1. A 1600Kg van moving at 20ms^{-1} crashes into the back of a small car of mass 700Kg traveling in the same direction at 15ms^{-1}.

a) What is the velocity of the two vehicles immediately after the collision of the lock together?
b) How much kinetic energy is transferred to other forms in the collision?
Ans: (a) 18.478 m/s (b) 6098.04 J

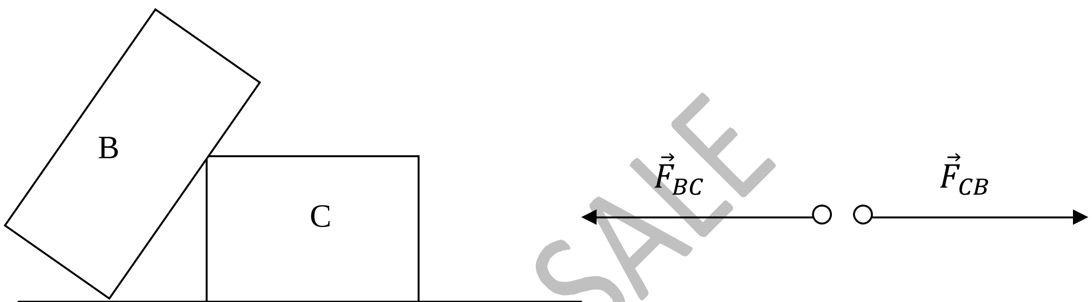
2. In an experiment using a linear air track a, 0.5Kg rider travelling to the right at 2ms^{-1} collides with a stationary rider of mass 1Kg. Find the velocities of the two riders after the collision if:
    a. It is perfectly elastic Ans: V1 = -0.667 m/s V2 = 1.33 m/s
    b. It is totally inelastic. Ans: 0.667 m/s

3. The National Transportation Safety Board is testing the crash-worthiness of a new car. The 2300Kg vehicle moving at 15 m/s is allowed to collide with a bridge abutment, which stops it in 0.56 s. What is the magnitude of the average force that acts on the car during the impact? Ans: 61607.1 N
4. A 150 g baseball pitched at a speed of 40 m/s is hit straight back to the pitcher at a speed of 60 m/s. What is the magnitude of the average force on the ball if the bat is in contact with the ball for 5.0 m/s? Ans: 3,000 N
5. Two 2.0Kg bodies, A and B collide. The velocities before the collision are $\overrightarrow{V_A} = 15i + 30j$ and $\overrightarrow{V_B} = -10i + 5.0j$. After the collision, $\overrightarrow{V_A} = -5.0j + 20j$. All speeds are given in meters per second. (a) What is the final velocity B? (b) How much Kinetic energy is gained or lost in the collision? Ans: (a) 10i + 15j (b) 525 J

NEWTON'S THIRD LAW OF MOTION

When two objects interact, the forces on the objects from each other are always equal in magnitude and opposite in direction.

Fig. 13



Consider the two objects shown in Fig. 13 box B is leaning on crate C, the third law can be written for this arrangement as a scalar relation.

$F_{BC} = F_{CB}$ i.e. equal magnitude

As a vector relation,

$\overrightarrow{F_{BC}} = \overrightarrow{F_{CB}}$ i.e. equal magnitude and opposite direction

Where F_{BC} is the force on the box from the crate

F_{CB} is the force on the crate from the box.

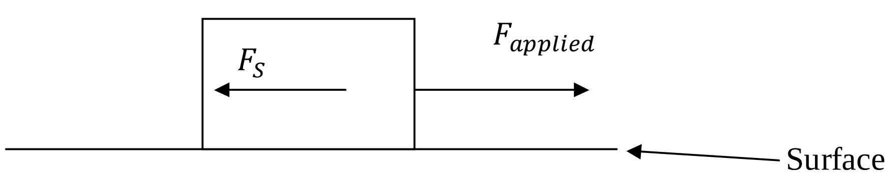
FRICTION

Friction is the force exerted by a surface as an object moves across it or makes an effort to move across it. From this definition, friction is called to play only when an object moves or tries to move across a surface.

Consider an object placed on a surface as shown in fig. 13. If there is no movement of the object, though there is an applied force $F_{applied}$, in the direction shown, then

Frictional force $F_S = F_{applied}$ (i.e. if a=0)

Fig. 14



The implication of the above equation is that F_s is equal to the applied force ($F_{applied}$) as long as there is no movement of the object. Fig. 15 is an illustration of the relationship between the applied force and static friction F_s. The relation is broken just as the object is about to move. At this point, we have what is called maximum static friction $F_{s,max} = \mu_s F_N$, where μ_s is called coefficient of static friction and F_N=mg is the normal reaction or weight of the object.

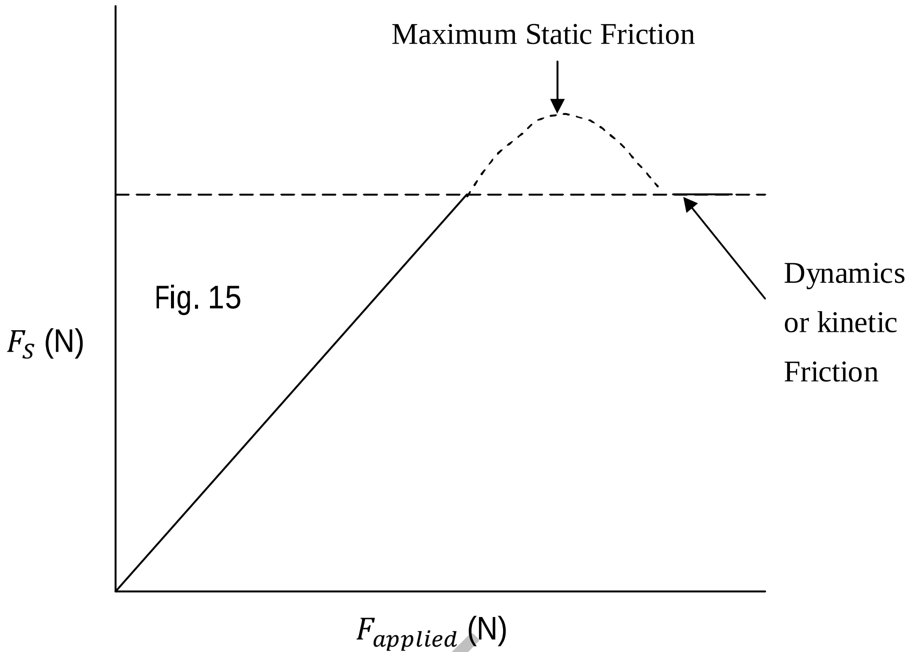


Fig. 15

The moment the object starts to move, dynamic friction or kinetic friction is called into play.

Dynamic friction $F_K = \mu_k F_N$, where μ_k is the coefficient of dynamic friction.

Static Frictional force acts between surfaces at rest when a force is applied to make them slide over each other. Kinetic frictional force act between surface as they slide over each other.

Factors upon which friction depends

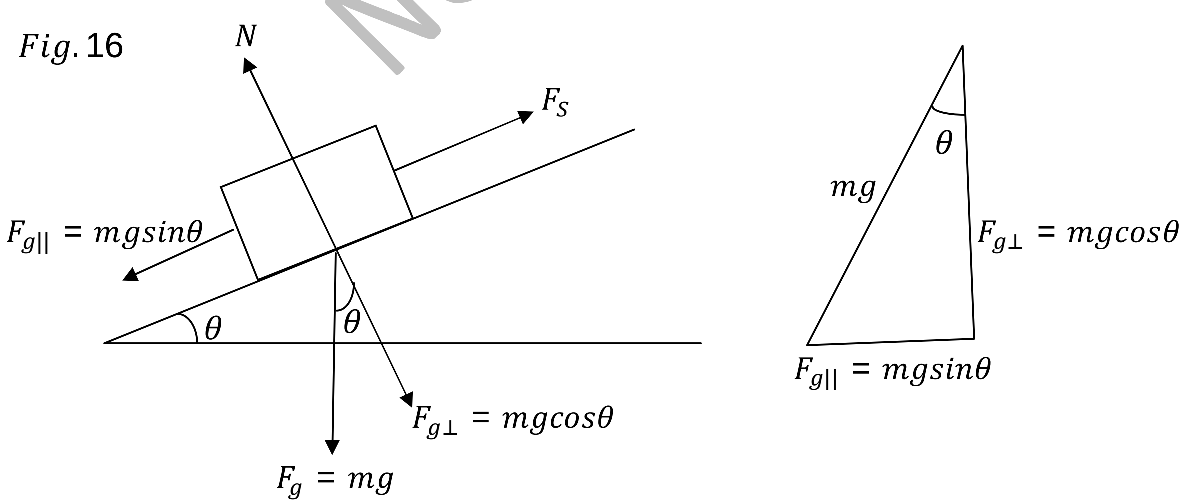
Friction depends on:

1. The nature of the two surfaces in contact
2. In the case of an object placed on a surface, friction depends on the weight of the object.

Note: Friction does not depend on the amount of surface area in contact or the relative speed of the bodies.

Inclined Plane

$Fig.$ 16



Inclined plane can as well be treated like horizontal surface in which forces act along the surface. Fig. 16 shows an object placed on an inclined plane and the forces. In this case, not only forces parallel to the surface and their directions are to be considered.

To be considered are the forces acting along the surface of the inclined plane, parallel component of the weight of the object $F_{g11} = mgsin\theta$ and the friction $F_s = \mu N$ (N=Normal Reaction). The right angle triangle on the right of the figure shows the weight and the parallel (F_{g11}) and vertical ($F_{g\perp}$) components of the weight.

If no external force acts on the object, the object will start to slide when F_{g11} is equal to the maximum static friction (i.e. $F_{s,max}$). At that point, one can estimate the coefficient of static friction of the inclined plane.

$$F_{s,max} = F_{g11} = mgsin\theta$$

But $F_{s,max} = \mu_s N$ and $N = F_{g\perp} = mgcos\theta$ (i.e. from 3rd law)

$$\mu_s mgcos\theta = mgsin\theta$$

$$\mu_s = \frac{mgsin\theta}{mgcos\theta} = \frac{sin\theta}{cos\theta} = tan\theta$$

$$\mu_s = tan\theta$$

WORK AND ENERGY

Work is done when an object is moved through a distance in the direction of the applied force, while energy is the ability to do work. The unit of work and that of energy is in Joule (J).

Work done by a constant force

Suppose an object moves from an initial point x_i to a final point x_f so that the displacement of the point the force acts on is positive i.e.

$$\Delta x = F_{applied} - x_i > 0$$

Then, workdone $W = F_{applied} . \Delta x$

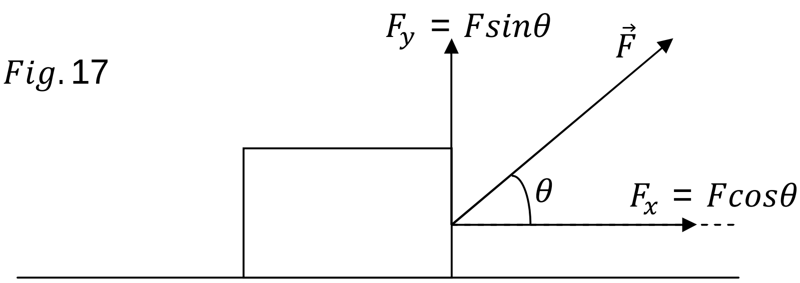
Work is a scalar quantity and the unit is in (J).

Work done against friction, $W_{friction} = -\mu_K mg\Delta x$

Work done by a constant force applied at an angle

If the direction of the applied force is at an angle θ to the horizontal, then the x-component of the applied force is responsible for the motion of the object along the horizontal.

$Fig.$17



Work done $W = F_x \, X \, \Delta x$

Conservation of Energy: The law of conservation of energy states that the total energy of a closed system is constant.

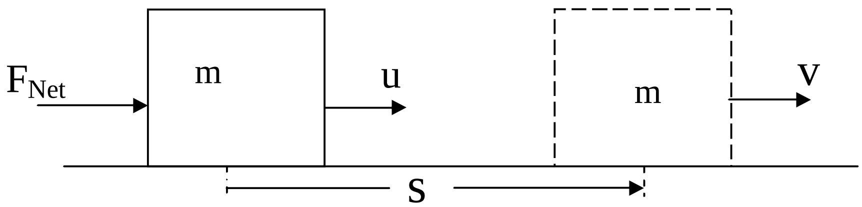
WORK AND KINETIC ENERGY

From the definition of work, applied force must have moved an object through a distance – meaning that it causes the object's speed to change. Hence, the work done on an object can be expressed as a function of the object's final speed v and mass m i.e. Kinetic energy of the object.

$$K.E. = \frac{1}{2}mv^2$$

WORK-ENERGY THEOREM

Fig. 18



Consider the diagram in Fig. 18 above, assuming a net force F_{Net} in the direction indicated causes an object of mass m to move through a distance S. Then from the Newton's 2^{nd} law of motion,

$$\overrightarrow{F_{Net}} = m\vec{a},$$

For a motion under constant acceleration, we write

$$2ax = v^2 - u^2 \; as \; \frac{v^2 - u^2}{2}$$

Where a is the acceleration and x or s is the distance or displacement.

From the depth of work,

$$W_{net} = F_{net}.S = mas = m\frac{v^2 - u^2}{2}$$

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = KE_f - KE_i = \Delta KE$$

We can therefore say that total net work done is equal to the change in Kinetic energy.

Simple Harmonic Motion

1 Object

To determine the period of motion of objects that are executing simple harmonic motion and to check the theoretical prediction of such periods.

2 Apparatus

Assorted weights and spheres, clamps, meter stick, spring, stand, stopwatch, protractor, string, motion detector and interfaced computer.

3 Theory

Simple harmonic motion refers to a type of movement that is found in a number of apparently dissimilar situations. Three such situations will be studied in this experiment. Harmonic motion is that which repeats itself at certain intervals; one such interval is called the period of motion. A specific type of periodic motion, simple harmonic motion, has a rather straightforward mathematical representation,

$$x = A\cos(\omega t + \delta) \tag{1}$$

where A is the amplitude of motion, ω is the angular frequency, t is the elapsed time, and δ is a phase factor identifying at what point in the cycle we chose $t = 0$. The angular frequency is related to the period through the relationship

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \tag{2}$$

If an object undergoes motion as described above, the object's velocity and acceleration as functions of time will be,

$$\begin{aligned} v &= -A\omega\sin(\omega t + \delta) \\ a &= -A\omega^2\cos(\omega t + \delta) \end{aligned} \tag{3}$$

One can now see the necessary condition for an object to undergo simple harmonic motion. Notice in equation 3, that the acceleration as a function of time is proportional to $A\cos(\omega t + \delta)$ which is nothing other than the position of the object as a function of time. Substitution yields,

$$a = -\omega^2 x \tag{4}$$

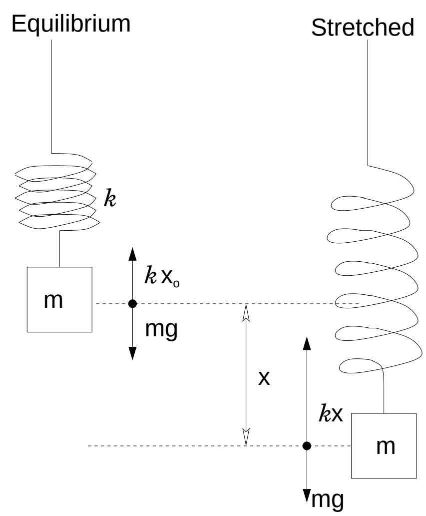
An object undergoing simple harmonic motion must always have an acceleration directed in the opposite direction of the object's displacement from equilibrium and the magnitude of the acceleration is always proportional to the objects displacement. In other words, the acceleration is always directed towards equilibrium and gets proportionately larger in magnitude as the object moves further from equilibrium. Since the net force is always in the direction of an object's acceleration, it is often said that simple harmonic motion requires a restoring net force, a net force directed towards equilibrium, which is proportional to the displacement from equilibrium. If one can show that the acceleration is equal to a negative constant times the displacement from equilibrium, the motion will be simple harmonic and the angular frequency of the motion is the square root of the magnitude of that constant.

3.1 Part 1 Mass on a Spring

It was found experimentally by Robert Hooke that to within elastic limits (no permanent deformation) a spring will be stretched a distance proportional to the applied force. Mathematically this is stated as

$$F = -k \cdot stretch \tag{5}$$

where F is the force applied to the spring and k is a constant of proportionality, often called the *spring constant*, which is a measure of the elasticity of the spring. The negative sign shows that the force exerted by the spring is always in the opposite direction of the stretch of the spring. If a mass is at rest, suspended from the spring, the mass is in static equilibrium. The only forces acting on the mass are a gravitational force down, mg, and the spring force up, kx_o, where x_0 is the



stretch of the spring at equilibrium. If the mass is now displaced from that equilibrium position by some displacement x, the net force acting on the mass would be the extra force due to this further displacement x. Newton's second law would yield

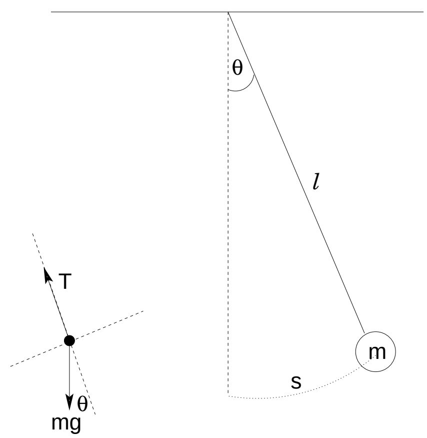
$$F_{net} = -kx = ma \quad or \quad a = -\frac{k}{m}x \tag{6}$$

after solving for the acceleration. Notice that this is just the form which indicates motion which is simple harmonic. The resulting displacement of the mass from the equilibrium position as a function of time must be given by equation 1. The constant in equation 6 can be used in conjunction with equation 2 to show that for the oscillating motion of a mass on a vertical spring

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{7}$$

3.2 Part 2 Simple Pendulum

A similar type of reasoning will allow one to derive the period of motion for a simple pendulum. An example of a simple pendulum is a mass attached to a massless string of length, ℓ, which in turn is attached to a fixed object. Assume the string to be pulled away from the vertical by a small angle θ; the mass attached to the string is displaced through a circular arc of length s. Applying



Newton's 2^{nd} law to the object in the direction of motion gives

$$F_{net} = ma = mg\sin(\theta) \tag{8}$$

for the magnitude of the net force acting on the mass in the direction of motion. Notice that the net force is always directed toward the equilibrium position at the bottom of the pendulum's swing. Also, for small angles, $\sin(\theta) \simeq \theta = \frac{s}{\ell}$, where s is the displacement of the mass from equilibrium. Solving equation 8 for the acceleration a yields

$$a = -\frac{gs}{\ell} \tag{9}$$

where the minus sign indicates that the net force is in the opposite direction of the displacement. This result is the same form as equation 4, which was the condition for simple harmonic motion. The expected period of this motion would be

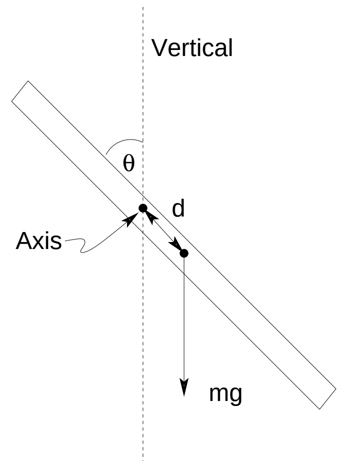
$$T = 2\pi\sqrt{\frac{\ell}{g}} \tag{10}$$

where ℓ is the length of the pendulum and g is the acceleration due to gravity.

If the angle of displacement is not small, the $\sin\theta$ cannot be reasonably approximated by θ and one has a much more complicated expression

$$T = 2\pi\sqrt{\frac{\ell}{g}}\left[1 + \frac{\left(\sin\frac{\theta}{2}\right)^2}{2^2} + \frac{1}{2^2}\frac{3^2}{4^2}\left(\sin\frac{\theta}{2}\right)^4 + \ldots\right] \tag{11}$$

The ... means "and so on forever."



3.3 Part 3 Physical Pendulum

A similar result is obtained for the physical pendulum. This is a situation where an object of finite mass and size is rotating about a fixed point on the object itself. In this situation, one considers the rotational equivalent of Newton's Second Law

$$\tau_{net} = I\alpha \tag{12}$$

where τ_{net} is the net torque acting on the object, I is the object's moment of inertia about the fixed point and α is the angular acceleration. Once again, it is clear that the torque will be a restoring torque since any displacement will produce motion back toward a vertical orientation of d (see figure below). Note that

$$\tau_{net} = Fr_{\perp} = mgr_{\perp} = mgd\sin(\theta) \tag{13}$$

where m is the mass of the object, P is the pivot point, C is the center of mass, d is the distance between those two points and θ is the angle of displacement of d from the vertical. If once again, the angle of displacement is small, $\sin\theta \simeq \theta$ and

$$\alpha = -\frac{mgd}{I}\theta \tag{14}$$

where again, the negative sign is due to the direction of the net torque. The restoring torque is proportional to the angular displacement and one will have simple harmonic motion. Again, by comparing equation 14 with equation 4 and using equation 2, the period of this simple harmonic motion is determined as

$$T = 2\pi\sqrt{\frac{I}{mgd}} \tag{15}$$

where the only term to be determined is I, the moment of inertia of the physical pendulum about the pivot point P. The moment of inertia is the rotational equivalent for the mass of an object. It will, in general, depend on the mass of the object and the location of the pivot point. About the object's center of mass, the moment of inertia will be a minimum; however, as the pivot point recedes from the center of mass, the object will better resist changes in its rotational motion meaning I has increased. The parallel axis theorem allows us to find the moment of inertia of an object about some point that isn't its center of mass by

$$I_P = I_{c.m.} + md^2 \tag{16}$$

where m is the object's mass, d is the distance between point P and the center of mass and the moment of inertia about the center of mass is known. For the bar used in part 3 of the experiment, the moment of inertia is then

$$I_P = \frac{mL^2}{12} + md^2 \tag{17}$$

where L is the length of the bar. Again, if the angle of initial displacement is not small, a correction similar to that made for the case of a simple pendulum is in order,

$$T = 2\pi\sqrt{\frac{I}{mgd}}\left[1 + \frac{\left(\sin\frac{\theta}{2}\right)^2}{2^2} + \frac{1}{2^2}\frac{3^2}{4^2}\left(\sin\frac{\theta}{2}\right)^4 + \ldots\right] \tag{18}$$

4 Procedure

1. Set up a simple pendulum. Pull the string aside so that it makes an angle of 30° with the vertical. This is called the initial angle. Let the object swing and record the time for ten full oscillations. Record the length of the string. This is the distance from the point where the string is attached to the center of mass of the object used as the pendulum. Repeat this four times for the same object, same length, and same initial angle.

   Change the initial angle to 5° and repeat the procedure. Then repeat the above procedure for initial angles of 10°, 15°, and 45°.

2. To determine the spring constant k of a spring, hang a $150\,g$ mass on a spring, then record the additional stretching due to $30\,g$ more, then $60\,g$, $90\,g$, $120\,g$, $150\,g$, and $180\,g$.

3. Setup the computer to take data using the motion detector. You may load settings from the file *shm*. Remember, the yellow motion detector plug goes in the leftmost input. Place the motion detector on the bench top, facing up, and orient the spring so that it is hanging directly above the motion detector. Place $300\,g$ total on the end of the spring. Tape the masses to the mass hanger, and also tape the mass hanger hook to the bottom of the spring, so that nothing can fall off and possibly damage the motion sensor. Adjust the support bar so that the equilibrium position is at least $50\,cm$ above the motion detector. Record graphs for several periods of the motion resulting from displacing the mass $10\,cm$ and releasing it. Make copies of these graphs. Take a good look at these three graphs to better understand the relationships between the physical quantities position, velocity and acceleration for an object undergoing simple harmonic motion. Do the graphs look sinusoidal?

4. Record the following information from the graphs. When the mass is at its maximum positive displacement, what is its velocity, what is its acceleration? When the mass passes back through equilibrium, what is its velocity and acceleration. Repeat at the maximum negative displacement and at the following equilibrium point. From the graphs, determine the period of the simple harmonic motion. Measure and record the period at 3 different points for each of the three graphs. Also measure and record the amplitude of the motion, the maximum speed of the object and the maximum acceleration.

5. Using a stopwatch, record the time for 20 oscillations of the mass on the spring.

   Repeat this timing process for five different masses, recording the time for twenty oscillations for each mass.

6. Construct a physical pendulum so that it is able to swing about a pivot point. The physical pendulum in this case is a meter stick. Record the length and mass of this object. Also record the distance from its pivot point to its center. Then pull it from the vertical so that it makes an initial angle of 30°. Release it and time 10 oscillations. Repeat the above measurement for an initial angle of 5°.

   Change the pivot point of the object and repeat the above procedure. Record the location of the new pivot point. Then repeat the entire process for three other pivot points.

5 Calculations

1. Calculate the theoretical period for one oscillation of the simple pendulum by using equation 10. Compare this with the experimentally (stopwatch) determined period (mean, error and range) as measured directly.

2. Use your data from procedure step 2 to determine the spring constant of your spring. To do this, construct a graph of added weight (weight of the additional mass added to the initial $150\,g$) vs additional stretch. Does this graph appear to be linear? Should it? Determine the best fit line to the data using a linear regression program. According to equation 5, the slope of this line should be related to spring constant of the spring. Determine k from your best fit line. Is the y intercept of the best fit line what it should be? Is Hooke's law valid for your spring?

3. Calculate the average of the 9 values for the period of the oscillating mass measured from your motion detector graphs in step 4 of the procedure. Using this value for the period and your measured amplitude, calculate the expected maximum speed and the expected maximum acceleration as predicted by equation 3. How do these values compare to those measured from the graphs?

4. Using the values for T from the stopwatch measurements, construct a graph of period vs mass. Also construct a graph of T^2 vs total mass on spring. Do these graphs look like they should? According to equation 7 the period squared graph should be a straight line with slope of $\frac{4\pi^2}{k}$. Using the value of k extracted from the graph in step 2 of calculations, calculate a theoretical value for the slope of your new graph. Use a linear regression program to find the best fit line to the data. Do the slope and y intercept of this line agree with your expected values?

5. Turning to the physical pendulum data, use equation 15 to calculate theoretical values for the period of your physical pendulum for each value of d. Which set of data, 5° or 30°, agrees best with the theoretical predictions? Explain.

6 Questions

1. If you chose the center of the meter stick as the pivot point, what would the period be? Give a physical (non-mathematical) explanation.
2. If spring A has a spring constant which is 8 times spring B's spring constant, what is the ratio of their periods? Which is the stiffer spring?
3. Why doesn't your period squared vs mass graph for the spring go through the origin? Explain.

Simple Harmonic Motion Data Sheet

Part 1 - Simple Pendulum

Length (cm): ____________________

Trial	5°	10°	15°	30°	45°
1					
2					
3					
4					
5					

Which of the five initial angles should give results that most closely agree with the theoretical predictions? Why?

Part 2 - Mass on a spring

Additional Mass (g)	Additional Stretch (cm)

Added Mass (g)	Time 20 Osc. (sec)

Simple Harmonic Motion Data Sheet

Period (*sec*)				

Displacement	Velocity	Acceleration
Max. Pos. Disp		
Equil. Moving Down		
Max. Neg. Disp		
Equil. moving Up		

Amplitude	
Max. Speed	
Max Acceleration	

Part 3 - Physical Pendulum

Pendulum Mass	
Pendulum Length	

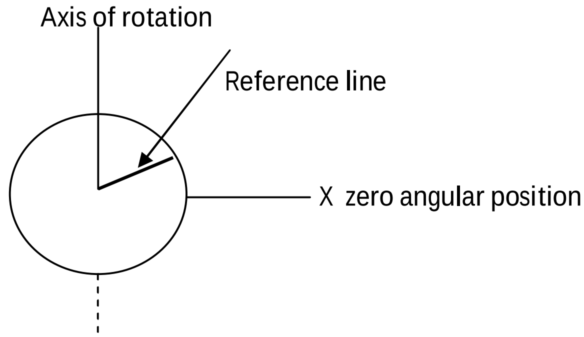
Distance From Center to pivot	Time for 10 oscillations	
	30°	5°

RIGID BODY DYNAMICS

A rigid body is a body that can rotate with all its parts locked together and without any change in its shape.

Rotation of a rigid body about a fixed axis

A fixed axis means that the rotation occurs about an axis that does not move.



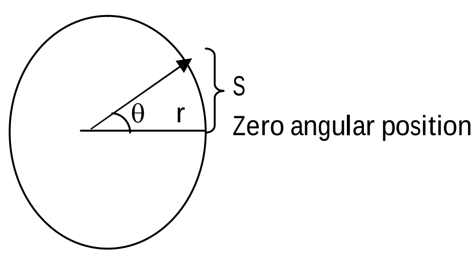
Consider a disk rotating about an axis shown in the diagram above with reference line fixed on the desk and x is on the zero angular position.

We can define the following.

<u>Angular Position, θ</u>

This is the angle of the reference line relative to a fixed direction which is taken as the zero angular position in the diagram above.

θ is measured in radian.



from the diagram

$$\sin\theta = \frac{s}{r}$$

1 revolution = 360^0 = 2π rad. For small θ, $\sin\theta \cong \theta$

$$\theta = {}^{s}\!/_{r}$$

S = length of are along a circle.

r = radius of that circle.

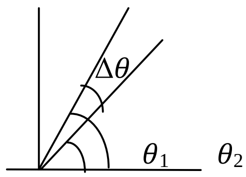
1 rad = 57.3^0 = 0.159 rev

Angular Displacement $\Delta\theta$

If a body rotates about the rotation axis with the angular position of the reference line changing from θ_1 to θ_2, the resulting angular displacement $\Delta\theta$ is given by

$$\Delta\theta = \theta_2 - \theta_1$$

$\Delta\theta$ is positive for counter clockwise direction a d – ve for clockwise direction.



Angular velocity W

Consider a rotating body with angular position θ_1 at time t_1, and angular position θ_2 at time t_2. Average velocity was is given by

$$W_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

Also, Instantaneous angular velocity W

$$W = lim \quad \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

W is measured in $rads^{-1}$ or $rev.s^{-1}$

Angular Acceleration α

A changing angular velocity means that an angular acceleration is occurring.

Average angular acceleration of the rotating body in the interval t_1 and t_2 is defined as

$$\alpha_{av} = \frac{w_2 - w_1}{t_2 - t_1} = \frac{\Delta w}{\Delta t}$$

Instantaneous angular acceleration α

$$\alpha = \lim \frac{\Delta w}{\Delta t} = \frac{dw}{dt}$$

α is measured in rad s^{-2} or rev. s^{-2}

Examples

1. After being turned on, a turntable reaches its rated angular speed of 45.0 rev/min in a time interval of 4.10s what is the average angular acceleration in rad/s^2?

2. An automatic drier spins wet clothes at an angular speed of 65 rev/min. starting from rest the drier reaches its operating speed with an average angular acceleration of 7.0 rads^{-1}. How long (in seconds) does it take the driver to come up to this speed?

3. During a time interval t the flywheel of a generator turns through the angle θ = at + bt^2 – ct^4, where a, b and c are constants. Write expressions for the wheel's (a) angular velocity and (b) angular acceleration.

4. A diver makes 2.5 revolutions on the way from a 10m high platform to the water. Assuming zero vertical velocity find the driver's average angular velocity during a dive.

Ans. 11 rads^{-1}

Rotation with Constant Angular Acceleration.

Equations

$$w = w_0 t + \frac{1}{2}\alpha t^2$$

$$\theta - \theta_0 = w\,t + \frac{1}{2}\alpha t^2$$

$$w^2 = w_0^2 + 2\alpha\left(\theta - \theta_0\right)$$

$$\theta - \theta_0 = \frac{1}{2}\left(w_0 + w\right)t$$

$$\theta - \theta_0 = wt - \frac{1}{2}\alpha t^2$$

Q1. The angular speed of an automobile engine is increased at a constant rate from 1200 rev/min to 3000 rev/min in 2s.

(a) What is its angular acceleration in revolutions per minute squared?

(b) How many revolutions does the engine make during this 12 seconds interval?

Q2. A record turntable rotating at 33½ rev/min slows down and stops in 30s after the motor is turned off.

(a) Find its (constant) angular acceleration in rev min^{-2}

(b) How many revolutions does it make in this time?

Q3. A disk initially rotating at 120 rads^{-1} is slowed down with a constant angular acceleration of magnitude 4.0 rads^{-2}

(a) How much time does the disk take to stop?

(b)Through what angle does the disk rotate during that time?

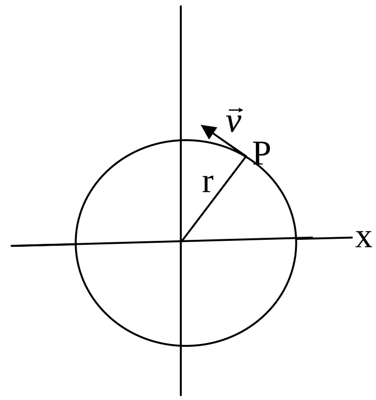
Linear and Angular Variables

If a reference line on a rigid body rotates through angle θ about an axis

$$s = \theta r \qquad 1 \qquad \theta \text{ in radian}$$

Differentiating the equation above gives us the speed

$$\frac{ds}{dt} = \frac{d\theta}{dt} r$$



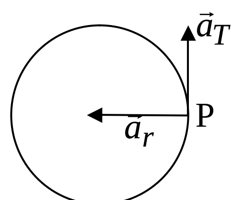
But $\frac{ds}{dt}$ = V the linear velocity and $\frac{d\theta}{dt}$ = w the angular speed

$$V = wr \qquad 2 \quad , \qquad w \text{ in rads}^{-1}$$

Please <u>Note:</u> pts with greater r have greater v.

Accelleration

$$a = \frac{dv}{dt}$$



Differentiating (2) with respect to t

$$\frac{dv}{dt} = \frac{dw}{dt} r \qquad 3$$

$$a_t = \alpha r$$

$\vec{a}_T$ =Tangential acceleration

$\vec{a}_r$ = radial component

Also radial acceleration a_r

$$a_r = \frac{v^2}{r} \qquad 4$$

Put equ (2) in to equ (4)

$$a_r = w^2 r$$

Kinetic Energy of Rotation

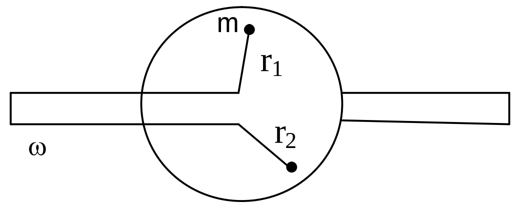
A rotating body possesses K.E, because its constituent points are in motion. If the point's mass is m as shown in the diagram below, K.E of a single point can be written as

$$K.E = \frac{1}{2} m v_T^2$$

where v_T *is* the tanjential speed or linear speed

$$K.E = \frac{1}{2} m v_T^2 = \frac{1}{2} m r^2 w^2$$

.



The K.E of the entire rotating body is the sum of K.Es of the points

Rotational K.E = $\sum \frac{1}{2} mr^2 w^2 = \frac{1}{2}\left(\sum mr^2\right) w^2$

$\sum mr^2 = I$ and is called moment of inertial of the body. It is a constant for a particular rigid body and a particular rotating axis.

Therefore Rotational Kinetic Energy

$$K.E = \frac{1}{2} Iw^2$$

where W is in $rads^{-1}$ and K.E in Joule (J).

For a rigid body undergoing both rotational and translated motion, the total K. E of the body is the sum of its translational and rotational kinetic energies.

Total K.E = $\frac{1}{2} mv^2 + \frac{1}{2} Iw^2$

<u>Moment of inertial *I*</u>

Movement of Inertia

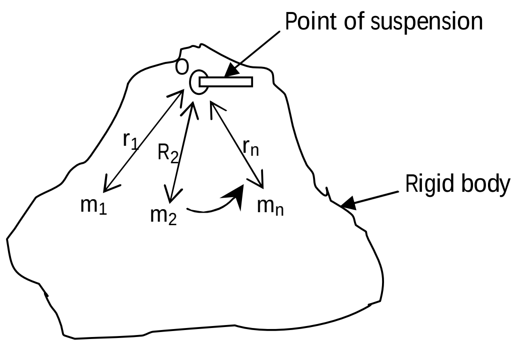


Fig.: Suspended Rigid Body

Consider a rigid body rotating about O, containing particle masses m_1, m_2, m_3, ---- m_n and at distances r_1, r_2, r_3 ---- r_n from O.

The sun of the products of mass of each particle of the body and the square of its perpendicular distance from the axis of rotation is called moment of inertia I.

$I = m_1 r_1^2 + m_2 r_2^2 \ldots\ldots m_{1n} r_n^2$

$I = \sum_{n=1}^{n} m_{1n} r_n^2$,

Unit of I = kgm^2

Total rotational kinetic energy $(ke) = \frac{1}{2}m_1 r_1^2 \omega^2 + \frac{1}{2}m_2 r_2^2 \omega^2 \ldots\ldots m_n r_n^2 \omega^2$

$$i.e.\, ke = \frac{1}{2}\omega^2 \sum_{n=1}^{n} m_n r_n^2$$

$$i.e.\, ke = \frac{1}{2}I\omega^2 \equiv \frac{1}{2}mV^2$$

(≡: means compare with)

$\rightarrow I \equiv m$

$\omega = \ angular\ velocity$

Moment of inertia is analogous to mass but the man differences are:

(a) Its value own varies for a body while mass is constant

(b) Its unit is kgm^2 but kg is for mass.

(c) It depends on how mass is distributed in a body but mass is not

(d) It exists for a rotating rigid body but mass does not depend on rotation

Determination of I for Various forms/Shapes of Bodies

(a) Moment of Inertia I of uniform rod about axis through its centre

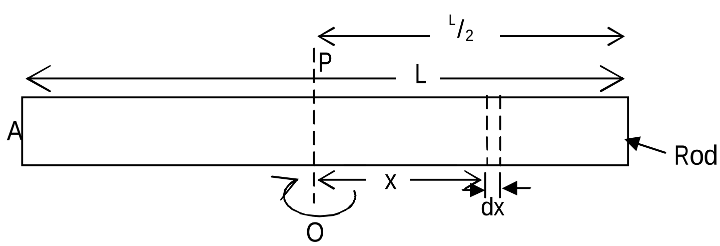


Fig: Rod, rotating about axis O at its centre

Mass of rod =m

Length of rod =L

X – Sec. Area = A

Volume density = ρ

Let the rod be divided into slices, dx thick

Volume of slice = Adx

$\therefore$ Mass of the slice (from V ρ) = Adx ρ

$\therefore$ I for the slice shown about OP (from mx^2) = Apx^2dx

$\therefore$ Total l for the whole rod = $A\rho \int_{-\frac{1}{2}}^{\frac{1}{2}} x^2 \, dx$

$$= A\rho \left[\frac{x^3}{3}\right]_{-\frac{1}{2}}^{\frac{1}{2}}$$

$$I = \frac{Am}{AL}\left[\frac{L^3}{24} \pm \frac{L^3}{24}\right]$$

$$i.e. \mathrm{I} = \frac{\mathrm{M}L^2}{12}$$

(b) I of uniform rod about axis through one end

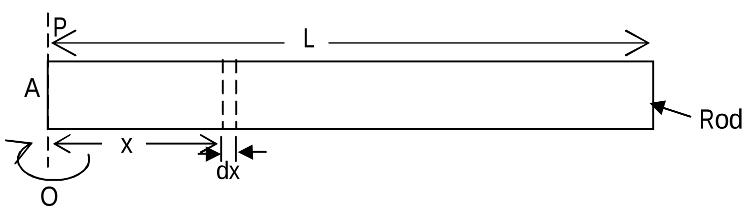


Fig: Rod, rotating about axis O at one end

Mass of rod =m

Length of rod =L

Location of the dx from the axis of rotation = X

Sectional area =A

Mass/vol. = ρ

Slice's width =dx

Volume of slice =Adx

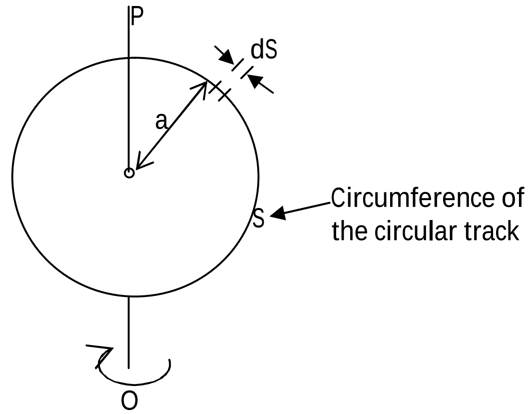
Mass of slice = ρAdx

$\therefore$ I for slice about PO = ρAdx X^2

$\therefore$ Total I for the whole rod is = $A\rho \int_0^L X^2 dx$

$$\mathrm{I} = \frac{mL^3}{3}$$

(c) I of ring about axis through its centre



Mass of ring = m

Fig.: Circular Ring rotating about its centre

Length of ring = S = 2πa

Mass/length of slice (i.e. linear density) = ρ

Slice's length = dS

Mass of slice = ρdS

I of slice = ρdS a^2

$$Total\ I = \rho a^2 \int_0^{2\pi a} dS$$

$$= [S]_0^{2\pi a}$$

$$i.e.\ I = ma^2$$

(d) I of circular disc about axis through its centre

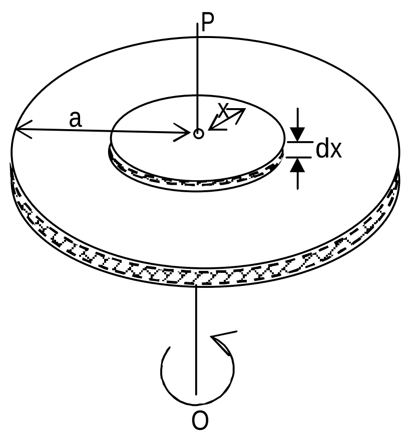


Fig.: Circular disc rotating about its centre

Let ρ =m/A

i.e. for the entire disc, ρ = m/πa^2

i.e. m = ρ πa^2

Area of slice (i.e. in pipe form) =2πxdx

Mass of slice = 2πxρdx

Moment of inertia of slice = $2\pi x\, x^2 dx\rho = 2\pi\, x^3 \rho dx$

Total MI of disc $= 2\pi\rho \int_0^a x^3\, dx$

$$= 2\pi\rho \left[\frac{x^4}{4}\right]_0^a$$

$$= \frac{ma^2}{2}$$

(e) MI of Hollow cylinder about axis through its centre

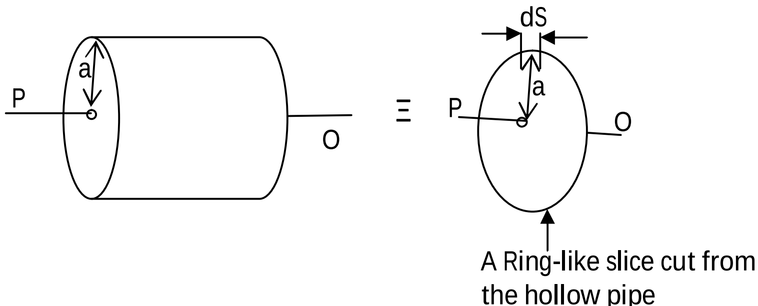


Fig.: Hollow pipe rotating about its centre

We can see that the slice is a ring. So We can adopt ring procedure and generalize thus:

i.e. $I = m_1a^2 + m_2a^2$ …………….

(f) MI of solid cylinder about axis through its centre

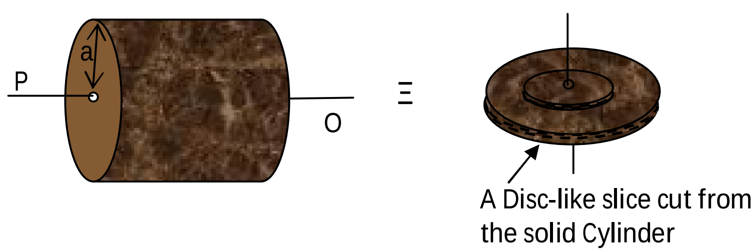


Fig.: Solid Cylinder rotating about its centre

We can see that the slice is a disc. So

We can adopt disc procedure and generalize, thus:

$$I = \frac{m_1a^2}{2} + \frac{m_2a^2}{2} \ldots\ldots\ldots\ldots$$

$$i.e.\ total\ I = \frac{Ma^2}{2}$$

EX 1. Prove the MI of Lamina about axis through its centre parallel to its plane

EX2. Prove the MI of solid sphere about axis through its diameter

Hints:

Ex 1 ρ= Mass/Length

Length = 2a, breadth = 2b

Ans: $I = Ma^3/3$ or $Mb^3/3$

Ex 2.

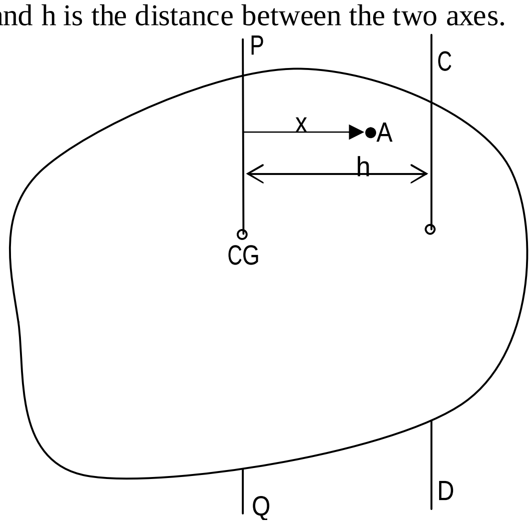
Slice is a solid cylinder in shape

Radius of sphere = a

ρ= Mass/Volume
Ans: $I = 2ma^2/5$

Parallel axes theorem

The moment of inertia of a body about any axis is equal to the moment of inertia IG about a parallel axis through the centre of gravity of the body plus Mh2, where M is the mass of the body and h is the distance between the two axes.



Let I = MI about CD

I_G= MI about PQ, axis through point of the center of gravity CG,

PQ is parallel to CD and h is perpendicular distance away from CD

Let a unit Particle A of mass m be at distance x from PQ axis.

Moment of inertia I of A about CD = $m (h – x)^2$

i.e. $I = \Sigma m (h – x)^2$

$I = \Sigma mh^2 + \Sigma mx^2 – \Sigma 2mhx$ ---- (1)

But $\Sigma mh^2 = h^2\Sigma m = Mh^2$ (M = total mass) ---- (2)

$\Sigma Mx^2 = I_G$ ---- (3)

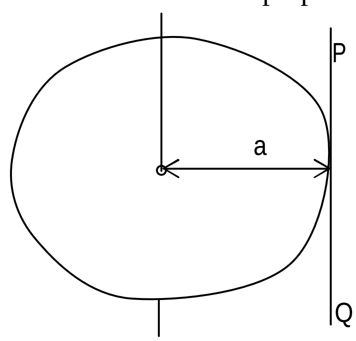
$\Sigma 2mhx = 2h\Sigma mx = 0$ ----(4), since Σmx =Sum of moment about CG = 0,

Using equations 2,3 and 4 in equation 1

i.e. $I = I_G + Mh^2$ (Parallel axis law proved).

e.g.

What is the moment of I of a disc, whose radius is 'a' and mass is m, about an axis PQ through a point on its circumference perpendicular to its plane (Its $I_G = Ma^2/2$)



$I_{PQ} = I_G + Mh^2$ ….parallel axis theorem

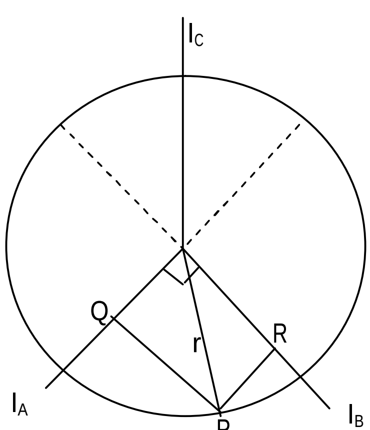
So, $I = ma^2/2 + ma^2$
i.e. $I_{PQ} = 3ma^2/2$

<u>Ex.</u>

What is I of a solid cylinder about an axis along its height, if $I_G = ma^2/2$?

Perpendicular axes Theorem

The moment of inertia I of a body about any axis is the sum of the moment if inertia about any mutually perpendicular axes. This theorem is most useful when considering a body which is of regular form (i.e. Symmetrical about two out of three axes). If the moment of inertia about these axes is then the third may be calculated.



A and B are mutually perpendicular

and are through symmetry of the body.

From $I_C = \sum mr^2$

But $r^2 = (PQ)^2 + (PR)^2$

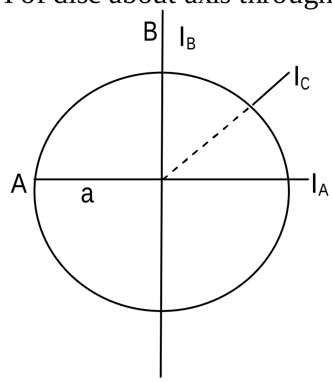
i.e $I = \Sigma m [(PQ)^2 + (PR)^2]$

$= \Sigma m (PQ)^2 + \Sigma M (PR)^2$

i.e. $I_C = I_A + I_B$ ……. perpendicular axis theorem.

E. g.

Find the M I of disc about axis through its diameter



$I_C = I_A + I_B$

I_C = MI through its centre perpendicular to its plane

i.e. through its centre of mass.

i.e. $I_C = \frac{ma^2}{2}$

i.e. MI about A is $\frac{1}{2}$ of $\frac{1}{2} ma^2$

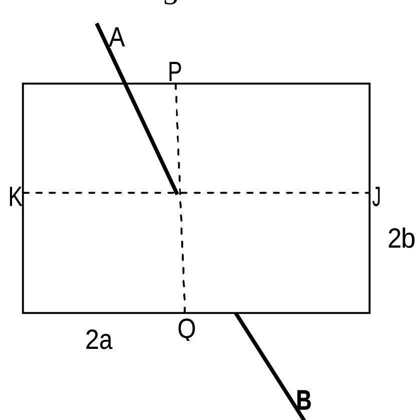
And the rest half is MI about B.

Axes A and B are parallel to the plane of the disc.

Thus, MI through diameter = $\frac{1}{4} ma^2$

EX. Find the MI of lamina shown about axis AB through its centre if

$I_{JK} = \frac{mb^2}{3}$ and $I_{PQ} = \frac{ma^2}{3}$



Radius of Gyration

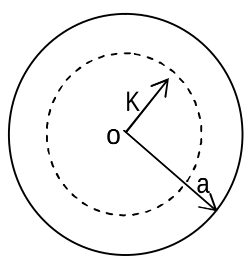
If we write M of I as mk^2, k = distance called radius of gyration of the body. Thus, radius of gyration is the defined as the distance from the centre of rotation to the point where the mass can be considered to be concentrated.

E.g. 1. For disc $I = \frac{ma^2}{2}$, write expression for its radius of gyration as

2

$$\frac{ma^2}{2} \Xi \text{ mk}^2$$

$$i.e.\ k = \left(\frac{a^2}{2}\right)^{\frac{1}{2}}$$



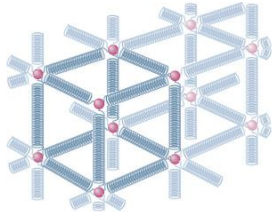
(2) Given that I of rod = $\frac{mL^2}{3}$ if its length is 1m and mass is 50g calculate its radius of gyration

$$\frac{mL^2}{3} = \text{mK}^2$$

$$K = \left(\frac{L^2}{3}\right)^{\frac{1}{2}} \qquad = \left(\frac{1^2}{3}\right)^{\frac{1}{2}}$$

=0.58m

EX. If I of sphere is $\frac{2ma^2}{5}$, given that m = 2 g and its radius is 20 cm calculate the radius of gyration.

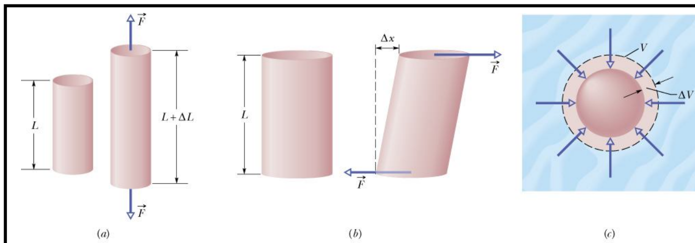


Elasticity

Metallic solids consist of a large number of atoms positioned on a regular three-dimensional lattice as shown in the figure. The lattice is repetition of a pattern (in the figure this pattern is a cube).

Each atom of the solid is a well-defined equilibrium distance from its nearest neighbors. The atoms are held together by interatomic forces that can be modeled as tiny springs. If we try to change the interatomic distance the resulting force is proportional to the atom displacement from the equilibrium position. The spring constants are large and thus the lattice is remarkably rigid. Nevertheless all "rigid" bodies are to some extent elastic, which means that we can change their dimensions slightly by pulling, pushing, twisting, or compressing them. For example, if you suspend a subcompact car from a steel rod 1 m long and 1 cm in diameter, the rod will stretch by only 0.5 mm. The rod will return to its original length of 1 m when the car is removed. If you suspend two cars from the rod the rod will be permanently deformed. If you suspend three cars the rod will break.

(1)



$\boxed{\text{stress} = \text{modulus} \times \text{strain}}$

In the three figues above we show the three ways in which a solid might change its dimensions under the action of external deforming forces. In fig. a the cylinder is stretched by forces acting along the cylinder axis. In fig. b the cylinder is deformed by forces perpendicular to its axis. In fig. c solid placed in a fluid under high pressure is compressed uniformly on all sides. All three deformation types have stress in common (**defined as deforming force per unit area**).

These stresses are known as tensile/compressive for fig.a, shearing for fig. b, and hydraulic for fig. c.

The application of stress on a solid results in strain, which takes different form for the three types of strain.

Stress is related to strainvia the equation: stress = modulus X strain

(2)

Lateral strain

Consider a rod which becomes longer after undergoing tensile stress. The lateral dimensions of the rod become shorter since the volume remains constant.

If the diameter D of the rod is reduced by ΔD

Suppose the final diameter is D'

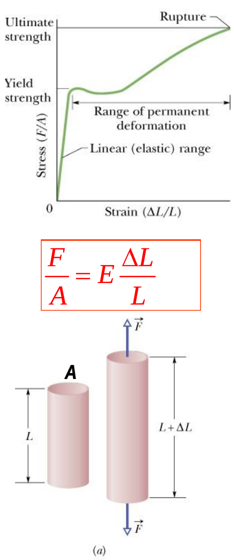
Lateral strain on the rod is $\frac{D'-D}{D}$ or $\frac{-\Delta D}{D}$

Ratio of lateral strain to the normal strain is called Poisson's ratio given as

$$\nu = \frac{\text{lateral strain}}{\text{normal strain}} = \frac{\varepsilon_L}{\varepsilon_n} = \frac{\frac{-\Delta D}{D}}{\frac{\Delta L}{L}}$$

Poisson's ratio is dimensionless and value lies between 0.1 and 0.4 for most materials.

(3)



$$\boxed{\frac{F}{A} = E\frac{\Delta L}{L}}$$

Tensile stress is defined as the ratio $\frac{F}{A}$ where A is the solid area.

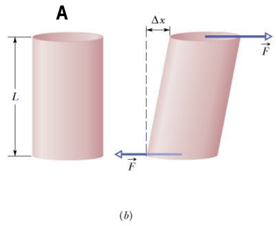
Strain (symbol S) is defined as the ratio $\frac{\Delta L}{L}$ where ΔL is the change in the length L of the cylindrical solid.
Stress is plotted versus strain in the upper figure.
For a wide range of applied stresses the stress-strain relation is linear and the solid returns to its original length when the stress is removed. This is known as the elastic range. If the stress is increased beyond a maximum value known as the yield strength S_y the cylinder becomes permanently deformed. If the stress continues to increase, the cylinder breaks at a stress value known as ultimate strength S_u.

For stresses below S_y (elastic range) stress and strain are connected via the equation

$\frac{F}{A} = E\frac{\Delta L}{L}$. The constant E (modulus) is known as Young's modulus.

Note: Young's modulus is almost the same for tension and compression.
The ultimate strength S_u may be different.

(4)



$$\boxed{\frac{F}{A} = G\frac{\Delta x}{L}}$$

Shearing. In the case of shearing deformation, strain is defined as the dimensionless ratio $\frac{\Delta x}{L}$. The stress/strain equation has the form:

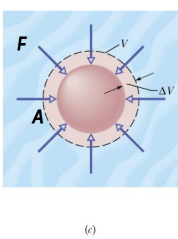
$$\frac{F}{A} = G\frac{\Delta x}{L}$$

The constant G is known as the shear modulus.

Also, the shear strain $\frac{\Delta x}{L} = \tan\theta$

Where θ is the angle of deformation

(5)



$$\boxed{p = B\frac{\Delta V}{V}}$$

Hydraulic Stress. The stress is this case is the pressure $p = \frac{F}{A}$ that the surrounding fluid exerts on the immersed object. Here A is the area of the object. In this case strain is defined as the dimensionless ratio $\frac{\Delta V}{V}$ where V is the volume of the object and ΔV the change in the volume due to the fluid pressure. The stress/strain equation has the form: $p = B\frac{\Delta V}{V}$. The constant B is known as the bulk modulus of the material.

(6)

Force constant

Under elastic deformation for both normal stress and shear stress, ratio of stress/strain is constant

i.e.
$$\frac{\sigma_n}{\varepsilon_n} = E$$
$$\sigma_n = E\varepsilon_n$$
$$\frac{F_n}{A} = E\frac{\Delta L}{L}$$
$$F_n = AE\frac{\Delta L}{L}$$
$$F_n = k\Delta L$$

Where $k = \frac{AE}{L}$ Force constant or stiffness of the material in N/m

Thus increase in length is proportional to the tensile force *Fn*.

$F_n = k\Delta L$ is a statement of Hooke's law

(7)

Fluids

In this section we will explore the behavior of fluids. In particular we will study the following:

Static fluids:

Pressure exerted by a static fluid

Methods of measuring pressure

Pascal's principle

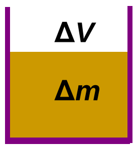
Archimedes' principle, buoyancy

Real versus ideal **fluids in motion:** fluids

Equation of continuity

Bernoulli's equation

(8)



$$\boxed{\rho = \frac{\Delta m}{\Delta V}}$$

Fluids

As the name implies, a fluid is defined as a substance that can flow. Fluids conform to the boundaries of any container in which they are placed. A fluid cannot exert a force tangential to its surface. It can only exert a force perpendicular to its surface. Liquids and gases are classified together as fluids to contrast them with solids. In crystalline solids the constituent atoms are organized in a rigid three-dimensional regular array known as the "lattice."

Density :

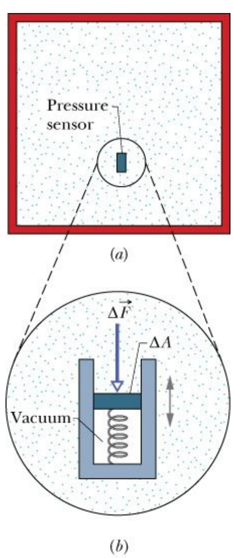
Consider the fluid shown in the figure. It has a mass Δm and volume ΔV. The density (symbol ρ) is defined as the ratio of the mass over the volume: $\rho = \dfrac{\Delta m}{\Delta V}$.

SI unit: kg/m^3

If the fluid is homogeneous, the above equation has the form $\rho = \dfrac{m}{V}$.

(9)

Pressure



$$\boxed{p = \frac{F}{A}}$$

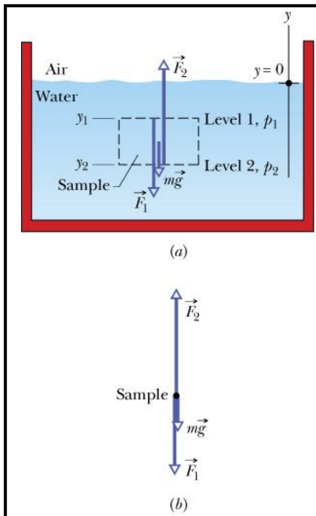
Consider the device shown in the insert of the figure, which is immersed in a fluid-filled vessel. The device can measure the normal force F exerted on its piston from the compression of the spring attached to the piston. We assume that the piston has an area A. The pressure p exerted by the fluid on the piston is defined as $p = \dfrac{F}{A}$.

The SI unit for pressure, $\dfrac{\text{N}}{\text{m}^2}$, is known as the pascal (symbol: Pa). Other units are the atmosphere (atm), the torr, and the lb/in^2. The atm is defined as the average pressure of the atmosphere at sea level:

1 atm = 1.01×10^5 Pa = 760 Torr = 14.7 lb/in^2.

Experimentally it is found that the pressure p at any point inside the fluid has the same value regardless of the orientation of the cylinder. The assumption is made that the fluid is at rest.

(10)



$$\boxed{(p_2 - p_1) = \rho g (y_1 - y_2)}$$

$$\boxed{p = p_0 + \rho g h}$$

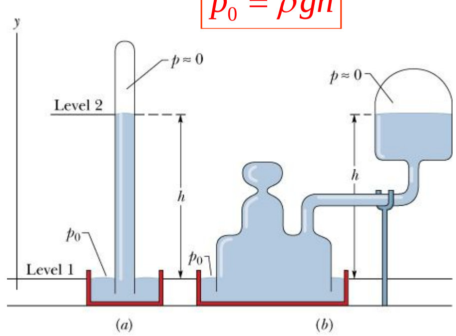
Fluids at Rest

Consider the tank shown in the figure. It contains a fluid of density ρ at rest. We will determine the pressure difference $p_2 - p_1$ between point 2 and point 1 whose y-coordinates are y_2 and y_1, respectively. Consider a part of the fluid in the form of a cylinder indicated by the dashed lines in the figure. This is our "system" and it is at equilibrium. The equilibrium condition is:
$F_{y,\text{net}} = F_2 - F_1 - mg = 0$. Here F_2 and F_1 are the forces exerted by the rest of the fluid on the bottom and top faces of the cylinder, respectively. Each face has an area A:
$F_1 = p_1 A,\ \ F_2 = p_2 A,\ \ m = \rho V = \rho A (y_1 - y_2)$.
If we substitute into the equilibrium condition we get:
$p_2 A - p_1 A - \rho g A (y_1 - y_2) = 0 \rightarrow (p_2 - p_1) = \rho g (y_1 - y_2)$.
If we take $y_1 = 0$ and $h = -y_2$ then $p_1 = p_0$ and $p_2 = p$.
The equation above takes the form $p = p_0 + \rho g h$.

(11)

Note: The difference $p - p_0$ is known as "**gauge pressure.**"

$$\boxed{p_0 = \rho g h}$$

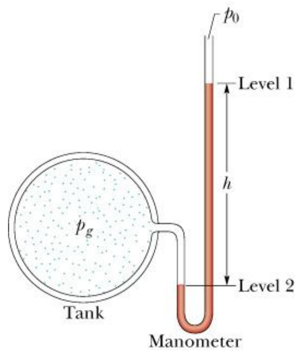


The Mercury Barometer

The mercury barometer shown in fig. a was constructed for the first time by Evangelista Toricelli. It consists of a glass tube of length approximately equal to 1 meter. The tube is filled with mercury and then it is inverted with its open end immersed in a dish filled also with mercury.

Toricelli observed that the mercury column drops so that its length is equal to h.
The space in the tube above the mercury can be considered as empty.
If we take $y_1 = 0$ and $y_2 = h$ then $p_1 = p_0$ and
$(p_2 - p_1) = \rho g (y_1 - y_2) \rightarrow p_0 = \rho g h.$
We note that the height h does not depend on the cross-sectional area A of the tube. This is illustrated in fig. b. The average height of the mercury column at sea level is equal to 760 mm.

(12)



$\boxed{p_g = \rho g h}$

The Open - Tube Manometer

The open-tube manometer consists of a U-tube that contains a liquid. One end is connected to the vessel for which we wish to measure the gauge pressure. The other end is open to the atmosphere.

At level 1: $y_1 = 0$ and $p_1 = p_0$

At level 2: $y_2 = -h$ and $p_2 = p$

$p_2 = p_1 + \rho g h \rightarrow p - p_0 = \rho g h \rightarrow$

$p_g = \rho g h$

If we measure the length h and if we assume that g is known, we can determine p_g.

The gauge pressure can take either positive or negative values.

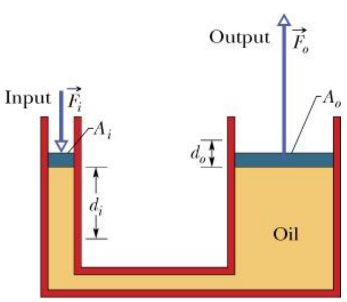
(13)

Pascal's Principle and the Hydraulic Lever

$\boxed{F_o = F_i \frac{A_o}{A_i}}$

Pascal's principle can be formulated as follows:

A change in the pressure applied to an enclosed incompressible liquid is transmitted undiminished to every portion of the fluid and to the walls of the container.



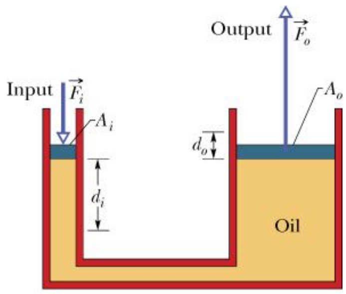
Consider the enclosed vessel shown in the figure, which contains a liquid. A force F_i is applied downward to the left piston of area A_i.

As a result, an upward force F_o appears on the right piston, which has area A_o. Force F_i produces a change in pressure $\Delta p = \frac{F_i}{A_i}$. This change will also appear on the right piston. Thus we have:

$$\Delta p = \frac{F_i}{A_i} = \frac{F_o}{A_o} \rightarrow F_o = F_i \frac{A_o}{A_i}$$

If $A_o > A_i \rightarrow F_o > F_i$

(14)



The Hydraulic Lever; Energy Considerations

The hydraulic lever shown in the figure is filled with an incompressible liquid. We assume that under the action of force F_i the piston to the left travels downward by a distance d_i. At the same time the piston to the right travels upward by a distance d_o. During the motion we assume that a volume V of the liquid is displaced at both pistons:

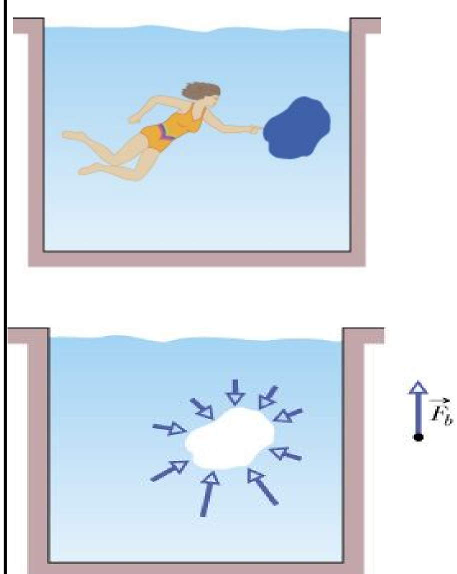
$$V = A_i d_i = A_f d_f \rightarrow d_o = d_i \frac{A_i}{A_o}$$ **Note:** Since $A_o > A_i \rightarrow d_o < d_i$.

The output work $W_o = F_o d_o = \left(F_i \frac{A_o}{A_i}\right)\left(d_i \frac{A_i}{A_o}\right)$.

Thus $W_o = F_i d_i = W_i$. The work done on the left piston by F_i is equal to the work done by the piston to the right in lifting a load placed on it.

With a hydraulic lever a given force F_i applied over a distance d_i can be transformed into a larger force F_o applied over a smaller distance d_o.

(15)



(a)

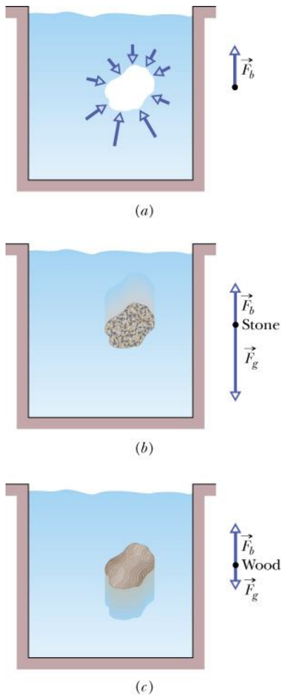
$\boxed{F_b = \rho_f g V}$

Buoyant Force

Consider a very thin plastic bag that is filled with water. The bag is at equilibrium thus the net force acting on it must be zero. In addition to the gravitational force $\vec{F}_g$ there exists a second force $\vec{F}_b$ known as "**buoyant force,**" which balances $\vec{F}_g$: $F_b = F_g = m_f g$.

Here m_f is the mass of the water in the bag. If V is the bag volume we have $m_f = \rho_f g V$. Thus the magnitude of the buoyant force $F_b = \rho_f g V$. $\vec{F}_b$ exists because the pressure on the bag exerted by the surrounding water increases with depth. The vector sum of all the forces points upward, as shown in the figure.

(16)



Archimedes' Principle

Consider the three figures to the left. They show three objects that have the same volume (V) and shape but are made of different materials. The first is made of water, the second of stone, and the third of wood. The buoyant force F_b in all cases is the same: $F_b = \rho_f g V$. This result is summarized in what is known as "**Archimedes' Principle.**"

When a body is fully or partially submerged in a fluid a buoyant force $\vec{F}_b$ is exerted on the body by the surrounding fluid. This force is directed upward and its magnitude is equal to the weight $m_f g$ of the fluid that has been displaced by the body.

We note that the submerged body in fig. a is at equilibrium with $F_g = F_b$. In fig. b $F_g > F_b$ and the stone accelerates downward. In fig. c $F_b > F_g$ and the wood accelerates upward.

(17)

Ideal Fluids : The motion of real fluids is very complicated and not fully understood. For this reason we shall discuss the motion of an **ideal fluid,** which is simpler to describe. Below we describe the characteristics of an ideal fluid.

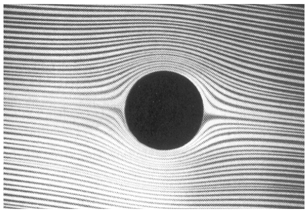
1. Steady flow. The velocity $\vec{v}$ of the moving fluid at any fixed point does not change with time. This type of flow is known as "laminar."

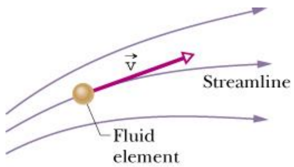
2. Incompressible flow. The assumption is made that the moving fluid is incompressible, i.e., its density is uniform and constant.

3. Nonviscous flow. Viscosity in fluids is a measure of how resistive the fluid is to flow. Viscosity in fluids is the analog of friction between solids. Both mechanisms convert kinetic energy into thermal energy (heat). An object moving in a nonviscous fluid experiences no drag force.

4. Irrotational flow. A small particle that moves with the fluid will not rotate about an axis through its center of mass.

(18)

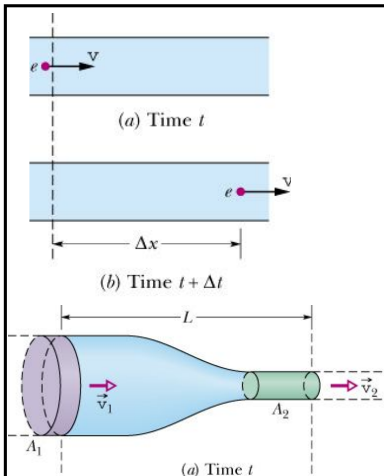




Streamlines

The flow of a fluid can be made visible by adding a tracer. In the case of a liquid the tracer can be a dye. An example is given in the picture to the left.
In the case of gas, smoke particles can be used as a tracer. Each visible tracer particle follows a ***streamline,*** which is a path that a fluid element would take. Three such streamlines are shown in the figure to the left. The velocity $\vec{v}$ of a fluid element is always tangent to a streamline, in the same way that the velocity of a moving object is tangent to the path at any point. Two streamlines cannot intersect. If they did, then two different velocities, each corresponding to the two streamlines at the intersection point, could be defined. This would be physically meaningless.

(19)



Equation of Continuity

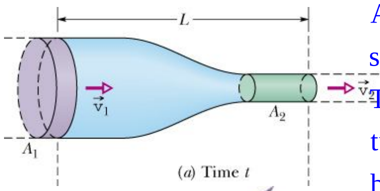
In this section we consider the flow of a fluid through a tube whose cross-sectional area A is not constant. We will find the equation that connects the area A with the fluid speed v.
Consider a fluid element "e" that moves with speed v through a tube of cross-sectional area A.
In a time interval Δt the element travels a distance $\Delta x = v\Delta t$ as shown in fig. *b*. The fluid volume ΔV is given by the equation $\Delta V = A\Delta x = Av\Delta t$.

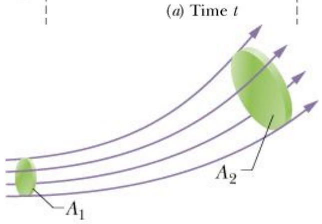
Assume that the fluid of volume ΔV and speed v_1 enters the tube from its left end. The cross-sectional area on the left is A_1. The same volume exits at the right end of the tube that has cross-sectional area A_2: $\Delta V = A_1 v_1 \Delta t = A_2 v_2 \Delta t \rightarrow$
$A_1 v_1 = A_2 v_2$. The equation of continuity is based on the assumption that the fluid is incompressible.

(20)

$$\boxed{A_1 v_1 = A_2 v_2}$$

$$\boxed{R_V = Av = \text{a constant}}$$





If we solve the equation of continuity for v_2 we get:

$v_2 = v_1 \dfrac{A_1}{A_2}$. If $A_2 < A_1$ then $v_2 > v_1$.

In other words, if the tube narrows the fluid speeds up.
The opposite is also true: If $A_2 > A_1$ then $v_2 < v_1$.
At points where the tube becomes wider, the fluid slows down.

The equation of continuity is also true for a tube of flow, which is a section of the fluid bounded by streamlines. This is so because streamlines cannot cross, and therefore all the fluid inside the tube remains within its boundary.

We refine the **volume flow rate** $R_V = \dfrac{\Delta V}{\Delta t} = \dfrac{vA\Delta t}{\Delta t} = vA.$

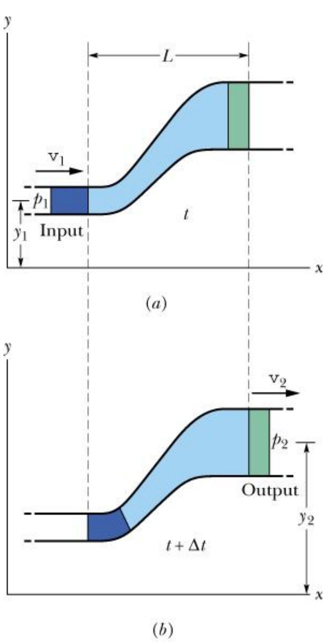
In a similar fashion we define the **mass flow rate :**

$$R_m = \frac{\Delta m}{\Delta t} = \frac{\rho \Delta V}{\Delta t} = \rho R_V = \rho A v.$$

The continuity equation can be written in the form: $R_V = Av =$ a constant.

(21)

Bernoulli's Equation



Consider an ideal fluid flowing through the tube shown in the figure. A fluid volume ΔV enters to the left at height y_1 with speed v_1 under pressure p_1. The same volume exits at the right end at height y_2 with speed v_2 under pressure p_2. We apply the work-kinetic energy theorem:

$W = \Delta K$ **(eq. 1).** The change in kinetic energy

$$\Delta K = \frac{\Delta m v_2^2}{2} - \frac{\Delta m v_1^2}{2} = \frac{\rho \Delta V}{2}\left(v_2^2 - v_1^2\right) \quad \textbf{(eq. 2).}$$

The work W has two terms: one term $\left(W_g\right)$ from the gravitational force and a second $\left(W_p\right)$ from the pressure force: $W = W_g + W_p$. $\quad W_g = -\Delta m g\left(y_2 - y_1\right)$

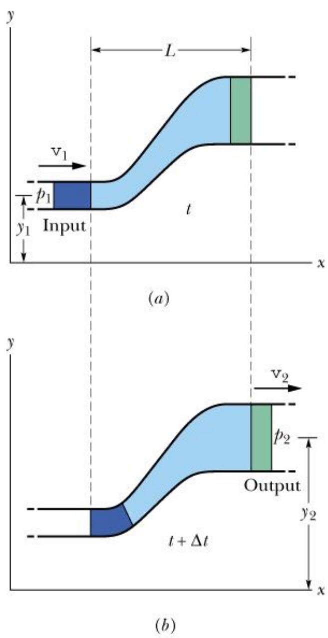
$W_g = -\rho g \Delta V\left(y_2 - y_1\right) \quad W_p = p_1 A_1 \Delta x_1 - p_2 A_2 \Delta x_2$

$W_p = p_1 \Delta V - p_2 \Delta V = -\left(p_2 - p_1\right)\Delta V$

$W = -\rho g \Delta V\left(y_2 - y_1\right) - \left(p_2 - p_1\right)\Delta V$ **(eq. 3)**

(22)

$$p_1 + \frac{\rho v_1^2}{2} + \rho g y_1 = p_2 + \frac{\rho v_2^2}{2} + \rho g y_2$$



Bernoulli's Equation

$W = \Delta K$ **(eq. 1)**

$\Delta K = \frac{\rho \Delta V}{2}\left(v_2^2 - v_1^2\right)$ **(eq. 2)**

$W = -\rho g \Delta V\left(y_2 - y_1\right) - \left(p_2 - p_1\right)\Delta V$ **(eq. 3)**

If we substitute ΔK from eq. 2 and W from eq. 3 into eq. 1 we get:

$$\frac{\rho \Delta V}{2}\left(v_2^2 - v_1^2\right) = -\rho g \Delta V\left(y_2 - y_1\right) - \left(p_2 - p_1\right)\Delta V$$

If we rearrange the terms we have:

$$p_1 + \frac{\rho v_1^2}{2} + \rho g y_1 = p_2 + \frac{\rho v_2^2}{2} + \rho g y_2$$

For the special case in which $y_1 = y_2$ we have:

$p_1 + \frac{\rho v_1^2}{2} = p_2 + \frac{\rho v_2^2}{2}$. This equation states that:

If the speed of a fluid element increases as the element travels along a horizontal streamline, the pressure must decrease. (23)

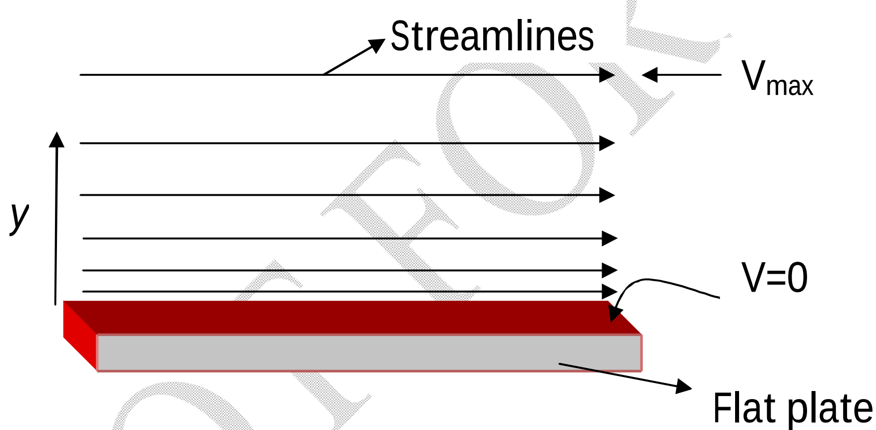
Viscosity

Viscosity is an internal property of a fluid that offers resistance to flow. For example, pushing a spoon with a small force moves it easily through a bowl of water, but the same force moves mashed potatoes very slowly.

The **viscosity** of a fluid is a measure of its resistance to gradual deformation by shear stress or tensile stress. For liquids, it corresponds to the informal concept of "thickness". For example, honey has a much higher viscosity than water.

Viscosity is a property arising from collisions between neighboring particles in a fluid that are moving at different **velocities**. When the fluid is forced through a tube, the particles which comprise the fluid generally move more quickly near the tube's axis and more slowly near its walls: therefore some **stress**, (such as a **pressure** difference between the two ends of the tube), is needed to overcome the friction between particle layers and keep the fluid moving. For the same velocity pattern, the stress required is proportional to the fluid's viscosity.

That is, unlike solid which moves all in one piece, different layers of the fluid move with different velocities. Consider a laminar flow of a fluid over a flat plate as shown below.



On the surface of the plate velocity V = 0.

At the topmost layer, the fluid has maximum velocity (V_{max}).

Since the fluid layers are moving with different velocities, then friction is set up.

This friction in fluid is termed viscosity.

Fluid friction is due to the attractive intermolecular forces between the fluid particles in one layer and those in the adjacent layer. As shown in the figure above velocity increases gradually in *y* direction (i.e. perpendicular to the surface of the plate).

The frictional force (viscous force) F between two adjacent fluid layers is directly proportional to the velocity gradient($\frac{dv}{dx}$) and the area of contact between the two layers.

$$F = \eta A\frac{dv}{dy} = \eta A\frac{v}{y}$$

The proportionality factor μ in this formula is the viscosity (specifically, the **dynamic viscosity**) of the fluid.

The ratio $\frac{v}{y}$ is called the ***rate of shear deformation*** or ***shear velocity***, and is the derivative of the fluid speed in the direction perpendicular to the plates. Isaac Newton expressed the viscous forces by the differential equation

$$\tau = \frac{F}{A} = \eta\frac{dv}{dy}$$

where $\tau = \frac{F}{A}$ and $\frac{dv}{dy}$ is the local shear velocity. This formula assumes that the flow is moving along parallel lines and the y axis, perpendicular to the flow, points in the direction of maximum shear velocity. This equation can be used where the velocity does not vary linearly with y, such as in fluid flowing through a pipe.

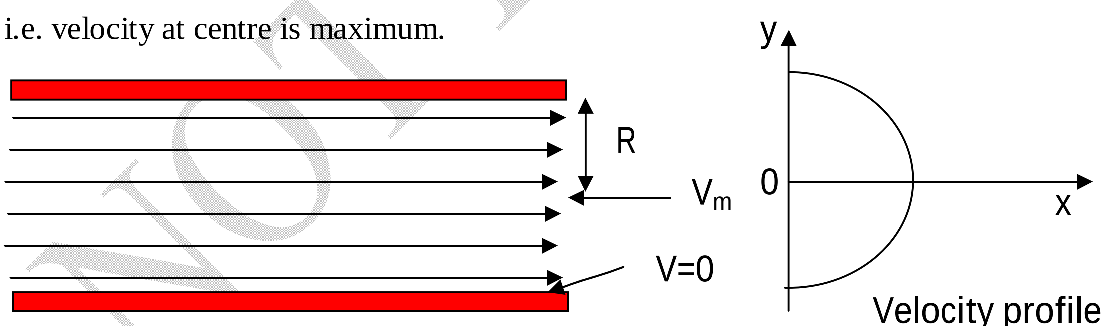
η has the unit N.s/m^2 or kg/ms.

Other unit is poise. 1 poise = 0.1 N.s/m^2.

- Viscosity is independent of pressure (except at very high pressure); and
- Viscosity tends to fall as temperature increases

Consider a fluid flowing through a pipe as shown below. The fluid payer next to the wall of the pipe is at rest (i.e. v =0), and the fluid velocity increases towards the centre of the pipe.

i.e. velocity at centre is maximum.



Relationship between the velocity (v) and the distance (y) from the centre of the pipe is parabolic given as

$$v = \frac{1}{4\eta}(R^2 - y^2)\frac{\Delta p}{\Delta L}$$

Where R = radius of the pipe and $\frac{\Delta p}{\Delta L}$ is the pressure gradient along the pipe.

The rate of heat flow Q is related to the pressure gradient and viscosity according to the equation

$Q = -\frac{\pi R^4}{8\eta}\frac{\Delta p}{\Delta L}$ (Poiseuille's Law).

Stoke's Law and Terminal Velocity

When any object rises or falls through a fluid it will experience a viscous drag, whether it is a parachutist or spacecraft falling through air, a stone falling through water or a bubble rising through fizzy lemonade. The mathematics of the viscous drag on irregular shapes is difficult; we will consider here only the case of a falling sphere. The formula was first suggested by Stokes and is therefore known as Stokes' law.

Consider a sphere falling through a viscous fluid. As the sphere falls so its velocity increases until it reaches a velocity known as the terminal velocity. At this velocity the frictional drag due to viscous forces is just balanced by the gravitational force and the velocity is constant (shown by Figure 2).

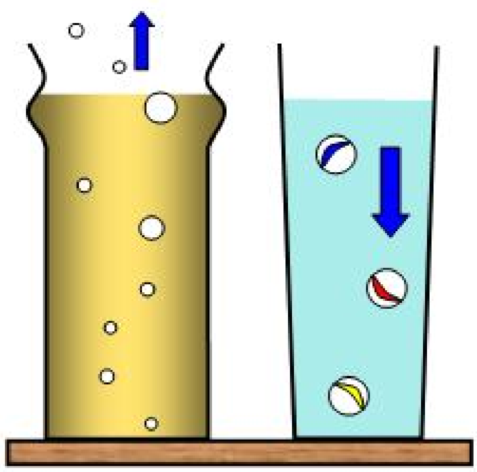


Figure 1

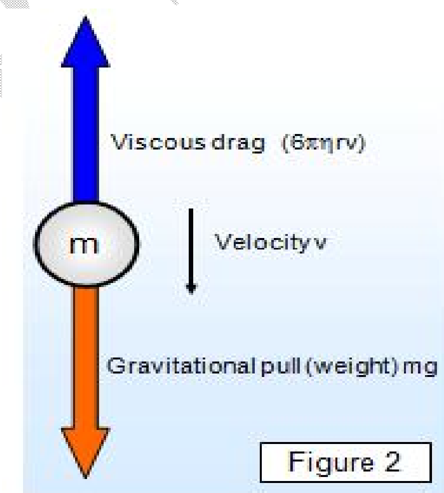


Figure 2

Let r = radius of the sphere
η = fluid viscosity
v = terminal velocity

The viscous drag on the sphere is given as
$F = kr^x \eta^y v^z$

Using dimensional analysis
$MLT^{-2} = L^x(ML^{-1}T^{-1})^y(LT^{-1})^z$

From this

$x = 1, y = 1, z = 1$

The constant k is known to be 6π

Therefore the viscous drag (Frictional force) F = 6πηrv   (Stokes' law)

If the density of the material of the sphere is **ρ** and that of the liquid **σ**, then

Effective gravitational force = weight - upthrust $= m_1 g - m_2 g$

Where m_1 (mass of the sphere) = density x volume = **ρV**

M_2 (mass of fluid displaced) = density x volume = **σ V**

Effective gravitational force = **ρVg - σVg = Vg(ρ – σ)**

$$= \frac{4\pi r^3}{3}(\rho - \sigma)g$$

At the terminal speed:

$$6\pi\eta r v = \frac{4\pi r^3}{3}(\rho - \sigma)g$$

And

$$v = \frac{2r^2 g}{9\eta}(\rho - \sigma)$$

And viscosity

$$\eta = \frac{2r^2 g}{9v}(\rho - \sigma)$$

One can conclude that

1. the frictional drag is smaller for large spheres than for small ones
2. theterminal velocity of a large sphere is greater than that for a small sphere of the same material.

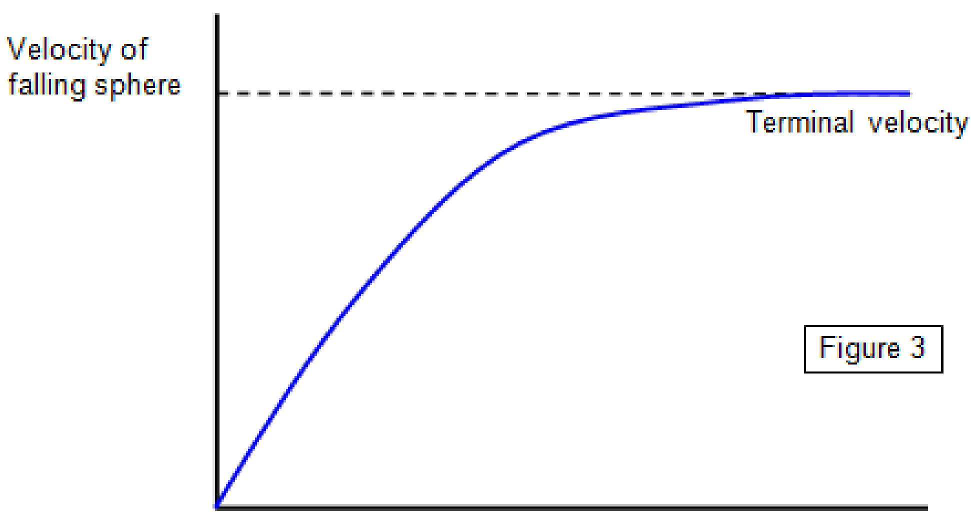


Figure 3 shows how the velocity of an object will increase with time as it falls through a viscous fluid.

Surface tension

Property of a liquid surface that causes it to act like a stretched elastic membrane. Its strength depends on the forces of attraction among the particles of the liquid itself and with the particles of the gas, solid, or liquid with which it comes in contact. Surface tension allows certain insects to stand on the surface of water and can support a razor blade placed horizontally on the liquid's surface, even though the blade may be denser than the liquid and unable to float. Surface tension results in spherical drops of liquid, as the liquid tends to minimize its surface area.

At liquid-air interfaces, surface tension results from the greater attraction of water molecules to each other (due to cohesion) than to the molecules in the air (due to adhesion). The net effect is an inward force at its surface that causes water to behave as if its surface were covered with a stretched elastic membrane. Because of the relatively high attraction of water molecules for each other, water has a high surface tension (72.8 millinewtons per meter at 20°C) compared to that of most other liquids. Surface tension is an important factor in the phenomenon of capillarity.

Surface tension preventing a paper clip from submerging.

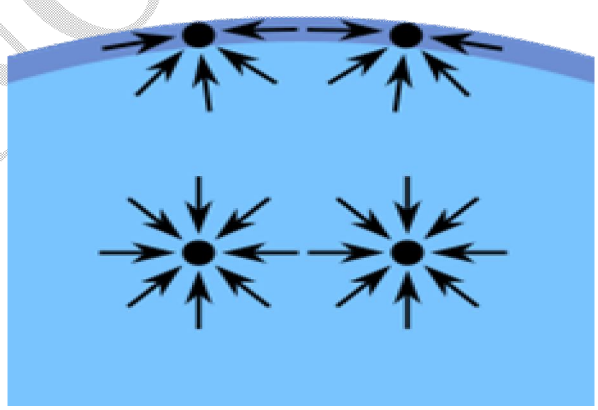


Diagram of the forces on molecules of a liquid

Surface tension has the is defined as force per unit length along a direction perpendicular to the force, or of energy per unit area. The two are equivalent—but when referring to energy per unit of area, people use the term surface energy—which is a more general term in the sense that it applies also to solids and not just liquids.

Mathematically surface tension is

$\gamma = \frac{F}{L}$ (N/m)

Surface tension γ varies from one liquid to another. For a given liquid, γ decreases with increasing temperature.

Effects of surface tension

Water

Several effects of surface tension can be seen with ordinary water:

A. Beading of rain water on a waxy surface, such as a leaf. Water adheres weakly to wax and strongly to itself, so water clusters into drops. Surface tension gives them their near-spherical shape, because a sphere has the smallest possible surface area to volume ratio.

Figure 4 beading on a leaf

B. Formation of drops occurs when a mass of liquid is stretched.

Figure 5. Water dripping from a tap

C. Flotation of objects denser than water occurs when the object is nonwettable and its weight is small enough to be borne by the forces arising from surface tension. For example, water striders use surface tension to walk on the surface of a pond. The surface of the water behaves like an elastic film: the insect's feet cause indentations in the water's surface, increasing its surface area.

C. Water striders stay atop the liquid because of surface tension

Floating objects

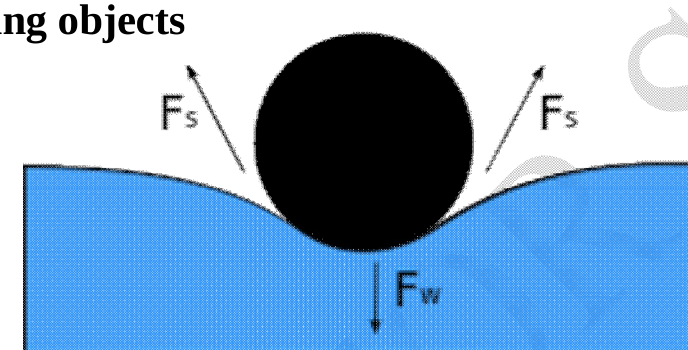


Figure 5. Floating object

When an object is placed on a liquid, its weight F_w depresses the surface, and is balanced by the surface tension forces on either side F_s, which are each parallel to the water's surface at the points where it contacts the object. Notice that the horizontal components of the two F_s arrows point in opposite directions, so they cancel each other, but the vertical components point in the same direction and therefore add up to balance F_w.

Contact angles

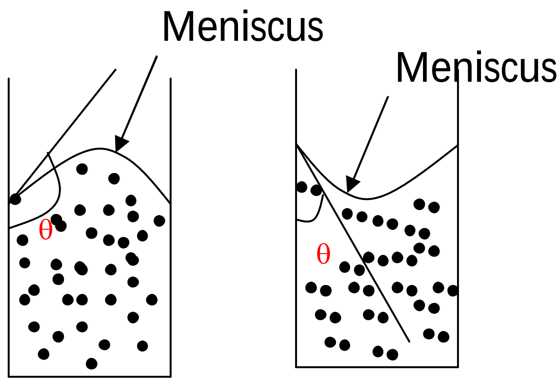
Where the two surfaces meet, they form a contact angle,θ , which is the angle the tangent to the surface makes with the solid surface. The diagram to the right shows two examples. Tension forces are as shown in the figure below. The liquid surface is called meniscus which may curve downwards or upwards depending on which of the two forces (cohesive or adhesive frces) dominates.

If cohesive force > adhesive force

Contact angle $\theta > 90^o$ (we have convex meniscus, i.e. meniscus curves upward) Figure 7a.

If cohesive force < adhesive force

Contact angle $\theta < 90^o$ (we have concave meniscus, i.e. meniscus curves upward). Figure 7b.



$\theta > 90^o$ (7a) $\theta < 90^o$ (7b)

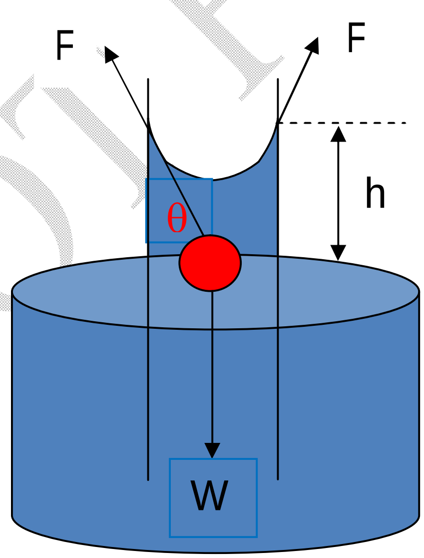
If a very thin test tube is inserted in a liquid reservoir, there is a different between the levels of the liquid inside and outside the tube.

If $\theta < 90^o$, liquid level rises higher inside the tube e.g. water in a glass tube.
If $\theta > 90^o$, liquid level depresses inside the tube e.g. mercury-in-glass tube.

Two forces act on the tube inside the liquid:

1. Upward force F due to surface tension which acts around the circles of contact between the meniscus and the glass tube.
2. The downward pull of gravity F_w (weight of the liquid column)

Consider the figure below



At equilibrium

$$F\cos\theta = Mg = \rho V g$$
$$= \rho \pi r^2 h g$$

Where $\pi r^2 h$ is the volume of the liquid, r is the radius of the tube.

But $\gamma = \frac{F}{L}$, where L is the circumference of the circle of contact angle along which the surface tension acts.

Therefore $\gamma = \frac{F}{2\pi r}$,

And $F = 2\pi r\gamma$

Equation (1) then becomes $2\pi r\gamma\, cos\theta = \rho\pi r^2 hg$

$$h = \frac{2\gamma \cos\theta}{\rho g r}$$

h varies inversely as the radius of the tube.

PHY115 PAST QUESTIONS

1. During a race on an oval track, a car travels at an average speed of 200km/hr. Determine its average velocity at the end of its third lap. a. 55.6m/s **B. zero** c.40m/s d. 102m/s

2. A marble rolling with speed 20cm/s rolls off the edge of a table that is 80cm high. How far horizontally from the table edge does the marble strike the floor a. 81cm **B. 8.1cm** c. 2.7cm d. 27cm

3. Calculate the horizontal Inertia of a wheel that has a kinetic energy of 2400J when rotating at 602rev/min. a. $32kgm^2$ b. $11kgm^2$ c. $42kgm^2$ **D. $12.3kgm^2$**

4. Water at 20^0C flows through a horizontal pipe of radius 1.0cm. If the flow velocity at the center is 0.2m/s find the pressure drop along a 4m section of the pipe due to viscosity $1.6x10^{-3}NSM^{-2}$ **A. $0.32NM^{-2}$** b.$320NM^{-2}$ c. $32NM^{-2}$ d. pressure does not change

5. Castor oil at 20^0C has a co-efficient of viscosity $2.42NSm^{-2}$ and a density $940kgm^{-3}$. Calculate the terminal velocity of a steel ball of radius 2.0mm falling under gravity. (Density of steel = $7800kgm^{-3}$). a. 0.041m/s b. 0.324m/s **C. 0.025m/s** d. 0.121m/s

6. A circular disc of mass 20kg and radius 15cm is mounted in an horizontal cylindrical axle of radius 1.5cm. Calculate the K.E of the disc after 1.2sec if s force 12N is applied tangentially to the axle. a. 121J **B.29J** c. 43J d. 33J

7. The flywheel of a stationary engine has a moment of Inertia of $60kgm^2$, what is the K.E if its angular acceleration is $2rads^{-2}$? a. 45J b. 18J c.36J **D. 60J**

8. What is the length of a simple pendulum whose period is exactly 1sec at a point where $g=9.80m/s^2$? **A. 0.25m** b. 3.29m c. 0.33sm d. 0.50m

9. Find the moment of inertia of a rod 4cm in diameter and 2m long of mass 8kg about an axis perpendicular to the rod and passing through one end.

10. A wire is subjected to tensile stress of $4.5x10^9Nm^{-2}$. Calculate the young modulus if the length increases by 6% of its original length. a. $75x10^7Nm^{-2}$ b. $75x10^{-9}Nm^{-2}$ **C. $75x10^{10}Nm^{-2}$** d. $7.5x10^{-9}Nm^{-2}$

11. A 7.0g bullet moving horizontally at 200m/s strikes and passes through a 150g tin can sitting on a post, just after impact, ,the can has a horizontal speed of 180m/s. What was the bullet's speed after hitting the can? **A.161m/s** b. 205m/s c. 102m/s d. 355m/s

12. A cube of wood of side 10m floats in water with 4.5cm of its depth below the surface, and with its sides vertical; what is the density of the wood? a. $0.33g/cm^3$ **B. $0.45g/cm^3$** c. $3.2kg/m^3$ d. $3.2g/cm^3$

13. A rectangular box rests on a flat surface and has weight of 100N. Calculate the pressure on the surface if the base has an area of $2m^2$. a. $25N/m^2$ b. $200N/m^2$ **C. $0.5x10^2N/m^2$** d. $45N/m^2$

14 A rectangular box rests on a flat surface and has weight of 100N. Calculate the total weight if another body is placed on it and the pressure on the surface of the base is $60N/m^2$ if it has an area of $2m^2$ a. 12N **B. 20N** c.120N d. 50N

15. The rubber cord of a catapult has a cross sectional area of 1.0mm^2 and total un-stretched length of 10cm. It is stretched to 12cm and then released to project a missile of mass 5.0g. Calculate the velocity of the projection taking young modulus for rubber as 5.0x10^{8}N/m^2 a. 1.2m/s **B. 20m/s** c. 200m.s d. 33m/s

16. A solid sphere of mass 20kg rolls without slipping down a 30^0 slope. Calculate the acceleration and the frictional force needed to prevent slipping. a. 1.20m/s^2, 25N b. 4.20m/s^2, 32N **C. 3.50m/s^2, 28N** d. 3.0m/s^2, 25N

17. Water flows steadily through a horizontal pipe of varying cross-section. At one place the pressure is 130kPa and the speed is 0.6m/s. Determine the pressure at another place in the same pipe where the speed is 9.0m/s a. 65kPa b. 40kPa c. 110kPa **D. 90kPa**

18. What volume of water will escape per minute from an open-top tank through an opening 3.0cm in diameter that is 5.0m below the water level . **A. 0.42m^3/min** b. 6.1 m^3/min c. 4.8 m^3/min d.5.5 m^3/min

19. Compute the average speed of water in a pipe having an internal diameter of 5.0cm and delivering 2.5m^3 of water per hour a. 44m/s **B.0.3m/s** c.2m/s d. 11 m/s

20. Find the flow in liters/sec of a non –viscous fluid through an opening 0.5cm^2 in area and 2.5m below the water level in an open tank surrounded by air a. 2lit/sec b. 0.5lit/sec **C.0.35 lit/sec** d. 0.22lit/sec

21. A 12g bullet is accelerated from rest to a speed of 700m/s as it travels 20.0cm in a gun barrel. Assuming the acceleration to be constant, how large was the accelerating force a. 12KN b. 18.7KN c.16.5KN **D.14.7KN**

22. A 60kg woman walks up a flight of stairs that connects two floors 3.0m apart. How much lifting work is done by the woman? **A.1.8kJ** b.0.7kJ c.12J d. 18J

23. How long will it take for 500ml of water to flow through a 15cm long and 3.0mm internal diameter pipe if the pressure down the pipe is 4.0kPa? The viscosity of water is 0.8cP a. 12secs b. 2.5secs **C. 7.5secs** d.11secs

24. A missile was to be launched at angle 30^0 to the horizontal at an initial velocity is to hit a target 5km away. Calculate V_0 a. 121m/s b. 2km/s c. 454m/s **D.237m/s**

25. A 2.0kg block of wood rests on a long table top, a 500g bullet moving horizontally with speed of 150m/s is shot into the block and sticks in it. The block then slides 270cm along the table and stops. Find the speed of the block just after impact **A. 0.37m/s** b.1.2m/s c. 0.44m/s d. 0.91m/s

26. Find the moment of inertia of a rod 4cm in diameter and 2m long of mass 8kg about an axis perpendicular to the rod and passing through one end a. 12kgm^2 b. 9 kgm^2 c. 14 kgm^2 **D.10.67 kgm^2**

27. The size of the contact angle Ɵ of any liquid with the wall of the container depends on the following except A. the type of the liquid

28. Find the angle between two vectors A= 2i + 3j-4k and B= i-2j+3k a. 12.48^0 **B.66.61^0** c. 43.44^0 d.54.24^0

29.Two objects of mass M_1 and M_2 placed side by side at a distance r apart have the magnitude of their force of attraction on each other? a. $F=(GM_1)/M_2r$ b. $F=(M_1M_2)/r^2$ c. $F= M_2/M_1$ **D $F=(GM_1M_2)/r^2$**

30. A footballer lobs a football at an angle of 40^0 to the horizontal with an initial speed of 20m/s, what is are the greatest height attained and the time of flight a. 23m, 3s b. 4.3m, 5.4s **C.8.4m, 2.6m/s** d.14m, 3.5s

31 A circular plate of radius 0.5m start from rest with constant angular acceleration. 20secs latter its angular velocity increased to 150rad/s, calculate the angle through which the plate has turned a. 1220rad **B.1500 rad** c. 1600rad d. 243 rad

32 Water moving with a speed of 5.0m/s through a pipe with cross sectional area of $4.0cm^2$ gradually descends 10m as the pipe increases in area to $8cm^2$. Calculate the speed of flow and pressure at the lower level if the pressure at the upper level is $1.50x10^5$Pa a. 3.9m/s, $1.9x10^4$ b. 2.7m/s, $2.9x10^4$ **C. 2.5m/s, $2.6x10^5$** d. 4.3m/s, $2.6x10^4$

33.A body of mass 2kg moving with initial speed of 10m/s in a horizontal frictionless plane is pulled 4m by a force of magnitude 25N in the 10m/s in the direction of the initial velocity. What is the final speed a. 32m/s b. 98m/s c. 18.4m/s **D. 14.1m/s**

34. An object of mass 25 is projected at an angle 30^0 to the horizontal with an initial speed of 120m/s. calculate the horizontal range and the time taken to reach max height a.432m, 4sec b. 1221m 6.**5sec C. 1272m, 6.12secs** d. 674m , 5.99secs

35. Just before striking the ground a 200kg mass has 400J of K.E if friction can be ignored from what height was it dropped a. 34m **B.20m** c.45m d.12m

36. A student observes that $0.24m^3$ of water flows out of a horizontal pipe in 1min if the internal diameter of the pipe is 0.05, calculate the speed of the water in the pipe **A.20m/s** b. 23m/s c. 12m/s d.34m/s

37. At a certain point in a pipeline where the diameter is 30cm, the speed of water flow is 1.5m/s. What is the speed of flow at a point where the diameter is 1cm a. $1.22x10^2$m/s b. $2.01x10^3$m/s c. $1.34x10^2$m/s **D. $1.35x10^3$m/s**

38 A pipe of internal diameter 0.04m is joined to another pipe of internal diameter 0.05mm. if the pipe is horizontal and the flow rate of water in the pipe is $4x10^{-3}m^3/s$. find the pressure drop between the 2 pipes a.$1.22x10^2$Pa b. $2.01x10^3$Pac. $1.34x10^2$Pa **D. $1.37x10^3$Pa**

39 What is the dimension for Torque a. $M^{-1}LT^{-2}$**B.ML^2T^{-2}** c.$MT^{-2}S^{-1}$ d. MLT^{-1}

40. If an object moves such that its position (in meters) at any time is $X=20t+15t+15t^2$. Calculate its instantaneous velocity a.120m/**s B.135m/s** c. 200m/s d. 90m/s

41. A man's brain is approximately 0.33 above his heart, if the density of human blood is $1.05x10^3kg/m^3$ determine the pressure required to circulate a. $2.7x10^3N/m^2$ b. $5.7x10^3N/m^2$ c. $1.3x10^2N/m^2$ **D. $3.4x10^3N/m^2$**

42. How much work is done in pumping 1.4cm^3 of water through a 13m internal diameter pipe if the difference in pressure at the two ends of the pipe is 1x10^{5}N/m^2? **A.1.4x10^{5}J** b. 3.6x10^{5}J c. 2.3x10^{4}J d. 3.6x10^{3}J

43. A man kicks a stationary ball of mass 200g giving it a speed of 100cm/s. What impulse is imparted to the ball? a. 2.3kgm/s b. 4.1kgm/s **C. 0.2kgm/s** d. 1.2kgm/s

44 Calculate the force required to accelerate a 1300kg car from rest to a speed of 20m/s in a distance of 80m. a. 1200N **B. 3200N** c. 2400N d. 1600N

45. A bullet of mass 0.002kg is fired with a velocity of 800m/s into a soft wood of mass 2kg, lying in a smooth surface. What is the final velocity if the collision is inelastic? **A. 0.799m/s** b.1.11m/s c. 0.524m/s d. 0.622m/s

46. Which of the following is/are true of friction? **(i)** Limiting friction is proportional to normal reaction **(ii)** dynamic friction is greater than static friction **(iii)** friction and the horizontally applied force are not equal when acceleration is not zero **(iv)** Friction opposes motion

a. (i), (ii) & (iv) b. (i), (iii) & (iv) c. (i), (ii) & (iv) **D. All of the above**

47. When a body is in equilibrium, the vector sum of the forces on it is zero. This is

a. vector sum law b. Polygon law of vector **D. Newton's first** d. Fleming rule

48. When a force acts on an object, it produces acceleratino which may be given as a = F/m where F is force on the obect and m is the mass of the object. This is otherwise known as

a. First law of motion **B. Newton second law** c. third law d. all of the above

49. A fluid of density 1760kgm^-3 and velocity 0.084 Nsm^-2 flows through a pipe of radius 25cm with an average speed of 2.5/s. Calculate the Reynolds's number R and decide whether the flow is turbulent or laminar. a. 1926, turbulent **B. 1926, laminar** c. 2619, unpredictable d. 2619, turbulent

50. All these are true of friction except a. Friction is zero when applied force b. Friction and applied forces are equal in magnitude when acceleration is zero c. coefficient of static friction is greater than coefficient of dynamic friction **D. Limiting friction is not proportional to normal reaction.**

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