

Neglecting density the incompressible Navier-Stokes equations are given as

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = \frac{\partial}{\partial x_j} \left[(\nu + \nu_T) \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] - \frac{\partial p}{\partial x_i} \quad (1)$$

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (2)$$

Take the divergence of (1). The terms with constant ν will then disappear due to continuity and we get

$$\frac{\partial}{\partial x_i} \left(u_j \frac{\partial u_i}{\partial x_j} \right) = \frac{\partial}{\partial x_i} \left(\frac{\partial}{\partial x_j} \left[\nu_T \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] \right) - \frac{\partial^2 p}{\partial x_i \partial x_i} \quad (3)$$

Multiply by testfunction q and integrate over domain:

$$\int_{\Omega} q \frac{\partial}{\partial x_i} \left(u_j \frac{\partial u_i}{\partial x_j} \right) = \int_{\Omega} q \frac{\partial}{\partial x_i} \left(\frac{\partial}{\partial x_j} \left[\nu_T \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] \right) - \int_{\Omega} q \frac{\partial^2 p}{\partial x_i \partial x_i} \quad (4)$$

Integrate first term by parts:

$$\int_{\Omega} q \frac{\partial}{\partial x_i} \left(u_j \frac{\partial u_i}{\partial x_j} \right) = \int_{\Gamma} q u_j \frac{\partial u_i}{\partial x_j} n_i - \int_{\Omega} \frac{\partial q}{\partial x_i} u_j \frac{\partial u_i}{\partial x_j} \quad (5)$$

Second term:

$$\begin{aligned} \int_{\Omega} q \frac{\partial}{\partial x_i} \left(\frac{\partial}{\partial x_j} \left[\nu_T \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] \right) &= \int_{\Gamma} q \frac{\partial}{\partial x_j} \left[\nu_T \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] n_i \\ &\quad - \int_{\Omega} \frac{\partial q}{\partial x_i} \frac{\partial}{\partial x_j} \left[\nu_T \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] \end{aligned} \quad (6)$$

Rewrite second term on the right, remove one term due to continuity

$$\int_{\Omega} \frac{\partial q}{\partial x_i} \frac{\partial}{\partial x_j} \left[\nu_T \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] = \int_{\Omega} \frac{\partial q}{\partial x_i} \frac{\partial \nu_T}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \frac{\partial q}{\partial x_i} \nu_T \frac{\partial^2 u_i}{\partial x_j \partial x_j} \quad (7)$$

Rewrite second term on the right again

$$\int_{\Omega} \frac{\partial q}{\partial x_i} \nu_T \frac{\partial^2 u_i}{\partial x_j \partial x_j} = \int_{\Gamma} \frac{\partial q}{\partial x_i} \nu_T \frac{\partial u_i}{\partial x_j} - \int_{\Omega} \frac{\partial}{\partial x_j} \left(\frac{\partial q}{\partial x_i} \nu_T \right) \frac{\partial u_i}{\partial x_j} \quad (8)$$

Puh, but now I think you should be able to recognize all terms from NSCoupled.