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A genetic algorithm-based approach to calculate the optimal configuration of ultrasonic sensors in a 3D position estimation system

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Abstract

This paper provides a genetic algorithm-based approach to calculate the optimal placement of receivers in a novel 3D position estimation system that uses a single transmitter and multiple receivers. The novelty in the system is the use of the difference in the times of arrival (TOAs) of an ultrasonic wave from the transmitter to the different receivers fixed in 3D space. This is a different approach to traditional systems that use the actual times of flight (TOFs) from the transmitter to the different receivers and triangulate the position of the transmitter. The new approach makes the system more accurate, makes the transmitter independent of the receivers and does not require the need of calculating the time delay term that is inherent in traditional systems due to delays caused by the electronic circuitry. This paper presents a thorough analysis of receiver configurations in the 2D and 3D systems that lead to singularities, i.e. locations of receivers that lead to formulations that cannot be solved due to a shortage of information. It provides guidelines of where not to place receivers so as to get a robust system, and further, presents a detailed analysis of locations that are optimal, i.e. locations that lead to the most accurate estimation of the transmitter positions. The results presented in this paper are not only applicable to ultrasonic systems but all systems that use wave theory, e.g. infrared, laser, etc. This work finds applications in virtual reality cells, robotics, guidance of indoor autonomous vehicles and vibration analysis.

Keywords: Ultrasonics; 3D positioning; Optimal configuration; Genetic algorithms

1. Introduction

Sensor placement is a critical issue in numerous fields, since an optimal configuration can lead to the use of minimum number of sensors, increased accuracy, and simpler yet robust systems. Padula and Kincaid [1] give a comprehensive survey of optimization strategies for sensor placement. They also include actuator placement as part of the same survey, and give numerous examples of applications in the aerospace industry [2–5]. Smart structures that use embedded sensors and actuators rely heavily on the optimal placement of sensors and actuators [6]. Naimimohasses et al. [7] reports on the optimal sensor placement problem in the process industry. Oh and No [8] report on the sensor placement problem for the safe operation of nuclear reactors.

Numerous techniques have been used for optimal sensor placement problems, and have been widely reported in the literature. Some use trial and error methods, some intuitive placement or heuristic

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recipes, and others systematic optimization methods. The use of genetic algorithms (GAs) is gaining popularity in terms of finding optimal solutions for complex sensor/actuator placement problems [9–13].

Figueroa and Mahajan [14] had developed an ultrasonic 3D position estimation system that used the times of flight (TOFs) from a single transmitter to numerous receivers to triangulate the position of the transmitter. They had used an analytical approach based on the Lagrange Multiplier method for the constrained minimization problem to extract the optimal receiver configuration from the linear formulation. The system was designed for the robotics industry, i.e. to track the end-effector of a fixed robot or guide and navigate an autonomous mobile robot in an indoor environment. This is an important application for position estimation systems [15–18], and still remains an active area of research. There are numerous 3D position estimation systems that use wave energy. 3D position estimation using wave sources is typically done by triangulating the position of the wave source using the actual times of flight (TOFs) to the various receivers. Low cost ultrasonic 3D position estimation systems have been developed that use actual TOFs from the transmitter to the receivers [14,19,20].

Wehn and Belanger [19] point out that the main obstacle to precise range measurements is the presence of air turbulence and convection currents. They present a stochastic model for a 3D system. The system by Figueroa and Mahajan [14,20] accounted for the problems stated above by including the speed of sound as an unknown in the system formulation, hence it was updated at each ranging operation. The problem was described by a set of linear equations presented in matrix form, and simply involved the solution of five (or more, depending on the number of receivers being used) simultaneous linear equations, thereby allowing for high update rates. There are two problems with this type of formulation. The first one deals with synchronizing the wave bursts from the transmitter so as to measure the TOFs to the receivers. This measured TOF incorporates delays from the electronic circuitry used to condition the signal at the receiver, delay inherent to the signal detection method to acknowledge reception of the pulses at the receivers, and the acousto-electro-mechanical delay associated with the transducers. All these can be grouped together and called the system time delay that has to be subtracted from the measured TOFs to obtain the actual TOFs. Identification of this time delay term is the second problem.

Mahajan and Walworth [21] developed a formulation that takes the differences in the times of arrival (TOAs) to the various receivers, which circumvents both the above mentioned problems. Firstly, one need not know when the pulse left the transmitter since only the differences in the times of the received signals are recorded. Secondly, since the time delay term is the same for all receivers (this is an assumption based on using the same components for all the receiver circuits), then it automatically cancels out once the difference is taken for any two TOFs. Using the difference in the TOAs addresses both the problems but is sensitive to the location of the receivers and hence, creates a need for a detailed analysis on the receiver configuration geometry.

As mentioned earlier, the problem of adequate placement of receivers in space has been analyzed by Figueroa and Mahajan [14] and Ray and Luck [22] but does not apply to the difference in TOAs formulation. This formulation implicitly contains the position of the transmitter. The singularity condition keeps on changing with the position of the transmitter. Thus the singularity condition has to be exhaustively checked for all the positions of the transmitter in the entire workspace. Analytical optimization methods could not be used due to the complexity of the problem. Hence, genetic algorithms (GAs) are used to solve the optimization problem. The objective is to find the geometric locations of the receivers so as to obtain well-behaved non-singular matrices.

The formulation considers the 2D (transmitter and receivers are in single plane) and the 3D (transmitter and receivers are distributed in space) case. In both the cases two different formulations are presented, one, that uses an externally calculated speed of sound, and the second, that treats the speed of sound as an unknown, and calculates it every time. Both have their advantages and disadvantages, and may be used depending on factors such as environmental conditions, accuracy required, number of receivers available, etc. The greatest advantage of including the speed of sound as an unknown is that the system can automatically compensate for changes in the speed of sound, but now requires an additional receiver, and hence a greater number of equations to solve.

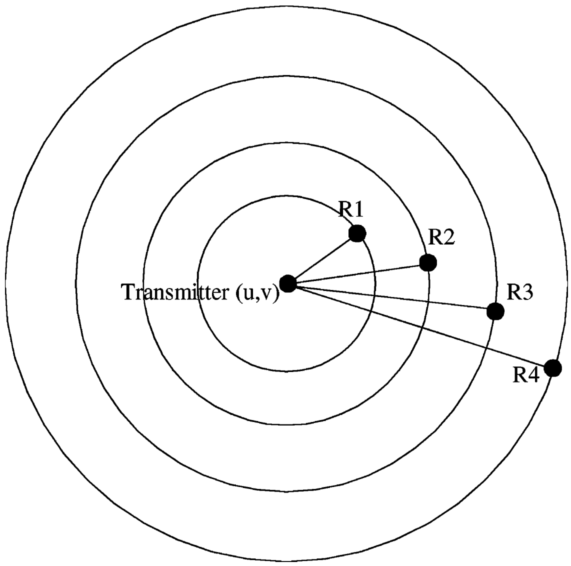

Fig. 1. The formulation.

2. The formulation

For the 2D formulation [21], the transmitter is located at an unknown position (u, v) and the receivers are randomly located at known positions: R1 (x_1, y_1), R2 (x_2, y_2), R3 (x_3, y_3) and R4 (x_4, y_4). Actually three receivers should be sufficient for triangulation, but the equations will be non-linear. Four receivers are used to linearize the equations (see Fig. 1). The signal sent by the transmitter arrives at the receivers (TOA) at times T_1, T_2, T_3 and T_4. The difference between the times of arrival (TOAs) are determined as

$$\Delta T_{12} = T_2 - T_1, \qquad \Delta T_{13} = T_3 - T_1,$$
$$\Delta T_{14} = T_4 - T_1$$

The first receiver to sense the signal is considered to be R1 and is at a distance d from the transmitter. If the speed of sound is c, the 2D linear formulation with a known speed of sound becomes

$$\begin{bmatrix} 2x_1 - 2x_2 & 2y_1 - 2y_2 & -2c\Delta T_{12} \\ 2x_1 - 2x_3 & 2y_1 - 2y_3 & -2c\Delta T_{13} \\ 2x_1 - 2x_4 & 2y_1 - 2y_4 & -2c\Delta T_{14} \end{bmatrix} \begin{bmatrix} u \\ v \\ d \end{bmatrix} = \begin{bmatrix} c^2\Delta T_{12}^2 + x_1^2 + y_1^2 - x_2^2 - y_2^2 \\ c^2\Delta T_{13}^2 + x_1^2 + y_1^2 - x_3^2 - y_3^2 \\ c^2\Delta T_{14}^2 + x_1^2 + y_1^2 - x_4^2 - y_4^2 \end{bmatrix}. \tag{1}$$

It can be written as

$$\mathbf{A} \times \boldsymbol{\mu} = \mathbf{B},$$

where $\mathbf{A}$ and $\mathbf{B}$ are known matrices and vector $\boldsymbol{\mu}$ is to be determined. This case requires a minimum of four receivers. The vector $\boldsymbol{\mu}$ is solved for to obtain values for u, v which are the transmitter position, as well as d which is redundant information but is useful as a check value. Similarly, the governing equation for the 2D case with unknown speed of sound is

$$\begin{bmatrix} 2(x_1-x_2) & 2(y_1-y_2) & -2\Delta T_{12} & -2\Delta T_{12}^2 \\ 2(x_1-x_3) & 2(y_1-y_3) & -2\Delta T_{13} & -2\Delta T_{13}^2 \\ 2(x_1-x_4) & 2(y_1-y_4) & -2\Delta T_{14} & -2\Delta T_{14}^2 \\ 2(x_1-x_5) & 2(y_1-y_5) & -2\Delta T_{15} & -2\Delta T_{15}^2 \end{bmatrix} \begin{bmatrix} u \\ v \\ cd \\ c2 \end{bmatrix} = \begin{bmatrix} x_1^2 + y_1^2 - x_2^2 - y_2^2 \\ x_1^2 + y_1^2 - x_3^2 - y_3^2 \\ x_1^2 + y_1^2 - x_4^2 - y_4^2 \\ x_1^2 + y_1^2 - x_5^2 - y_5^2 \end{bmatrix}. \tag{2}$$

Notice, in this case the speed of sound (c) is solved for in every ranging operation. This makes the system robust in terms of changes in the speed of sound, but forces one to use another receiver.

The linear matrix formulations for the 3D case [21] with a known and an unknown speed of sound are

$$\begin{bmatrix} 2x_1 - 2x_2 & 2y_1 - 2y_2 & 2z_1 - 2z_2 & -2c\Delta T_{12} \\ 2x_1 - 2x_3 & 2y_1 - 2y_3 & 2z_1 - 2z_3 & -2c\Delta T_{13} \\ 2x_1 - 2x_4 & 2y_1 - 2y_4 & 2z_1 - 2z_4 & -2c\Delta T_{14} \\ 2x_1 - 2x_5 & 2y_1 - 2y_5 & 2z_1 - 2z_5 & -2c\Delta T_{15} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ d \end{bmatrix} = \begin{bmatrix} c^2\Delta T_{12}^2 + x_1^2 + y_1^2 + z_1^2 - x_2^2 - y_2^2 - z_2^2 \\ c^2\Delta T_{13}^2 + x_1^2 + y_1^2 + z_1^2 - x_3^2 - y_3^2 - z_3^2 \\ c^2\Delta T_{14}^2 + x_1^2 + y_1^2 + z_1^2 - x_4^2 - y_4^2 - z_4^2 \\ c^2\Delta T_{15}^2 + x_1^2 + y_1^2 + z_1^2 - x_5^2 - y_5^2 - z_5^2 \end{bmatrix} \tag{3}$$

and

$$\begin{bmatrix} 2(x_1-x_2) & 2(y_1-y_2) & 2(z_1-z_2) & -2\Delta T_{12} & -2\Delta T_{12}^2 \\ 2(x_1-x_3) & 2(y_1-y_3) & 2(z_1-z_3) & -2\Delta T_{13} & -2\Delta T_{13}^2 \\ 2(x_1-x_4) & 2(y_1-y_4) & 2(z_1-z_4) & -2\Delta T_{14} & -2\Delta T_{14}^2 \\ 2(x_1-x_5) & 2(y_1-y_5) & 2(z_1-z_5) & -2\Delta T_{15} & -2\Delta T_{15}^2 \\ 2(x_1-x_6) & 2(y_1-y_6) & 2(z_1-z_6) & -2\Delta T_{16} & -2\Delta T_{16}^2 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ cd \\ c2 \end{bmatrix} = \begin{bmatrix} x_1^2+y_1^2+z_1^2-x_2^2-y_2^2-z_2^2 \\ x_1^2+y_1^2+z_1^2-x_3^2-y_3^2-z_3^2 \\ x_1^2+y_1^2+z_1^2-x_4^2-y_4^2-z_4^2 \\ x_1^2+y_1^2+z_1^2-x_5^2-y_5^2-z_5^2 \\ x_1^2+y_1^2+z_1^2-x_6^2-y_6^2-z_6^2 \end{bmatrix}, \tag{4}$$

respectively.

3. Singularities

The unknown vector μ cannot be determined if the system is singular, i.e., the matrix **A** is singular and thus the inverse of **A** is undefined. **A** will be singular if the determinant of **A** is zero. The physical interpretation of the singularity is that more than one location of the transmitter in the workspace produces the same data in the receiver system (ΔTs). So, analytically it is impossible to estimate where the receiver is among the possible options. Since the system has been linearized for ease in solving, there will be some more situations for which the matrix **A** will be singular. In practice, the goal is to get a value of the determinant as further *away from zero* as possible.

One big problem is that the matrix **A** implicitly contains the position of the transmitter.

$$\Delta T_{12} = \frac{cT_2 - cT_1}{c} = \frac{\sqrt{(x_2-u)^2+(y_2-v)^2}}{c} - \frac{\sqrt{(x_2-u)^2+(y_2-v)^2}}{c}.$$

Thus, the matrix **A** is dependent on the position of the transmitter and hence the singular situations are also dependent on the position of the transmitter. This is a highly undesirable situation since every possible transmitter position would have its own singularity analysis. Now, the positions of the receivers should be chosen in such a way that the determinant of the matrix is not zero for any position of the transmitter in the entire specified workspace. This is the price one pays for using the difference in TOAs instead of actual TOFs. Of course there are many advantages too. As mentioned earlier, one need not hook up the transmitter with the computer and the accuracy and resolution of the overall system can be increased.

Figueroa and Mahajan [14] investigated the system using actual TOFs. The linear matrix formulation for that system is

$$\frac{1}{c^2}\begin{bmatrix} 1 & r_1^2 & x_1 & y_1 & z_1 \\ 1 & r_2^2 & x_2 & y_2 & z_2 \\ 1 & r_3^2 & x_3 & y_3 & z_3 \\ 1 & r_4^2 & x_4 & y_4 & z_4 \\ 1 & r_5^2 & x_5 & y_5 & z_5 \end{bmatrix} \begin{Bmatrix} p^2 \\ 1 \\ -2u \\ -2v \\ -2w \end{Bmatrix} = \begin{Bmatrix} \Delta T_{d1} \\ \Delta T_{d2} \\ \Delta T_{d3} \\ \Delta T_{d4} \\ \Delta T_{d5} \end{Bmatrix}.$$

It can be expressed as

$$\mathbf{A} \times \boldsymbol{\mu} = \mathbf{B}.$$

It can be seen that the matrix **A** is independent of the position of the transmitter (there are no time terms in it). Since **A** satisfies the equation of a sphere, the only condition to be satisfied by the five receiver for the matrix **A** to be non-singular is that all of them should not lie on a sphere. If this condition is satisfied, the transmitter can be anywhere in the entire 3D space and one can estimate its position. But, practically speaking, if the transmitter is far away as compared to the distance between the different receivers, the ΔTs may become almost equal and hence the receiver loses resolution even though **A** is non-singular.

To illustrate the above point, consider the 1D case for which two receivers are used to find the position of the transmitter in 1D (just a distance from any one receiver). If both the receivers are equidistant from the transmitter (T1), it is a situation of singularity as the difference in TOAs is zero and all the points (T1, T2) equidistant from both the receivers will give the same information, i.e. zero (Fig. 2).

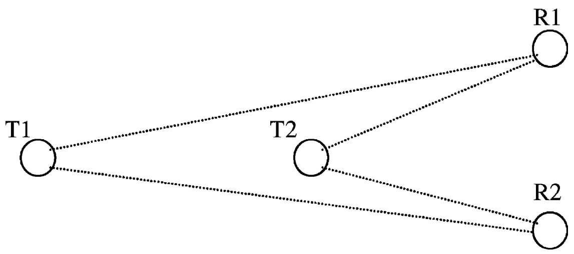

Fig. 2. Singularity positions.

For the 3D formulation with known speed of sound, the determinant of matrix **A** in Eq. (3) should not be zero for non-singularity. The following conditions must be satisfied to obtain a non-singular matrix:

1. All the five receivers cannot lie in the same line, they cannot even lie on a plane. This makes the first, second and the third column linearly dependent and thus the determinant becomes zero.
2. Even worse than the previous condition, the projection of the receivers on the *xy*, *yz* or *zx* plane should not lie in a line. This will produce linear dependency in two of the first three columns.
3. The first receiver and any two receivers cannot lie in a line. When the transmitter is at the same position as the first receiver, two rows become linearly dependent.
4. All the five receivers cannot lie on a sphere, the last column will be zero for the transmitter at the common center. This is the only singularity condition for the actual TOFs formulation [14] and condition 1 is a special case of this when the radius is infinite.

Similar conditions exist for the other three formulations. There will be many other situations where the matrix **A** becomes singular for certain position of the transmitter. No analytical solution has yet been developed, hence the only avenue to conduct an exhaustive search for all the transmitter positions for each receiver geometry was to use GAs.

4. Optimal locations by using genetic algorithms

Not only are the singularity conditions to be avoided, a receiver geometry is to be selected such that the determinant of the matrix **A** is as far away from zero as possible *for all potential transmitter positions* in a given workspace. For the same relative receiver geometry, if the receiver distances are enlarged, the resolution and accuracy of the system will increase and hence the determinant of matrix **A** will also increase. Thus our aim is to find the receiver position within a fixed circle for 2D or a sphere for 3D. The workspace is taken as a square and a cube for the 2D and the 3D cases, respectively, so that they can be easily divided into smaller squares/cubes. The receivers are placed within a fixed circle/sphere at the center of the square/cubic workspace. For a particular receiver geometry, the determinants are calculated at the center of all the small squares. The objective is to maximize the *minimum of the absolute value* (MAV) of the determinants in the entire work space. The optimum configuration of the receiver will be the one with the highest MAV.

The GA is a parallel, global search technique that simulates genetic reproduction and mutation in the natural selection process. A basic GA involves three types of operations: reproduction, crossover and mutation, which are repeatedly applied to a population of “chromosomes” or parameter strings. The flowchart (in Fig. 3) provides an overview.

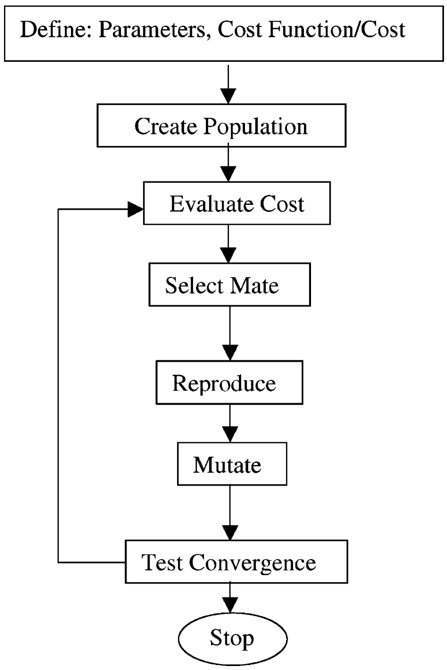

Fig. 3. GA flowchart.

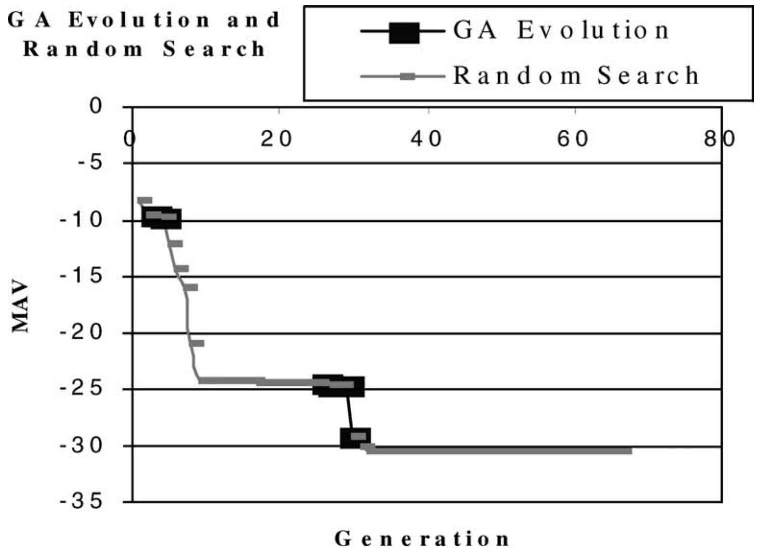

Fig. 4. GA evolution.

Continuous parameter GA is used in the paper to reduce the chromosome length and to avoid the Humming cliff diversion problem that is often encountered in binary parameter GAs [23]. The chromosomes are made up of x, y and z positions of all the receivers. The negative value of the MAV is considered as the cost function (optimality criteria). Weighted random paring is followed for selection, as cost weighting does not explore different regions. Offspring are produced in two different ways for odd and even generations. Two-point random crossover is used as it gives flexibility of two point center block crossover and one point crossover. For even generations interior blending (simple crossover) is done at the crossover point(s) and for the odd generations exterior blending (heuristic crossover) is followed. This allows search inside and outside the range specified by the parameter value of parent crossover points. If the difference in cost of the best and worst eligible parent is less than 5% of cost of the best eligible parent, no offsprings are produced and evolution totally relies on mutation. This is done because all of the almost same parents will not produce offspring any different than the parents. Thus, the chromosomes might be stuck at a point which may not even be a local minimum. In addition to all these techniques, a random search is also done near the genes of the best chromosome in each generation. This produces a better chromosome but the overall solution gets stuck in a local minimum and thus relies on the evolution of GA to move to better solution. Fig. 4 shows the cost of the best chromosome as it evolves. It can be seen that the random search does most of the movement but the GA ultimately helps the solution to move to the optimal minimum position.

It is a 12-dimensional problem for the 2D formulation with a known speed of sound and five receivers fixed in 3D space. This can be visualized as the transmitter moving in a single plane (e.g. mobile robot) and the receivers are fixed in 3D space (e.g. near the ceiling of the workspace for a transmitter fixed on a mobile robot). The cost surface is very undulating and cannot be easily shown, as it needs 12 dimensions. As a simplistic example, two surfaces are shown in Fig. 5 by changing only one dimension to give some idea about the highly undulating cost surface. The mutation rate is kept very high and 24 mutation points are selected and the same number or less mutated chromosomes are formed. Mutation is again done so as to take the total population to 48. This is particularly useful when the chromosomes are stuck in local minimas as shown in Fig. 4. And then as the most important step, the identical or near identical chromosomes are deleted. Identical chromosomes with low cost are the single biggest obstacles for evolution. Costs of all

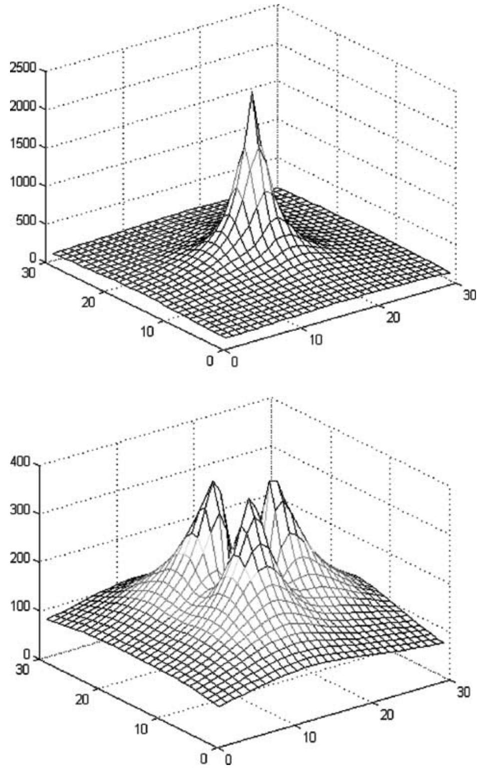

Fig. 5. Two cost surfaces for small changes in just 1D.

the new chromosomes are calculated at the end before moving to the next generation.

5. Results

The cost surface, as defined earlier, is the negative of the MAV. The negative sign is added since the GA has been developed to find the global minima. Number of generations required to reach the global solution depends on the complexity of the problem. Consider the 2D problem which has the transmitter motion and the receivers in the same plane as a guide to understand the more realistic problem of moving the receivers to 3D space. For the considered problem with a workspace of 150 cm × 150 cm and receivers within a circle of 10 cm radius, the MAV of optimum solution is 30.11. The GA reaches an optimal solution of 28.12 MAV in 30 generations. No significant evolution is seen after the 30th generation even though

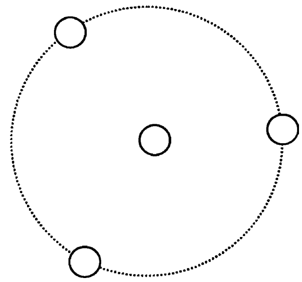

Fig. 6. Receiver configuration.

the GA ran till 60 generations. The configuration produced by the GA is considered near-optimum configuration. The optimum configuration is three receivers at the circumference of a circle forming an equilateral triangle and the fourth one at the center as shown in Fig. 6. Before using GA, a brute search was done manually to find the optimal receiver configuration.

Some of the search results are shown in Fig. 7 along with the axes and the receiver positions (denoted by the four black dots). The absolute value of the determinant between zero and a threshold is converted into a gray level image as shown. White represents any value above the threshold and black represent zero. Thus black regions are the unwanted singularity positions for the given receiver configuration and the transmitter moving in the entire workspace. Some near optimal configurations (0, 10), (0, 7.5), (−10, 0) and (7.5, 0) were discovered but there was no way to categorically state that these were the optimal configurations. The fourth picture in Fig. 7 is a desired result that shows white spaces within a working volume with no singularities for any transmitter position, and corresponds to the configuration discovered by the GA search.

This section deals with the more realistic 2D case with the receivers in 3D space. The optimum configuration is a regular tetrahedron (see Fig. 8). Tetrahedron is the only configuration in which four receivers can be placed furthest apart in a given sphere. The center of tetrahedron should not lie in the plane of the transmitter movement. The MAV increases as the tetrahedron is moved away from the plane, but the value saturates after some distance, as shown in Fig. 9. Thus it is preferable to keep the receivers 3 m away from the working plane. Note that the movement is perpendicular to the plane of the motion of the transmitter. When the tetrahedron is near the

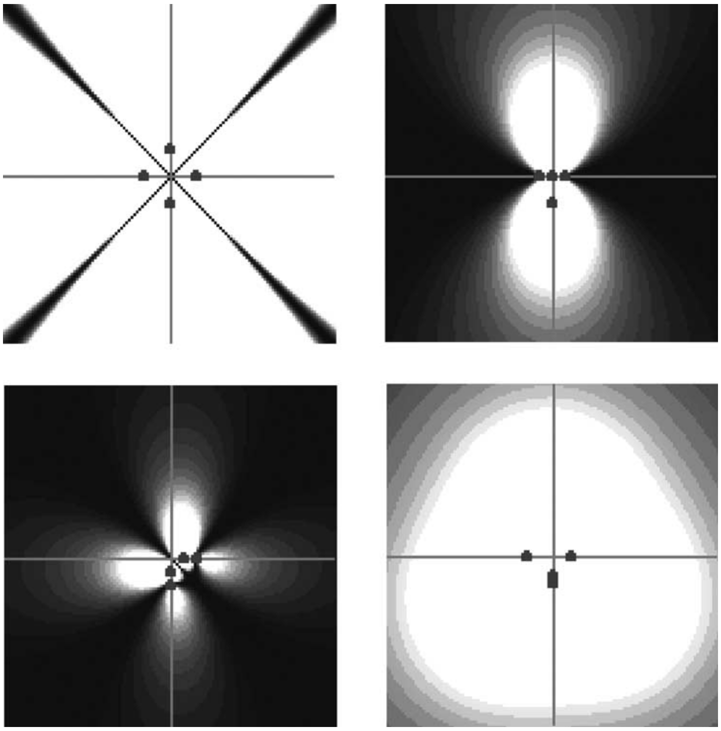

Fig. 7. Singularity search.

plane, the difference in TOAs are relatively small, as the spherical distance is small. It increases as the tetrahedron moves away from the plane as shown in Fig. 10. Rotating the tetrahedron does not have much effect on the determinant value, especially when it is away from the plane. At 10, 20, 30, 40 and 300 cm from the plane the determinant changes by only 15%,

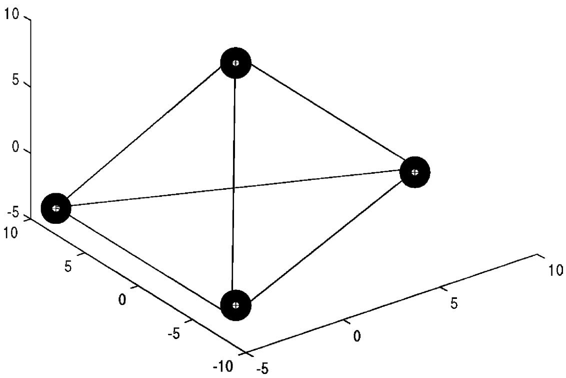

Fig. 8. Receiver configuration.

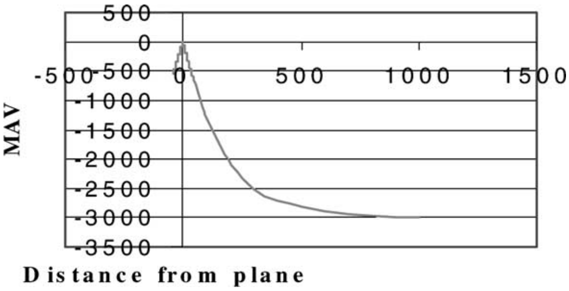

Fig. 9. MAV variation with distance.

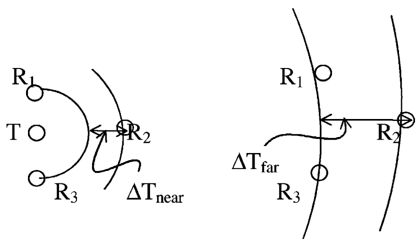

Fig. 10. ΔT variation with distance.

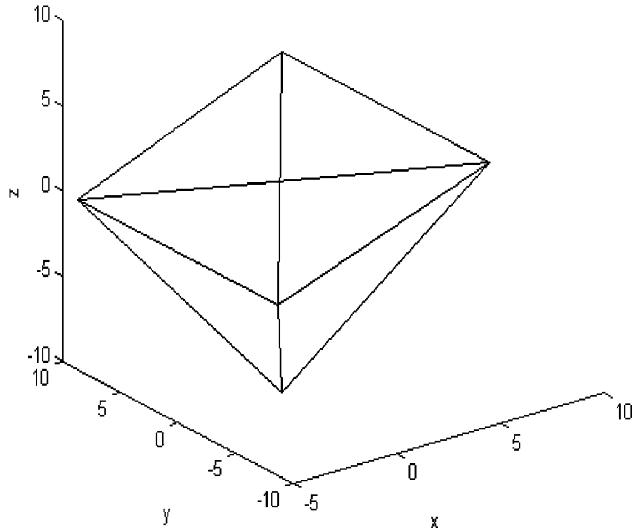

Fig. 11. Receiver configuration.

8%, 6%, 4% and 0.7%, respectively, because of the rotation. For the tetrahedron receiver geometry (0, 0, 10), (−4.71, 8.16, −3.33), (−4.71, −8.16, −3.33) and (9.43, 0, −3.33) at a distance of 3 m from the plane 45° counter clockwise rotation about *X*-axis and 30° clockwise rotation about *Y*-axis gives highest MAV.

With an unknown speed of sound, the 2D system with receivers in 3D needs five receivers. The optimum configuration is three receivers on a sphere forming an equilateral triangle on a plane passing through the center of the sphere. The other two receivers are also on the sphere, the furthest point on the two side of

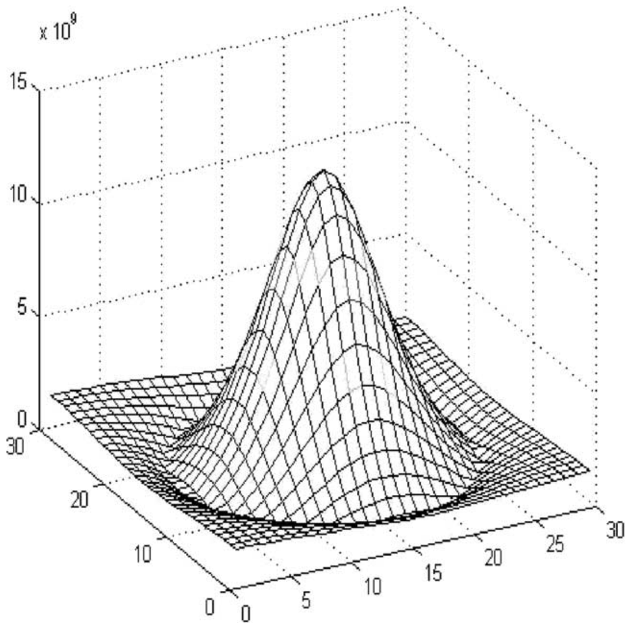

Fig. 12. Optimum cost surface.

the plane. Considering the coordinate system at the center of the sphere, one configuration can be (0, 0, 10), (10, 0, 0), (−5, 8.66, 0), (−5, −8.66, 0) and (0, 0, −10) as shown in Fig. 11. This is different from the configuration obtained by Figueroa and Mahajan [14], where more than three receivers cannot lie on a sphere. Again, like the previous case, the sphere (receiver configuration) should be away from the plane. This configuration also gives a similar graph. The center of the sphere on the plane gives a singularity condition. Fig. 12 shows the optimum cost surface for the receiver configuration placed 80 cm away from the transmitter plane. It can be seen that the cost surface is a dome shape which reaches a minimum value before increasing again as the transmitter moves away from the center of the workspace. Knowing the entire cost surface well in advance helps in the design of the positioning system. Hence, depending upon the workspace requirement, the receiver configuration can be moved still further away to increase the radius of the minimum cost surface circle.

Finally, let us consider a real-life problem of receivers in 3D with the transmitter also moving in a 3D space. The only constraint applied, for now, is that the speed of sound is known for this case. This system also needs five receivers. The optimum configuration for this formulation is similar to that of a 3D receiver configuration with the transmitter moving in 2D and a known speed of sound (refer to Fig. 8). An extra receiver is added at the center of the tetrahedron (0, 0, 0). The new configuration is illustrated in Fig. 13.

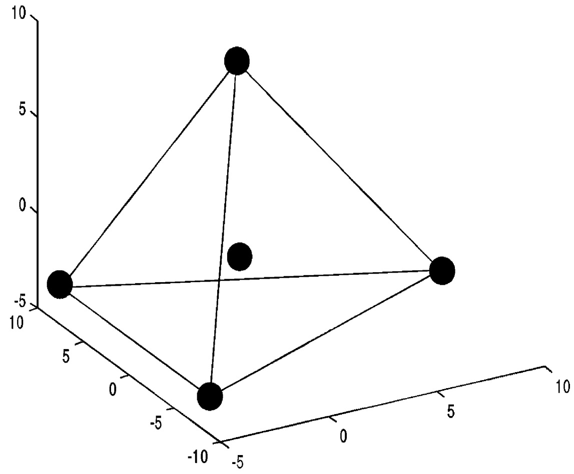

Fig. 13. Receiver configuration using five receivers.

The last formulation to be considered is the same as the previous system with the known speed of sound constraint removed, i.e., the speed of sound is unknown. This is the most complex situation but most often used for virtual reality or robotics applications.

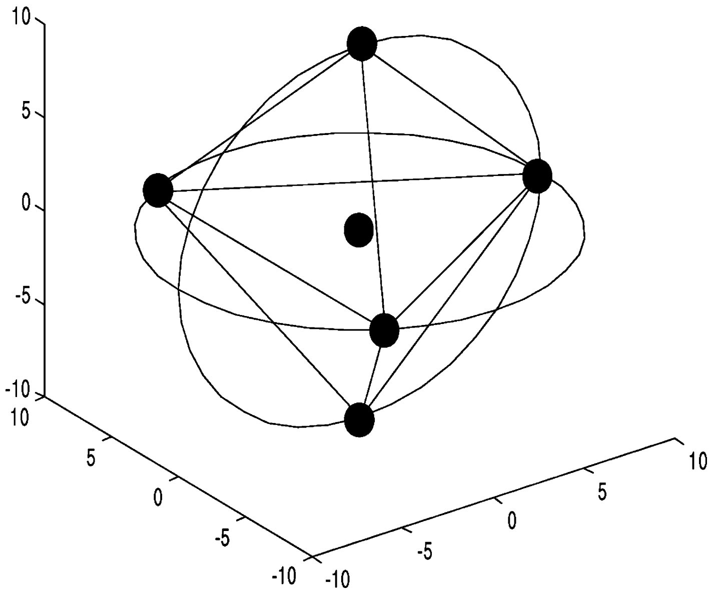

Fig. 14. Receiver configuration using six receivers.

Again, the optimum configuration for this formulation is similar to that of the 3D receiver configuration with the transmitter moving in 2D and an unknown speed of sound (refer to Fig. 11). An extra receiver is added at the center of the tetrahedron (0, 0, 0), and the resulting final configuration is shown in Fig. 14.

Knowing the optimal configuration, the next step is to install the receivers in this configuration. While installing the receivers there will always be some error. Moreover, there will be situations when the receivers have been disturbed by accident or some other reason. A procedure has been developed to use the same set-up for installation and calibration, but is beyond the scope of this paper.

6. Sensitivity analysis

Tracking the error in the position estimate for changes in ΔT and the receiver configuration provides the sensitivity of the system. Both ΔT and the receiver configuration play an important part in the accurate estimation of the position of the transmitter, hence, a sensitivity analysis needs to be done for both. Since the focus of this paper is the search for the optimal receiver configuration, only the sensitivity analysis for the receiver configuration is presented here. Error in the transmitter position estimate due to changes in the receiver configuration are calculated for the following four cases:

(a) Optimal receiver configuration.
(b) Sub-optimal receiver configuration (one of the receivers is moved from the optimal position).
(c) Sub-singular receiver configuration (one of the receivers is moved from the singular position).
(d) Singular receiver configuration.

The receivers are configured in a 20 cm $\times$ 20 cm $\times$ 20 cm volume with the center of the volume at the origin, and the transmitter is allowed to move up to 100 cm away from the nearest receiver. For the first case (a), the optimal configuration obtained from the

Table 1
Comparison of DTOA system for optimal, sub-optimal, sub-singular and singular receiver configurations

System	Receiver configuration (coordinates in cm)	Mean error (cm)	Mean error (%) (based on a mean position of 50 cm)
Optimal	R1: (0, 0, 10.0) R2: (10.0, 0, 0) R3: (−5.0, 8.66, 0) R4: (−5.0, −8.66, 0) R5: (0, 0, −10.0) R6: (0, 0, 0)	1.35	2.70
Sub-optimal	R1: (0, 0, 10.0) R2: (10.0, 0, 0) R3: (−5.0, 8.66, 0) R4: (−5.0, −8.66, 0) R5: (0, 0, −10.0) R6: (0, 0, 5.0)	2.05	4.10
Sub-singular	R1: (0, 0, 10.0) R2: (7.07, 7.07, 0) R3: (−7.07, 7.07, 0) R4: (−7.07, −7.07, 0) R5: (7.07, −7.07, 0) R6: (0, 0, −5.0)	10.77	21.54
Singular	R1: (0, 0, 100) R2: (7.07, 7.07, 0) R3: (−7.07, 7.07, 0) R4: (−7.07, −7.07, 0) R5: (7.07, −7.07, 0) R6: (0, 0, −10.0)	51.79	103.58

Gas, shown in Fig. 14 is used for the analysis. A 100 sets of readings are taken for known transmitter positions and the mean and percentage errors are calculated. The receiver positions are shown in the second column of Table 1, and the mean error for the 100 transmitter position estimations is shown in the third column. The mean position is 50 cm, hence the percentage error is calculated and shown in the fourth column. For the second case (b) one of the receivers (R6) is displaced by 5.0 cm away from the optimal position, and a 100 transmitter position estimations done. The mean and percentage errors are again calculated and shown in Table 1. For the third and fourth cases (c and d), a singular and sub-singular receiver configurations are used for the analysis, and the mean and percentage errors are calculated and shown in Table 1.

The mean error for case (a) is 1.35 cm, for case (b) is 2.05 cm, for case (c) is 10.77 cm and for case (d) is 51.79 cm. The mean percentage error for case (a) is 2.70%, for case (b) is 4.10%, for case (c) is 21.54% and for case (d) is 103.58%. The optimal configuration is shown to be the most accurate and the singular configuration the most inaccurate. The singular configuration has such high error rates because for most cases it finds completely wrong transmitter position estimations, hence the results are really meaningless, but the error rates do give an idea of the inaccuracies relative to the other configurations.

It can be seen that for a 5.0 cm displacement of a single receiver from the optimal configuration, the error in the position estimate increases by 0.7 cm, and the percentage error increases by 1.4%. Within a working volume of 20 cm × 20 cm × 20 cm, a 5.0 cm shift in a receiver position is considered a major shift, and the percentage error simply increases by 1.4%. Hence small shifts in the receiver positions about the optimal receiver configurations will have errors far less than 1.4%. The system is fairly robust for small shifts in receiver positions about the optimal receiver configuration.

7. Conclusions and recommendations

This paper presents a scheme to use GAs in the search for optimal configuration of receivers in a unique 3D position estimation system that uses the difference in TOFs from a single transmitter to various receivers. The focus of this paper is not the 3D system, but the placement of receivers so as to obtain well-behaved matrices in the linearized model. The problem of optimal configurations has been investigated before by many researchers, including one of the authors, but they have all been for systems that used the actual TOFs. It has been shown in this paper that the transmitter position plays a part in determining if the system is close to a singularity. GAs have been used due to the complexity of the problem to search for optimal configurations that lead to the entire workspace having no singular position as well as optimizing (maximizing) the value of the determinant of the characteristic matrix of the system. A sensitivity analysis has been done and the system has been shown to be fairly robust for small shifts in receiver positions about the optimal receiver configuration. This work has applications in any system that uses differences in the TOF information to estimate 3D positions using any type of wave energy, be it ultrasonic, laser or infrared.

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