Assessment of Inertial-Coupling Terms in EOMs mAEWing2 FEM 1.2 – Initial Results

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Intro and Objectives

- **Typically, inertial coupling between RB and Elastic DOFs is ignored.**
- **But the FEM for mAEWing2 has raised some questions - collinearity**
- **We want to**
  - **Model the inertial coupling presently assumed small ✔**
  - **Assess the effects of the coupling present – in process**

Review of EOM's

$$m\overset{\circ}{\mathbf{V}}_{CM} + m\left(\boldsymbol{\omega}_{B,I} \times \mathbf{V}_{CM}\right) = m\mathbf{g} + \mathbf{F}$$

Ang-momentum change

$$[\mathbf{J}]\overset{\circ}{\boldsymbol{\omega}}_{B,I} + \boldsymbol{\omega}_{B,I} \times [\mathbf{J}]\boldsymbol{\omega}_{B,I} + \left[\overset{\circ}{\mathbf{J}}\right]\boldsymbol{\omega}_{B,I} + \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\mathbf{h}_{i,j}\left(\dot{\eta}_i\dot{\eta}_j + \eta_i\ddot{\eta}_j\right) + \boldsymbol{\omega}_{B,I} \times \mathbf{h}_{i,j}\eta_i\dot{\eta}_j\right) = \mathbf{M}$$

$$M_{i,i}\ddot{\eta}_j + M_{i,i}\omega_i^2\eta_j - \sum_{j=1}^{n}\left(\overset{\circ}{\boldsymbol{\omega}}_{B,I} \bullet \mathbf{h}_{i,j}\eta_j + 2\boldsymbol{\omega}_{B,I} \bullet \mathbf{h}_{i,j}\dot{\eta}_j\right) - \frac{1}{2}\boldsymbol{\omega}_{B,I}^T\left[\overline{\Delta J_i}\right]\boldsymbol{\omega}_{B,I} = \mathbf{Q}_i, \quad i = 1 \ldots n$$

Ang Accel of MA **Coriolis** **Centrifugal Loading**

$$\mathbf{J} = \mathbf{J}_{Rig} + \Delta\mathbf{J} + \Delta^2\mathbf{J}$$

$$\overset{\circ}{\mathbf{J}} = \sum_{i=1}^{n}\left[\Delta J_i\right]\dot{\eta}_i + \sum_{i=1}^{n}\sum_{j=1}^{n}\left[\Delta^2 J_{i,j}\right]\eta_i\dot{\eta}_j$$

$$\Delta\mathbf{J} = \sum_{i=1}^{n}\left[\Delta J_i\right]\eta_i(t)$$

$$\overline{\Delta J_i} = \left[\Delta J_i\right] + \sum_{j=1}^{n}\left[\Delta^2 J_{i,j}\right]\eta_j$$

$$\Delta^2\mathbf{J} = \sum_{i=1}^{n}\left(\sum_{j=1}^{n}\left[\Delta^2 J_{i,j}\right]\eta_i(t)\eta_j(t)\right)$$

Red terms currently assumed small

Blue terms (+ $\mathbf{h}_{i,j}$) estimated by VT team
From FEM 1.2 results - $fn(m_i, \phi_i)$

The $h_{i,j}$ Rigid-Elastic Coupling Terms

- **As noted in intro, these terms are all zero for collinear mode shapes.**
- **But in FEMs 1.1 and 1.2, the mode shapes are not collinear.**
- **VT has determined that for FEM 1.2 , these $h_{i,j}$ coupling terms are all on the order of 10^{-26} lb-in^2 , compared to RB inertia terms ~ 10^3 lb-in^2**
- **At this point they will not be considered further**

Effect on Perturbation Dynamics in Level Flight

Linearizing about straight and level flight, the perturbation equations are

$$m\,\partial\overset{\circ}{\mathbf{V}}_{CM} + m\left(\partial\boldsymbol{\omega}_{B,I} \times \mathbf{V}_{\text{Ref}\,CM}\right) = m\left[\partial\mathbf{DCM}\right]\mathbf{g} + \partial\mathbf{F}, \quad \left[\mathbf{DCM}\right] = \text{ Direction-cosine matrix}$$

$$\left[\mathbf{J}_{\text{Ref}}\right]\partial\overset{\circ}{\boldsymbol{\omega}}_{B,I} = \partial\mathbf{M}, \quad \Rightarrow \quad \left[\mathbf{J}_{\text{Ref}}\right] = \left[\mathbf{J}_{Rig} + \Delta\mathbf{J}_{\text{Ref}} + \Delta^2\mathbf{J}_{\text{Ref}}\right]$$

$$M_{i,i}\,\partial\ddot{\eta}_i + M_{i,i}\omega_i^2\,\partial\eta_i = \mathbf{Q}_i, \quad i = 1\ldots n$$

$$\Delta\mathbf{J}_{\text{Ref}} = \sum_{i=1}^{n}\left[\Delta J_i\right]\eta_{i\,\text{Ref}}(t)$$

$$\Delta^2\mathbf{J}_{\text{Ref}} = \sum_{i=1}^{n}\left(\sum_{j=1}^{n}\left[\Delta^2 J_{i,j}\right]\eta_{i\,\text{Ref}}(t)\,\eta_{j\,\text{Ref}}(t)\right)$$

Note that $\mathbf{J}_{\text{Ref}}$ is a function of the structural deformation in trim

The Trim Solutions

Trim constraints:

$$0 = m\left[\mathbf{DCM}_{\text{Ref}}\right]\mathbf{g} + \mathbf{F}_{\text{Ref}} \qquad \Rightarrow L_{\text{Ref}} \approx mg$$

$$0 = \mathbf{M}_{\text{Ref}} \qquad \Rightarrow M_A + M_T = 0$$

$$M_{i,i}\omega_i^2\eta_{i\text{Ref}} = \mathbf{Q}_{i\text{ Ref}}, \quad i = 1 \ldots n \qquad \Rightarrow \mathbf{Q}_{i\text{ Ref}} - M_{i,i}\omega_i^2\eta_{i\text{Ref}} = 0 \ \text{ symmetric modes only}$$

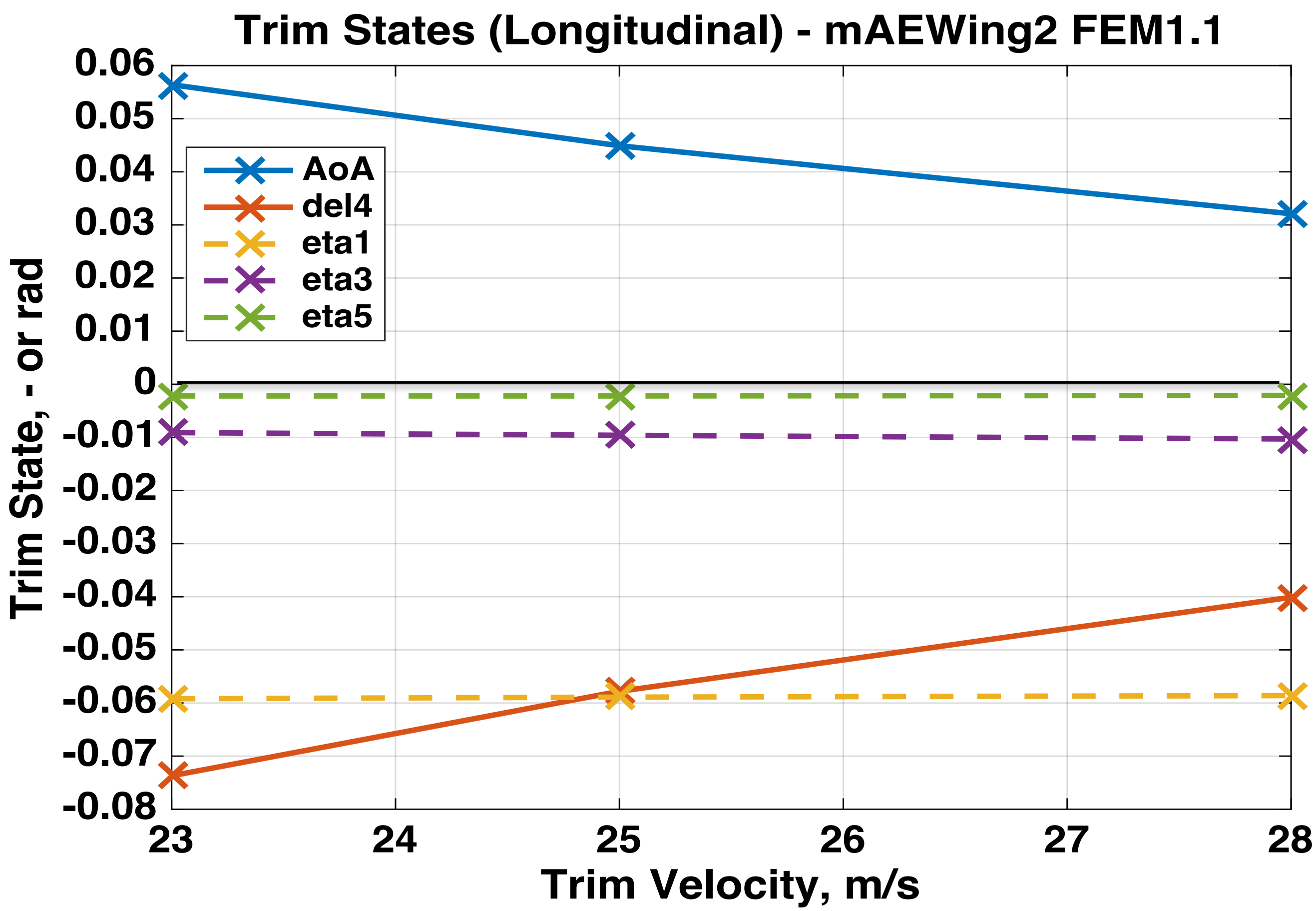
Or, in terms of the coefficients

$$\begin{bmatrix} C_{L_\alpha} & C_{L_\delta} & C_{L_{\eta 1}} & \cdots \\ C_{M_\alpha} & C_{M_\delta} & C_{M_{\eta 1}} & \cdots \\ C_{Q1_\alpha} & C_{Q1_\delta} & \left(C_{Q1_{\eta 1}} - \dfrac{M_{i,i}\omega_1^2}{q_\infty S_W \overline{c}}\right) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \alpha_{Trim} \\ \delta_{Trim} \\ \eta_{1\,Trim} \\ \vdots \end{bmatrix} = \begin{bmatrix} mg / q_\infty S_W - C_{L_0} \\ -C_{M_0} \\ 0 \\ \vdots \end{bmatrix}$$

Solve for the trim states

mAEWing2 Trim, Using Symmetric Flap 4

~ 3.5 deg



~ -4.5 deg

The Change in Inertia Tensor

Jrigid =

$$\begin{bmatrix} 29{,}283. & 1.1416e\text{-}05 & -328.7. \\ 1.1416e\text{-}05 & 4042. & -1.8735e\text{-}05 \\ -328.7. & -1.8735e\text{-}05 & 32{,}923. \end{bmatrix} \text{lb-in}^2$$

Theoretically zero
for symmetric vehicle

Jref 28 m/s =

$$\begin{bmatrix} 29{,}283. & -9.1365e\text{-}04 & -328.7 \\ -9.1365e\text{-}04 & 4042. & 2.0266e\text{-}03 \\ -328.7 & 2.0266e\text{-}03 & 32{,}923. \end{bmatrix} \text{lb-in}^2$$

Small terms affected

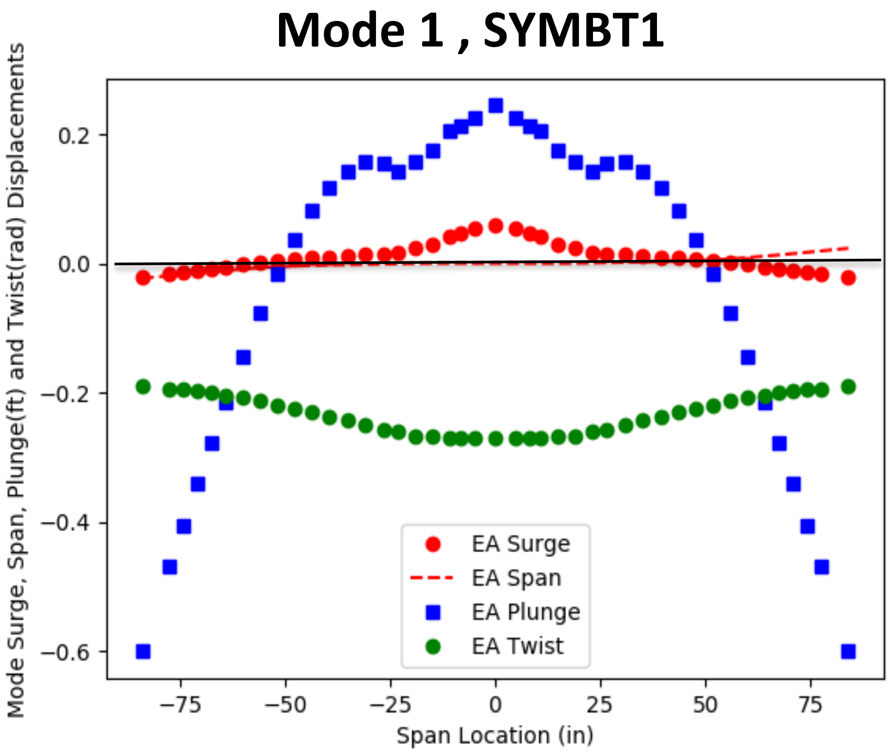
Delta J =

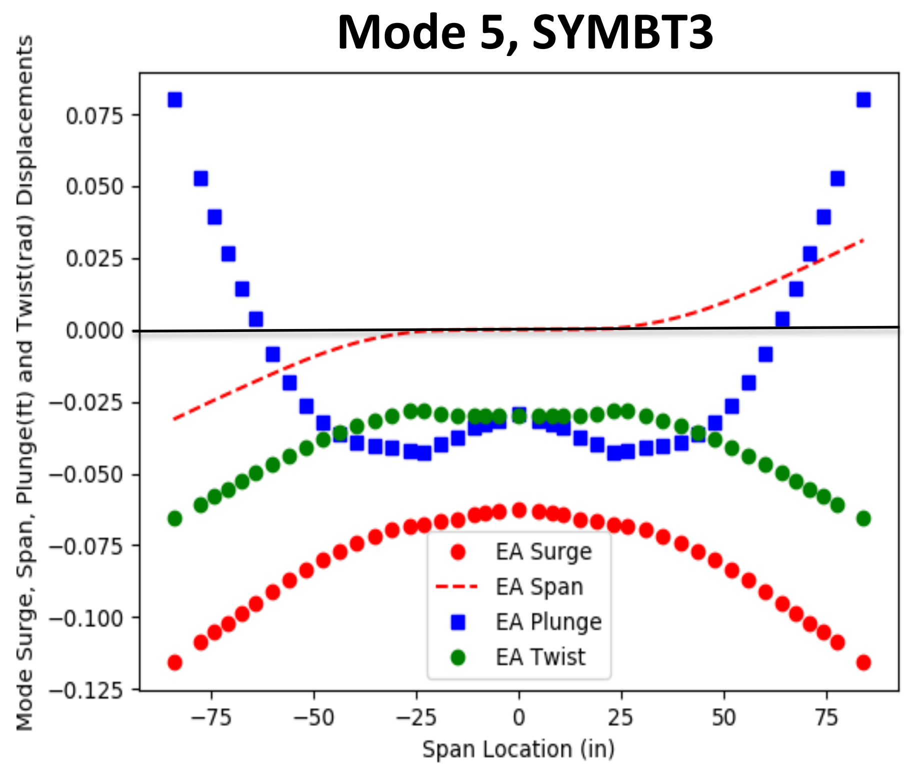
$$\begin{bmatrix} 4.0702e\text{-}03 & -9.2507e\text{-}04 & 2.0037e\text{-}03 \\ -9.2507e\text{-}04 & 4.1497e\text{-}03 & 2.0454e\text{-}03 \\ 2.0037e\text{-}03 & 2.0454e\text{-}03 & 8.4147e\text{-}05 \end{bmatrix} \text{lb-in}^2$$

Preliminary conclusion,
this effect remains small,
But effects on longitudinal
eigenvalues and flutter
will be further evaluated.

Reason: non-collinear mode shapes are of small magnitude

Example FEM 1.1 Mode Shapes





Mostly collinear, d_{max} = 0.6 ft

Not collinear, d_{max} = 0.075 ft

Conclusions

- So far, results support ignoring these effects in level flight
- Further eigenvalue analysis will be performed
- In future, need to continue monitoring FEM mode shapes
- Effects in maneuvering flight must also be monitored, but terms will likely remain negligible unless maneuvering is very aggressive.