Optimal Insulin Dosing for Glucose Regulation

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***Abstract*—This project explores how convex optimization techniques can be used to determine an optimal insulin dosing schedule for regulating blood glucose levels. A simplified linear model is used to represent how glucose levels change in response to insulin dosage and meal disturbances. The dosing problem is set up as a quadratic program (QP) that aims to keep glucose close to a target value while also minimizing how much insulin is used. Safety constraints are included to ensure insulin doses and glucose levels remain within realistic bounds. MATLAB's optimization tools are used to solve the problem, and simulated meal inputs are used to test the model. The results show that convex optimization provides a practical way to design stable glucose control strategies in biomedical applications.**

I. Introduction

Maintaining blood glucose levels within a safe range is essential for diabetic patients, as deviations can cause serious health complications. Optimized insulin dosing strategies can help improve glucose regulation by reducing human error and improving consistency. This project models the glucose–insulin interaction using a simplified linear dynamic system and applies convex optimization to determine an optimal insulin dosing schedule that minimizes deviation from a target glucose level while limiting insulin use.

Numerous studies have explored optimal hormone dosing using control theory and optimization methods. Work from Afroza Shirin has demonstrated that simplified glucose–insulin models can be used to evaluate insulin and glucagon control strategies through optimization. Standard convex optimization principles described by Boyd and Vandenberghe provide the mathematical tools necessary to formulate and evaluate these dosing problems.

The problem is formulated as a quadratic program with constraints on insulin dosage and glucose safety limits, solved using MATLAB's optimization tools. The model will be validated with simulated meal inputs and insulin responses to demonstrate the practical application of convex optimization in biomedical systems. The objective of this work is to implement and test this formulation in MATLAB to determine whether convex optimization can provide efficient glucose control within a biomedical environment.

II. System Model

A. Variables

The target glucose level G^* is set to 110 mg/dL, which is a commonly used target in glucose regulation studies. To begin setting up the optimization model, all of the key variables must be defined clearly. The variables used in the model are listed as follows:

- U_k — insulin dose at time step $k = 0, 1, \ldots, N-1$
- G_k — blood glucose level at time step $k = 0, 1, \ldots, N$
- G_0 — initial glucose level
- G^* — target glucose level
- M_k — meal disturbance input at time step k
- a, b, d — model coefficients for glucose–insulin dynamics
- $U_{\min}, U_{\max}$ — insulin dose bounds
- $G_{\min}, G_{\max}$ — safe glucose bounds
- α — weight on insulin usage in the objective function

B. Dynamic Model

The glucose–insulin interaction is represented using a simplified linear discrete-time model. Glucose evolves according to:

$$G_{k+1} = aG_k + bU_k + dM_k,$$

for $k = 0, 1, \ldots, N-1$. This model captures the effect of prior glucose levels, insulin dosing, and meal disturbances on future glucose dynamics.

The initial condition is given by:

$$G_0 = G_{\text{init}}.$$

C. Parameters

The parameters a, b, and d describe the sensitivity of glucose to its previous value, insulin input, and meal disturbances. The variable a represents the natural dynamic decay factor. Since $a < 1$, the glucose will naturally decay over time. The variable b represents how strongly insulin inputs affect glucose levels. Since $b < 0$, insulin inputs lower the glucose level. A larger magnitude of b means the insulin has a stronger effect. The variable d represents how meal inputs increase glucose levels. A larger value of d will cause bigger glucose spikes. The bounds $U_{\min}, U_{\max}$ restrict insulin to realistic values, and $G_{\min}, G_{\max}$ define safe blood glucose bounds. The variable α determines the trade-off between glucose tracking accuracy and insulin usage in the optimization problem.

III. Optimization Problem Formulation

The goal of this project is to determine an insulin dosing schedule that keeps blood glucose levels close to a desired target while keeping the insulin usage within safe limits. This section presents the objective function as well as the constraints of the optimization problem.

A. Objective Function

The objective of the optimization problem is to minimize the deviation of glucose from the target value, as well as the total insulin usage. The objective function is defined as:

$$J = \sum_{k=0}^{N-1} \left[(G_k - G^*)^2 + \alpha U_k^2 \right],$$

where the first term penalizes glucose tracking error and the second term penalizes excessive insulin dosing. The term α controls the trade-off between tracking performance and insulin usage. Because the objective contains quadratic terms in G_k and U_k, the problem becomes a quadratic program (QP).

B. Constraints

The optimization problem is restricted by the following constraints:

- System Dynamics:

$$G_{k+1} = aG_k + bU_k + dM_k, \quad k = 0, 1, \ldots, N-1$$

- Insulin Dose Limits:

$$U_{\min} \leq U_k \leq U_{\max}, \quad k = 0, 1, \ldots, N-1$$

- Glucose Safety Bounds:

$$G_{\min} \leq G_k \leq G_{\max}, \quad k = 0, 1, \ldots, N$$

- Initial Condition:

$$G_0 = G_{\text{init}}$$

These constraints ensure realistic doses, stable glucose levels, and adherence to the simplified linear model.

C. Quadratic Program

Since the objective function has quadratic terms in G_k and U_k, and all constraints are linear, the problem can be written in a compact form as a quadratic program (QP):

$$\min_U J(U) \quad \text{s.t.} \quad \begin{cases} G_{k+1} = aG_k + bU_k + dM_k, \\ U_{\min} \leq U_k \leq U_{\max}, \\ G_{\min} \leq G_k \leq G_{\max}, \\ G_0 = G_{\text{init}} \end{cases}$$

This compact form highlights the QP structure.

IV. Methodology

The problem is solved in MATLAB using `fmincon`. Although the problem can be written as a quadratic program, `fmincon` is utilized because it can directly handle the glucose dynamics as well as the safety constraints.

M_k is defined as a simulated meal sequence to represent how eating meals affect glucose levels. Higher values for M_k cause more significant glucose increases, whereas lower values simulate baseline fluctuations. Meal disturbances are introduced at selected time steps to represent realistic conditions.

Following this, the insulin sequence U_0 is initialized and the glucose simulation function is defined. The objective function is specified to penalize deviations from the target glucose G_*, excessive insulin use, and hypoglycemia risk. Lower and upper bounds on insulin dosing are enforced, and nonlinear constraints ensure that glucose remains between $G_{\min}$ and $G_{\max}$. With these components defined, `fmincon` is called to compute the optimal insulin plan U_k. The optimized sequence is then applied to the glucose dynamics model to generate the trajectory G_k. Finally, the trajectory is validated by plotting against the target and safety bounds and reporting the minimum and maximum glucose values.

V. Results

The optimized insulin inputs returned by `fmincon` were applied to the glucose model to evaluate system behavior. Figure 1 shows the resulting glucose trajectory.

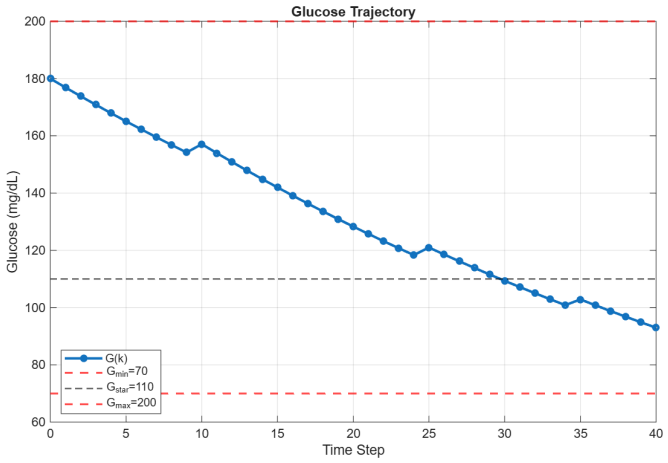

Fig. 1. Optimized Glucose Trajectory

The glucose remained within the safety bounds of $[70, 200]$ mg/dL, the minimum value was 92.97 mg/dL, and the maximum value was 180.00 mg/dL. The trajectory of the graph decreases smoothly from the initial condition and approaches the target level of 110 mg/dL. Meal disturbances can be seen in the graph as they produce small rises in the glucose level, which is also controlled effectively by the optimized insulin inputs.

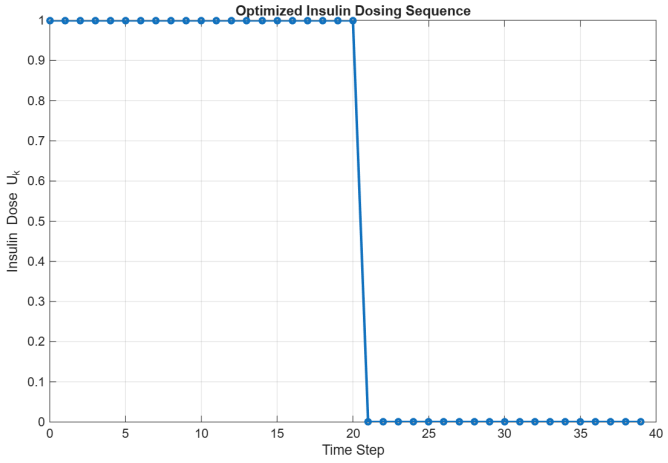

Fig. 2. Optimized Insulin Dosing Sequence

Figure 2 shows the optimized insulin dosing sequence. At the start, the optimizer applies the maximum allowed insulin to lower the high initial glucose level. As levels approach G_*, the insulin dose is reduced to zero, showing how the optimization model stabilizes the system.

VI. Analysis

The results show that the optimized dosing sequence keeps glucose levels within the safety bounds while also responding appropriately to meal disturbances. As the glucose level approaches the target range, the optimizer reduces the dose to zero due to the penalty on insulin usage in the objective function.

A key trade-off comes from the weight α, which determines how much the optimizer tracks the target. Smaller values produce tighter tracking but higher insulin usage. Larger values lead to more conservative behavior. The linear model also causes the system to settle slightly below the target, which is expected for this model.

The primary limitations include the usage of a linear model, simplified meal disturbances, and an open-loop approach. Despite these limitations, the results clearly show that convex optimization methods provide a stable insulin dosing schedule that effectively handles meal disturbances.

VII. Conclusion

This project demonstrated that convex optimization methods can be effectively utilized to create an optimized insulin dosing schedule that keeps glucose levels within safe limits while minimizing insulin usage. Using a simplified linear glucose–insulin model, the quadratic program was successful in regulating glucose levels from a high initial condition toward the optimal value of 110 mg/dL. The model showed robust performance, even with the added meal disturbances. The optimized dosing sequence remained within the predefined safe bounds and produced a stable glucose trajectory graph. These results indicate that convex optimization offers a practical method for exploring glucose control strategies. Future work for improvement would include implementing nonlinear models, multi-hormone control, or a closed-loop MPC implementation.

Individual Contribution

Christian Bammann is the sole contributor of this project, including the problem conceptualization, system modeling, optimization problem formulation, and MATLAB simulations.

References

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[2] S. Boyd and L. Vandenberghe. *Convex Optimization*. Cambridge University Press, 2004.