Error-correction Codes

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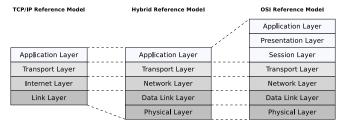
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Learning Objectives of this Slide Set

- Data Link Layer (part 2)
 - Framing (creating frames)
 - Character count in the header
 - Byte stuffing
 - Bit stuffing
 - Physical Layer coding violations
 - Framing in current computer networks
 - Error-detection codes
 - Two-dimensional parity-check code
 - The polynomial code Cyclic Redundancy Check (CRC)
 - Error-correction codes
 - Hamming code
 - Flow Control
 - Stop-and-wait protocol
 - Sliding window protocol

Data Link Layer

- Functions of the Data Link Layer
 - Sender: Pack packets of the Network Layer into frames
 - Receiver: Identify the frames in the bit stream from the Physical Layer
 - Ensure correct transmission of the frames inside a physical network from one network device to another one via error detection with checksums
 - Provide physical addresses (MAC addresses)
 - Control access to the transmission medium



- Devices: Bridge, Layer-2-Switch (Multiport-Bridge), Modem
- Protocols: Ethernet, Token Ring, WLAN, Bluetooth, PPP

- The Data Link Layer does among others...
 - identify the frames in the bit stream of the Physical Layer
 - pack the data packages of the Network Layer in frames
- It is required that the beginning of each frame is highlighted
 - Without markings, the receiver cannot detect the frame boundaries
- Different ways exist to mark the frames' borders
 - Character count in the header
 - Byte/Character stuffing
 - Bit stuffing
 - Line code violations of Physical Layer with illegal signals
- All these different procedures have advantages and drawbacks

Character Count in the Frame Header

- The header of the frame contains among others the character count in the frame
- An example for a Data Link Layer protocol that specifies the frames border with a length information in the frame header is the byte-oriented Digital Data Communications Message Protocol (DDCMP) of DECnet
- In each frame, the field Count contains the number of bytes payload inside the frame

8 Bits	14 Bits	2 Bits	8 Bits	8 Bits	8 Bits	16 Bits		16 Bits
SOH	Count	Flags	RESP	NUM	ADDR	CRC 1	Body	CRC 2
Start of Header	Number of bytes in payload			-	-	Checksum of header	Payload	Chacksum of payload

 Potential issue: If the field Count is modified during transmission, the receiver is unable to correctly detect the end of the frame

Byte/Character stuffing (1/2)

Framing

0000000000000

 A protocol, which highlights the frames border with special characters, is the byte-oriented (character-oriented) protocol Binary Synchronous Communication (BISYNC), which was invented by IBM in the 1960s

8 bits 8 bits 8 bits 8 bits 8 bits 16 bits SYN SOH Header STX Body **ETX** CRC Start of Synchronization Start of End of Payload characters Header Text Text

Control characters
 ("Sentinel characters")
 highlight the structure of
 the frames

- The start of a frame highlights the character SYN
- The start of the header highlights the character SOH (Start of Header)
- \bullet The payload is between STX (Start of text) and ETX (End of Text)
- If the payload (body) contains an ETX character, it it must be protected (escaped) by a stuffed DLE (Data Link Escape) in the Data Link Layer
- The DLE character is represented in the payload by sequence DLE DLE

Concepts of Flow Control

Byte/Character stuffing (2/2)

- The method is called Byte Stuffing or Character Stuffing, because the...
 - sender inserts (stuffs) extra characters into the payload
 - receiver removes the stuffed characters from the received payload, before passing it to the Network Layer
- Drawback of this method:
 - Strong relationship with the ASCII character encoding
 - More recent protocols of this layer no longer operate byte-oriented, but bit-oriented because this allows using any character encoding

Bit Stuffing

- When bit-oriented protocols are used, each frame begins and ends with a special bit pattern
- Examples: The protocol High-Level Data Link Control (HDLC) and the Point-to-Point Protocol (PPP), which implments HDLC
 - Each frame begins and ends with the sequence 01111110

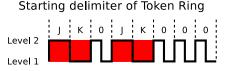


- If the HDLC protocol in the Data Link Layer...
 - of the sender discovers 5 consecutive 1-bits in the bit stream from the Network Layer, it stuffs a 0-bit in the outgoing bit stream
 - of the receiver discovers 5 consecutive 1-bits, followed by a 0-bit in the bit stream from the Physical Layer, it removes (destuffs) the 0-bit
- Advantages:
 - \bullet Ensures that the start/end sequence does not occur in the payload
 - Every character encoding can be used with this framing method

Line Code Violations

- Depending on the line code used in the Physical Layer, illegal signals can be used to highlight the frame boundaries
 - Example: Token Ring uses the Differential Manchester Encoding
 - With this line code, a signal level change occurs inside each bit cell





 If Token Ring is used, frames start with a byte (starting delimiter) which contains 4 line code violations

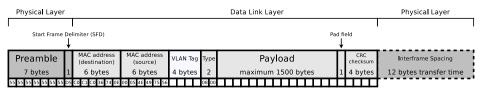
The second last byte (ending delimiter) of a Token Ring frame contains the same 4 line code violations as the starting delimiter

Framing in current Computer Networks (1/8) – Ethernet

Physical Layer					Data Link Layer				Physica l Layer	
	Start Frame	De	imiter (SFD)				Pa	d fi	eld	
	↓							<u></u>		
Prea	amble		MAC address (destination)	MAC address (source)	VLAN Tag	Type	Payload	Γ	CRC checksum	Interframe Spacing
7 1	bytes	1	6 bytes	6 bytes	4 bytes	2	maximum 1500 bytes	1	4 bytes	12 bytes transfer time
55 55 55	55 55 55 55	D5	C0 C1 C0 36 74 0E	00 05 4E 49 75 56	Ш	08 00		Γ	ПП	

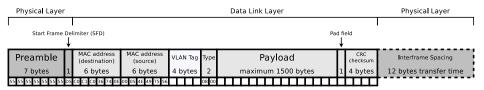
- Up-to-date Data Link Layer protocols (e.g. Ethernet and WLAN) work bit-oriented and not byte-oriented
 - Reason: This way, every character encoding can be used
- Preamble is a 7 bytes long bit sequence 101010 ... 1010
 - Is used in bus networks (topologies) to synchronize the receiver with the clock and to identify clearly the beginning of the frame
 - Is followed by the SFD (1 byte) with the bit sequence 10101011

Framing in current Computer Networks (2/8) – Ethernet



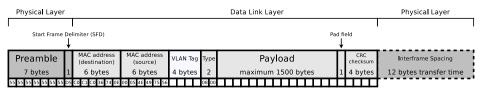
- The fields for the physical addresses (MAC addresses) of sender and destination are 6 bytes long each
- The 4 bytes long optional VLAN tag contains, among others. . .
 - a 12 bits long VLAN ID (\Longrightarrow see slide set 4)
 - and a 3 bits long field for the priority information
- The field Type contains the information what protocol is used in the next upper protocol layer
- If IPv4 is used, the field Type has value 0x0800
- If IPv6 is used, the field Type has value 0x86DD
- If the payload contains an ARP message, the field Type has value 0x0806

Framing in current Computer Networks (3/8) – Ethernet



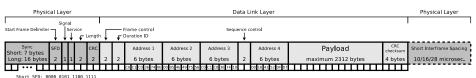
- Minimum size of an Ethernet frame: 72 bytes
- Maximum size (including preamble and SFD): 1526 bytes
- The VLAN tag increases the maximum size by 4 bytes
- Each frame can contain a maximum of 1500 bytes payload
 - With the Pad field, the frame length can be increased to the minimum frame size (72 bytes) when needed
 - This is required to get the collision detection via CSMA/CD working (⇒ slide set 6)
- The last field contains a checksum (32 bits) for all fields, except the preamble and SFD

Framing in current Computer Networks (4/8) – Ethernet



- The Interframe Spacing or Interframe Gap is the minimum idle period between the transmission of Ethernet frames via the transmission medium
- The minimum idle period is 96 bit times (12 bytes)
 - It is 9.6 microseconds when using 10 Mbps Ethernet
 - It is 0.96 microseconds when using 100 Mbps Ethernet
 - It is 96 nanoseconds when using 1 Gbps Ethernet
- Some network devices allow to reduce the Interframe Spacing period
 - Benefit: Better data rate is possible
 - Drawback: For the receiver it may become impossible to detect the frames' borders (\improx the number of collisions may rise)

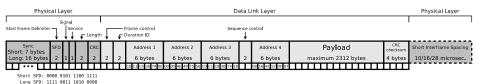
Framing in current Computer Networks (5/8) – WLAN



Long SFD: 1111 0011 1010 0000

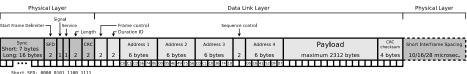
- In Physical layer, the protocol adds...
 - a preamble to synchronize the receiver including a SFD
 - The Short Preamble Format is an optional standard which is not supported by all devices
 - a Signal field, which specifies the data rate (Mbps)
 - a Service field, which may contains some information about the modulation method used
 - a Length field, which specifies the time in microseconds which is required to transmit the frame
 - a CRC field, which contains a checksum over the fields Signal, Service and Length

Framing in current Computer Networks (6/8) – WLAN



- Maximum size of a WLAN frame (only the DLL part): 2346 bytes
- The field Frame Control (2 bytes) contains several smaller fields
 - Among other things, here is the protocol version specified and it is specified what sort of frame (e.g. data frame or beacon) this frame is and if the frame is encrypted with the WEP method
- The field Duration ID (2 bytes) contains a duration value for the update of the counter variable **Network Allocation Vector** (NAV)

Framing in current Computer Networks (7/8) – WLAN



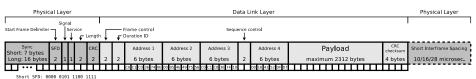
Long SFD: 1111 0011 1010 0000

- The content of the 4 address fields depends on whether the WLAN network runs in infrastructure mode or ad-hoc mode and if the frame has been sent from a terminal device or an Access Point
- The MAC addresses are called Basic Service Set Identifier (BSSID) in this context

Possible values (MAC addresses) in the address fields

- MAC of sender (Source Address)
- MAC of receiver (Destination Address)
- MAC of next station on the route to the destination (Receiver Address)
- MAC of last station which forwarded the frame (Transmitter Address)

Framing in current Computer Networks (8/8) – WLAN



Long SFD: 1111 0011 1010 0000

- The field Sequence Control (2 bytes) consists of a fragment number (4 bits) and a sequence number (12 bits)
 - If a frame has been split into several fragments, the sequence number is equal for all fragments
- The final field contains a CRC checksum (32 bits) for all fields, except the payload

Failure Causes

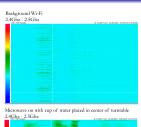
- During the transmission of bit sequences, errors may occur
- Failures are typically caused by...
 - Signal deformation
 - Attenuation of the transmission medium
 - Noise
 - Thermal or electronic noise
 - Crosstalk
 - Unwanted influence neighboring channels (e.g. capacitive coupling)
 - Capacitive coupling increases with increasing frequency

The next slide presents an example for such a cause of error

Short-time disturbances

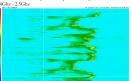
- Multiple bits in a row are incorrect (burst error)
- Cosmic radiation
- Defective or insufficient insulation
- It is important that errors are at least detected
 - Depending on the application, error correction may also be necessary

"Why my microwave makes me lose Wi-Fi connection"





Microwave on with cup of water placed on edge of turntable 2.4Ghz - 2.5Ghz



This is a graph of the 2.4Ghz to 2.5Ghz spectrum over a period of 20 seconds of microwave usage The horizontal direction is frequency, the vertical is time, color is power. Blue was around -80db and yellow around -60db

The top graph is the baseline and on it you can see some Wi-Fi packets around the 2.3Ghz and 2.45Ghz-2.6Ghz range. Following is the first test I did where I placed a ceramic mug filled with water directly in the center of the turn table so that it didn't shift its xy position inside the microwave very much. The last graph is from when I put the ceramic mug of water on the very edge of the turn table so that it would have the most xy motion inside the microwave.

To address people concerned for my safety because my microwave is leaking: It isn't, microwaves can't be perfectly insulated and they have a fuck load of power to dampen. Inside the microwave, it's like 1000w of radiated power which is like 60db. Outside the microwave, I only measured at most -60db. That is about 0.00000001W of radiation power. Granted the antenna I used wasn't perfect so the actual ambient power level was higher. But still vastly below any amount of power that could cause harm.

Source: https://www.reddit.com/r/dataisbeautiful/comments/8xu89b/why_my microwave makes me lose wifi connection/

Error-detection Codes

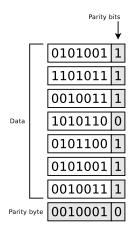
- For error detection, the sender attaches a checksum at each frame
 - This way, frames with errors are detected and thrown away by the receiver

The Data Link Layer protocols do not provide the functionality, that the receiver can request previously discarded frames at the sender

This functionality provides the Transport Layer protocol TCP (\Longrightarrow slide set 9)

- Possible ways to detect errors:
 - Two-dimensional parity-check code
 - The polynomial code Cyclic Redundancy Check (CRC)

Two-dimensional Parity-check Code (1/2)



- This method is well-suited, especially when the 7-bit US-ASCII character encoding is used
- For each 7-bit section, an additional parity bit is calculated and attached to balance out the number of 1 bits in the byte
 - If the protocol defines even parity, the parity bit is used to obtain an even number of 1 bits in every byte
 - If odd parity is desired, the parity bit is used to obtain an odd number of 1 bits in every byte
 - ⇒ one-dimensional parity-check code

Two-dimensional Parity-check Code (2/2)



- For each byte exists, in addition to the parity bits, an additional parity byte
 - The content of the parity byte is calculated over each byte of the frame
 - ⇒ two-dimensional parity-check code
- All 1-bit, 2-bit and 3-bit errors and most of the 4-bit errors can be detected via two-dimensional parity-check codes

Source: Computernetzwerke, Larry L. Peterson, Bruce S. Davie, dpunkt (2008)

 BISYNC is an example for a protocol, which uses this method for the transmission of data, encoded with the 7-bit US-ASCII character encoding

Cyclic Redundancy Check (CRC)

- Bit sequences can be written as polynomials with the coefficients 0 and 1
- ullet A frame with k bits is considered as a polynomial of degree k-1
 - The most significant bit is the coefficient of x^{k-1}
 - The next bit is the coefficient of x^{k-2}
 - ...
- Example: The bit sequence 10011010 corresponds to this polynomial:

$$M(x) = 1 * x^7 + 0 * x^6 + 0 * x^5 + 1 * x^4 + 1 * x^3 + 0 * x^2 + 1 * x^1 + 0 * x^0$$

= $x^7 + x^4 + x^3 + x^1$

 Sending and receiving messages can be imagined as an exchange of polynomials

Polynomials in mathematics

A polynomial is an expression which consists variables and coefficients and non-negative integer exponents

Cyclic Redundancy Check (Method)

- The Data Link Layer protocol specifies a **generator polynomial** C(x)
 - The choice of the generator polynomial determines, which error types can be detected
- C(x) is a polynomial of degree k
 - If e.g. $C(x) = x^3 + x^2 + x^0 = 1101$, then k = 3
 - Therefore, the degree of the generator polynomial is 3

The degree of the generator polynomial is equal to the number of bits minus one

- If for a frame, the CRC checksum need to be calculated, n 0 bits are appended to the frame
 - n corresponds to the degree of the generator polynomial

Selection of common Generator Polynomials

Name	C(x)	Application
CRC-5	$x^5 + x^2 + x^0$	USB
CRC-8	$x^8 + x^2 + x^1 + x^0$	ISDN
CRC-32	$x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^{8} + x^{7} + x^{5} + x^{4} + x^{2} + x^{1} + x^{0}$	Ethernet

Example of Cyclic Redundancy Check (1/4)

Generator polynomial: 100110

- The generator polynomial has 6 digits
 - Therefore, five 0 bits are appended

Frame (payload):	10101
Frame with appended 0 bits:	1010100000

 The frame with the appended 0 bits is divided from the left only via XOR by the generator polynomial

```
• 1 XOR 1 = 0, 1 XOR 0 = 1, 0 XOR 1 = 1, 0 XOR 0 = 0
```

- Always start with the first common 1
- The remainder (remainder polynomial) is the checksum

The sender calculates the checksum

```
1010100000

100110||||

-----v||

110000||

100110||

-----v|

101100|

100110|
```

10100 = Remainder

Example of Cyclic Redundancy Check (2/4)

- The checksum is appended to the payload
 - The length of the remainder must be *n* bits
 - n is the degree of the generator polynomial
- Result: 10100 will be appended to the frame
- Transmitted frame including checksum (code polynomial): 1010110100

Generator polynomial:	100110
Frame (payload):	10101
Frame with appended 0 bits:	1010100000
Remainder (remainder polynomial):	10100
Transferred frame (code polynomial):	1010110100

fill the remainder

- The remainder must consist n bits and n is the degree of the generator polynomial
- If the remainder is shorter than n, it must be filled with zeros
- Example: The remainder is 101 and n is 5, then the checksum to appended is 00101

Example of Cyclic Redundancy Check (3/4)

Transferred frame (code polynomial):	1010110100
Generator polynomial:	100110

The receiver verifies if the transmission was error-free

- The receiver of the frame is able to verify, if the frame did arrive error-free
- With a division (only via XOR) through the generator polynomial, transmissions with errors are detected
 - For division with XOR, always start with the first common 1
- If the remainder of the division is 0, then the transmission was error-free

1010110100 100110||| ----vv|| 110101|| 10011011 ----v 1001101 100110

Example of Cyclic Redundancy Check (4/4)

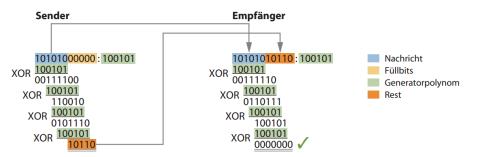
Transferred frame (code polynomial):	1 <mark>1</mark> 10110100
Generator polynomial:	100110
Correct Transmission:	1010110100

- If the transmitted frame contains a defective bit, the remainder of the division via XOR not 0
- This method identifies all 1-bit errors, every odd number of defective bits, and all burst errors of length n
 - n is the degree of the generator polynomial

Sources

- Hompel, Büchter, Franzke. Identifikationssysteme und Automatisierung. Springer. 2008. P 219-221
- Meinel, Sack Digitale Kommunikation: Vernetzen, Multimedia, Sicherheit. Springer. 2009. P.122-124

1110110100
100110
v
111010
100110
v
111001
100110
v
111110
100110
v
110000
100110
10110



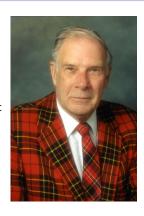
Der CRC-Prüfwert ist der Rest der Division der Nachricht durch das Generatorpolynom. Dieselbe Rechnung mit der Nachricht plus angehängtem CRC ergibt beim Empfänger 0, wenn kein Übertragungsfehler aufgetreten ist.

Source: CRCs berechnen mit Intrinsics. Oliver Lau. c't 9/2013. P.184-185

Error-correction Codes

Image Source: http://amturing.acm.org/images/hamming-3.jpg

- Transmission errors can be detected via CRC checksums
 - If it is important also to correct errors, the data to be transmitted must be encoded in a way, that error-correction is possible
- Error correction can be realized via Hamming code
 - Named after the mathematician Richard Wesley Hamming (1915-1998)



Richard Hamming (1915 - 1998)

None of the established Data Link Layer protocols provide error correction

They all provide only error detection!

In this course, we discuss a $\underbrace{\text{simplified version}}_{\text{Hamming-ECC}}$ of the Hamming Code which is sometimes called Hamming-ECC

```
http://www.mathcs.emory.edu/-cheung/Courses/455/Syllabus/2-physical/errors-Hamming.html
http://de.wikipedia.org/w/index.php?title=Fehlerkorrekturverfahren
Andrew Tanenbaum. Computernetzwerke. Pearson (2000). 3rd edition. P. 208-209
```

- The bits of a bit sequence are numbered starting with 1 from the left
 - Bits, which are powers of 2 (1, 2, 4, 8, 16, etc.) are parity bits
 - The remaining bits are the payload
- Example:
 - 8 bits payload: 01001100

```
Position: 1 2 3 4 5 6 7 8 Payload: 0 1 0 0 1 1 0 0
```

Position: 1 2 3 4 5 6 7 8 9 10 11 12
Data to be transmitted: ? ? 0 ? 1 0 0 ? 1 1 0 0

• The sender calculates the parity bits values

Hamming Code – Procedure: Parity Bits Values (1/2)

- Each bit position in the transmission is assigned to a position number
- In this example, the position number is a 4-digit number, because we have 4 parity bits
- Examples:

. . .

Hamming Code – Procedure: Parity Bits Values (2/2)

- Next, XOR of the values of those positions (in 4-digit format), where the payload is a 1 bit in the transmission, is calculated
 - In the example it is position 5, position 9 and position 10

- The result are the values of the parity bits
 - These are inserted into the transmission

Position: 1 2 3 4 5 6 7 8 9 10 11 12 Data to be transmitted: 0 1 0 1 1 0 0 0 1 1 0 0

- The receiver can verify if a bit sequence is correct
 - It calculates XOR of the calculated and received parity bits
 - The parity bits are position 1, 2, 4 and 8

```
Received data: 1 2 3 4 5 6 7 8 9 10 11 12 0 1 0 1 1 0 0 0 1 1 0 0
```

```
0101 Position 5
1001 Position 9

XOR 1010 Position 10
-----
0110 Parity bits calculated

XOR 0110 Parity bits received
------
= 0000 => Correct transmission
```

Hamming Code – Procedure: Check Data (2/2)

```
Received data: 1 2 3 4 5 6 7 8 9 10 11 12
```

```
1001
           Position 9
XOR 1010
           Position 10
    0011
           Parity bits calculated
XOR 0110
           Parity bits received
    0101
           => Bit 5 is defective!
```

- Possible results of the calculation:
 - Position number of the modified bit
 - 0 if the transmission was correct
- If ≥ 2 bits have been changed, the only statement that can be made is, that bits have been changed at all
 - The positions can not be determined this way

Conclusion for Error Detection Codes

- Encoding data with error correction codes is useful, where transmission retries cannot be requested
 - Example: The network only consists of a simplex channel
- Typically, only error detection codes are used
 - If an error is detected, the frame is just thrown away
 - None of the established Data Link Layer protocols provide error correction
 - The Data Link Layer protocols provide only error detection!
 - The Transport Layer protocol TCP requests again transmissions with errors
- If only error detection is supported, and not error correction, lesser parity bits need to be transmitted
 - For this reason, error detection is more efficient than error correction

See the model calculation for error detection/correction on the next slide

Model calculation for error detection/correction

- Block size: 1.000 bits
- 1 Megabit data (1,000 blocks) shall be transferred
- Error detection
 - A single parity bit at the end of each block is enough to detect if the block contains errors
 - This increases the size of each block by 1 bit to 1.001 bits
 - After 1,000 blocks, an extra block (1.001 bits) need to be transmitted
 - Therefore, 1 Megabit data requires 2,001 parity bits
- Error correction (Hamming code)
 - To correct errors in 1.000 bits data, 10 parity bits are required
 - The parity bits are positions 1, 2, 4, 8, 16, 32, 64, 128, 256 and 512
 - Therefore, 1 Megabit data requires 10,000 parity bits

Source

- 8 bits (10011010) of payload exist
 - What is the data to be transmitted (including payload and parity bits)
- Verify that the following messages were transmitted correctly
 - 00111101
 - 101110100010
 - 001101100100
 - 0001101100101101
- ⇒ Exercise sheet 3

Framing

- Via flow control, the receiver controls the transmission speed of the sender dynamically, and ensures this way the integrity of the transmission
 - Receivers with poor performance are not flooded with data, they are unable to handle
 - As a result data would be lost
 - During transmission, lost data is transmitted again
- Procedure: Transmission retries occur when they are needed
 - Basic mechanisms:
 - Acknowledgements (ACK) as feedback
 - Timeouts
 - Concepts of flow control:
 - Stop-and-Wait (⇒ see slide set 9)
 - Sliding-Window (⇒ see slide set 9)
- Ethernet does not implement flow control mechanisms on Data Link Layer
- In practice, if flow control is required, it is provided by the Transport Layer protocol TCP
- Flow Control in the Data Link Layer will not be discussed in this course