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MAT10, 2. gr. time

Delvis integrasjon

produktregel: $(uv)' = u'v + uv'$ | $\int VS = \int HS$

$\int (uv)' = \int u'v + \int uv'$ | $VS = HS$

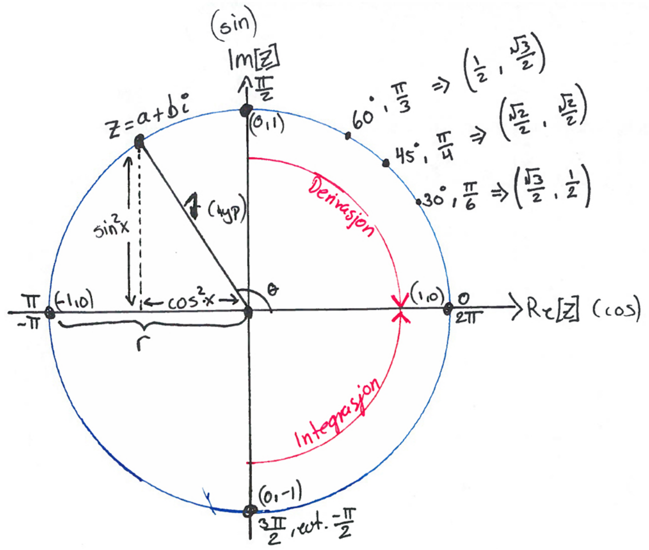
$\int u'v + \int uv' = \int (uv)'$ | $\int (uv)' = uv$

$\int u'v + \int uv' = uv$ | Flytter over

I $\int uv' = uv - \int u'v$ (formelen for delvis integrasjon)

Derivasjon og integrasjon av trigonometriske funksjoner

II



III Taylor-rekker

i) Definer $P_n(x) = \sum_{i=0}^{n} \frac{f^{(i)}(a)(x-a)^i}{i!}$ og $R_{n+1}(x) = \frac{f^{(n+1)}(c)(x-a)^{n+1}}{(n+1)!}$

ii) x nær a = ...

$f^{(0)}(x) =$	$f^{(0)}(x=a) =$
$\vdots$	$\vdots$
$f^{(n)}(x) =$	$f^{(n)}(x=a) =$
$f^{(n+1)}(x) =$	$f^{(n+1)}(x=c) =$

Til restleddet

iii) Finn c's øvregrense ($= \max(a; x)$)

$\Rightarrow c \in [x; a]$ (x a)

øvregrense

$\Rightarrow c \in [a; x]$ (a x)

iv) Sett inn c's øvre grense i

$f^{(n+1)}_{(c)}$ (nøyaktig svar); og

$M \approx \text{roundup}(f^{(n+1)}_{(c)})$

$\left|f^{(n+1)}_{(c)}\right| \leq |M|$

iv fortsatt)

$\left|f^{(n+1)}_{(c)}\right| \leq |M| \quad \Big| \cdot \frac{|(x-a)^{n+1}|}{(n+1)!}$

$\frac{\left|f^{(n+1)}_{(c)} \cdot (x-a)^{n+1}\right|}{(n+1)!} \leq \frac{|M| \cdot |(x-a)^{n+1}|}{(n+1)!} \quad \Big| VS = |R_{n+1}(x)|$

$|R_{n+1}(x)| \leq \frac{|M| \cdot |(x-a)^{n+1}|}{(n+1)!}$

#1a)

$$\int 7x e^{4x} \, dx$$

I

$u = 7x \qquad v' = e^{4x}$

$u' = 7 \qquad v = \frac{1}{4}e^{4x}$

$$= 7x \cdot \frac{1}{4}e^{4x} - \frac{7}{4}\int e^{4x} \, dx$$

Utleder

$$= \frac{7}{4}xe^{4x} - \frac{7}{4} \cdot \frac{1}{4}e^{4x} + C$$

Utleder

$$= \frac{7}{4}xe^{4x} - \frac{7}{16}e^{4x} + C$$

Ikke et must, men det er en fordel å renskrive ved å faktorisere mest mulig. En tommelfingerregel er å minimere antall brøker. Så sett $\frac{7}{16}$ utenfor istedenfor $\frac{7}{4}$

$$= \frac{7}{16}e^{4x}(4x - 1) + C$$

#1b

$$\int x^3 \ln x \, dx$$

I

$u = \ln|x|$ $\quad$ $v' = x^3$

$u' = \frac{1}{x}$ $\quad$ $v = \frac{1}{4}x^4$

$$= \frac{1}{4}x^4 \cdot \ln|x| - \frac{1}{4}\int x^3 dx$$

Utleder

$$= \frac{1}{4}x^4 \ln|x| - \frac{1}{16}x^4 + C$$

Faktoriserer

$$= \frac{1}{16}x^4\left(4\ln|x| - 1\right) + C$$

#1c)

$$\int x^2 \cos x \, dx$$

I og II

$u = x^2$ $\quad v' = \cos x$

$u' = 2x$ $\quad v = \sin x$

$$= x^2 \sin x - \int 2x \sin x \, dx$$

I og II. Har allerede brukt u og v, så vi må bruke andre variabler.

$w = 2x$ $\quad r' = \sin x$

$w' = 2$ $\quad r = -\cos x$

$$\cancel{x^2 \sin x + 2x\cos x - 2}$$

$$= x^2 \sin x - \left(2x \cdot [-\cos x] - \int 2 \cdot [-\cos x] \, dx\right)$$

Rydder opp i fortegn

$$= x^2 \sin x + 2x\cos x - 2\int \cos x \, dx$$

Utleder

$$= x^2 \sin x + 2x \cos x - 2\sin x + C$$

$$\int \cos x \sin x \, dx$$

variabelskifte: velger passende u ved å se på (II): – Derivasjon går med klokka. Velger vi å derivere $u = \cos x$, får vi $-\sin x$. Vi har ikke $-\sin x$. Men om vi deriverer $u = \sin x$, får vi $\cos x$. Den har vi. Så vi velger å substituere $\sin x$.

$u = \sin x$

$\frac{du}{dx} = \cos x \iff du = \cos x \, dx$

~~setter inn~~. Omskriver her kun denne ene gangen så jeg har alle med.

$$= \int \underbrace{\sin x}_{u} \cdot \underbrace{\cos x \, dx}_{du}$$ | substituerer

$$= \int u \, du$$ | Utleder

$$= \frac{1}{2} u^2 + C$$ | $u = \sin x$

$$= \frac{1}{2} \sin^2 x + C$$

#1c)

$$\int x(\ln x)^2 dx$$

I
$u = (\ln x)^2 \qquad v' = x$

$u' = 2(\ln x)^1 \cdot \frac{1}{x} \qquad v = \frac{1}{2}x^2$

kjerneregel

$$= \frac{1}{2}x^2 \cdot (\ln x)^2 - \int 2\ln x \cdot \frac{1}{x} \cdot \frac{1}{2}x^2 dx$$ Renskriver

$$= \frac{1}{2}x^2 (\ln x)^2 - \int x \ln x \, dx$$

I
$w = \ln x \qquad r' = x$

$w' = \frac{1}{x} \cdot 1 \qquad r = \frac{1}{2}x^2$

$$= \frac{1}{2}x^2(\ln x)^2 - \left(\frac{1}{2}x^2 \ln x - \frac{1}{2}\int \frac{\cancel{x^2}}{x} dx\right)$$ ~~Utleder~~ Renskriver

$$= \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x + \frac{1}{2}\int x \, dx$$ Utleder

$$= \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2 + C$$ Faktoriserer

$$= \frac{1}{4}x^2\left(2[\ln x]^2 - 2\ln x + 1\right) + C$$

#1f)

$$\int \arctan(x)\,dx$$

Det står egentlig ikke arctan (x), men arctan(x) · 1. Setter inn

$$= \int \arctan(x) \cdot 1\,dx$$

I

$u = \arctan(x)$ $\quad v' = 1$

$u' = \dfrac{1}{x^2+1}$ $\quad v = x$

$$= x\arctan(x) - \int \frac{x}{x^2+1}\,dx$$

Variabelskifte:

$w = x^2+1$

$\frac{dw}{dx} = 2x \Leftrightarrow dw = 2x\,dx \Leftrightarrow \frac{1}{2}dw = x\,dx$

Setter inn

$$= x\arctan(x) - \frac{1}{2}\int \frac{1}{w}\,dw$$

Utleder

$$= x\arctan(x) - \frac{1}{2}\ln|w| + C$$

$w = x^2+1 > 0 \ \forall\ x \in \mathbb{R}$, så ~~vi endrer~~ absoluttketstegnet er overflødig. Endrer til $\ln(\ldots)$

$$= x\arctan(x) - \frac{1}{2}\ln(x^2+1) + C$$

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I

#1g) $\int x \arctan x \, dx$

$u = \arctan(x)$ $\quad v' = x$

$u' = \frac{1}{x^2+1}$ $\quad v = \frac{1}{2}x^2$

$= \frac{1}{2}x^2 \arctan(x) - \frac{1}{2}\int \frac{x^2}{x^2+1}\,dx$ $\quad \Big| \quad x^2 = x^2+1-1$

$= \frac{1}{2}x^2 \arctan(x) - \frac{1}{2}\int \frac{x^2+1-1}{x^2+1}\,dx$ $\quad \Big|$ Deler opp og $\frac{x^2+1}{x^2+1} = 1$

$= \frac{1}{2}x^2 \arctan(x) - \frac{1}{2}\int 1\,dx - (-1)\int \frac{1}{x^2+1}\,dx$ $\quad \Big|$ Utleder

$= \frac{1}{2}x^2 \arctan(x) - \frac{1}{2}x + \arctan(x) + C$ $\quad \Big|$ Faktoriserer

$\cancel{= \frac{1}{2}\left(x^2 \arctan(x) - x + \arctan x\right) + C}$

$$= \frac{1}{2}\arctan(x)\left(x^2+1\right) - \frac{1}{2}x + C$$

4.1 h)

$$\int \arcsin x \, dx$$

står egentlig $\arcsin x \cdot 1$

$$= \int \arcsin x \cdot 1 \, dx$$

I

$u = \arcsin x$ $\qquad v' = 1$

$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$ $\qquad v = x$

$$= x \arcsin x - \int \frac{x}{\sqrt{1-x^2}}$$

variabelskifte

$w = 1 - x^2$

$\frac{dw}{dx} = -2x \iff dw = -2x\,dx \iff -\frac{1}{2} dw = x\,dx$

$$= x \arcsin x - \left(-\tfrac{1}{2}\right) \int \frac{1}{\sqrt{w}} \, dw$$

$\frac{1}{\sqrt{w}} = \frac{1}{w^{1/2}} = w^{-1/2}$

$$= x \arcsin x + \frac{1}{2} \int w^{-1/2} \, dw$$

Utleder

$$= x \arcsin x + \frac{1}{2} \cdot \left(2 w^{1/2} \right) + C$$

~~Bytter opp og~~ $w = 1 - x^2$

$$\underline{\underline{= x \arcsin x + (1 - x^2)^{1/2} + C}}$$

#1 i

$\int \arccos x \, dx$ | Igjen, dets kiv $\arccos x \cdot 1$

$= \int \arccos x \cdot 1 \, dx$ | (I) $u = \arccos(x)$ $v' = 1$

$u' = \frac{-1}{(1-x^2)^{1/2}}$ $v = x$

$u' = -(1-x^2)^{-1/2}$

$= x\arccos(x) - (-1)\int x(1-x^2)^{-1/2} dx$ | Var.skifte:

$w = 1-x^2$

$\frac{dw}{dx} = -2x \Leftrightarrow dw = -2x dx \Leftrightarrow \frac{-1}{2} dw = x dx$

$= x\arccos(x) + 1 \cdot \left(\frac{-1}{2}\right)\int w^{-1/2} dw$ | Utleder

$= x\arccos(x) - \frac{1}{2} \cdot 2 w^{1/2} + C$ | ~~Renskriver og~~ $w = 1-x^2$

$$= x\arccos(x) - (1-x^2)^{1/2} + C$$

#1 j)

$$\int \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$$

var. skifte:

$u = \arcsin(x)$

$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}} \Leftrightarrow du = \frac{1}{\sqrt{1-x^2}}\,dx$

$= \int u\,du$ | Utleder

$= \frac{1}{2}u^2 + C$ | $u = \arcsin(x)$

$$= \frac{1}{2}\arcsin^2 x + C$$

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#1 K)

$$\int e^{-\sqrt{x}}dx \quad \Big| \quad \sqrt{x} = x^{1/2}$$

$$= \int e^{-x^{1/2}}dx$$

var. skifte:

$u = -x^{1/2}$

$\frac{du}{dx} = -\frac{1}{2}x^{-1/2} \Leftrightarrow du = -\frac{1}{2}x^{-1/2}dx \Leftrightarrow -2x^{1/2}du = dx \quad \Big| -x^{1/2} = u$

$2u\,du = dx$

$$= \int 2ue^{u}du$$

I

$v = 2u \qquad w' = e^{u}$

$v' = 2 \qquad w = e^{u}$

$$= 2ue^{u} - 2\int e^{u}du \quad \Big| \text{ utleder}$$

$$= 2ue^{u} - 2e^{u} + C \quad \Big| \text{ faktoriserer og } u = -x^{1/2}$$

$$= 2e^{-x^{1/2}}\left(-x^{1/2} - 1\right) + C \quad \Big| \text{ faktoriserer}$$

$$= -2e^{-x^{1/2}}\left(x^{1/2} + 1\right) + C$$

#1 L)

$$\int x^3 \cdot \sqrt{1+x^2}\,dx$$ | $\sqrt{1+x^2} = (1+x^2)^{1/2}$

$$= \int x^3 \cdot (1+x^2)^{1/2}\,dx$$

Var. skifte:

$u = 1+x^2$

$\frac{du}{dx} = 2x \Leftrightarrow du = 2x\,dx \Leftrightarrow \frac{1}{2}du = x\,dx$

omskriver før vi setter inn. Husk: $x^2 = x^2+1-1$

$$= \int (x^2+1-1) \cdot (1+x^2)^{1/2} \cdot x\,dx$$ | setter inn $u = 1+x^2$ og $\frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\int (u-1) \cdot u^{1/2}\,du$$ | løser opp

$$\cancel{= \frac{1}{2}\int u^{3/2}\,du - \frac{1}{2}\int u^{1/2}\,du}$$ | ~~Utled~~

$$\cancel{= \frac{2\cdot 1}{5\cdot 2}u^{5/2} - \frac{2\cdot 1}{3\cdot 2}u^{3/2} + C}$$

$$= \frac{1}{2}\left(\int u^{3/2}\,du - \int u^{1/2}\,du\right)$$ | Utled

$$= \frac{1}{2}\left(\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C_1\right)$$ | Løser opp ( ) og $u = 1+x^2$

$$= \frac{1}{5}(1+x^2)^{5/2} + \frac{1}{3}(1+x^2)^{3/2} + C \quad , \quad C = \frac{1}{2}C_1$$

Kunne faktorisert her, men uttrykket hadde ikke blitt noe penere, så jeg lar det bare være.

#2 a)

$$\int_0^9 \frac{dx}{\sqrt{1+\sqrt{x}}}$$

Renskriver

$$= \int_{x=0}^{x=9} \left(1+x^{1/2}\right)^{-1/2} dx$$

Var.skifte:

- $u = 1 + x^{1/2}$
- $\frac{du}{dx} = \frac{1}{2}x^{-1/2} \iff 2x^{1/2}du = dx \quad \Big| \quad x^{1/2} = x^{1/2} + 1 - 1$

$2\left(x^{1/2}+1-1\right)du = dx \quad \Big| \quad x^{1/2} + 1 = u$

$2(u-1)du = dx$

- Nye integrasjonsgrenser: $\int_{u=1+0^{1/2}=1}^{u=1+9^{1/2}=4}$

$$= \int_{u=1}^{u=4} u^{-1/2} \cdot 2(u-1)du$$

Deler opp

$$= 2\left(\int_1^4 u^{1/2}du - \int_1^4 u^{-1/2}du\right)$$

Utleder

$$= 2\left(\left[\frac{2}{3}u^{3/2}\right]_1^4 - \left[2u^{1/2}\right]_1^4\right)$$

Utleder

$$= 2\left(\left[\frac{2}{3}\cdot 4^{3/2} - \frac{2}{3}\cdot 1^{3/2}\right] - \left[2\cdot 4^{1/2} - 2\cdot 1^{1/2}\right]\right)$$

Σ

$$= \frac{16}{3}$$

#2b)

$$\int_{x=0}^{x=1} \frac{\text{arcos}(x)}{\sqrt{1-x^2}}$$

Var. skifte:

- $u = \arccos(x)$
- $\frac{du}{dx} = \frac{-1}{\sqrt{1-x^2}} \Leftrightarrow (-1)\,du = \frac{1}{\sqrt{1-x^2}}\,dx$
- Nye grenser: $\int_{u=\arccos(0)=\frac{\pi}{2}}^{u=\arccos(1)=0}$

$$= \int_{u=\frac{\pi}{2}}^{u=0} \cancel{(-1)}\ \cancel{u\,du}\ u \cdot (-1)du \quad \Big| \text{ Utleder}$$

$$= \left[-\frac{1}{2}u^2\right]_{\frac{\pi}{2}}^{0} \quad \Big| \text{ Utleder}$$

$$= \left(-\frac{1}{2}\cdot 0^2\right) - \left(-\frac{1}{2}\cdot\left[\frac{\pi}{2}\right]^2\right) \quad \Big| \ \Sigma$$

$$= \underline{\underline{\frac{\pi^2}{8}}}$$

#2c)

$$\int_0^1 \frac{x^3+1}{x^2-4}\,dx$$

Polynomdivisjon siden grad i teller er høyere enn grad i nevner.

$$(x^3+0x^2+0x+1):(x^2-4) = x + \frac{4x+1}{x^2-4}$$

$$\frac{-(x^3 \qquad -4x)}{4x+1}$$

Setter inn ~~og deler opp brøken~~

$$= \cancel{\int_0^1 x\,dx}$$

$$= \int_0^1 \left(x + \frac{4x+1}{x^2-4}\right)dx$$

Deler opp og delbrøkoppspalting.

$$\frac{4x+1}{x^2-4} = \frac{4x+1}{(x-2)(x+2)} \doteq \frac{A}{(x-2)} + \frac{B}{(x+2)} \quad \Big|\cdot(x-2)(x+2)$$

$$4x+1 = A(x+2) + B(x-2)$$

Kan skrives som to ligninger (epler og pærer)

$$\begin{aligned} 4x &= Ax + Bx \\ 1 &= 2A - 2B \end{aligned} \Rightarrow \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 9/4 \\ 7/4 \end{pmatrix}$$

$$= \cancel{\int_0^1}$$

Setter inn

$$= \int_0^1 x\,dx + \frac{9}{4}\int_0^1 \frac{1}{x-2}\,dx + \frac{7}{4}\int_0^1 \frac{1}{x+2}\,dx$$

Utleder

$$= \left[\frac{1}{2}x^2 + \frac{9}{4}\ln|x-2| + \frac{7}{4}\ln|x+2|\right]_0^1$$

Utleder

$$= \left(\frac{1}{2}\cdot 1^2 + \frac{9}{4}\cdot\ln|1-2| + \frac{7}{4}\ln(1+2)\right) - \left(\frac{1}{2}\cdot 0^2 + \frac{9}{4}\ln|0-2| + \frac{7}{4}\ln|0+2|\right)$$

utleder og $\ln|1-2| = \ln 1 = 0$ og $\ln|0-2| = \ln 2$

$$= \left(\frac{1}{2} + 0 + \frac{7}{4}\ln 3\right) - \left(\frac{9}{4}\ln 2 + \frac{7}{4}\ln 2\right)$$

Σ

$$= \frac{1}{2} + \frac{7}{4}\ln 3 - 4\ln 2$$

#2 d)

$$\int_{x=0}^{x=2} \frac{x^3+1}{x^2+4}\,dx$$

Polynomdivisjon:

$$(x^3+0x^2+0x+1):(x^2+4) = x + \frac{(-4)x+1}{x^2+4}$$
$$\frac{-(x^3 \qquad +4x)}{-4x+1}$$

Setter inn og deler opp

$$= \int_{x=0}^{x=2} x\,dx - 4\int_{x=0}^{x=2}\frac{x}{x^2+4}\,dx + \int_{x=0}^{x=2}\frac{1}{x^2+4}\,dx$$

Vi starter først med det 2. integralet.
var. skifte:

- $u = x^2+4$
- $\frac{du}{dx} = 2x \Leftrightarrow \frac{1}{2}du = x\,dx$
- nye grenser: $\int_{u=0^2+4=4}^{u=2^2+4=8}$

$$= \int_{x=0}^{x=2} x\,dx - \frac{4}{2}\int_{u=4}^{u=8}\frac{1}{u}\,du + \int_{x=0}^{x=2}\frac{1}{x^2+4}\,dx$$

Vi har fått et hint om at $\int\frac{1}{x^2+1}dx = \arctan(x)+C$

- Vi trenger altså 1 der nede i nevneren. Så vi faktoriserer
  så $\int\frac{1}{x^2+4}dx = \int\frac{1}{4(\frac{x^2}{4}+1)}dx = \frac{1}{4}\int\frac{1}{\frac{x^2}{4}+1}$
- Fra hintet ser vi også at vi trenger et kvadrat i nevner. Kan $\frac{x^2}{4}$ omskrives til et kvadrat? Vel, $2^2=4$,
  så $\frac{x^2}{4} = \frac{x^2}{2^2} = \left(\frac{x}{2}\right)^2$. ~~Om vi setter v~~
- Så var. skifte:
  $v = \frac{x}{2}$
  $\frac{dv}{dx} = \frac{1}{2} \Leftrightarrow$ ~~$\frac{1}{2}dv$~~ $2dv = dx$,
  evt $\frac{1}{2}dv = \frac{1}{4}dx$ om vi vil fjerne $\frac{1}{4}$ foran integralet.
- Nye grenser: $\int_{v=\frac{0}{2}=0}^{v=\frac{2}{2}=1}$

Setter inn

$$= \int_{x=0}^{x=2} x\,dx - 2\int_{u=4}^{u=8}\frac{1}{u}\,du + \frac{1}{2}\int_{v=0}^{v=1}\frac{1}{v^2+1}\,dv$$

Utleder

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$$= \left[\frac{1}{2}x^2\right]_0^2 - 2\left[\ln|u|\right]_4^8 + \frac{1}{2}\left[\arctan(v)\right]_0^1$$

~~Setter inn~~
Utleder

$$= \left(\frac{1}{2}2^2 - \frac{1}{2}0^2\right) - 2\left(\ln 8 - \ln 4\right) + \frac{1}{2}\left(\arctan(1) - \arctan(0)\right)$$

- $\ln(a^b) = b\ln(a)$
- $\ln 8 = \ln 2^3 = 3\ln 2$
- $\ln 4 = \ln 2^2 = 2\ln 2$
- $\arctan(1) = \frac{1}{4}\pi$
- $\arctan(0) = 0$
- Σ

$$= 2 - 2\ln 2 + \frac{1}{8}\pi$$

$\boxed{1}$ $n=3$, $a=0$ Må bruke ~~2 og~~ (II) og (III)

x nær a = 0

a)

$f(x) = \sin x$	$f(0) = \sin 0 = 0$
$f'(x) = \cos x$	$f'(0) = \cos(0) = 1$
$f''(x) = -\sin x$	$f''(0) = -\sin(0) = 0$
$f'''(x) = -\cos x$	$f'''(0) = -\cos(0) = -1$
$f^{(4)}(x) = \sin x$	$f^{4}(c) = \sin(c)$

$$P_3(x) = \sum_{i=0}^{3} \frac{f^{(i)}(a)(x-a)^i}{i!} = \frac{0(x)^0}{0!} + \frac{1\cdot x^1}{1!} + \frac{0\cdot x^2}{2!} + \frac{(-1)\cdot x^3}{3!}$$ Utleder

$$\underline{P_3(x) = x - \frac{1}{6}x^3}$$

$$R_4(x) = \frac{f^{(3+1)}(c)\cdot(x-a)^4}{(3+1)!} = \underline{\underline{\frac{\sin(c)\cdot x^4}{24}}}$$

$$\underline{\underline{f(x) = P_3(x) + R_4(x) = x - \frac{1}{6}x^3 + \frac{\sin(c)\cdot x^4}{24}}}$$

Først, litt utregninger:

$g(x) = \arctan x$

$g'(x) = \frac{1}{x^2+1} = (x^2+1)^{-1}$

$g''(x) = (-1)(x^2+1)^{-2}\cdot 2x = (-2x)(x^2+1)^{-2}$

$g'''(x) = (-2)(x^2+1)^{-2} + (-2x)(-2)(x^2+1)^{-3}(2x)$

$= (-2)(x^2+1)^{-2} + 8x^2(x^2+1)^{-3}$ | felles nevner

$\cancel{= \frac{-2}{(x^2+1)^2} + \frac{8x^2}{(x^2+1)^3}}$ | ~~Felles nevner~~

$\cancel{= \frac{-2(x^2+1)+8x^2}{(x^2+1)^3}}$ | ~~Σ~~

$\cancel{= \frac{6x^2+2}{(x^2+1)^3} = (6x^2-2)}$

$= (-2)(x^2+1)(x^2+1)^{-3} + 8x^2(x^2+1)^{-3}$ | ~~Σ~~ faktorisere

$\cancel{= (-2x^2-2+8x^2)(x^2+1)^{-3}}$ | Σ

$\cancel{= 6x^2}$

$= (-2x^2-2+8x^2)(x^2+1)^{-3}$ | Σ

$= (6x^2-2)(x^2+1)^{-3}$

g

$g^{(4)}(x) = (12x)(x^2+1)^{-3} + (6x^2-2)\cdot(-3)(x^2+1)^{-4}\cdot(2x)$ | ~~felles~~ ~~forenkle og~~ felles nevner

$g^{(4)}(x) = (12x)(x^2+1)(x^2+1)^{-4} \cancel{+ 6x} - 6x\cdot(6x^2-2)(x^2+1)^{-4}$ | utleder

$g^{(4)}(x) = (\cancel{24}\,12x^3+12x)(x^2+1)^{-4} - (36x^3 - 12x)(x^2+1)^{-4}$ | Σ

$g^{(4)}(x) = (-24x^3+24x)(x^2+1)^{-4}$ | faktorisere

$g^{(4)}(x) = \frac{24x(\cancel{24}\,1-x^2)}{(x^2+1)^4}$

x nær $a=0$

$g(x) = \arctan x$

$g'(x) = \frac{1}{x^2+1}$

$g''(x) = \frac{-2x}{(1+2x)^2}$

$g'''(x) = \frac{6x^2-2}{(x^2+1)^3}$

$g^{(4)}(x) = \frac{24x(1-x^2)}{(x^2+1)^4}$

$g(0) = 0$

$g'(0) = \frac{1}{0+1} = 1$

$g''(0) = 0$

$g'''(0) = \frac{6\cdot 0-2}{(0+1)^3} = -2$

$g^4(c) = \frac{24c(1-c^2)}{(c^2+1)^4}$

$g(x) = P_3(x) + R_4(x)$

~~setter~~

$P_3(x) = \sum_{i=0}^{3} \frac{g^{(i)}(a)(x-a)^i}{i!}$

$R_4(x) = \frac{g^{(4)}(c)(x-a)^4}{4!}$

$g(x) = \frac{0 \cdot (x-0)^0}{0!} + \frac{1\cdot(x-0)^1}{1!} + \frac{0\cdot(x-0)^2}{2!} + \frac{(-2)(x-0)^3}{3!} + \frac{24c(1-c^2)}{(c^2+1)^4} \cdot \frac{(x-0)^4}{24}$ | Renskriver

$$g(x) = \underbrace{x - \frac{1}{3}x^3}_{P_3(x)} + \underbrace{\frac{c(1-c^2)}{(c^2+1)^4}x^4}_{R_4(x)}$$

X når $a=0$

c)

$h(x) = \ln(1+x)$ | $h(0) = \ln 1 = 0$

$h'(x) = \frac{1}{1+x} = (1+x)^{-1}$ | $h'(0) = (1+0)^{-1} = 1$

$h''(x) = (-1)(1+x)^{-2}$ | $h''(0) = (-1)(1+0)^{-2} = -1$

$h'''(x) = 2(1+x)^{-3}$ | $h'''(0) = 2(1+0)^{-3} = 2$

$h^{4}(x) = (-6)(1+x)^{-4}$ | $h^{(4)}(c) = (-6)(1+c)^{-4}$

$h(x) = P_3(x) + R_4(x)$ | Setter inn

$$h(x) = \frac{0\cdot(x-0)^0}{0!} + \frac{1\cdot(x-0)^1}{1!} + \frac{(-1)(x-0)^2}{2!} + \frac{2(x-0)^3}{3!} + \frac{(-6)}{(1+c)^4}\cdot\frac{(x-0)^4}{4!}$$ | Renskriver

$$h(x) = \underbrace{x - \frac{1}{2}x^2 + \frac{1}{3}x^3}_{P_3(x)} - \underbrace{\frac{1}{4\cdot(1+c)^4}x^4}_{R_4(x)}$$

22

$$j(x) = \frac{1+x}{1-x} = \frac{(1+x)+(1-1)}{1-x} = \frac{2-(1-x)}{(1-x)} = \frac{2}{1-x} - 1$$

x nær a=0

$f(x) = 2(1-x)^{-1} - 1$ | $j(0) = 2(1-0)^{-1} - 1 = 1$

$j'(x) = 2(-1)(1-x)^{-2} \cdot (-1) = 2(1-x)^{-2}$ | $j'(0) = 2(1-0)^{-2} = 2$

$j''(x) = 2 \cdot (-2)(1-x)^{-3}(-1) = 4(1-x)^{-3}$ | $j''(0) = 4(1-0)^{-3} = 4$

$j'''(x) = -12(1-x)^{-4}(-1) = 12(1-x)^{-4}$ | $j'''(0) = 12(1-0)^{-4} = 12$

$j^{(4)}(x) = -48(1-x)^{-5}(-1) = 48(1-x)^{-5}$ | $j^{(4)}(c) = 48(1-c)^{-5}$

$j(x) = P_3(x) + R_4(x)$ | Setter inn

$$j(x) = \frac{1 \cdot (x-0)^0}{0!} + \frac{2(x-0)^1}{1!} + \frac{4(x-0)^2}{2!} + \frac{12(x-0)^3}{3!} + \frac{48}{(1-c)^5} \cdot \frac{(x-0)^4}{4!}$$ | Renskriver

$$j(x) = \underbrace{1 + 2x + 2x^2 + 2x^3}_{P_3(x)} + \underbrace{\frac{2}{(1-c)^5} x^4}_{R_4(x)}$$

23 #2

$$\frac{d}{dx}\left[\arctan(x)\right] = \frac{1}{1+x^2}$$

Formelen for uendelig geometrisk rekke:

$$\frac{a}{1-k} = a + ak + \dots + ak^{n-1} \quad , \quad |k| < 1$$

Setter inn med $a = 1$ og $k = -x^2$ siden $\frac{1}{1-(-x^2)} = \frac{1}{1+x^2}$

$$\frac{d}{dx}\left[\arctan(x)\right] = 1 + (-x^2) + (-x^2)^2 + (-x^2)^3 + \dots$$ | Løser opp

$$\frac{d}{dx}\left[\arctan(x)\right] = 1 - x^2 + x^4 - x^6 + \dots$$ | $\int VS = \int HS$

$$\int \frac{d}{dx}\left[\arctan(x)\right]dx = \int (1 - x^2 + x^4 - x^6 + \dots)\,dx$$

$$\arctan(x) = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots + C$$

Ønsker å finne C. om vi setter $x = 0$,

er $\arctan(0) = 0 + 0 + \dots + C$ | $\arctan(0) = 0$

$0 = C$.

Så vi fjerner C-leddet.

$$\underline{\underline{\arctan(x) = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots}}$$

#3

$P_3(x)$ til $\sin(x)$ har en feil som er mindre enn 0,0001. → (merk: i absoluttverdi)

Videre, fra 1a fant vi at $R_4(x) = \dfrac{\sin(c)\cdot x^4}{24}$

a) $|\sin x| \leq 1$

$$\left|R_4(x)\right| = \left|\frac{1}{24}\sin(c)x^4\right| \leq \left|\frac{1}{24}\cdot 1\cdot x^4\right| < 0{,}0001 \quad \Big| \ \left|\frac{1}{24}\right| = \frac{1}{24}$$

$$\frac{1}{24}|x^4| < 0{,}0001 \quad \Big| \cdot 24 \text{ og } |x^4| = |x|^4$$

$$|x|^4 < 0{,}0024 \quad \Big| \sqrt[4]{VS} < \sqrt[4]{HS}$$

$$|x| < 0{,}0024^{1/4} \approx 0{,}221337$$

b) $|\sin x| \leq |x|$

$$\left|R_4(x)\right| = \left|\frac{1}{24}\sin(c)x^4\right| \leq \left|\frac{1}{24}\cdot x\cdot x^4\right| < 0{,}0001 \quad \Big| \ \left|\frac{1}{24}\right| = \frac{1}{24}$$

$$\frac{1}{24}|x^5| < 0{,}0001 \quad \Big| \cdot 24 \text{ og } |x^5| = |x|^5$$

$$|x|^5 < 0{,}0024 \quad \Big| \sqrt[5]{VS} < \sqrt[5]{HS}$$

$$|x| < 0{,}0024^{1/5} \approx 0{,}299256$$

#4 går vi ikke gjennom siden det verken er pensum eller er gitt noe løsningsforslag.