Exercise 1 a)

The first-order conditions are that (by the proof)

- $\frac{\partial \mathcal{L}}{\partial x_i} = 0 \quad \forall \; i \in 3, 4$ (since we don't have non-negativity restrictions)
- $\frac{\partial \mathcal{L}}{\partial \lambda} = 0$ since we have an equality in the restriction
- We can rewrite these conditions to the following: $\nabla F = \sum_{j=1}^{m} \lambda_j \nabla G_j$
- Since we have only one constraint, we have by plugging in the values, that

$$\begin{pmatrix} 2 \\ 4 \\ -6 \\ 5 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 2 \\ -3 \\ 2 \end{pmatrix}$$

- It's clear to see that $\lambda = 2$.
- Since the above expression holds, we can ~~here~~ conclude that the necessary FOCs holds
- Please note that the constraint qualification is satisfied because $\nabla G \neq \vec{0}$.
- The statement is true.

Exercise 1b

- Again, we have the requirement that

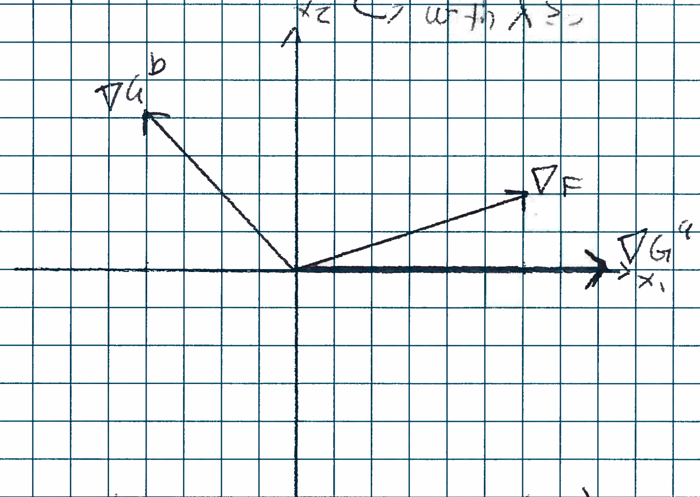
$$\nabla F = \sum_{j=1}^{m} \lambda_j \nabla G^j, \quad \lambda_j \geq 0 \ \forall j = 1, \dots, m$$

because we have a $\leq$ restriction in a max problem.

- ~~Since we have~~ only now with

- Also note that we don't have non-negativity restrictions.

- So for the FOC to be satisfied, we need that ∇F is linearly dependent on the active restrictions, with $\lambda \geq 0$



- So we have $\begin{pmatrix}3\\1\end{pmatrix} = \lambda_1 \begin{pmatrix}4\\0\end{pmatrix} + \lambda_2 \begin{pmatrix}-2\\2\end{pmatrix}$.

- Using row reduction, I find that $\begin{pmatrix}\lambda_1\\\lambda_2\end{pmatrix} \begin{pmatrix}1\\\frac{1}{2}\end{pmatrix} \gg 0$

- $\begin{pmatrix}3\\1\end{pmatrix} = 1\begin{pmatrix}4\\0\end{pmatrix} + \frac{1}{2}\begin{pmatrix}-2\\2\end{pmatrix}$

- $\begin{pmatrix}3\\1\end{pmatrix} = \begin{pmatrix}3\\1\end{pmatrix}$

- The equality holds, so x^* satisfies the necessary FOCs.

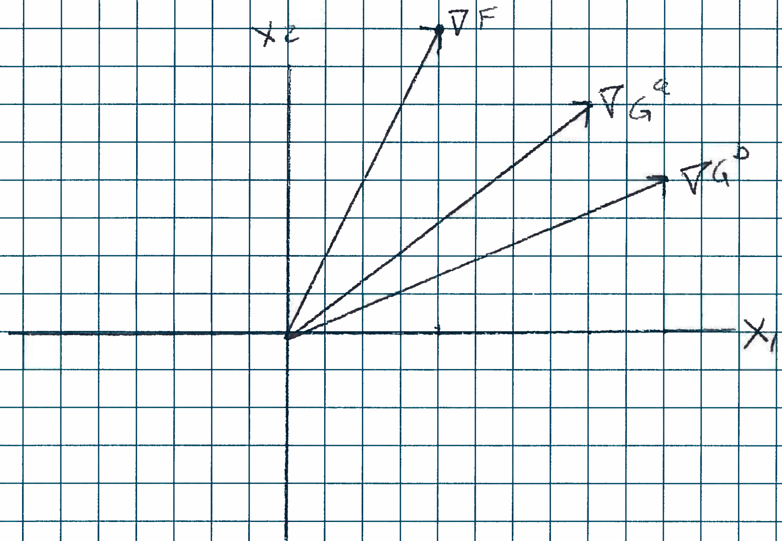
So the statement is true.

Exercise 1.

- Again, we use the requirement that

$$\nabla F = \sum_{j=1}^{m} \lambda_j \nabla G^j \;, \quad \lambda_j \geq 0 \;\; \forall j = 1, \dots, m$$

- Illustration:



- It is clear from the illustration that ∇F is linearly dependent on ∇G^a and ∇G^b but ~~only~~ iff one of the λ's are negative because $\nexists$ ~~combination~~ there doesn't exist any positive combination of ∇G^a and ∇G^b that can give ∇F. So the statement must be false.
(then ∇F should lie in between ∇G^a and ∇G^b)

- ~~O~~ Proof using our conditions, we have that

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 16/7 \\ -10/7 \end{pmatrix}.$$ $\lambda_2 < 0$, so $\bar{x}$ does not satisfy our FOC conditions.

$\therefore$ the statement is false.

Exercise 1d

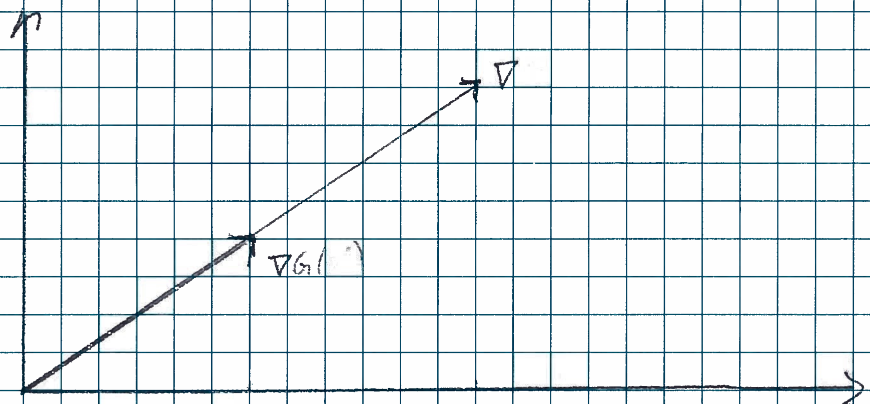
- First of all, I want to find λ^*.

We know that $\nabla F(x^*) = \lambda^* \nabla G(x^*)$,

$$\Rightarrow \begin{pmatrix} 6 \\ 4 \end{pmatrix} = \lambda^* \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$

- Thus, $\lambda^* = 2$.

- Illustration.



- Subquestion 1d ii)

- ~~Since the question~~

- $H^F = \begin{pmatrix} 12 & 4 \\ 4 & 8 \end{pmatrix}$. This is clearly PD. Proof:

– The determinant of this at $k = 1$ is $\det(12) = 2 > 0$ ✓

– The determinant of this at $k = 2$ is $\det\begin{pmatrix} 12 & 4 \\ 4 & 8 \end{pmatrix} = 80 > 0$ ✓

$\therefore$ the quadratic form of the objective ~~value~~ function is PD.

Subquestion 1d ii)

- Since we are now examining the problem itself around x^*, we must evaluate the bordered Hessian
- that is, we need to evaluate $B = \begin{pmatrix} 0 & \nabla G^T \\ \nabla G & H_{\mathcal{L}} \end{pmatrix}$ in which

$$H_{\mathcal{L}} = H_F - \lambda H_G$$

$\uparrow$ Lagrange

- Plugging in H_F, $\lambda = 2$, and H_G, we get

$$H_{\mathcal{L}} = \begin{pmatrix} 12 & 4 \\ 4 & 8 \end{pmatrix} - 2\begin{pmatrix} 16 & 2 \\ 2 & 1 \end{pmatrix}$$

- Which gives

$$H_{\mathcal{L}} = \begin{pmatrix} -2 & 0 \\ 0 & -16 \end{pmatrix}$$

- Plugging in $H_{\mathcal{L}}$ and $\nabla G(x^*)$, we get

$$B = \begin{pmatrix} 0 & 3 & 2 \\ 3 & -20 & 0 \\ 2 & 0 & -16 \end{pmatrix}$$

- We then evaluate for $k = \underline{2}, 3, \ldots, n$, with $k = 2$ representing a 3×3 matrix (which is all of the matrix B).
- $k = 2$ gives $\det\begin{pmatrix} 0 & 3 & 2 \\ 3 & -20 & 0 \\ 2 & 0 & -16 \end{pmatrix} = 224$
- Test for ND gives $(-1)^{2 \leftarrow k} \cdot 224 = 224 > 0$ ✓

- test of PD gives $(-1)\cdot 2\cdot 4 = -4 \not> 0$.

- Since it looks like it is ND, we have a strict local maximum for problem $P_{\frac{3}{3}}$ in x^*.

- The statement is true.

Subquestion 1d. iii)

- The statement is false.
- The value function clearly shows that if you increase the right-hand side of the constraint, the increase in the objective value equal the shadow price λ.
- This is easily shown multiple ways. For example taking the derivative and solving this will ~~display~~ prove that $V'(\cdot; c) = \lambda$, with V being the value function. Another way is deriving the Lagrange, which would give $\frac{\partial \mathcal{L}}{\partial c} = 0 + \frac{\partial}{\partial c}\left[\lambda \cdot (c - G(x))\right] = \lambda$.
- We know that $\lambda = 2$. Since there are only one
- constraint, meaning we won't have issues that another constraint is binding and $G(\cdot) = c$, we know for certain that a marginal increase of c will increase the value of F with $\lambda = 2$.
- The statement is false.

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Kandidatens nr/Candidate number 401006

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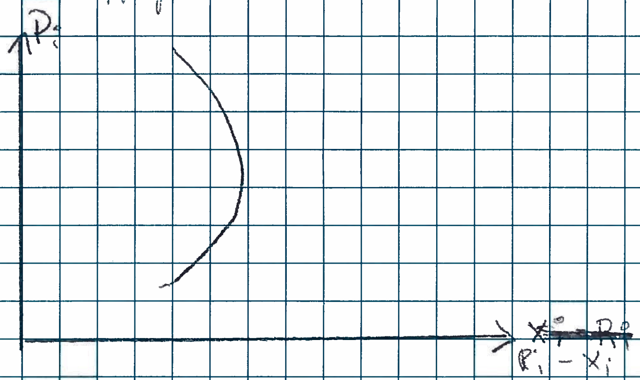
Question 2c

- The supply curve can be illustrated as follows:



- If a household faces an increased price of good 1, it may be inclined to supply more of good 1 to the market because it can buy relatively more of good 2 from supplying one unit more of good 1 (i.e. financing the purchase of good 2) than it could under the initial prices.

- However, a ~~sup~~ person prefers a mixed basket ~~sep~~ of goods instead of a specialized one, so as the price of good 1 continues to increase, supplying more of good 1 (i.e. consuming less of good 1) and consuming more of good 2 makes the basket more specialized in favor of good 2

- This means that we, at higher price levels want to supply less of good 1 and consume more of it ourselves.

- However, P_1 is now so high that we can still consume high levels of good 2.

- a great example is labor supply.
  you have some time constraint,
  and you work more when wages go up (more supply)
  However, if you work 23 hours per day,
  and wages go up you're thinking you
  have enough money and therefore spend
  more time relaxing. However, if (i.e. reducing supply)
  you're earning the bare minimum to
  survive, this is not the case.

- This effect can easily be proven with
  the slutsky equation.

Norges Handelshøyskole / Norwegian School of Economics

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Kandidatens nr/Candidate number ....

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Side/Page ....

Question 2b

- If the MRS is equal to the MRS of another utility function for all x_1, x_2, then the utility functions describe the same preferences over bundle (x_1, x_2).

Original utility function.

- $$MRS_{12} = \frac{-U_2(x_1, x_2)}{U_1(x_1, x_2)}$$

Plugging in for MRS

$$MRS_{12} = \frac{-2(x_1 x_2 + [illegible]) \cdot x_1}{2(x_1 x_2 + [illegible]) \cdot x_2}$$

$$MRS_{12} = \frac{-[illegible]}{x_2}$$

- The utility function $U^{A'}(x_1, x_2) = U^{B'}(x_1, x_2) = x_1 \cdot x_2$ has the same MRS:

$$MRS_{12} = \frac{-U_2(x_1, x_2)}{U_1(x_1, x_2)} = \frac{-x_1}{x_2}$$

∴ a simple utility function that describes the same preferences over bundle is $U^{A'}(\bar{x}) = U^{B'}(\bar{x}) = x_1 x_2$

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Total sum pages (write on the first page)

Kandidatens nr/Candidate number 401006

Side/Page 8

Question 7c

- We must first solve the utility maximization problem since $V(\vec{p}, y) = U^*(x_1, \dots)$
- I will use the simpler utility function from 7b.
- We have ~~that~~ $\max x_1 x_2$ s.t. $p_1 x_1 + p_2 x_2 \leq y$

- $\mathcal{L} = x_1 x_2 + \lambda(y - p_1 x_1 - p_2 x_2)$

- Note that for $y, p_1, p_2 \in \langle 0, \infty \rangle$, either or both of $x_i = 0$ would obviously not solve the maximization problem since $U = \prod_{i=1}^{2} x_i$. So $x_i > 0$, $i = 1, 2$.

- Furthermore, we don't have a bliss point here, so increasing y would increase U. ~~(...)~~ Therefore, $\lambda > 0$.

- That gives us the following FOCs (since $x_1, x_2, \lambda > 0$)

$$\frac{\partial \mathcal{L}}{\partial x_1} = 0: \quad x_2 - \lambda p_1 = 0$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = 0: \quad x_1 - \lambda p_2 = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0: \quad y - p_1 x_1 - p_2 x_2 = 0$$

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- We can rearrange and get

$$\cancel{\lambda}\ \lambda = \frac{X_2}{P_1} = \frac{X_1}{\ } \quad \Big| \ P_1 = P_2$$

$$2X = P_1 X_1$$

- Plugging into budget constraint gives

$$y = 2P_1X_1 \quad \text{and} \quad y = P_2X_2$$

$$\underline{\underline{X_1^* = \frac{y}{2P_1} \quad \text{and} \quad X_2^* = \frac{y}{2P_2}}}$$

[redacted]

- $V(\vec{P}, y) = U^*(\bar{x}) \quad \Big| \ U(\ ) = X_1^* X_2^*$ is ~~s~~ something

$$V(\vec{P}, y) = \frac{y}{2P_1} \cdot \frac{\ }{2P}$$

$$\underline{V(\vec{p}, y) = \frac{y^2}{4P_1P_2}}$$

- Since $U^A(\vec{x}^*) = U^B(\vec{x}^*) = X_1 X_2$, we have that

$$V(\vec{P}, y) = V^A(\ , y) = V^B(\vec{P}, y).$$

$$\therefore \ \underline{V^A(\vec{P}, y) = \frac{(y^A)^2}{4P_1P_2}}$$

0/0

- $$\frac{\partial V^A(p, y^A)}{\partial p_1} = \frac{\partial}{\partial p_1}\left[(y^A)^2 \cdot (4p_1p_2)^{-1}\right] = \frac{-(y^A)^2 \cdot 4p_2}{(4p_1p_2)^2}$$

$$\frac{\partial V^A(p, y^A)}{\partial p_2} = \frac{\partial}{\partial p_2}\left[(y^A)^2 (4p_1p_2)^{-1}\right] = \frac{-(y^A)^2 \cdot 4p_1}{(4p_1p_2)^2}$$

$$\frac{\partial V^A(p, y^A)}{\partial y^A} = \frac{\partial}{\partial y^A}\left[(y^A)^2 \cdot \frac{1}{4p_1p_2}\right] = \frac{2y^A}{4p_1p_2} = \frac{y^A}{2p_1p_2}$$

- Checking whether Roy's identity holds

$$x_i^{*A} = \frac{-V_i^A(p, y)}{V_{y^A}^A(p, y^A)}$$

- substituting gives

good 1:

$$\frac{y}{2p_1} = \frac{-\left(\frac{-(y^A)^2 4p_2}{(4p_1p_2)^2}\right)}{\left(\frac{2y^A}{4p_1p_2}\right)}$$

$$\frac{y}{2p_1} = \frac{1}{2}\frac{(y^A)^2}{y^A} \cdot \frac{4p_2 \cdot 4p_1p_2}{(4p_1p_2)^2}$$

$$\frac{y}{2p_1} = \frac{y}{2} \cdot \frac{1}{p_1}$$

$$\frac{y}{2p_1} = \frac{y}{2p_1}$$ holds for good 1.

good 2:

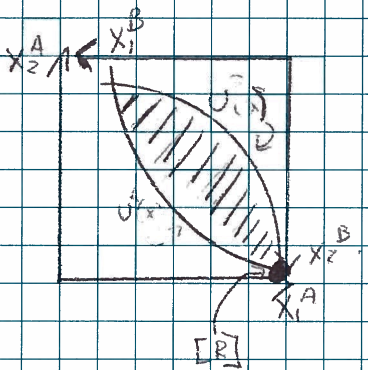
$$\frac{y}{2p_2} = \frac{-\left(\frac{-(y^A)^2 4p_1}{(4p_1p_2)^2}\right)}{\frac{2y^A}{4p_1p_2}}$$

identical calculations as with good 1 (only indices changed)

$$\frac{y^A}{2p_2} = \frac{y}{2p_2}$$ so Roy's identity holds

Question 2d

- Let's illustrate this with an Edgeworth box.



- the endowment ~~is~~ lies in the lower right corner
- the indifference curve that achieves the same utility as the endowment is also illustrated.

- The core is the shaded area between the indifference curves.
- the meaning with the core is to show for which allocations it would be meaningful for a party to trade/exchange goods.
- This means that a party can not be worse off by trading goods – i.e. can not ~~come~~ reach a lower utility level!
- therefore all allocations $(x_1^A, x_2^A, x_1^B, x_2^B)$ in the shaded region lie in the core.

- We can therefore describe the core as the set $C = \{ x \in X : U(\vec{x}^j) \geq U(\vec{e}^j) \ \forall j \in \{A, B\} \}$

(the set of all feasible allocations such that the utility level with allocation achieved by person j is higher than or equal to the ~~allocation~~ utility level achieved from not trading (i.e. consuming the endowments) $\}$, and that this must hold for ~~all~~ person A and B).

(continues)

- However, note the initial endowment is 0 of one good.
- If we assume the same utility function as in the previous subquestion this means a consumer would have achieved utility level 0 from the endowment.

~~All~~ Therefore, all(!) ~~$x \geq 0$~~ combinations of $x_1^A, x_2^A, y_1^B, y_2^B \geq 0$ would lie in the core.

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Side #15

dag /

Denne kolonnen er forbeholdt sensorene

Question 2e

good

- The excess demand functions are $E_i(p) = X_i - q_i - R_i, \forall i$ where X_i is demand for good i, and q_i and R_i is supply of good i — q_i is supply from production and R_i is endowment.

- Since we have two persons in the economy, we can rewrite the excess demand function to

$$\forall \text{ good } i.\quad E_i(p) := \sum_{j \in \{Persons\}=\{A,B\}} \left(X_i^j - q_i^j - R_i^j \right)$$

- ~~I will not consider the input here since it is used in production and not consumption.~~

- That is, u

$$\cancel{E_1(p) = X_1^A + X_1^B - R_1^A - R_1^B - q_1^A - q_1^B}$$

$$\forall i.\ E^i(p) := X_i^A(p) + X_i^B(p) - q_i^A - q_i^B - R_i^A - R_i^B$$

$\cancel{E}$

- We know $\bar{R}^A = \vec{0}$ and $R^B = (0, 0, 8)$.

Furthermore we know that $q_i^B = \vec{0}$ since

B owns the firm. And we also have q_3 and R_3 only

- So

$$\begin{cases} E_1(p) := X_1^A(p) + X_1^B(p) - q_1^A(p) \\ E_2(p) := X_2^A(p) + X_2^B(p) - q_2^A(p) \\ \cancel{E_3(p) := X_3^A(p) + X_3^B(p) - q_3^A(p)} \\ \cancel{E_3(p) = X_3(p) + X} \quad - \\ E_3(p) := -q_3^A(p) - R_3^B \end{cases}$$

Question 7f

- We know that excess demand needs to be 0.

$$q_1^*(p) = x_1^A(p) + x_1^B(p)$$

$$q_2^*(p) = x_1^A(p) + x^B(p)$$

$$q_3^* = 8$$

- We know that $-MRT_{ij} = \frac{-p_j}{p_i}$ in optimum.

With ~~p_1 and p_3~~ $i = 3$, $p_3 = 1$, we get that

$-MRT_{3j} = -p_j$ | $-MRT_{31} = \frac{-\phi_j(q)}{\phi_3(q)} = \frac{-\phi_j(q)}{1} = -\phi_j(q)$

- So we get that $-\phi_j(q) = -p_j \Leftrightarrow \phi_j(q) = p_j$, $j = 1, 2$.

$$\begin{cases} j = 1 & \frac{1}{2} q_1 = p_1 \Leftrightarrow q_1^* = 2p_1 \\ j = 2 & \frac{1}{2} q_2 = p_2 \Leftrightarrow q_2^* = 2p_2 \end{cases}$$

- We know that $\left(\frac{1}{2} q_1\right)^2 + \left(\frac{1}{2} q_2\right)^2 = 8$ in optimum.

- We see that they prefer x_1 and x_2 equally as much ($u = x_1 \cdot x_2$)

so $q_1 = q_2 = q$.

$2\left(\frac{1}{2} q\right)^2 = 8 \Leftrightarrow \left(\frac{1}{2} q\right)^2 = 4 \Leftrightarrow$ ~~$q_1 = 4$~~ $q_1 = q_2 = 4$

- We know that $q_1 = 2p_1$ and $q_1 = 4$
  $q_2 = 2p_2$ $q_2 = 4$

- That gives $\underline{p_1 = p_2 = 2}$

∴ Price vector is $\vec{p}(2, 1)$

- $y^A = 2 \cdot 1 + 2 \cdot 4 = 8$
  $y^B = 2 \cdot 1 \cdot 8 = 8$

- They have the same income and face the same prices and have the same utility functions, so they will have identical consumption of

$X_i^j = 2 \;\forall$ good i person j.