ECE/MSE 446/546 Quiz 2
SOLUTIONS Source Code: `https://github.com/christiancopic/ECE546_quiz2_sln`

Problem 1. *Derive the pulling rate in the Czochralski crystal growth process.*

Solution. Heat balance equation:

$$L * \frac{dm}{dt} + k_L * \frac{T}{x_1} * A_1 = k_s * \frac{T}{x_2} * A_2 \tag{1}$$

L = Latent heat of fusion
$\frac{dm}{dt}$ = Rate of amount of Si freezing per unit time
k_L = thermal conductivity of melt
A_1 = cross sectinal area of isotherm at x_1
$\frac{dT}{dx_1}$ = temperature gradient at isotherm x_1
k_s = thermal conductivity of solid
A_2 = cross sectional area of isotherm at x_2
$\frac{dT}{dx_2}$ = temprature gradient at isotherm x_2

The rate by which the crystal is pulled out of the melt is

$$\frac{dm}{dt} = v_p * A * N \tag{2}$$

N = density of Si
v_p = Pull rate of crystal

Put (2) into (1), with the assumption that $A_1 = A_2 = A$

$$\implies L * v_p * A * N + k_L * \frac{dT}{dx_1} * A = k_s * \frac{dT}{dx_2} * A$$

$$\implies L * v_p * A * N = k_s * \frac{dT}{dx_2} * A$$

$$\implies v_{pMAX} = \frac{k_s}{LN} * \frac{dT}{dx_2} \tag{3}$$

Heat loss due to radiation is given by Stefan-Boltzmann Law

$$dQ = (2 * \pi * r * dx) * (\sigma * \epsilon * T^4) \tag{4}$$

$2 * \pi * r * dx$ = Radiating surface area
σ = Stefan-Boltzmann constant
ϵ = Emissivity of Si

The heat conducted up the crystal is given by

$$Q = k_s * (\pi * r^2) * \frac{dT}{dx} \tag{5}$$

Differentiate (5) with respect to x

$$\implies \frac{dQ}{dx} = k_s * (\pi * r^2) * \frac{d^2T}{dx^2} + \pi * r^2 * \frac{dT}{dx} * \frac{dk_s}{dx} \cong k_s(\pi * r^2) * \frac{d^2T}{dx^2}$$

$$\implies \frac{d^2T}{dx^2} - \frac{2 * \sigma * \epsilon}{k_s * r} * T^4 = 0 \tag{6}$$

Thermal conductivity of Si below 1000 °C is given by

$$k_s = k_m * \frac{T_m}{T}$$

where k_m is the thermal conductivity of Si at melting point T_m

$$\implies \frac{d^2T}{dx^2} - \frac{2 * \sigma * \epsilon}{k_m * r * T_m} * T^5 = 0 \tag{7}$$

Solve the differential equation (7) for T. Then differentiate T with respect with x and evaluate it at x=0 to get:

$$V_{pMAX} = \frac{1}{L * N} * \sqrt{\frac{2 * \sigma * \epsilon * k_m * T_m^5}{3 * r}}$$

Problem 2. *Derive the expression for the segregation coefficient in the Czochralski crystal growth process.*

Solution. Equilibrium segregation coefficient

$$k_o = \frac{C_S}{C_L}$$

C_S = Equilibrium concentration of the dopant in the solid
C_L = Equilibrium concentration of the dopant in the liquid near interface

Doping concentration of liquid is

$$C_L = \frac{S}{M_o - M}$$

S = Amount of dopant remaining in the melt
M_o = Initial weight of grown crystal
M = Crystal weight

For incremental amount of weight dM of crystal, the corresponding reduction in dopant from the melt is

$$-dS = C_S * dM$$

Combining our equations we yield

$$\frac{dS}{S} = -k_o * \frac{dM}{M_o - M}$$

Integrate and simplify to get

$$C_S = C_o * k_o * (1 - \frac{M}{M_o})^{k_o - 1}$$

Problem 3. *List the steps involved in the fabrication of a silicon wafer for logic device applications.*

Solution.

1. Wafer production
2. Wafer slicing
3. Surface polishing
4. Oxidation
5. Photolithography
6. Etching
7. Doping
8. Thin film deposition
9. Annealing
10. Metallization
11. Chemical Mechanical polishing
12. Testing
13. Dicing
14. Packaging