ECE 441a/541a Homework 3 University of Arizona

Reading Assignments:

Shinners: Chapters 2, 3, and 4.

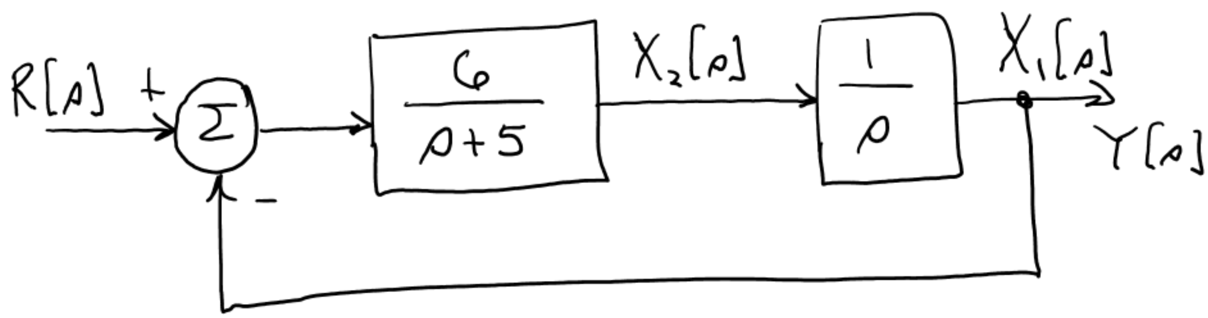
Doyle, Francis, and Tannenbaum: Chapter 3.

Please enter your answers using the D2L Quiz for Homework #3.

1. Find a state space representation, where the state vector is

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix},$$

the reference input is $r(t)$, and the output is $y(t)$.



2. Find an Input Feedforward state space representation for the given transfer function.

$$\frac{Y[s]}{U[s]} = \frac{20s + 1}{(s+1)(s+4)(s+10)}$$

3. Problem 2.67 in Shinners. (See page 179 of textbook or text below.)

2.67. A nuclear reactor has been operating in equilibrium for a long period of time at a high thermal-neutron flux level, and is suddenly shut down. At shutdown, the density of iodine 135 (I) is 5×10^{16} atoms per unit volume and that of xenon 135 (X) is 2×10^{15} atoms per unit volume. The equations of decay are given by the following state equations:

$$\dot{I}(t) = -0.1I(t), \quad \dot{X}(t) = -I(t) - 0.05X(t).$$

(a) Determine the state transition matrix using Eq. (2.256).

(b) Determine the system response equations, $I(t)$ and $X(t)$.

(c) Determine the half-life time of I 135. The unit of time for $\dot{I}$ and $\dot{X}$ is hours.

4. Problem 2.72 in Shinners. (See page 181 of textbook or text below.)

2.72. A very interesting ecological problem is that of rabbits and foxes in a controlled environment. If the number of rabbits were left alone, they would grow indefinitely until the food supply was exhausted. Representing the number of rabbits by $x_1(t)$, their growth rate is given by

$$\dot{x}_1(t) = Ax_1(t).$$

However, rabbit-eating foxes in the environment change this relationship to the following:

$$\dot{x}_1(t) = Ax_1(t) - Bx_2(t),$$

where $x_2(t)$ represents the fox population. In addition, if foxes must have rabbits to exist, then their growth rate is given by

$$\dot{x}_2(t) = -Cx_1(t) + Dx_2(t),$$

(a) Assume that $A = 1$, $B = 2$, $C = 2$, and $D = 4$. Determine the state transitions matrix for this ecological model.

(b) From the state transition matrix, determine the response of this ecological model when $x_1(0) = 100$ and $x_2(0) = 50$. Explain your results.

5. For the following systems, find the transfer function using MATLAB. Also, determine the poles and zeros of each transfer. You should be able to use some combination of the following MATLAB functions: 'ss2tf( )', 'ss( )', 'tf( )', 'pole( )', 'zero( )', and 'roots( )'.

   a. $\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -100 & 0 \\ -10 & 2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)\,,\ y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$

   b. $\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)\,,\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$

   c. $\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -4 & -2 & 0 \\ 2 & -1 & 0 \\ 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(t)\,,\ y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$

6. Problem 2.74 in Shinners. (See page 182 of textbook or text below.)

2.74. Substances $x_1(t)$ and $x_2(t)$ are involved in the reaction of a chemical process. The state equations representing this reaction are as follows:

$\dot{x}_1(t) = -8x_1(t) + 4x_2(t),$

$\dot{x}_2(t) = 4x_1(t) - 2x_2(t).$

(a) Determine the state transition matrix of this chemical process.

(b) Determine the response equations of this system, $x_1(t)$ and $x_2(t)$ when

$x_1(0) = 2,$

$x_2(0) = 1,$

(c) At what value of time will the amount of substances $x_1(t)$ and $x_2(t)$, be equal? The unit of time for $x_1(t)$ and $x_2(t)$ is hours.