Linear Regression

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June 2, 2025

Linear Regression

A statistical technique used to find the relationship between **features** and a **label**. Possible correlations: **Positive, Negative, Non-linear, No Correlation**.

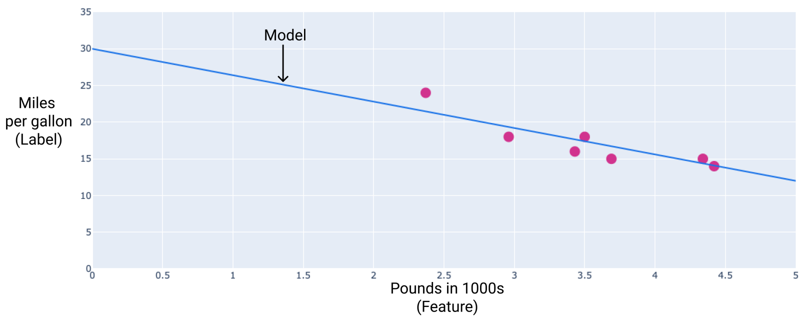

Figure 1: Linear Regression Plot

Linear Regression Equation

$$\hat{y} = b + w_1 x_1 \tag{1}$$

where:
$\hat{y}$: predicted label (output)
b: bias/parameter is the same concept as the y-intercept in algebra
w_1: weight/parameter is the same concept as slope m in algebra
x_1: feature (input)

During training, the model calculates the w_1 and b that produce the best model.

Models with Multiple Features

$$\hat{y} = b + w_1 x_1 + w_2 x_2 + \cdots + w_n x_n \tag{2}$$

$x_1, \ldots, x_5 = \{$Pounds, Displacement, Acceleration, Number of Cylinders, Horsepower$\}$

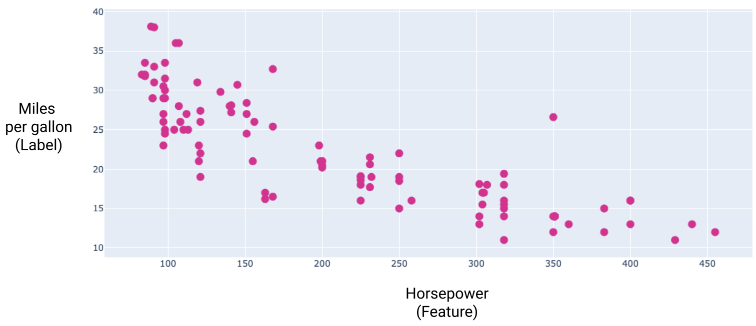

Figure 2: Horsepower Plot

Loss Function

A metric that describes how wrong a model's predictions are. It measures the distance between the model's predictions and the actual values. The objective is to minimize the loss, making it to its lowest possible value.

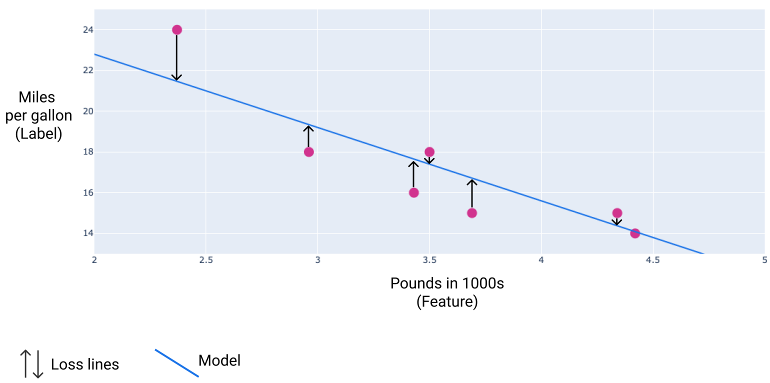

Figure 3: Loss Lines

Distance of Loss

Loss focuses on the distance, not the value. If the difference is a negative value, we need to remove the sign.
Common methods to remove the sign:

- Get the absolute value of the difference of errors
- Square the difference of errors

Error

The difference between the actual and predicted values

$$error = y - \hat{y} \tag{3}$$

Types of Loss

Loss Type	Definition	Equation (y)
L_1 loss	Sum of the absolute values of the errors	$\sum \lvert y - \hat{y} \rvert$
L_2 loss	Sum of the squared difference of the errors	$\sum (y - \hat{y})^2$
Mean Absolute Error	Average of L_1 losses across N	$\frac{1}{N} \sum \lvert y - \hat{y} \rvert$
Mean Squared Error	Average of L_2 losses across N	$\frac{1}{N} \sum (y - \hat{y})^2$

It is recommended to use MAE or MSE.

Gradient Descent

Hyperparameters

References

- Linear Regression - Google Developers
- Machine Learning Glossary - Google Developers