Planar Quads in free-form Surfaces

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Abstract

This chapter will discuss briefly the state of the art related to the Planar Quads Meshes, and how they are implemented in architecture. Therefore we will show the solutions behind the available softwares and the different techniques related to variantes researches achieved within this topic in the last twenty years. In addition to that we will try to cover all the techniques, by implementing them using the available tools.

1 Introduction

- Planar Quads(Definition)
- Statics-Sensitive Layout(Definition)
- Exhibit Properties
- Asthetical Properties

2 Background

- Different techniques:
- Related works(State of the art)

3 Case Study

3.1 Architectural Implmentations (Real Architecture Projects)

- Railway station B.Schneider (Liu et al. 2006) - Conical meshes.
- Roof over the Bosch Areal courtyards (Glymph et al. 2004) - Grid dome as translational surface with planar mesh.

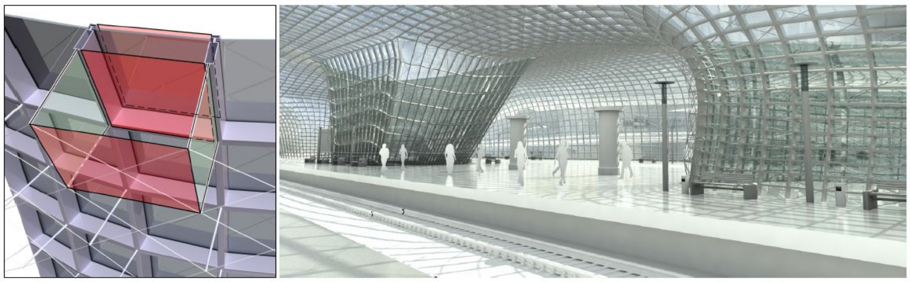

Figure 1: Railway station

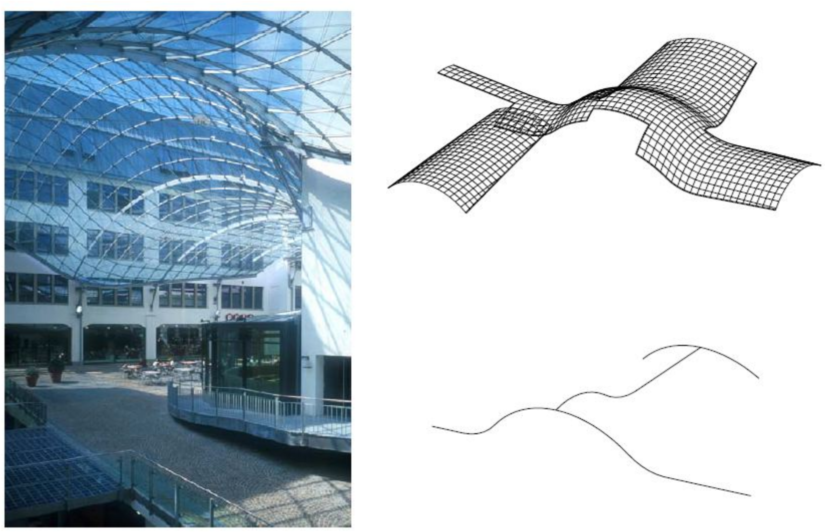

Figure 2: Roof over the Bosch courtyards

3.2 Software Solutions

- Evolute
- Kangaroo2

3.3 Techniques

3.3.1 Free-Form Surfaces

3.3.1.1 PQ Meshes as Conical Meshes

3.3.1.1.1 What are conical meshes in theory ?

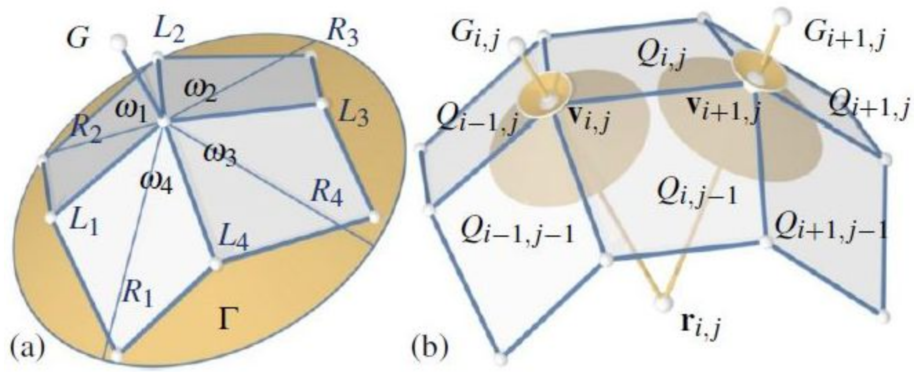

Figure 3: Faces Configuration of a Conical Ceshes

3.3.1.1.2 The relation between PQ meshes and conjugate networks:

Conjugate Surface Tangents as seen in fig. 5

Various Conjugate Networks on Surfaces as seen in fig. 6

3.3.1.1.3 PQ Perturbation:

- Quad mesh as input with vertices $\mathbf{v}_{i,j}$
- minimally perturb the vertices into a new positions while maintaining the PQ meshes properties.
- Penalty Linear fonctunality combination:

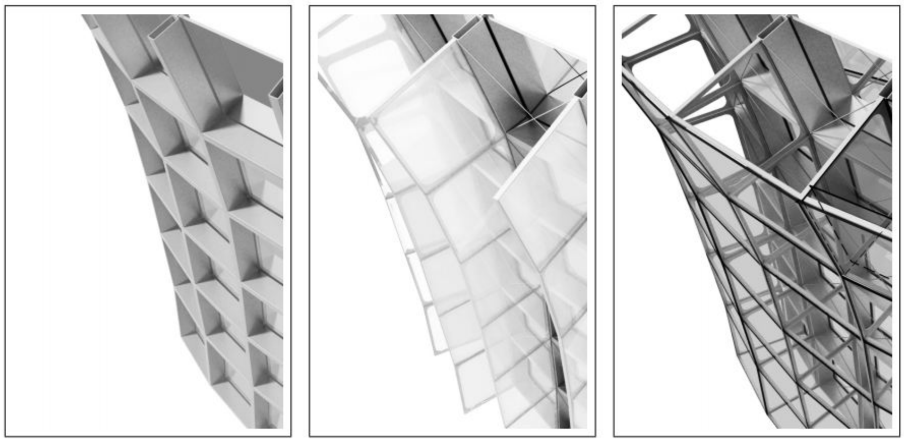

Figure 4: Constant offset of a Conical Mesh

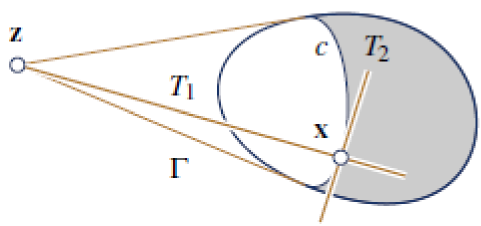

Figure 5: Conjugacy Via Shadow Contours

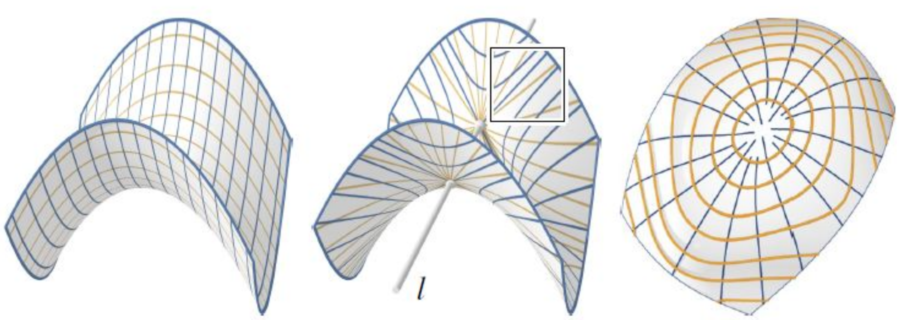

Figure 6: Various Conjugate Networks

1. $Q_{i,j}$ is convex if and only if the sum of angles is equals to 2π:

$$c_{pq} = {\alpha^1}_{i,j} + ... + {\alpha^4}_{i,j} - 2\pi = 0. \tag{1}$$

   Another Planarity Constraints on all the $\mathbf{v}i, j$, while computing the unit vectors along the egdes of each face consider the determinant on each edge equals to zero: ${\lambda^T}_{det} c_{det} = 0$.

2. Two energies terms:

   - Fairness (which simplifies the bending energies in the rows and columns on each polygon of the mesh):

$$f_{fair} = \sum_{i,j} [(\mathbf{v}_{i+1,j} - 2\mathbf{v}_{i,j} + \mathbf{v}_{i-1j})^2 + (\mathbf{v}_{i,j+1} - 2\mathbf{v}_{i,j} + \mathbf{v}_{i,j-1})^2].$$

   - Closeness:

     While the polygons are defined as squares, this fonction minimizes the distance between the original surface Φ and the vertices $\mathbf{v}i, j$ of the perturbed mesh, where $\mathbf{y}i, j$. are the closest points to the mesh, otherwise the undifined squares are set to zero :

$$f_{close} = \sum_{i,j} ||\mathbf{v}_{i,j} - \mathbf{y}_{i,j}||^2.$$

3. SQP(Sequential Quadratic Programming)

   As subject to the constraints above the Langrangian functions is written as follow:

$$f_{PQ} = w_1 f_{fair} + w_2 f_{close} + {\lambda^T}_{pq} c_{pq} + {\lambda^T}_{det} c_{det} \tag{2}$$

   The SQP minimizes the fairness and closeness subject to the planarity constraints of f_{PQ}. While w_1 and w_2 are user spicified constants to control fairness and closeness.

   SQP works only for up to 1000 vertices per mesh

4. Therefore another function by combining the constraints in {eq. 1} by summing up the angles on all the polygons. In addition to that a final function is added to minimize the objectives in {eq. 2}.

3.3.1.1.4 Subdiving PQ meshes:

Using a quad based subdivision algorithm such as Catmull-Clark and Doo-Sabin. after subdividing the corse mesh, another PQ perturbation is made as seen in fig. 7

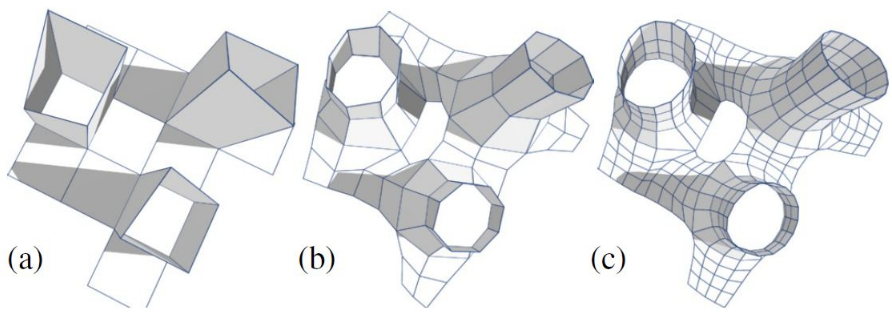

Figure 7: Corse Mesh Subdivision and PQ perturbation sequence

3.3.1.2 PQ Meshes by Conjugate Direction Fields (CDF)(Liu et al. 2011)

The main study of a CDF is to define a correct smoothness measure to allow $\pm k/4$. The study of conugate direction fields allow the presence of singularities with indices $\pm k/4(k \in Z)$

3.3.1.2.1 CDF on Triangle Meshes

- What is a CDF on a triangular face f_i? as seen in fig. 8

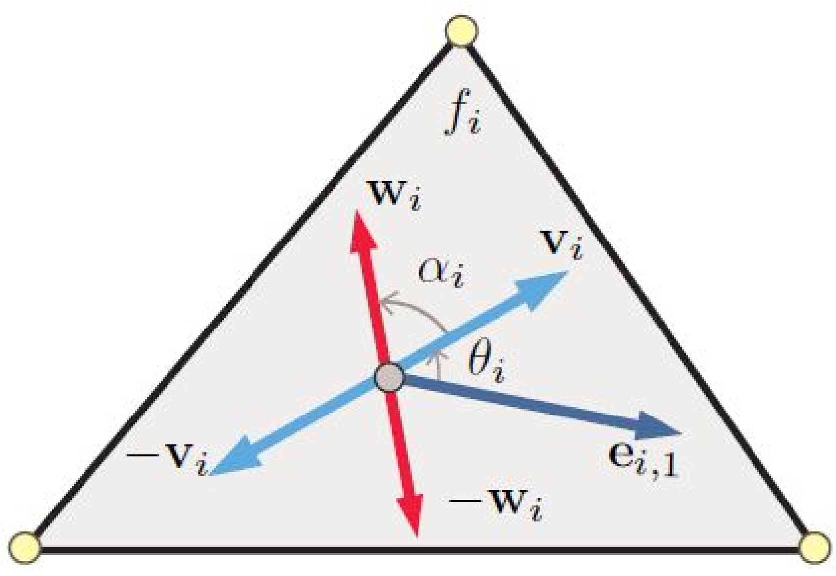

Figure 8: CDF on a Triangular face

Smooth Surface $S \subset \mathbb{R}^3$,

v_p, w_p are conjugate if and only if they are treated as two vectors in $\mathbb{R}^3$, $e_{i,1}$ is the principal direction.

$\{v_i, w_i, -v_i, -w_i\}$ four vectors that can be parametrized by $\{\theta, \alpha\}$ where: θ_i is the oriented angle between $e_{i,1}$ and v_i and α_i is the oriented angle between $ei_{,1}$ and v_i. Therefore:

$$v_i = (cos\theta_i, sin\theta_i)^T$$

$$w_i = (cos(\theta_i + \alpha_i), sin(\theta_i + \alpha_i))^T$$

- Smoothness of a CDF
  - The smoothness is computed at each edge e_{ij} incident to two triangles faces f_i, f_j
  - The angle difference is computed between the associated direction vectors which are called discrete connection to measure the change of the conjugate directions. In addition to that two angle differences need to be computed since at each face f there are two directions.

$$C_1(e_{ij}), C_1(e_{ij}).$$

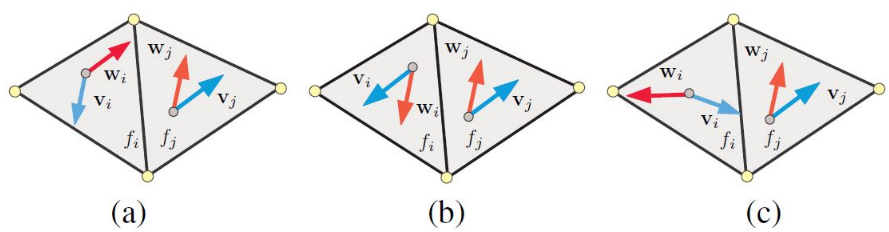

Figure 9: difference between smoothed and none smoothed CDF

- Index of singularities

3.3.1.2.2 CDF Optimization

3.3.1.2.3 PQ Mesh Generation

3.3.1.2.4 CDF Optimization

3.3.1.3 PQ Meshes by Level Set Method (Zadravec, Schiftner, and Wallner 2010)

Allows the presence of singularities with indices $\pm k/2 (k \in Z)$

3.3.1.4 PQ Meshes by statics-sensitive layout (Schiftner and Balzer 2010)

3.3.1.4.1 Quad dominant mesh by a stress directional field

3.3.2 Translational Surfaces

References

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