

CSE 015: Discrete Mathematics  
Fall 2020  
Homework #8  
Solution

Christian Duan  
Lab CSE-015-10L

December 14, 2020

**1. The Division Algorithm:**

(a)  $21 \operatorname{div} 4 = 5$   
 $21 \bmod 4 = 1$

$$21 = (4 \cdot 5) + 1$$
$$21 = 21$$

(b)  $13 \bmod 5 = 3$   
 $13 \operatorname{div} 5 = 2$

$$13 = (5 \cdot 2) + 3$$
$$13 = 13$$

(c)  $-12 \operatorname{div} 5 = -2$   
 $-12 \bmod 5 = -2$

$$-12 = (5 \cdot -2) + (-2)$$
$$-12 = -12$$

**2. Modular Arithmetic:  $m = 13$**

(a)  $4 +_m 11$   
 $4 +_{13} 11 = 15 \bmod 13 = 2$

(b)  $4 \cdot_m 11$   
 $4 \cdot_{13} 11 = 44 \bmod 13 = 5$

(c)  $23 +_m 54$   
 $23 +_{13} 54 = 77 \bmod 13 = 12$

$$\begin{aligned}
 \text{(d) } & 7 \cdot_m (11 +_m 6) \\
 & 7 \cdot_{13} (11 +_{13} 6) = 7 \cdot_{13} (17 \bmod 13) \\
 & 7 \cdot_{13} (11 +_{13} 6) = 7 \cdot_{13} (4) = 28 \bmod 13 = 2
 \end{aligned}$$

### 3. Trial Division for Prime Numbers:

- 683

2, 3, 5, 7, 11, 13, 17, 19, and 23 are the only primes that do not exceed  $\sqrt{683}$ .

Because 683 is not divisible by 2, 3, 5, 7, 11, 13, 17, 19, or 23, it follows that 683 **IS** a prime.

### 4. Shift Cipher:

- Original Text:  
STUDY FOR THE FINAL
- Encrypted Text:  
Z AKEGMVYG OLGMPUHS