CSE 015: Discrete Mathematics
Fall 2020
Homework #4
Solution

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Lab CSE-015-10L

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1. **Set Operations:**

   (a) $A \cup B$: There is a set of UCM students in the universal set of UCM students who are either registered in CSE015, who live in the Merced county, or are registered in CSE015 and in live in the Merced county.

   (b) $A \cap C$: There is a set of UCM students registered in CSE015 and there is a set of UCM students who are freshmen are both in the universal set of UCM students.

   (c) $C \setminus B$: There is a set of UCM students who are freshmen but who do not live in the Merced country in the universal set of UCM students.

   (d) $\overline{A}$: There is a set of UCM students in the universal set of UCM students who are not registered in CSE015.

   (e) $A \cap B \cap C$: There is a set of UCM students registered in CSE015, who live in Merced county, and who are freshmen in the universal set of UCM students.

2. **Cartesian Product:**

   (a) $C \times A$: {(True, 1), (False, 1), (True, 2), (False, 2), (True, 3), (False, 3), (True, 4), (False, 4)}

   (b) $B \times B$: {(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)}

   (c) $B \times A \times C$: {(a, 1, True), (a, 1, False), (a, 2, True), (a, 2, False), (a, 3, True), (a, 3, False), (a, 4, True), (a, 4, False), (b, 1, True), (b, 1, False), (b, 2, True), (b, 2, False), (b, 3, True), (b, 3, False), (b, 4, True), (b, 4, False), (c, 1, True), (c, 1, False), (c, 2, True), (c, 2, False), (c, 3, True), (c, 3, False), (c, 4, True), (c, 4, False)}

3. **Composite Cartesian Product:**

   (a) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

   - $A \times (B \cup C) = \{(a, b)|a \epsilon A \wedge b \epsilon B \cup C\}$
     $A \times (B \cup C) = \{(a, b)|(a, b) \epsilon A \times B \vee (a, b) \epsilon A \times C\}$
     $A \times (B \cup C) = (A \times B) \cup (A \times C)$

   $\therefore A \times (B \cup C) = (A \times B) \cup (A \times C)$ is **TRUE**.

4. **Relations:**

   (a) R_1 is symmetric and transitive. R_1 is symmetric as some elements are switching the places of the ordered pair. R_1 is also transitive as (a, b) leads to (b, a) and also leads from (b, a) to (a, a).

   (b) R_2 is transitive and antisymmetric. R_2 is transitive as the one element goes to a different element such as element a goes to element b and element b goes to element c. R_2 is also antisymmetric as all the elements are unique and are not reflexive or each other or are not symmetric elements.

   (c) R_3 is none of the former as there are repeating occurrences of elements such as (a, b).

   (d) R_4 is symmetric, antisymmetric, and transitive.R_4 is symmetric as the elements in the set switch places in the ordered pairs. R_4 is also antisymmetric as the elements are all unique and are not reflexive of each other. R_4 is also transitive as the elements lead into one another; such as element a to element a and element a to element a.

5. **Functions:**

   (a) $f(m, n) = 2m - n$ is surjective. This function is surjective because an ordered pair will have an image to correspond to the first variable. Such as (0, x), if these variables were to be substituted, there would be an image of $-x$. Therefore, $f(m, n) = 2m - n$ is **Surjective**.

   (b) $f(m, n) = m^2 - n^2$ is not surjective. For a function to be surjective, for every value of B, there must be an A value corresponding to it. The function $m^2 - n^2$ cannot output 2 as the difference of perfect squares. Therefore, $f(m, n) = m^2 - n^2$ is **NOT SURJECTIVE**.

   (c) $f(m, n) = |m| - |n|$ is surjective. This function is surjective as there is an image for both nonnegative integers and negative integers. If there are integers inputted into the function, there would be an image that correlates to the integer substituted in the function. Therefore, $f(m, n) = |m| - |n|$ is **SURJECTIVE**.

   (d) $f(m, n) = m^2 - 4$ is not surjective. For a function to be surjective, for every value of B, there must be an A value corresponding to it. The function $m^2 - 4$ cannot output any integers that are smaller than -4. Therefore, $f(m, n) = m^2 - 4$ is **NOT SURJECTIVE**.