



# CSE 15: Discrete Mathematics

## Homework 3

Fall 2020

### Preliminary Notes

- **This homework must be solved individually.** You can discuss your ideas with others, but when you prepare your solution you must work individually. Your submission must be yours and yours only. No exceptions, and be reminded that the CSE academic honesty policy discussed in class will be enforced.
- Your solution must be exclusively submitted via CatCourses. Pay attention to the posted deadline because **the system automatically stops accepting submissions when the deadline passes. Late submissions will receive a 0.** You only need to submit the PDF and you have to use the template file provided in CatCourses. Please note that the system does not allow to submit any other file format. Do not submit the  $\text{\LaTeX}$ source of your solution.
- As you are still learning  $\text{\LaTeX}$ , it is understandable that you do not yet fully master all details. Focus your attention on determining the correct answers. **Note that if in your  $\text{\LaTeX}$ submission you embed screenshots or scans of your handwritten solution those will not be graded.** You are encouraged to collaborate with other students to determine how to best format your submission or improve your  $\text{\LaTeX}$ skills.
- Start early.

## 1 Rules of Inference

What is the argument form for the following argument? If we assume the premises are true, is the conclusion true?

$$\begin{array}{l} \text{If Jane does not fly, then she is not a bird.} \\ \text{Jane is a bird.} \\ \hline \therefore \text{Jane flies.} \end{array}$$

Suggestion: start by rewriting the premises using propositions and logical symbols and introducing appropriate propositional variables.

## 2 More Rules of Inference

For each of the following arguments, write which rule of inference is used.

- Bats can fly and are mammals. Therefore bats are mammals.
- Pigs are mammals or birds. Pigs are not birds. Therefore pigs are mammals.

- c) Jack is a CSE major. Jack is a freshmen. Therefore Jack is a CSE major and a freshmen.
- d) Mary is a CSE major. Therefore Mary is a CSE major or Mary is a History major.
- e) If I go hiking, I will sweat a lot. If I sweat a lot, I will lose weight. Therefore, if I go hiking, I will lose weight.

### 3 Checking arguments

For each of the following arguments, state if they are correct or not and explain why. Just stating Correct/Not Correct will not give you any point, even if your answer is correct. Assume that all premises (anything stated before “Therefore”) are valid.

- a) If it is sunny, then I will go swimming. It is not sunny. Therefore I will not go swimming.
- b) If it is Sunday, then I will go to the park. I will not go to the park. Therefore it is not Sunday.
- c) I will pass the class if and only if I score at least 60% on the final exam. I scored 55% on the final exam. Therefore I will not pass the class.

### 4 Proofs by Contraposition

During one past lecture we have proved the following theorem:

if  $n$  is an integer and  $n^2$  is odd, then  $n$  is odd.

If you recall, we first tried a direct proof, but it did not work out. Therefore, we tried a proof by contraposition, and that worked. Rewrite the above theorem using the contraposition formula we used to prove the theorem. Note: you just have to rewrite the theorem statement, you do not have to prove it.

### 5 Proof by Cases

A type of proof we have not used yet is the *proof by cases*. If we want to prove that  $p \rightarrow q$ , it is at times convenient to rewrite  $p$  as a disjunction of simpler propositions, i.e.,  $p = p_1 \vee p_2 \vee \dots \vee p_n$  and then show that  $(p_1 \vee p_2 \vee \dots \vee p_n) \rightarrow q$ . This last implication, although apparently more complex, is often simpler because of the following tautology:

$$[(p_1 \vee p_2 \vee \dots \vee p_n) \rightarrow q] \leftrightarrow [(p_1 \rightarrow q) \wedge (p_2 \rightarrow q) \wedge \dots \wedge (p_n \rightarrow q)]$$

Here, each of the  $p_i \rightarrow q$  is typically simpler to prove than  $p \rightarrow q$ . Prove the relationship above is a tautology (to keep things simple, let  $n = 3$ , although the result holds for an arbitrary  $n$ .)