CSE 015: Discrete Mathematics
Fall 2020
Homework #3
Solution

Christian Duan
Lab CSE-015-10L

September 30, 2020

1. **Question 1:**

   (a) Modus Tollens: $(\neg q \land (p \to q)) \to \neg p$
   The conclusion is TRUE.

2. **Question 2:**

   (a) Simplification: $(p \land q) \to p$

   (b) Disjunctive Syllogism: $((p \lor q) \land \neg p) \to q$

   (c) Conjunction: $((p) \land (q)) \to (p \land q)$

   (d) Addition: $p \to (p \lor q)$

   (e) Hypothetical Syllogism: $((p \to q) \land (q \to r)) \to (p \to r)$

3. **Question 3:**

   (a) This argument is not correct in this case as there is no rule of inference that shows the argument is valid. If "if it is sunny" is set as p and "I will go swimming" is set as q, there is no rule where there exists a $\neg p$, or "it is not sunny". Therefore the argument is **NOT CORRECT**.

   (b) This argument is correct in this case as we can use Modus Tollens to verify if the argument is valid or not, where "I will not go to the park" is $\neg q$ and "if it is Sunday" is p. When using Modus Tollens, it results in $\neg p$, or "it is not Sunday". Therefore, the argument is **CORRECT**.

   (c) This argument is correct in this case as we can use Modus Tollens to verfiy if the argument is valid or not, where "I will pass the class" is set as p, "if I score at least 60 percent on the final exam" is set as q, and "I scored 55 percent on the final exam" is set as $\neg q$. By using Modus Tollens, the result is $\neg p$, or "I will not pass the class". Therefore, the argument is **CORRECT**

4. **Question 4:**

   (a) $p : n$ is an integer and n^2 is odd
   $q : n$ is odd
   $\neg p : n$ is an integer and n^2 is even
   $\neg q : n$ is even

   $p \to q \equiv \neg q \to \neg p$

   if n is even, then n is an integer and n^2 is even.

5. **Question 5:**

p_1	p_2	p_3	q	$(p_1 \vee p_2 \vee p_3) \to q$
F	F	F	F	T
F	F	F	T	T
F	F	T	F	F
F	F	T	T	T
F	T	F	F	F
F	T	F	T	T
F	T	T	F	F
F	T	T	T	T
T	F	F	F	F
T	F	F	T	T
T	F	T	F	F
T	F	T	T	T
T	T	F	F	F
T	T	F	T	T
T	T	T	F	F
T	T	T	T	T

p_1	p_2	p_3	q	$p_1 \to q$	$p_2 \to q$	$p_3 \to q$	$(p_1 \to q) \wedge (p_2 \to q) \wedge (p_3 \to q)$
F	F	F	F	T	T	T	T
F	F	F	T	T	T	T	T
F	F	T	F	T	T	F	F
F	F	T	T	T	F	T	T
F	T	F	F	T	T	T	F
F	T	F	T	T	F	T	T
F	T	T	F	T	T	F	F
F	T	T	T	T	T	T	T
T	F	F	F	F	T	T	F
T	F	F	T	T	T	T	T
T	F	T	F	F	T	F	F
T	F	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	T	F	T	T	T	T	T
T	T	T	F	F	F	F	F
T	T	T	T	T	T	T	T

$[(p_1 \vee p_2 \vee p_3) \rightarrow q] \rightarrow [(p_1 \rightarrow q) \wedge (p_2 \rightarrow q) \wedge (p_3 \rightarrow q)]$
T
T
T
T
T
T
T
T
T
T
T
T
T
T
T
T

- By the use of truth tables, the two expressions have exactly the same end results. Therefore, the expression $[p_1 \vee p_2 \vee ...p_n] \leftrightarrow [(p_1 \rightarrow q) \wedge (p_2 \rightarrow q) \wedge ...(p_n \rightarrow q)]$, is a tautology.