CSE 015: Discrete Mathematics
Fall 2020
Homework #2
Solution

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Lab CSE-015-10L

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1. **Question 1:**

   (a) $P(2) : 2 < 2^3 \equiv 2 < 8$. Therefore $P(2)$ is TRUE.

   (b) $P(-1) : -1 < -1^3 \equiv -1 < 1$. Therefore $P(-1)$ is FALSE.

   (c) $\forall x P(x)$: $P(2)$ is $2 < 2^3 = 2 < 8$, therefore true. $P(1)$ is $1 < 1^3 = 1 < 1$, therefore false. So $\forall x P(x)$ is FALSE.

   (d) $\exists x P(x)$: $P(2)$ is $2 < 2^3 = 2 < 8$, therefore true. $P(1)$ is $1 < 1^3 = 1 < 1$, therefore false. So $\exists x P(x)$ is TRUE.

   (e) $\exists ! x P(x)$: $P(0)$ is $0 < 0^3 = 0 < 0$, therefore false. $2 < 2^3 = 2 < 8$, therefore true. So $\exists ! x P(x)$ is TRUE.

2. **Question 2:**

   (a) $\neg \forall x (S(x) \rightarrow M(x))$

   (b) $\forall x (S(x) \oplus M(x))$

   (c) $\exists x (S(x) \wedge \neg M(x))$

3. **Question 3:**

   (a) $\forall x(A(x) \wedge B(x))$ is NOT equivalent to $\forall x(A(x) \rightarrow B(x))$ as their truth values are not identical. Thus, the proposition is NOT logically equivalent.

   $A(x) = p$
   $B(x) = q$

p	q	$p \wedge q$	$p \rightarrow q$
F	F	F	T
F	T	F	F
T	F	F	T
T	T	T	T

4. **Question 4:**

   (a) $\exists x \forall y A(x, y)$: $\exists x \forall y A(0, 1) = (0)(1) = 0$. Since there exists a x value that makes the formula true, $\exists x \forall y A(x, y)$ is TRUE.

   (b) $\exists x \exists y B(x, y)$: $\exists x \exists y B(0, 0) = 0 + 0 = 0$. Since there exists a x value and y value that makes the formula true, $\exists x \exists y B(x, y)$ is TRUE.

   (c) $\forall x \exists y A(x, y)$: $\forall x \exists y A(2, 0) = (2)(0) = 0$. Since there exists a y value that makes the formula true, $\forall x \exists y A(x, y)$ is TRUE.

   (d) $\exists x \forall y(A(x, y) \wedge B(x, y))$: $\exists x \forall y(A(1, 0) \wedge B(1, 0)) = (1)(0) = 0$ and $1 + 0 \neq 0$. Since $A(x, y)$ is true and $B(x, y)$ is false, $\exists x \forall y(A(x, y) \wedge B(x, y))$ is FALSE.

   (e) $\exists x \exists y(A(x, y) \wedge \neg B(x, y)$: $\exists x \exists y(A(0, 1) \wedge \neg B(0, 1)) = (0)(1) = 0$ and $0 + 1 \neq 0$. Since $A(x, y)$ is true and $\neg B(x, y)$ is true, $\exists x \exists y(A(x, y) \wedge \neg B(x, y)$ is TRUE.

5. **Question 5:**

   (a) $\neg \exists x \exists y(P(x) \rightarrow Q(x)) \equiv \forall x \forall y((P(x) \wedge \neg Q(x))$

   (b) $\neg \exists y(\exists x A(x, y) \vee \forall x B(x, y)) \equiv \forall y(\forall x \neg A(x, y) \wedge \exists x \neg B(x, y))$