CSE 15: Discrete Mathematics
Homework 4
Fall 2020

Preliminary Notes

- **This homework must be solved individually.** You can discuss your ideas with others, but when you prepare your solution you must work individually. Your submission must be yours and yours only. No exceptions, and be reminded that the CSE academic honesty policy discussed in class will be enforced.

- Your solution must be exclusively submitted via CatCourses. Pay attention to the posted deadline because **the system automatically stops accepting submissions when the deadline passes. Late submissions will receive a 0**. You only need to submit the PDF and you have to use the template file provided in CatCourses. Please note that the system does not allow to submit any other file format. Do not submit the LaTeXsource of your solution.

- By now you should have become somewhat familiar with LaTeX. You still will not be penalized for poor typesetting, but it is in your own interest to prepare your submission in a way that is easy to understand, so try using the appropriate LaTeXsymbols. If you do not know how to type a certain math symbol, search on the Internet and you will quickly find the answer.[1] **If in your LaTeXsubmission you embed screenshots or scans of your handwritten solution those will not be graded.** You are encouraged to collaborate with other students to determine how to best format your submission or improve your LaTeXskills.

- Start early.

1 Set Operations

Consider the following sets and let the universe set U be the set of UCM students.

- A is the set of UCM students registered in CSE015;

- B is the set of UCM students who live in Merced county;

- C is the set of UCM students who are freshmen.

First note that the above rules give clear criteria to establish membership in the sets, even if those criteria are expressed in English and not using formulas or quantified predicates.
Write the definition of the following sets using English language sentences similar to those used to define the sets given above.

a) $A \cup B$

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[1]see https://www.caam.rice.edu/~heinken/latex/symbols.pdf for example.

b) $A \cap C$

c) $C \setminus B$

d) $\overline{A}$

e) $A \cap B \cap C$

2 Cartesian Product

Consider the following three finite sets:

- $A = \{1, 2, 3, 4\}$

- $B = \{a, b, c\}$ (where a, b, c are letters of the English alphabet)

- $C = \{True, False\}$

Write the following Cartesian products (you need to list *all* the elements in these sets)

a) $C \times A$

b) $B \times B$

c) $B \times A \times C$

3 Composite Cartesian Products

Let A, B, and C be sets. Is the following equality true or false?

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

You must provide a justification for your answer. Simply stating true/false will not give any partial credit.

4 Relations

Let $A = \{a, b, c, d\}$ where a, b, c, d are letters of the English alphabet. The following relations are defined over $A \times A$. For each relation state if it is reflexive, symmetric, anti-symmetric, transitive, or none of the former. Each answer must be justified. Answers without justification will receive no partial grade, even if correct.

a) $R_1 = \{(a, b), (a, c), (a, a), (b, a), (c, a)\}$

b) $R_2 = \{(a, b), (b, b), (b, c), (c, c), (a, c)\}$

c) $R_3 = \{(a, b), (d, c), (c, a), (c, d), (a, b)\}$

d) $R_4 = \{(a, a), (b, b), (c, c)\}$

5 Functions

Determine if $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is a surjective function if:

a) $f(m, n) = 2m - n$

b) $f(m, n) = m^2 - n^2$

c) $f(m, n) = |m| - |n|$ (here $|x|$ is the absolute value of x)

d) $f(m, n) = m^2 - 4$