CSE 15: Discrete Mathematics
Homework 2

Fall 2020

Preliminary Notes

- **This homework must be solved individually.** You can discuss your ideas with others, but when you prepare your solution you must work individually. Your submission must be yours and yours only. No exceptions, and be reminded that the CSE academic honesty policy discussed in class will be enforced.
- Your solution must be exclusively submitted via CatCourses. Pay attention to the posted deadline because **the system automatically stops accepting submissions when the deadline passes. Late submissions will receive a 0**. You only need to submit the PDF and you have to use the template file provided in CatCourses. Please note that the system does not allow to submit any other file format. Do not submit the LATEXsource of your solution.
- As you are still learning LATEX, it is understandable that you do not yet fully master all details. Focus your attention on determining the correct answers. **Note that if in your LATEXsubmission you embed screenshots or scans of your handwritten solution those will not be graded.** You are encouraged to collaborate with other students to determine how to best format your submission or improve your LATEXskills.
- Start early.

1 Quantifiers

Consider the domain of integer numbers, and let $P(x)$ be the statement $x < x^3$. For each of the following formulas determine their truth values. You must justify your answers. Simply stating true or false will give you no credit, even if your answer is correct.

a) $P(2)$

b) $P(-1)$

c) $\forall x P(x)$

d) $\exists x P(x)$

e) $\exists! x P(x)$

2 Translating English Sentences into Formulas

Consider the following two predicates:
$S(x)$: x is a student in CSE015;
$M(x)$: x plays a musical instrument.
Let the domain be the set of all people. Using the above predicates, translate the following sentences into formulas using the appropriate quantifiers and operators.

a) Not every student in CSE015 plays a musical instrument.

b) A person is either a student in CSE015 or plays a musical instrument, but not both.

c) There exists at least one student in CSE015 who does not play a musical instrument.

3 Logical Equivalence

Let $A(x)$ and $B(x)$ be two predicates defined over the same domain (non empty). Is this a valid logical equivalence or not?

$$\forall x(A(x) \land B(x)) \equiv \forall x(A(x) \rightarrow B(x))$$

You must justify your answer. Simply stating yes or no will give you no credit, even if your answer is correct.

4 Nested Quantifiers

The following two statements are defined for real numbers:
$A(x, y)$ is the statement $xy = 0$;
$B(x, y)$ is the statement $x + y = 0$.

For each of the following formulas determine their truth values. You must justify your answers. Simply stating true or false will give you no credit, even if your answer is correct.

a) $\exists x \forall y A(x, y)$

b) $\exists x \exists y B(x, y)$

c) $\forall x \exists y A(x, y)$

d) $\exists x \forall y (A(x, y) \land B(x, y))$

e) $\exists x \exists y (A(x, y) \land \neg B(x, y))$

5 Negating Formulas with Nested Quantifiers

De Morgan's laws allow to rewrite the negation of a disjunction or a conjunction in a logical equivalent formula where the negation appears in front of the atomic propositions and not in front of a compound proposition:

$$\neg(a \land b) \equiv \neg a \lor \neg b \qquad\qquad \neg(a \lor b) \equiv \neg a \land \neg b$$

We have also seen how we can use so-called *De Morgan's laws for quantifiers* to rewrite negations of sentences with quantifiers in a logically equivalent form where the negation operator does not appear in front the quantifier:

$$\neg\forall x P(x) \equiv \exists x \neg P(x) \qquad\qquad \neg\exists x P(x) \equiv \forall x \neg P(x)$$

Using these two forms of De Morgan's laws it is possible write the negation of formulas with nested quantifiers in a logically equivalent form where the negation never appears in front of a quantifier or of an expression involving logical connectives. For example,

$$\neg\forall x \exists y (P(x,y) \wedge Q(x,y)) \equiv \exists x \forall y (\neg P(x,y) \vee \neg Q(x,y))$$

Note that in the second expression the negation operator only appears in front of $P(x, y)$ and in front of $Q(x, y)$.
Using De Morgan's laws, write the negation of the following statements so that the negation never appears in front of a quantifier or of an expression involving logical connectives.

a) $\exists x \exists y (P(x) \rightarrow Q(y))$

b) $\exists y (\exists x A(x,y) \vee \forall x B(x,y))$