CSE 015: Discrete Mathematics
Fall 2020
Homework #5
Solution

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1. **Mathematical Induction 1:**

   (a) $P(n) : 1^3 + 2^3 + 3^3 + ... + n^3 = (\frac{n(n+1)}{2})^2$

   (b) Basis Step:
   $P(1) : \sum_{i=1}^{1} i = 1 = (\frac{1(1+1)}{2})^2 = 1$

   (c) Induction Hypothesis:
   Let n = k:
   $P(k) : k^3 = (\frac{k(k+1)}{2})^2$

   Let n = k + 1:
   $P(k + 1) : (k + 1)^3 = (\frac{(k+1)(k+1)+1)}{2})^2$

   (d) Inductive Step:
   $P(k) \rightarrow P(k + 1)$

   $\sum_{i=1}^{k+1} i = 1^3 + 2^3 + 3^3 + ... + k + k + 1$
   $\sum_{i=1}^{k+1} i + (k + 1) = (\frac{k(k+1)}{2})^2 + (k + 1)$
   $= (\frac{k(k+1)2(k+1)}{2})^2$
   $= (\frac{(k+1)(k+2)}{2})^2$

$\sum_{i=1}^{k+1} i + (k+1)^3 = (\frac{k(k+1)}{2})^2 + (k+1)^3$
$= \frac{k^2(k+1)^2}{4} + (k+1)^3$
$= \frac{k^2(k+1)^2+4(k+1)^3}{4}$
$= \frac{(k+1)^2(k^2+4k+4)}{4}$
$= \frac{(k+1)^2(k(k+2)+2(k+2))}{4}$
$= \frac{(k+1)^2(k+2)^2}{4}$
$= (\frac{(k+1)(k+2)}{2})^2$

$\therefore P(n)$ is true since P(k) is $(\frac{(k+1)(k+2)}{2})^2$ and P(k+1) also equates to $(\frac{(k+1)(k+2)}{2})^2$.

2. **Mathematical Induction 2:**

   (a) $P(n) : 0 + 2 + 4 + ... + (2n-2) = n^2 - n$

   Let n = 4:
   $P(4) : 0 + 2 + (2(4) - 2) = 4^4 - 4$
   $P(4) : 0 + 2 + (8 - 2) = 16 - 4$
   $P(4) : 0 + 2 + (8 - 2) = 12$

   (b) Basis Step:
   $P(n) : 0 + 2 + 4 + ... + (2n-2) = n^2 - n$
   $P(2) : 0 + 2 = 2^2 - 2$
   $P(2) : 2 = 2$

   Induction Step: $0 + 2 + 4 + ... + 2(k-1) + 2((k+1)-1) = 2k$
   $k^2 - k + 2k = k^2 + k$

   This is the same as $P(k+1) = ((k+1)^2 - (k+1) = k^2 - k$

3. **Mathematical Induction 3:**

   (a) The statement P(2) is 2 factorial is less than 2 squared.

   (b) Basis Step:
   $P(2) : 2! < 2^2$
   $P(2) : 2 < 4$

   (c) Induction Hypothesis:
   Let n = k:
   $P(k) : k! < k^k$

   Let n = k + 1:
   $P(k+1) : (k+1)! < (k+1)^{k+1}$