CSE 015: Discrete Mathematics
Fall 2020
Homework #7
Solution

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1. **Asymptotic Notation:**

   (a) $f(n) = 178n + 45 \to n$
   Since $f(n) = 178n + 45 \to n$, and n grows slower than n^2. $\therefore$ **Yes**, since $f(n)$ is $O(n^2)$.

   (b) $f(n) = n\log n + 12 \to n\log n$
   Since $f(n) = n\log n + 12 \to n\log n$, and $n\log n$ grows slower than n^2. $\therefore$ **Yes**, since $f(n)$ is $O(n^2)$.

   (c) $f(n) = 34n^2 + 34n + 34 \to n^2$
   Since $f(n) = 34n^2 + 34n + 34 \to n^2$, and n^2 is equal to n^2. $\therefore$ **Yes**, since $f(n)$ is $O(n^2)$ and (n^2) is $O(f(n))$.

   (d) $f(n) = \sqrt{n} + 2 \to \sqrt{n}$
   Since $f(n) = \sqrt{n} + 2 \to \sqrt{n}$, and $\sqrt{n}$ grows slower than n^2. $\therefore$ **Yes**, since $f(n)$ is $O(n^2)$.

   (e) $f(n) = 0.0001n^3 + 72n \to n^3$
   Since $f(n) = 0.0001n^3 + 72n \to n^3$, and n^3 grows faster than n^2, $\therefore$ **No**, since (n^2) is $O(f(n))$.

2. **Asymptotic Notation:**

   - $\log n$
   - $n \log n$
   - $n^2 \log n$
   - $\sqrt{n}$
   - n
   - n^2
   - n^4
   - 2^n
   - 3^n

3. **Asymptotic Growth:**

   - Computer A: 10^6 operations/sec
     Computer B: 10^8 operations/sec

     Time Complexities:
     $f_1(n) = 5n^2 + 34n + 12$
     $f_2(n) = 10n + 4$
     $f_3(n) = 2^n$

     Computer A

     $\frac{10^6 operations}{sec} * \frac{60 sec}{min} * \frac{60 min}{hr} = 3.6 \times 10^9$ operations/hr

     $f_1 : 5n^2 + 34n + 12 \leq 3.6 \times 10^9$
     $n = 26829.42$
     Computer A can solve an instance of a f_1 as large as 26,829 in an hour.

     $f_2 : 10n + 4n \leq 3.6 \times 10^9$
     $n = 3.6 \times 10^8$
     Computer A can solve an instance of a f_2 as large as 3.6×10^8 in an hour.

     $f_3 : 2^n \leq 3.6 \times 10^9$
     $n = 31.745349$
     Computer A can solve an instance of a f_3 as large as 32 in an hour.

Computer B

$\frac{10^8 operations}{sec} * \frac{60 sec}{min} * \frac{60 min}{hr} = 3.6 \times 10^{11}$ operations/hr

$f_1 : 5n^2 + 34n + 12 \leq 3.6 \times 10^{11}$
$n = 268324.75732$
Computer B can solve an instance of a f_1 as large as 268324 in an hour.

$f_2 : 10n + 4n \leq 3.6 \times 10^{11}$
$n = 3.6 \times 10^{10}$
Computer B can solve an instance of a f_2 as large as 3.6×10^{10} in an hour.

$f_3 : 2^n \leq 3.6 \times 10^{11}$
$n = 38.38921$
Computer B can solve an instance of a f_3 as large as 38 in an hour.