From Latent to Deep Latent Variable Models

A Study On Causal Effect Inference with CEVAE

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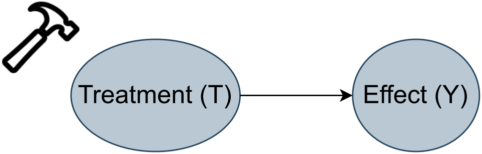
June 17, 2025

Objective: estimating causal effects

Estimate how a medical treatment T affects the health Y of a *random* patient.



$$P(Y = y | \mathrm{do}(T = t)) = P(Y = y | T = t)$$

$$\mathrm{ITE} = \mathbb{E}[(Y | \mathrm{do}(T = 1)] - \mathbb{E}[(Y | \mathrm{do}(T = 0)]$$

This is usually the case in a **randomized controlled trial** (RCT), where the treatment is randomly assigned to the patients.

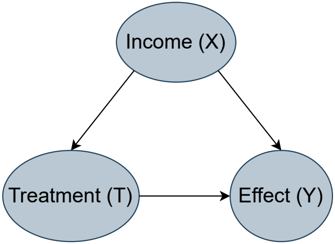
Confounder

In a observational study:

- **no** control over the treatment T assignment
  $\rightarrow$ there may be a confounder X (e.g. Income) that influences both variables

$$P(Y|\mathrm{do}(t)) \neq P(Y|t)$$

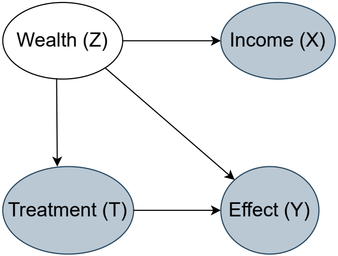
$$P(Y|\mathrm{do}(t), x) = P(Y|t, x)$$



If the confounder is observed even a linear rergession can do the job!

Latent confounder

But what if the confounder Z (e.g. Wealth) is **not** observed and the observed X is only a proxy of it?

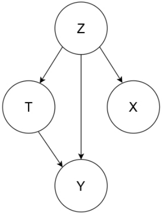


Idea: estimate the latent variable and condition on it

Vanilla Latent Variable Model

- **Generative model:**

$$Z \sim \mathcal{N}(0, 1)$$
$$X_j \sim \mathcal{N}(a\,z + b,\ \mathrm{diag}(\sigma_X^2))$$
$$T \sim \mathrm{Be}(\sigma(c\,z))$$
$$Y \sim \mathcal{N}(e\,t + f\,z,\ \sigma_Y^2)$$



- **Guide:** mean-field assumption over the observations
- **Problem:** we need to train a new model at test time!

From Latent to Deep Latent

Latent Variable Model (LVM):

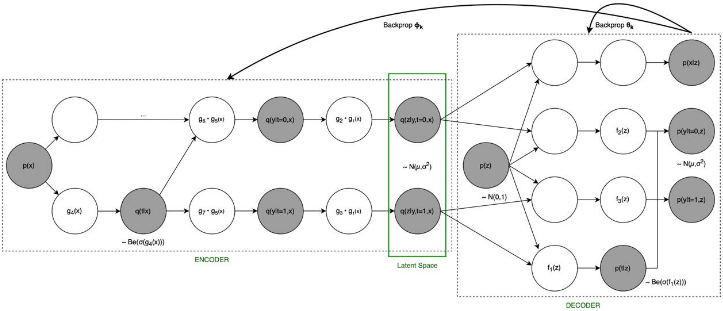
- Linear functions
- SVI + mini-SVI for new x
- Small number of parameters
- High interpretability

Deep Latent Variable Model (CEVAE):

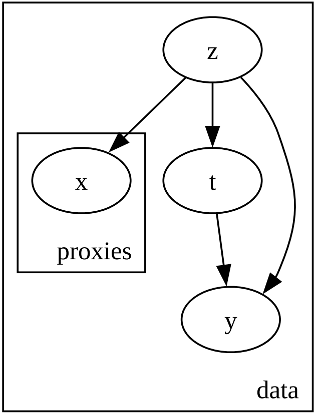
- MLP functions
- SVI + Amortized inference
- Large number of parameters
- High flexibility

CEVAE = LVM + MLPs + Amortized Inference
same causal idea, more flexibility and speed at test time.

Deep Latent Variable Model: CEVAE



Synthetic linear dataset



$$Z \sim \mathcal{N}(0,1)$$

$$a_j \sim \mathcal{U}(-10,10)$$

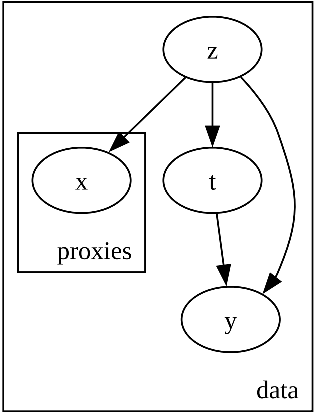
$$X_j \sim \mathcal{N}(a_j z, \sigma_X^2)$$

$$T|Z \sim \mathrm{Be}(\sigma(\beta z))$$

$$Y|T,Z \sim \mathcal{N}(z+t, \sigma_Y^2)$$

Remark: ITE is constant for all x!

Synthetic non linear dataset



$$Z \sim \mathcal{N}(0, 1)$$

$$a \sim \mathcal{U}(-10, 10)^d$$

$$X_1, ..., X_d \sim \mathcal{N}(a\,\mathsf{tanh}(z), \sigma_X^2 \mathbb{I})$$

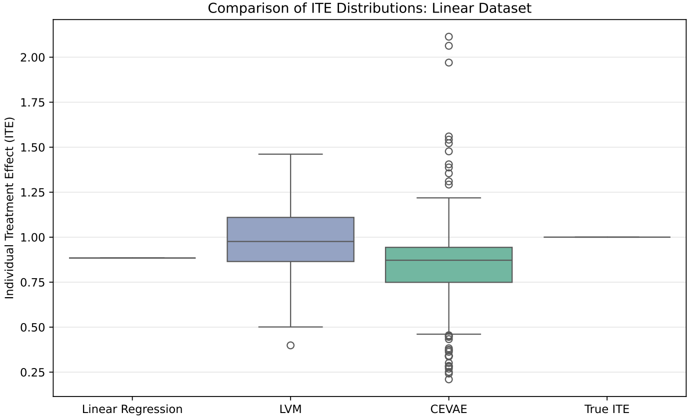
$$T|Z \sim \mathrm{Be}(\sigma(\beta z))$$

$$Y|T, Z \sim \mathcal{N}\left(\mathrm{NonLin}(z, t), \sigma_Y^2\right)$$

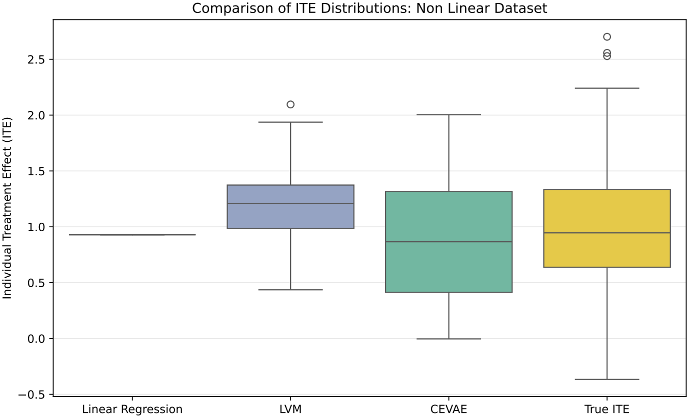
$$\mathrm{NonLin}(z, t) = \mathsf{sin}(z) + \frac{1}{2}z + t\left(1 + \frac{1}{2}z\right)$$

Remark: ITE depends on interaction between t and z!

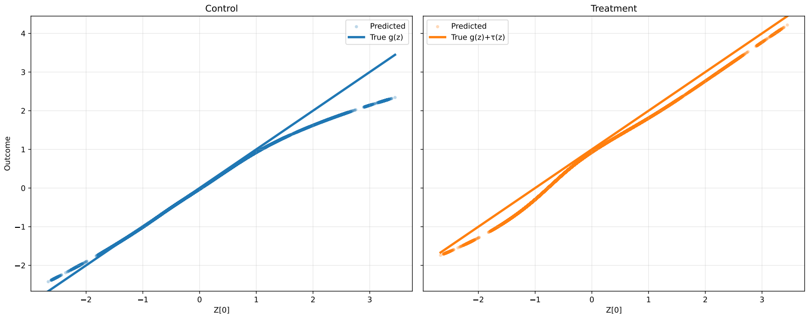
Results (1/2): Linear Dataset



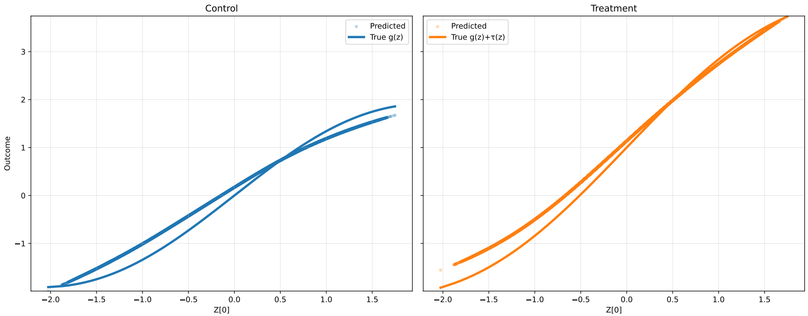
Results (2/2): Non Linear Dataset



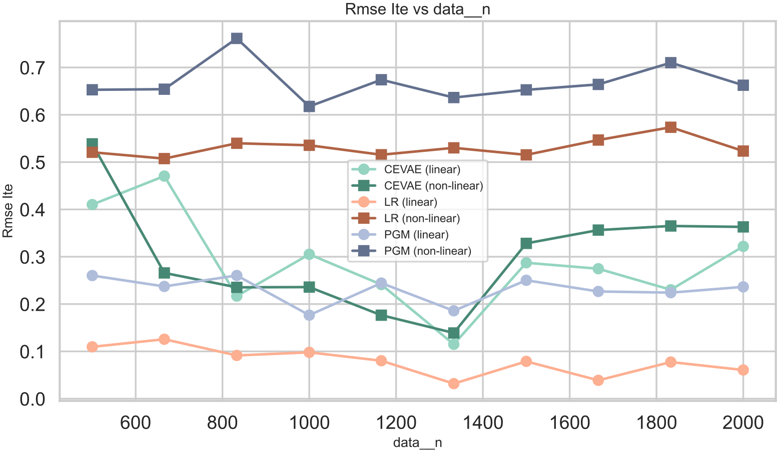
Internal Representation: Linear Dataset



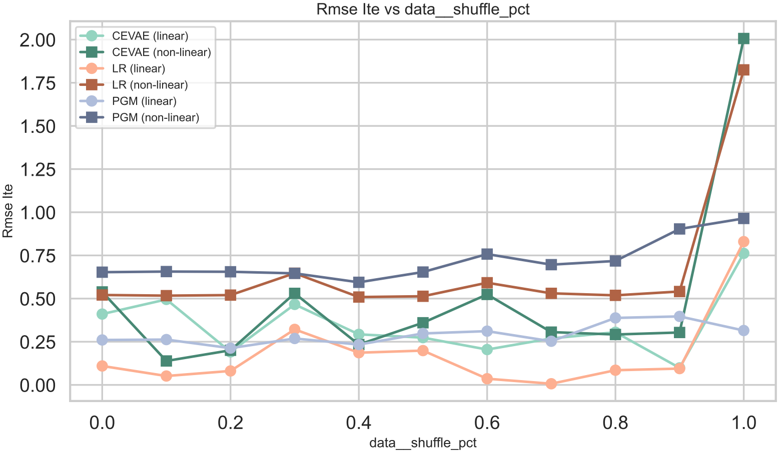
Internal Representation: Non Linear Dataset



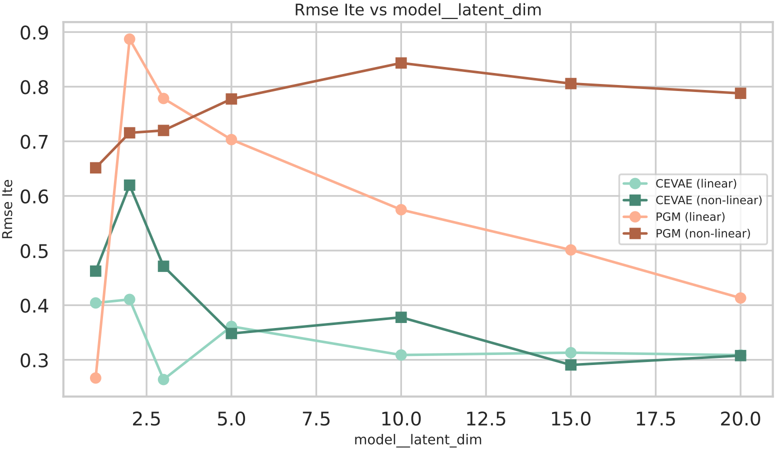
Experiment 1: Increasing the sample size



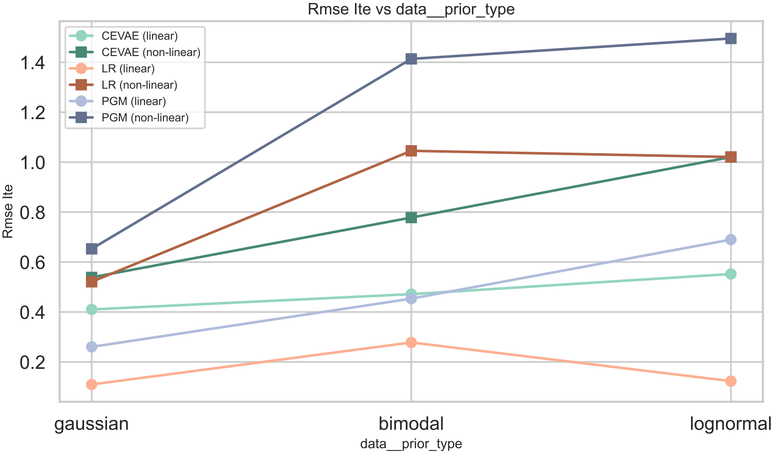
Experiment 2: Increasing decorrelation among proxies



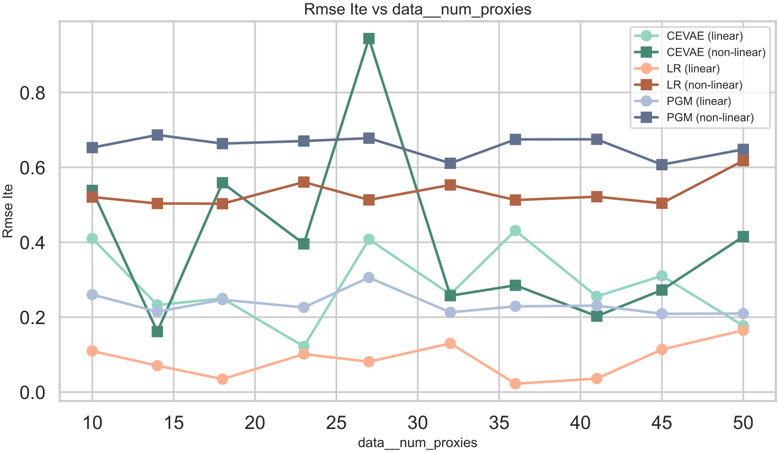
Experiment 3: Increasing latent dimension



Experiment 4: latent distribution misspecified



Experiment 5: Increasing the number of proxies



Conclusions

- **Linear synthetic data** – Linear models *benefit* from strong prior assumptions and recover the true effects almost perfectly; CEVAE offers no gain and adds variance.
- **Non-linear synthetic data** – With a tuned latent dimension, CEVAE outperforms linear baselines; if untuned it over-fits and the latent loses causal meaning.
- **Practical takeaway** – Use simple models when you can state strong causal assumptions; switch to CEVAE to relax those assumptions—provided you can afford heavier training and tuning.
- **Next steps** – Test on real datasets, explore more sophisticate variant of CEVAE (e.g. *TEDVAE*, *DCEVAE*, *ICEVAE*)

Thank You!