UNIVERSITÀ DEGLI STUDI DI TRIESTE

From Latent to Deep Latent Variable Models

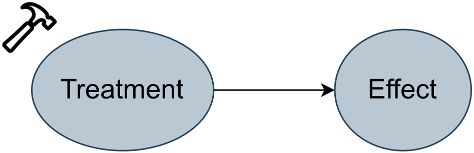
A Study On Causal Effect Inference with CEVAE

Valeria De Stasio, Christian Faccio, Giovanni Lucarelli

June 17, 2025

Objective: estimating causal effects

Estimate how a medical treatment T affects the health Y of a *random* patient.



$$P(Y = y|\text{do}(T = t)) = P(Y = y|T = t)$$

$$\text{ITE} = \mathbb{E}[(Y|\text{do}(T = 1)] - \mathbb{E}[(Y|\text{do}(T = 0)]$$

This is usually the case in a **randomized controlled trial** (RCT), where the treatment is randomly assigned to the patients.

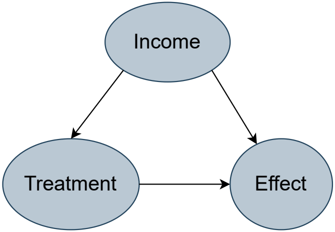
Confounder

In a observational study:

- **no** control over the treatment T assignment
  $\rightarrow$ there may be a confounder X (e.g. Income) that influences both variables

$$P(Y|\mathrm{do}(t)) \neq P(Y|t)$$

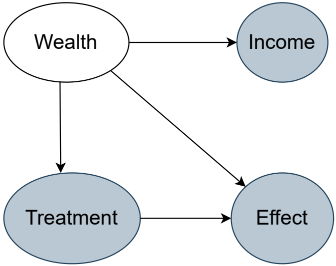
$$P(Y|\mathrm{do}(t), x) = P(Y|t, x)$$



If the confounder is observed even a linear rergession can do the job!

Latent confounder

But what if the confounder Z (e.g. Wealth) is **not** observed and the observed X is only a proxy of it?



Idea: estimate the latent variable and condition on it

Vanilla Latent Variable Model

- Assume parametric distributions
- Assume parametric relationships between variables

- Infer parameters using Stochastic Variational Inference (SVI)

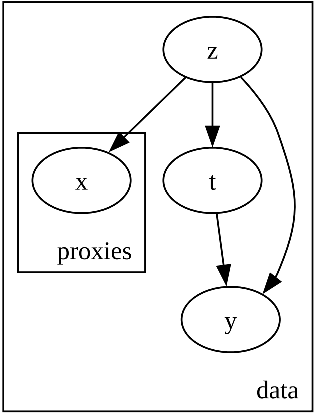
Problem: to infer from new data point x we need to train a new model at test time!

Deep Latent Variable Model: CEVAE

- Assume parametric distributions
- Assume parametric relationships between variables through Neural Networks

No need for test time training! **amortized inference**

Synthetic linear dataset



$$Z \sim \mathcal{N}(0, 1)$$

$$a_j \sim \mathcal{U}(-10, 10)$$

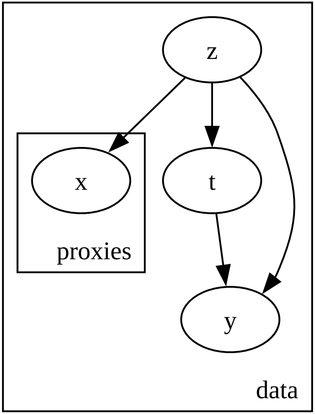
$$X_j \sim \mathcal{N}(a_j z, \sigma_X^2)$$

$$T|Z \sim \mathrm{Be}(\sigma(\beta z))$$

$$Y|T, Z \sim \mathcal{N}(z + t, \sigma_Y^2)$$

Remark: ITE is constant for all x!

Synthetic non linear dataset



$$Z \sim \mathcal{N}(0,1)$$

$$a \sim \mathcal{U}(-10,10)^d$$

$$\Sigma = \sigma_X^2[(1-\rho)\mathbb{I} + \rho J]$$

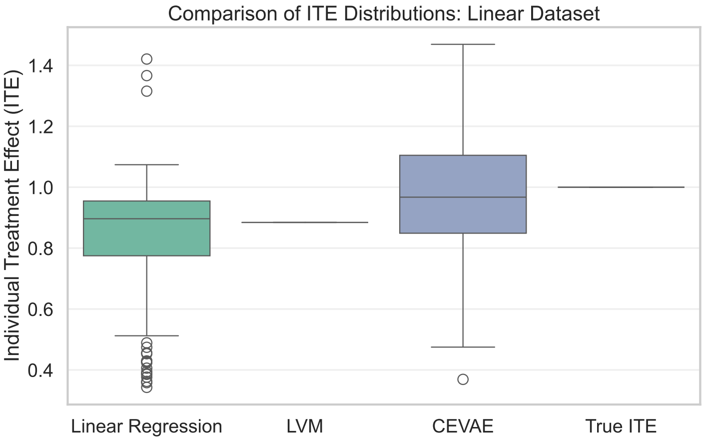
$$X_1, ..., X_d \sim \mathcal{N}(a \,\mathsf{tanh}(z), \Sigma)$$

$$T|Z \sim \mathrm{Be}(\sigma(\beta z))$$

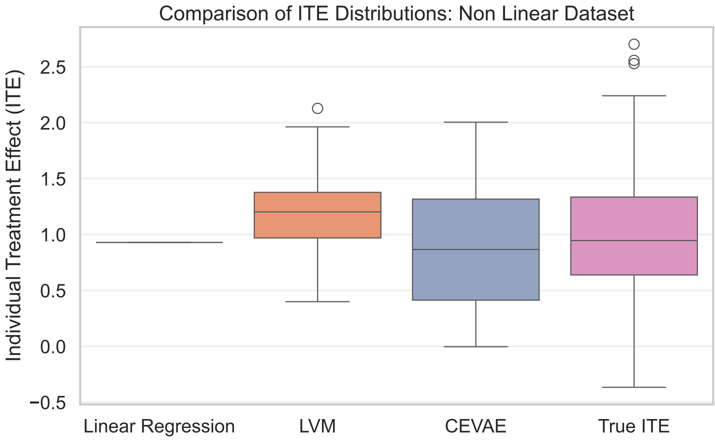
$$Y|T,Z \sim \mathcal{N}\left(\mathrm{NonLin}(z,t), \sigma_Y^2\right)$$

$$\mathrm{NonLin}(z,t) = \sin(z)+\frac{1}{2}z+t\left(1+\frac{1}{2}z\right)$$

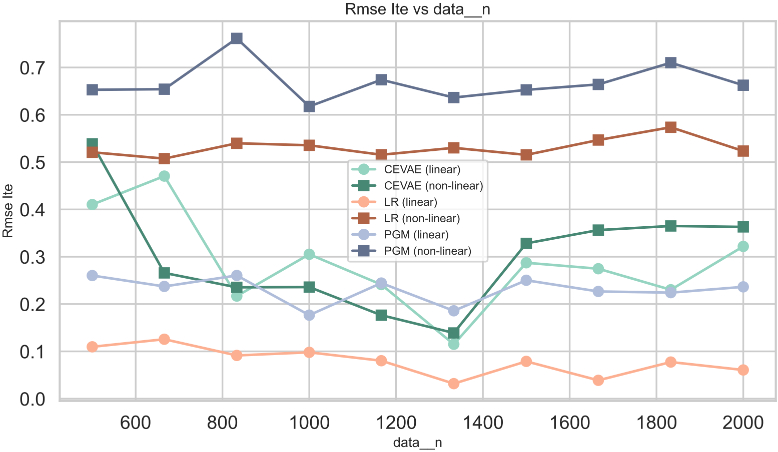
Results (1/2): Linear Dataset



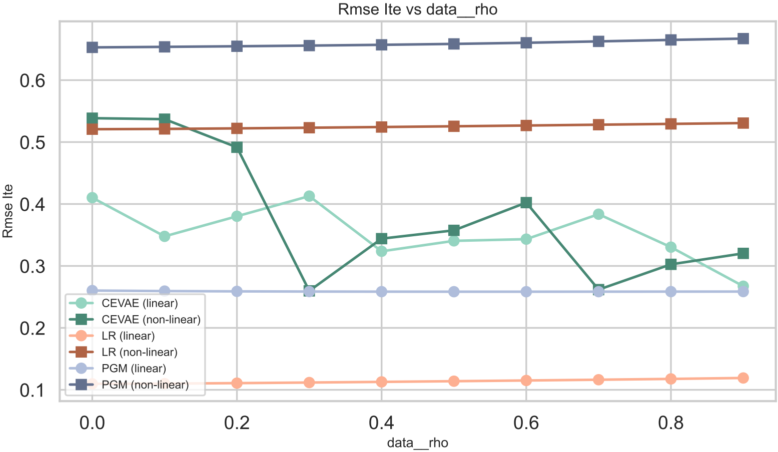
Results (2/2): Non Linear Dataset



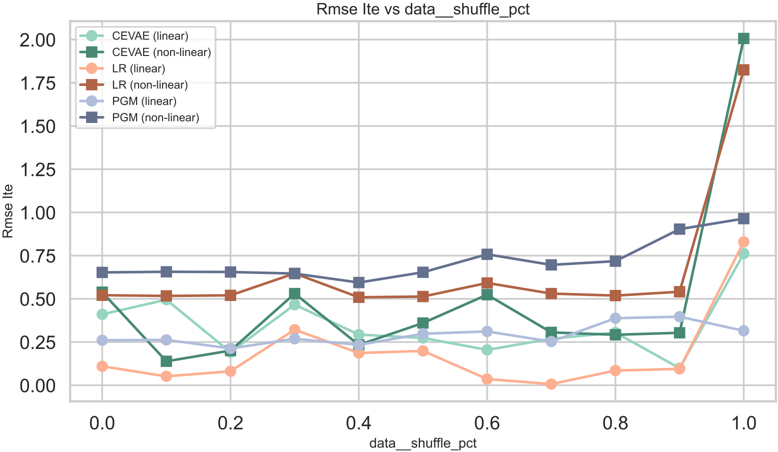
Experiment 1: Increasing the sample size

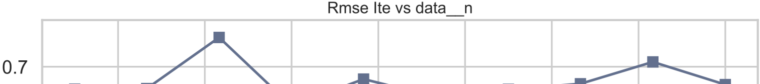


Experiment 2: Increasing correlation among proxies

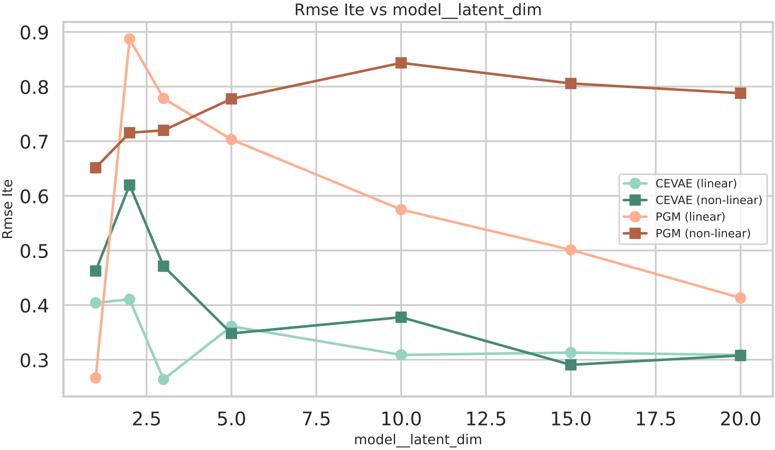


Experiment 3: Increasing decorrelation among proxies

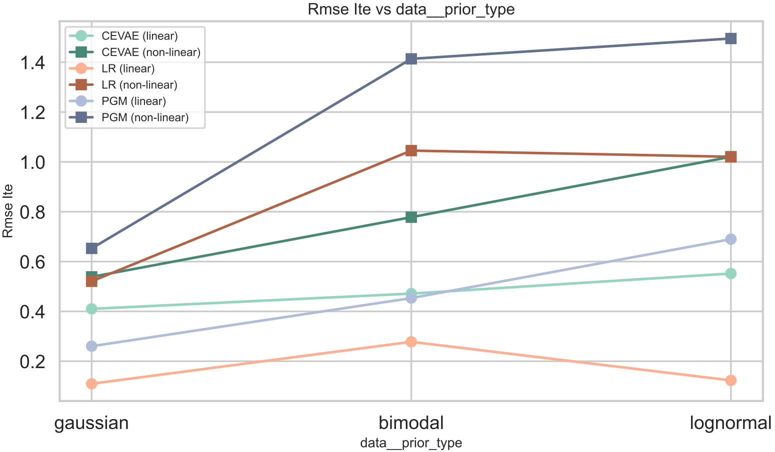




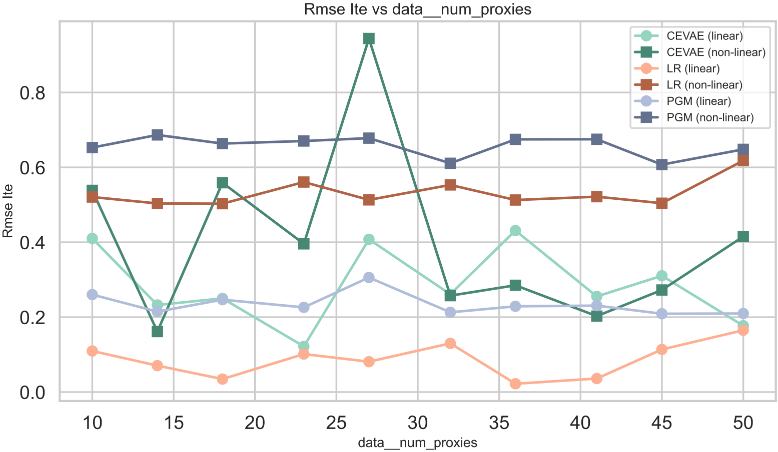
Experiment 4: Increasing latent dimension



Experiment 5: latent distribution misspecified



Experiment 6: Increasing the number of proxies



Conclusions

Pros:

- Don't require strong assumptions on the data
- No need training at test time
- Only model able to perform well on non-linear data

Cons:

- Computationally expensive
- Requires a lot of data

Thank You!