

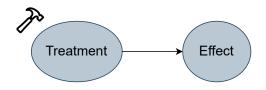
## From Latent to Deep Latent Variable Models

A Study On Causal Effect Inference with CEVAE

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### Objective: estimating causal effects

Estimate how a medical treatment *T* affects the health *Y* of a *random* patient.



$$P(Y = y | do(T = t)) = P(Y = y | T = t)$$

$$ITE = \mathbb{E}[(Y | do(T = 1)] - \mathbb{E}[(Y | do(T = 0)]]$$

This is usually the case in a **randomized controlled trial** (RCT), where the treatment is randomly assigned to the patients.

1

#### Confounder

In a observational study:

no control over the treatment T assignment
 → there may be a confounder X (e.g. Income) that influences both variables

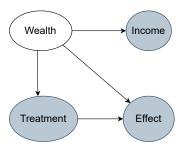


If the confounder is observed even a linear rergession can do the job!

2

#### Latent confounder

But what if the confounder *Z* (e.g. Wealth) is **not** observed and the observed *X* is only a proxy of it?



Idea: estimate the latent variable and condition on it

#### Vanilla Latent Variable Model

- · Assume parametric distributions
- · Assume parametric relationships between variables

Infer parameters using Stochastic Variational Inference (SVI)

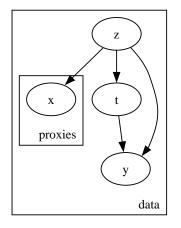
**Problem:** to infer from new data point *x* we need to train a new model at test time!

#### Deep Latent Variable Model: CEVAE

- · Assume parametric distributions
- Assume parametric relationships between variables through Neural Networks

No need for test time training! amortized inference

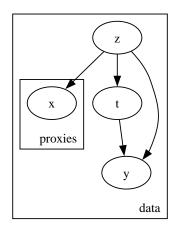
### Synthetic linear dataset



$$Z \sim \mathcal{N}(0,1)$$
 $a_j \sim \mathcal{U}(-10,10)$ 
 $X_j \sim \mathcal{N}(a_j z, \sigma_X^2)$ 
 $T|Z \sim \text{Be}(\sigma(\beta z))$ 
 $Y|T, Z \sim \mathcal{N}(z+t, \sigma_Y^2)$ 

**Remark:** ITE is constant for all x!

### Synthetic non linear dataset



$$Z \sim \mathcal{N}(0,1)$$

$$a \sim \mathcal{U}(-10,10)^d$$

$$\Sigma = \sigma_X^2[(1-\rho)\mathbb{I} + \rho J]$$

$$X_1, ..., X_d \sim \mathcal{N}(a \tanh(z), \Sigma)$$

$$T|Z \sim \text{Be}(\sigma(\beta z))$$

$$Y|T, Z \sim \mathcal{N}\left(\text{NonLin}(z,t), \sigma_Y^2\right)$$

$$\text{NonLin}(z,t) = \sin(z) + \frac{1}{2}z + t\left(1 + \frac{1}{2}z\right)$$

# Results (1/2): Linear Dataset

boxplot1

# Results (2/2): Non Linear Dataset

boxplot2

# Experiment: latent distribution misspecified

# Experiment: changing latent dimension

# Experiment: increasing the treatment effect

## Conclusions

