



**UNIVERSITÀ  
DEGLI STUDI  
DI TRIESTE**

# From Latent to Deep Latent Variable Models

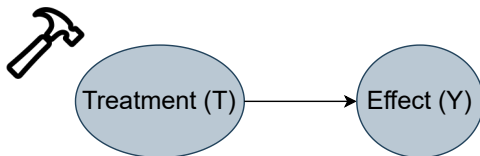
A Study On Causal Effect Inference with CEVAE

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Estimate how a medical treatment  $T$  affects the health  $Y$  of a *random* patient.



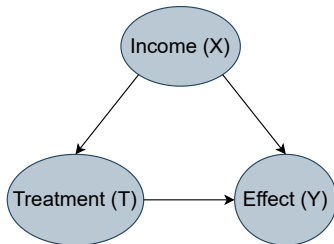
$$P(Y = y | \text{do}(T = t)) = P(Y = y | T = t)$$

This is usually the case in a **randomized controlled trial** (RCT), where the treatment is randomly assigned to the patients.

In a **observational** study **no control** over the treatment  $T$  assignment  
→ there may be a confounder  $X$  (e.g. Income) that influences both  $T$  and  $Y$ .

$$P(Y|\text{do}(t)) \neq P(Y|t)$$

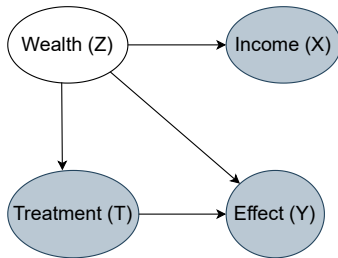
$$P(Y|\text{do}(t), x) = P(Y|t, x)$$



If the confounder is observed even a linear regression can do the job!

$$\text{ITE}(x) = \mathbb{E}[(Y|\text{do}(T = 1), x)] - \mathbb{E}[(Y|\text{do}(T = 0), x)]$$

But what if the confounder  $Z$  (e.g. Wealth) is **not** observed and the observed  $X$  is only a proxy of it?



**Idea:** estimate the latent variable and condition on it

# Methodology

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Two different synthetic datasets, relationships between variables are:

- **Linear** functions
- **Non-linear** functions

To address this problem:

- Linear Regression
- Latent Variable Model (LVM)
- Deep Latent Variable Model (CEVAE)

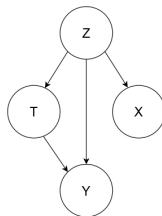
- **Generative model:**

$$Z \sim \mathcal{N}(0, 1)$$

$$X_j \sim \mathcal{N}(az + b, \text{diag}(\sigma_X^2))$$

$$T \sim \text{Be}(\sigma(cz))$$

$$Y \sim \mathcal{N}(et + fz, \sigma_Y^2)$$



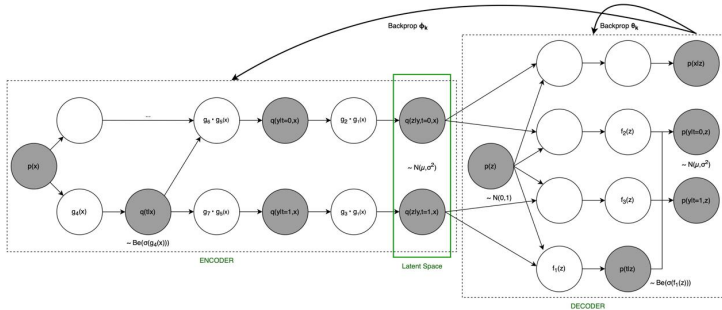
- **SVI** on both the generative parameters and the posterior distribution

## Latent Variable Model (LVM):

- Linear functions
- SVI + mini-SVI for new  $x$

## Deep Latent Variable Model (CEVAE):

- MLP functions
- SVI + Amortized inference



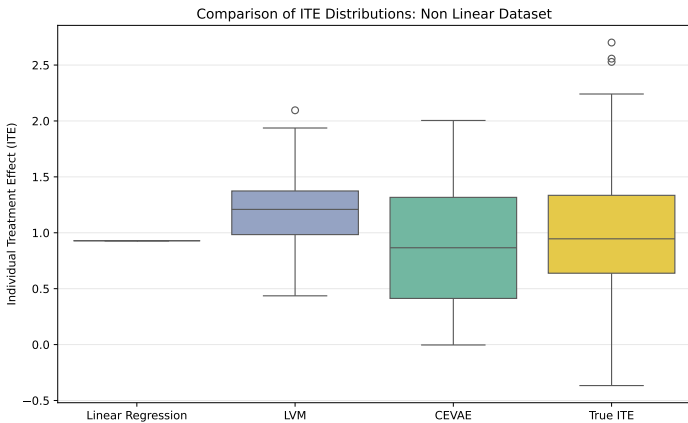


## Results

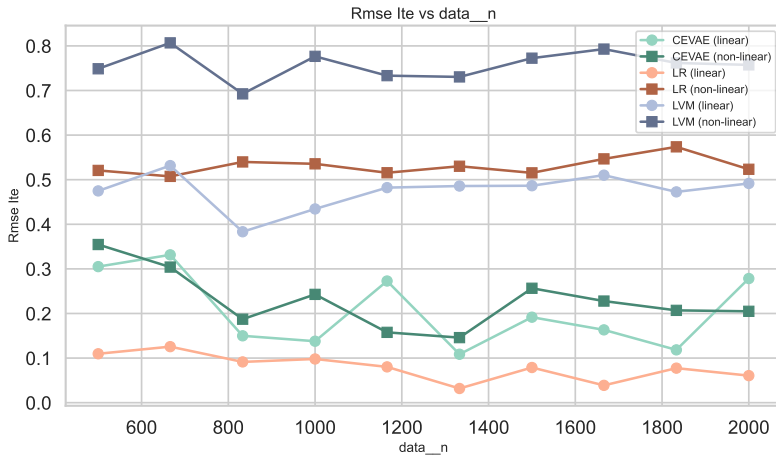
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Box plot showing the distribution of Individual Treatment Effect (ITE) for four methods: Linear Regression, LVM, CEVAE, and True ITE. The y-axis represents the ITE value, ranging from 0.25 to 2.00. Linear Regression shows a single value around 0.88. LVM shows a distribution with a median around 0.95 and whiskers extending from 0.5 to 1.45. CEVAE shows a distribution with a median around 0.85 and whiskers extending from 0.45 to 1.2. True ITE is a single line at 1.0. Outliers are indicated by open circles.

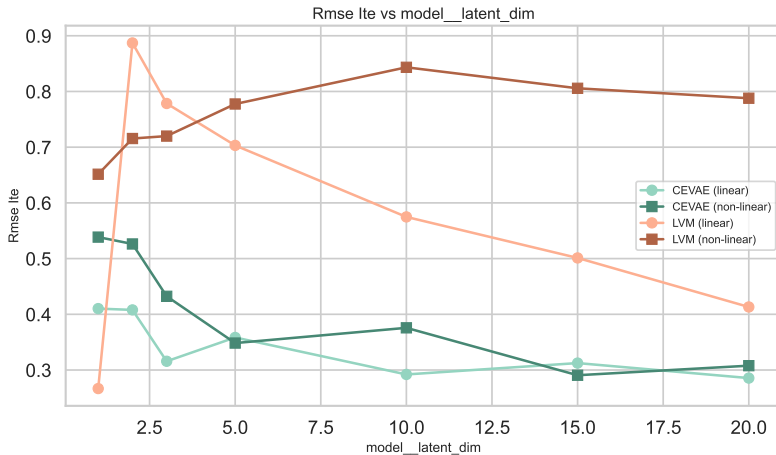
Method	Median	Q1	Q3	Min (Whisker)	Max (Whisker)	Outliers
Linear Regression	0.88	0.88	0.88	0.88	0.88	None
LVM	0.95	0.88	1.10	0.50	1.45	0.40
CEVAE	0.85	0.75	0.95	0.45	1.20	0.20, 0.22, 0.25, 0.28, 0.30, 0.32, 0.35, 0.38, 0.40, 0.42, 0.45, 0.48, 0.50, 0.52, 0.55, 0.58, 0.60, 0.62, 0.65, 0.68, 0.70, 0.72, 0.75, 0.78, 0.80, 0.82, 0.85, 0.88, 0.90, 0.92, 0.95, 0.98, 1.00, 1.02, 1.05, 1.08, 1.10, 1.12, 1.15, 1.18, 1.20, 1.22, 1.25, 1.28, 1.30, 1.32, 1.35, 1.38, 1.40, 1.42, 1.45, 1.48, 1.50, 1.52, 1.55, 1.58, 1.60, 1.62, 1.65, 1.68, 1.70, 1.72, 1.75, 1.78, 1.80, 1.82, 1.85, 1.88, 1.90, 1.92, 1.95, 1.98, 2.00, 2.02, 2.05, 2.08, 2.10, 2.12, 2.15, 2.18, 2.20, 2.22, 2.25, 2.28, 2.30, 2.32, 2.35, 2.38, 2.40, 2.42, 2.45, 2.48, 2.50, 2.52, 2.55, 2.58, 2.60, 2.62, 2.65, 2.68, 2.70, 2.72, 2.75, 2.78, 2.80, 2.82, 2.85, 2.88, 2.90, 2.92, 2.95, 2.98, 3.00, 3.02, 3.05, 3.08, 3.10, 3.12, 3.15, 3.18, 3.20, 3.22, 3.25, 3.28, 3.30, 3.32, 3.35, 3.38, 3.40, 3.42, 3.45, 3.48, 3.50, 3.52, 3.55, 3.58, 3.60, 3.62, 3.65, 3.68, 3.70, 3.72, 3.75, 3.78, 3.80, 3.82, 3.85, 3.88, 3.90, 3.92, 3.95, 3.98, 4.00, 4.02, 4.05, 4.08, 4.10, 4.12, 4.15, 4.18, 4.20, 4.22, 4.25, 4.28, 4.30, 4.32, 4.35, 4.38, 4.40, 4.42, 4.45, 4.48, 4.50, 4.52, 4.55, 4.58, 4.60, 4.62, 4.65, 4.68, 4.70, 4.72, 4.75, 4.78, 4.80, 4.82, 4.85, 4.88, 4.90, 4.92, 4.95, 4.98, 5.00, 5.02, 5.05, 5.08, 5.10, 5.12, 5.15, 5.18, 5.20, 5.22, 5.25, 5.28, 5.30, 5.32, 5.35, 5.38, 5.40, 5.42, 5.45, 5.48, 5.50, 5.52, 5.55, 5.58, 5.60, 5.62, 5.65, 5.68, 5.70, 5.72, 5.75, 5.78, 5.80, 5.82, 5.85, 5.88, 5.90, 5.92, 5.95, 5.98, 6.00, 6.02, 6.05, 6.08, 6.10, 6.12, 6.15, 6.18, 6.20, 6.22, 6.25, 6.28, 6.30, 6.32, 6.35, 6.38, 6.40, 6.42, 6.45, 6.48, 6.50, 6.52, 6.55, 6.58, 6.60, 6.62, 6.65, 6.68, 6.70, 6.72, 6.75, 6.78, 6.80, 6.82, 6.85, 6.88, 6.90, 6.92, 6.95, 6.98, 7.00, 7.02, 7.05, 7.08, 7.10, 7.12, 7.15, 7.18, 7.20, 7.22, 7.25, 7.28, 7.30, 7.32, 7.35, 7.38, 7.40, 7.42, 7.45, 7.48, 7.50, 7.52, 7.55, 7.58, 7.60, 7.62, 7.65, 7.68, 7.70, 7.72, 7.75, 7.78, 7.80, 7.82, 7.85, 7.88, 7.90, 7.92, 7.95, 7.98, 8.00, 8.02, 8.05, 8.08, 8.10, 8.12, 8.15, 8.18, 8.20, 8.22, 8.25, 8.28, 8.30, 8.32, 8.35, 8.38, 8.40, 8.42, 8.45, 8.48, 8.50, 8.52, 8.55, 8.58, 8.60, 8.62, 8.65, 8.68, 8.70, 8.72, 8.75, 8.78, 8.80, 8.82, 8.85, 8.88, 8.90, 8.92, 8.95, 8.98, 9.00, 9.02, 9.05, 9.08, 9.10, 9.12, 9.15, 9.18, 9.20, 9.22, 9.25, 9.28, 9.30, 9.32, 9.35, 9.38, 9.40, 9.42, 9.45, 9.48, 9.50, 9.52, 9.55, 9.58, 9.60, 9.62, 9.65, 9.68, 9.70, 9.72, 9.75, 9.78, 9.80, 9.82, 9.85, 9.88, 9.90, 9.92, 9.95, 9.98, 10.00, 10.02, 10.05, 10.08, 10.10, 10.12, 10.15, 10.18, 10.20, 10.22, 10.25, 10.28, 10.30, 10.32, 10.35, 10.38, 10.40, 10.42, 10.45, 10.48, 10.50, 10.52, 10.55, 10.58, 10.60, 10.62, 10.65, 10.68, 10.70, 10.72, 10.75, 10.78, 10.80, 10.82, 10.85, 10.88, 10.90, 10.92, 10.95, 10.98, 11.00, 11.02, 11.05, 11.08, 11.10, 11.12, 11.15, 11.18, 11.20, 11.22, 11.25, 11.28, 11.30, 11.32, 11.35, 11.38, 11.40, 11.42, 11.45, 11.48, 11.50, 11.52, 11.55, 11.58, 11.60, 11.62, 11.65, 11.68, 11.70, 11.72, 11.75, 11.78, 11.80, 11.82, 11.85, 11.88, 11.90, 11.92, 11.95, 11.98, 12.00, 12.02, 12.05, 12.08, 12.10, 12.12, 12.15, 12.18, 12.20, 12.22, 12.25, 12.28, 12.30, 12.32, 12.35, 12.38, 12.40, 12.42, 12.45, 12.48, 12.50, 12.52, 12.55, 12.58, 12.60, 12.62, 12.65, 12.68, 12.70, 12.72, 12.75, 12.78, 12.80, 12.82, 12.85, 12.88, 12.90, 12.92, 12.95, 12.98, 13.00, 13.02, 13.05, 13.08, 13.10, 13.12, 13.15, 13.18, 13.20, 13.22, 13.25, 13.28, 13.30, 13.32, 13.35, 13.38, 13.40, 13.42, 13.45, 13.48, 13.50, 13.52, 13.55, 13.58, 13.60, 13.62, 13.65, 13.68, 13.70, 13.72, 13.75, 13.78, 13.80, 13.82, 13.85, 13.88, 13.90, 13.92, 13.95, 13.98, 14.00, 14.02, 14.05, 14.08, 14.10, 1



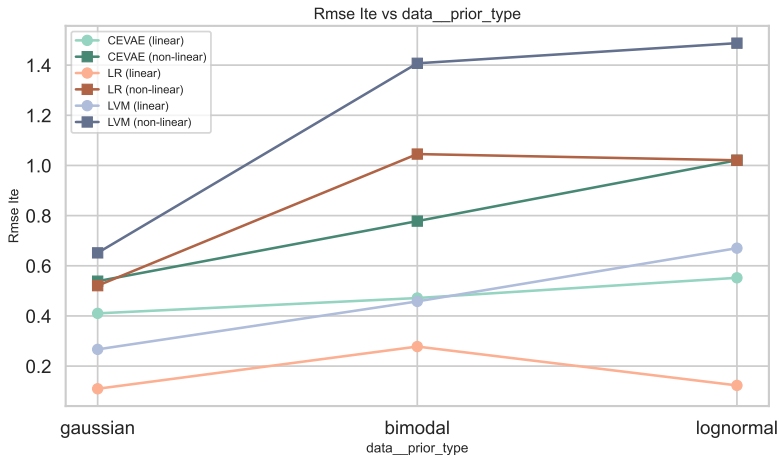
# Increasing the sample size



# Increasing latent dimension



# Latent distribution misspecified

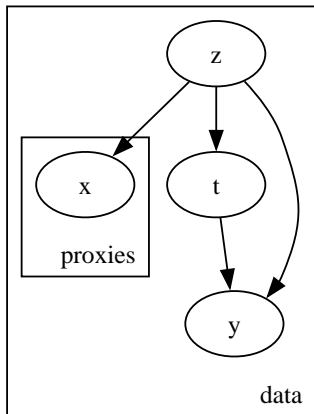


- **CEVAE** performs better on non-linear synthetic data, as more flexible
- Overcomplicates simple problems, e.g. linear dataset
- Does not need strong assumptions, being more robust than the LVM
- **Next steps** – Test on real datasets, explore more sophisticated variant of CEVAE (e.g. *TEDVAE*, *DCEVAE*, *ICEVAE*)

Thank You!



# Synthetic linear dataset



$$Z \sim \mathcal{N}(0, 1)$$

$$a_j \sim \mathcal{U}(-10, 10)$$

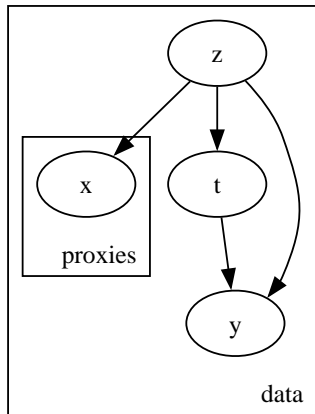
$$X_j \sim \mathcal{N}(a_j Z, \sigma_X^2)$$

$$T|Z \sim \text{Be}(\sigma(\beta Z))$$

$$Y|T, Z \sim \mathcal{N}(z + t, \sigma_Y^2)$$

**Remark:** ITE is constant for all  $x$ !

# Synthetic non linear dataset



$$Z \sim \mathcal{N}(0, 1)$$

$$a \sim \mathcal{U}(-10, 10)^d$$

$$X_1, \dots, X_d \sim \mathcal{N}(a \tanh(z), \sigma_X^2 \mathbb{I})$$

$$T|Z \sim \text{Be}(\sigma(\beta z))$$

$$Y|T, Z \sim \mathcal{N}(\text{NonLin}(z, t), \sigma_Y^2)$$

$$\text{NonLin}(z, t) = \sin(z) + \frac{1}{2}z + t \left(1 + \frac{1}{2}z\right)$$

**Remark:** ITE depends on interaction between  $t$  and  $z$ !