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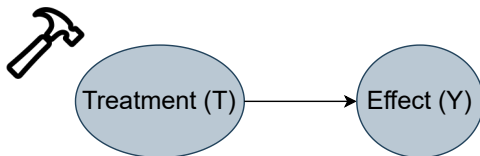
From Latent to Deep Latent Variable Models

A Study On Causal Effect Inference with CEVAE

Valeria De Stasio, Christian Faccio, Giovanni Lucarelli

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Estimate how a medical treatment T affects the health Y of a *random* patient.



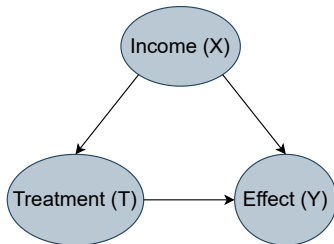
$$P(Y = y | \text{do}(T = t)) = P(Y = y | T = t)$$

This is usually the case in a **randomized controlled trial** (RCT), where the treatment is randomly assigned to the patients.

In a **observational** study **no control** over the treatment T assignment
→ there may be a confounder X (e.g. Income) that influences both T and Y .

$$P(Y|\text{do}(t)) \neq P(Y|t)$$

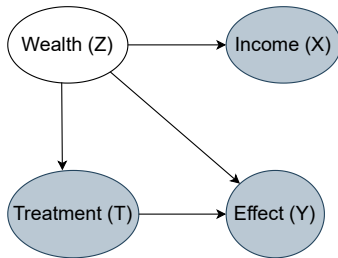
$$P(Y|\text{do}(t), x) = P(Y|t, x)$$



If the confounder is observed even a linear regression can do the job!

$$\text{ITE}(x) = \mathbb{E}[(Y|\text{do}(T = 1), x)] - \mathbb{E}[(Y|\text{do}(T = 0), x)]$$

But what if the confounder Z (e.g. Wealth) is **not** observed and the observed X is only a proxy of it?



Idea: estimate the latent variable and condition on it

Methodology

Two different synthetic datasets, relationships between variables are:

- **Linear** functions
- **Non-linear** functions

To address this problem:

- Linear Regression
- Latent Variable Model (LVM)
- Deep Latent Variable Model (CEVAE)

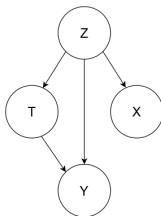
- **Generative model:**

$$Z \sim \mathcal{N}(0, 1)$$

$$X_j \sim \mathcal{N}(az + b, \text{diag}(\sigma_X^2))$$

$$T \sim \text{Be}(\sigma(cz))$$

$$Y \sim \mathcal{N}(et + fz, \sigma_Y^2)$$



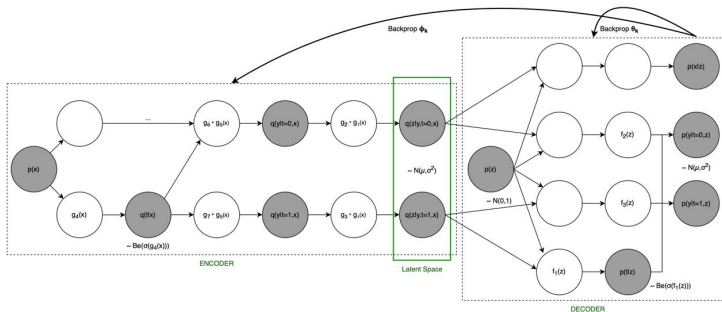
- **SVI** on both the generative parameters and the posterior distribution

Latent Variable Model (LVM):

- Linear functions
- SVI + mini-SVI for new x

Deep Latent Variable Model (CEVAE):

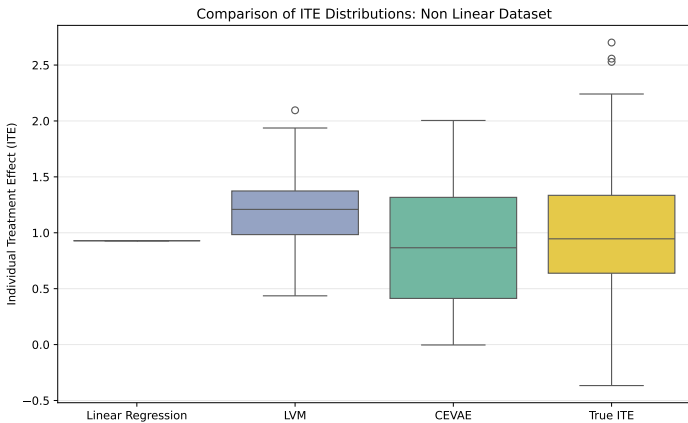
- MLP functions
- SVI + Amortized inference



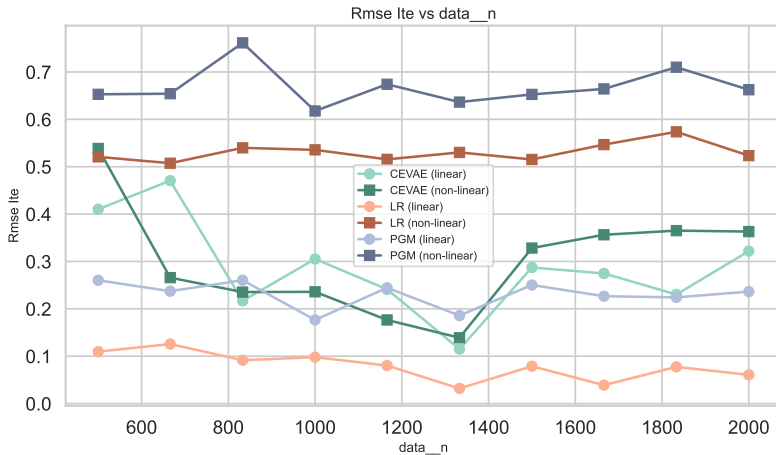
Results

Box plot showing the distribution of Individual Treatment Effect (ITE) for four methods: Linear Regression, LVM, CEVAE, and True ITE. The y-axis represents the ITE value, ranging from 0.25 to 2.00. Linear Regression shows a single value around 0.88. LVM shows a distribution with a median around 0.95 and whiskers extending from 0.5 to 1.45. CEVAE shows a distribution with a median around 0.85 and whiskers extending from 0.45 to 1.2. True ITE is a single line at 1.0. Outliers are indicated by open circles.

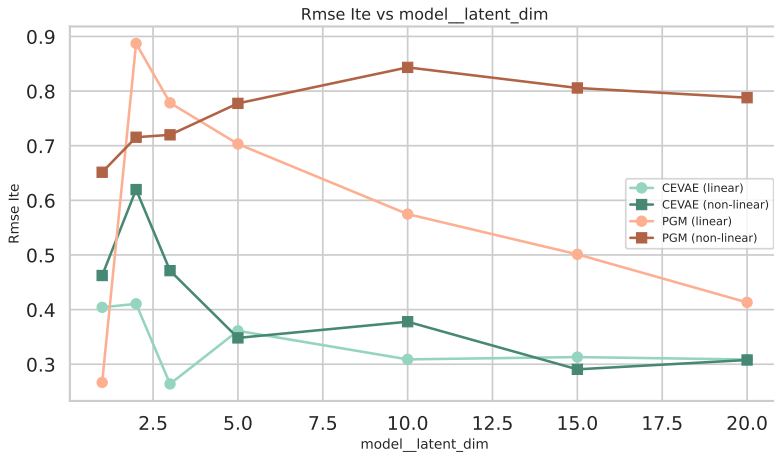
Method	Min (Whisker)	Q1	Median	Q3	Max (Whisker)	Outliers
Linear Regression	0.88	0.88	0.88	0.88	0.88	None
LVM	0.50	0.88	0.95	1.10	1.45	0.40
CEVAE	0.45	0.75	0.85	0.92	1.20	0.20, 0.22, 0.25, 0.28, 0.30, 0.32, 0.35, 0.38, 0.42, 0.45, 1.25, 1.28, 1.30, 1.32, 1.35, 1.38, 1.40, 1.42, 1.45, 1.48, 1.50, 1.52, 1.55, 1.58, 1.60, 1.62, 1.65, 1.68, 1.70, 1.72, 1.75, 1.78, 1.80, 1.82, 1.85, 1.88, 1.90, 1.92, 1.95, 1.98, 2.00, 2.02, 2.05, 2.08, 2.10, 2.12, 2.15, 2.18, 2.20, 2.22, 2.25, 2.28, 2.30, 2.32, 2.35, 2.38, 2.40, 2.42, 2.45, 2.48, 2.50, 2.52, 2.55, 2.58, 2.60, 2.62, 2.65, 2.68, 2.70, 2.72, 2.75, 2.78, 2.80, 2.82, 2.85, 2.88, 2.90, 2.92, 2.95, 2.98, 3.00, 3.02, 3.05, 3.08, 3.10, 3.12, 3.15, 3.18, 3.20, 3.22, 3.25, 3.28, 3.30, 3.32, 3.35, 3.38, 3.40, 3.42, 3.45, 3.48, 3.50, 3.52, 3.55, 3.58, 3.60, 3.62, 3.65, 3.68, 3.70, 3.72, 3.75, 3.78, 3.80, 3.82, 3.85, 3.88, 3.90, 3.92, 3.95, 3.98, 4.00, 4.02, 4.05, 4.08, 4.10, 4.12, 4.15, 4.18, 4.20, 4.22, 4.25, 4.28, 4.30, 4.32, 4.35, 4.38, 4.40, 4.42, 4.45, 4.48, 4.50, 4.52, 4.55, 4.58, 4.60, 4.62, 4.65, 4.68, 4.70, 4.72, 4.75, 4.78, 4.80, 4.82, 4.85, 4.88, 4.90, 4.92, 4.95, 4.98, 5.00, 5.02, 5.05, 5.08, 5.10, 5.12, 5.15, 5.18, 5.20, 5.22, 5.25, 5.28, 5.30, 5.32, 5.35, 5.38, 5.40, 5.42, 5.45, 5.48, 5.50, 5.52, 5.55, 5.58, 5.60, 5.62, 5.65, 5.68, 5.70, 5.72, 5.75, 5.78, 5.80, 5.82, 5.85, 5.88, 5.90, 5.92, 5.95, 5.98, 6.00, 6.02, 6.05, 6.08, 6.10, 6.12, 6.15, 6.18, 6.20, 6.22, 6.25, 6.28, 6.30, 6.32, 6.35, 6.38, 6.40, 6.42, 6.45, 6.48, 6.50, 6.52, 6.55, 6.58, 6.60, 6.62, 6.65, 6.68, 6.70, 6.72, 6.75, 6.78, 6.80, 6.82, 6.85, 6.88, 6.90, 6.92, 6.95, 6.98, 7.00, 7.02, 7.05, 7.08, 7.10, 7.12, 7.15, 7.18, 7.20, 7.22, 7.25, 7.28, 7.30, 7.32, 7.35, 7.38, 7.40, 7.42, 7.45, 7.48, 7.50, 7.52, 7.55, 7.58, 7.60, 7.62, 7.65, 7.68, 7.70, 7.72, 7.75, 7.78, 7.80, 7.82, 7.85, 7.88, 7.90, 7.92, 7.95, 7.98, 8.00, 8.02, 8.05, 8.08, 8.10, 8.12, 8.15, 8.18, 8.20, 8.22, 8.25, 8.28, 8.30, 8.32, 8.35, 8.38, 8.40, 8.42, 8.45, 8.48, 8.50, 8.52, 8.55, 8.58, 8.60, 8.62, 8.65, 8.68, 8.70, 8.72, 8.75, 8.78, 8.80, 8.82, 8.85, 8.88, 8.90, 8.92, 8.95, 8.98, 9.00, 9.02, 9.05, 9.08, 9.10, 9.12, 9.15, 9.18, 9.20, 9.22, 9.25, 9.28, 9.30, 9.32, 9.35, 9.38, 9.40, 9.42, 9.45, 9.48, 9.50, 9.52, 9.55, 9.58, 9.60, 9.62, 9.65, 9.68, 9.70, 9.72, 9.75, 9.78, 9.80, 9.82, 9.85, 9.88, 9.90, 9.92, 9.95, 9.98, 10.00, 10.02, 10.05, 10.08, 10.10, 10.12, 10.15, 10.18, 10.20, 10.22, 10.25, 10.28, 10.30, 10.32, 10.35, 10.38, 10.40, 10.42, 10.45, 10.48, 10.50, 10.52, 10.55, 10.58, 10.60, 10.62, 10.65, 10.68, 10.70, 10.72, 10.75, 10.78, 10.80, 10.82, 10.85, 10.88, 10.90, 10.92, 10.95, 10.98, 11.00, 11.02, 11.05, 11.08, 11.10, 11.12, 11.15, 11.18, 11.20, 11.22, 11.25, 11.28, 11.30, 11.32, 11.35, 11.38, 11.40, 11.42, 11.45, 11.48, 11.50, 11.52, 11.55, 11.58, 11.60, 11.62, 11.65, 11.68, 11.70, 11.72, 11.75, 11.78, 11.80, 11.82, 11.85, 11.88, 11.90, 11.92, 11.95, 11.98, 12.00, 12.02, 12.05, 12.08, 12.10, 12.12, 12.15, 12.18, 12.20, 12.22, 12.25, 12.28, 12.30, 12.32, 12.35, 12.38, 12.40, 12.42, 12.45, 12.48, 12.50, 12.52, 12.55, 12.58, 12.60, 12.62, 12.65, 12.68, 12.70, 12.72, 12.75, 12.78, 12.80, 12.82, 12.85, 12.88, 12.90, 12.92, 12.95, 12.98, 13.00, 13.02, 13.05, 13.08, 13.10, 13.12, 13.15, 13.18, 13.20, 13.22, 13.25, 13.28, 13.30, 13.32, 13.35, 13.38, 13.40, 13.42, 13.45, 13.48, 13.50, 13.52, 13.55, 13.58, 13.60, 13.62, 13.65, 13.68, 13.70, 13.72, 13.75, 13.78, 13.80, 13.82, 13.85, 13.88, 13.90, 13.92, 13.95, 13.98, 14.00, 14.02, 14.05, 14.08, 14.10, 14.12, 14.15, 14.18, 14.20, 14.22, 14.25, 14.28, 14.30, 14.32, 14.35, 14.38, 14.40, 14.42, 14.45, 14.48, 14.50, 14.52, 14.55, 14.58, 14.60, 14.62, 14.65, 14.68, 14.70, 14.72, 14.75, 14.78, 14.8



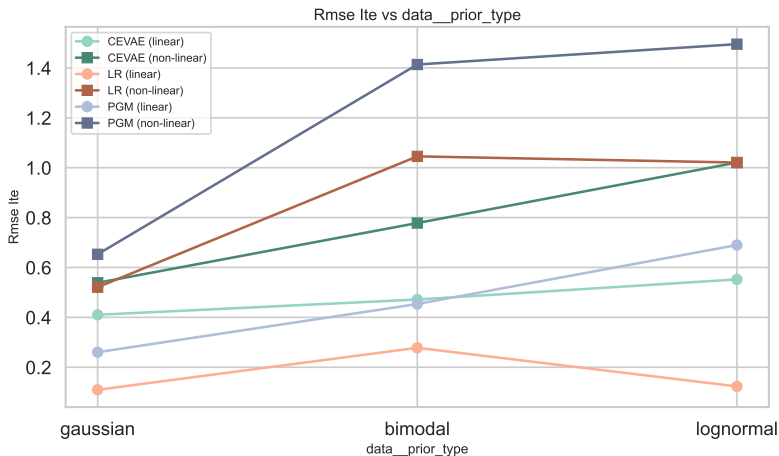
Increasing the sample size



Increasing latent dimension



Latent distribution misspecified

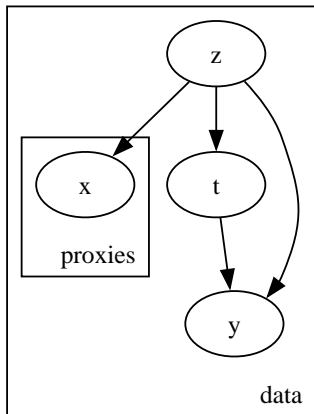


Conclusions

- **Linear synthetic data** – Linear models *benefit* from strong prior assumptions and recover the true effects almost perfectly; CEVAE offers no gain and adds variance.
- **Non-linear synthetic data** – With a tuned latent dimension, CEVAE outperforms linear baselines; if untuned it over-fits and the latent loses causal meaning.
- **Practical takeaway** – Use simple models when you can state strong causal assumptions; switch to CEVAE to relax those assumptions—provided you can afford heavier training and tuning.
- **Next steps** – Test on real datasets, explore more sophisticated variant of CEVAE (e.g. *TEDVAE*, *DCEVAE*, *ICEVAE*)

Thank You!

Synthetic linear dataset



$$Z \sim \mathcal{N}(0, 1)$$

$$a_j \sim \mathcal{U}(-10, 10)$$

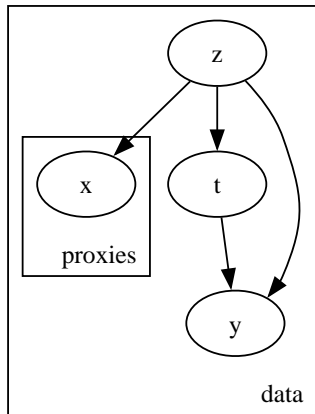
$$X_j \sim \mathcal{N}(a_j Z, \sigma_X^2)$$

$$T|Z \sim \text{Be}(\sigma(\beta Z))$$

$$Y|T, Z \sim \mathcal{N}(z + t, \sigma_Y^2)$$

Remark: ITE is constant for all x !

Synthetic non linear dataset



$$Z \sim \mathcal{N}(0, 1)$$

$$a \sim \mathcal{U}(-10, 10)^d$$

$$X_1, \dots, X_d \sim \mathcal{N}(a \tanh(z), \sigma_X^2 \mathbb{I})$$

$$T|Z \sim \text{Be}(\sigma(\beta z))$$

$$Y|T, Z \sim \mathcal{N}(\text{NonLin}(z, t), \sigma_Y^2)$$

$$\text{NonLin}(z, t) = \sin(z) + \frac{1}{2}z + t \left(1 + \frac{1}{2}z \right)$$

Remark: ITE depends on interaction between t and z !