Vorlesung 1 2023-10-18

Overview

Motivation:
Group Theory   math

rotation

teach toolbox

from spacetime
to Quantummechanic

place in between this lecture (Franck Laloe)

determinant 1   dimensions

applications in physics: particels SO(3), condensed matter SU(3), quantuminformation

matrix   orthogonal   unitary

transformations(symmetries) from spacetime goes to the Quantummechanics

classical symmetries = groups -> quantummechanics repesentation of same groups (Wigner, Weyl)
question: what is a elementary particle - representation of symmetries of spacetime
list represents have to be in QM

switch from one group to another

a teaser with examples

poisson brackets

$$\frac{\partial x}{\partial t} = -\{H, x\} = -\left(\frac{\partial H}{\partial x}\overset{=0}{\frac{\partial x}{\partial p}} - \frac{\partial H}{\partial p}\overset{=1}{\frac{\partial x}{\partial x}}\right) = -\frac{\partial H}{\partial p}$$

$$\frac{\partial p}{\partial t} = -\{H, p\}$$

H generates time evolution

function

energy conservation

$$= -\left(\frac{\partial H}{\partial x}\frac{\partial p}{\partial p} - \frac{\partial H}{\partial p}\frac{\partial p}{\partial x}\right)$$

play with change H with G $H \to G(x,p) = (pt - mx)\hat{u}$

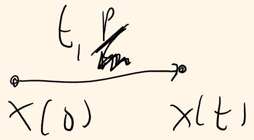
$$\dot{x} = \{G, x\} = \frac{\partial G}{\partial x}\frac{\partial x}{\partial p} - \frac{\partial G}{\partial p}\frac{\partial x}{\partial x} = -\frac{\partial G}{\partial p} = -t\hat{u}$$

$$\dot{p} = \{G, p\} = \frac{\partial G}{\partial x} = -m\hat{u}$$

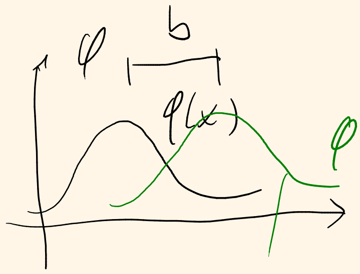
$$x' = x - ut$$
$$p' = p - mu$$

G generates Galilei Transformation

conserved quantity is initial center of mass position



translate function = shift in x



$\varphi'(x) = \varphi(x-b)$

shift in x $\quad \varphi'(x_b+b) = \max = \varphi(x_m)$

$x' = x - b$

generating translation

b very small $\quad \varphi'(x) \approx \varphi(x) - b\frac{\partial \varphi}{\partial x}\Big|_x = \varphi(x) + b\hat{\pi}\varphi\Big|_x$

$\hat{\pi} = -\frac{\partial}{\partial x}$

$\varphi' = \exp\left(-b\frac{\partial}{\partial x}\right)\varphi = \exp(b\hat{\pi})\varphi$

because the taylor expansion is the same as before

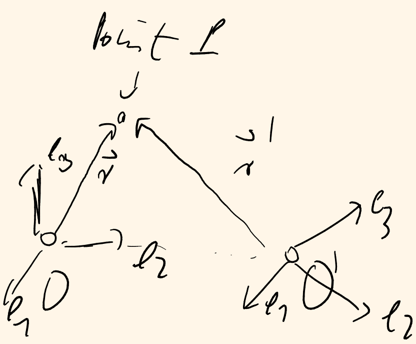
operators not as good as they are, so sometimes they use the original definition of displacement

in physics we usally use "nice" functions

det = +1 $\quad SO(3)$

Euclidean group E(3)= symmetry group of 3 space — translations $T(3)$, rotations $R(3)$ — subgroups

Combination (combined action)



$\vec{r}' = R\vec{r} + \vec{b}$ $\quad$ group element $(R, \vec{b})$

(3x3) matrix

coordinates are relativ and symmetries

inverse trafo $(R^{-1}, -R^{-1}\vec{b})$

$\begin{pmatrix}1 & \\ & 1\end{pmatrix} = \mathbb{1}$

$(\mathbb{1}, a)$ just translation

compute

$(R,b)^{-1} \circ (\mathbb{1}, a) \circ (R, b)\vec{r} \overset{!}{=} (\mathbb{1}, R^{-1}a)\vec{r}$

conjugation

represents a change of basis $\qquad \vec{r}' = R\vec{r} + \vec{b} \qquad \vec{r} = R^{-1}(\vec{r}' - \vec{b})$

$$(R, \vec{b})^{-1} \circ (\mathbb{1}, \vec{a}) \circ (R, \vec{b})\,\vec{r} \overset{!}{=} (\mathbb{1}, R^{-1}\vec{a})\,\vec{r}$$

$$R^{-1}(\vec{a} + (R\vec{r} + \vec{b})) - R^{-1}\vec{b} = \vec{r} + R^{-1}\vec{a}$$

$T(3)$ translation are a subgroup of E(3) normal divisor

an invariant subgroup (normal divisor - division)

ie $(\mathbb{1}, a) \circ (R, b) = (R, b) \circ (\mathbb{1}, R^{-1}a)$

$$E(3) = \underbrace{(E(3)/T(3))}_{\text{Rotions}} \otimes \underbrace{T(3)}_{\text{Commutative}}$$

non-Commutative

another example:

spin representations of SO(3)

rotate a spin 1/2 particle of angle ϑ

spin rotations matrix

$$R_3^{[1/2]}(\vartheta) = \begin{pmatrix} e^{-i\frac{\vartheta}{2}} & 0 \\ 0 & e^{i\frac{\vartheta}{2}} \end{pmatrix} = \exp\left(-\frac{i}{2}\vartheta\sigma_3\right) \in SU(2) \qquad e^{i\pi} = -1$$

$\vartheta \to 2\pi$

3-axes rotation

$$R_3(360°) = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

representation group

rotating a Q-Bit

topolical reason

only Spin 1/2 is the only allowed non trival cases