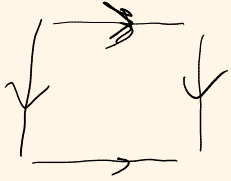


arrow is time evolution



group basics:

books: Wigner (group theory) 1959, group theory in a nut shell Zee (2019) for physis
for math people elementary introduction 2015 Hall

definition: $G \in a, b$
like "+" or "*" $a, b \mapsto a+b, a*b \text{ or } ab \in G$

 G -set

rules: neutral element e with $eg = g \quad \forall g \in G$

inverse g^{-1} with $g^{-1}g = e$

associative $(ab)g = a(bg)$ $eg = g$
 $(g^{-1}g)g = g^{-1}(gg) = g$

groups could be transformation, so the same rules for them
in general non commutative, $ab \neq ba$ "non-Abelian"

define "commutator"

$$C_{ab} = a^{-1}b^{-1}ab$$

$$\text{check: } e = (ba)^{-1}ba \\ = a^{-1}b^{-1}ba = a^{-1}ea = e$$

set of commutators, maybe it is a subset and it has some structures
commutator(derived) group

$$C_G = \{C_{ab} \mid a, b \in G\} \quad e \in C_G \text{ for } a, a^{-1}$$

product of numbers we use for the commutators

subgroup

$$\text{if } c, c' \in C_G, \text{ then } \left. \begin{array}{l} \text{also } cc' \in C_G \\ \text{and } c^{-1} \in C_G \end{array} \right\} \Rightarrow c^{-1}c' \in C_G$$

$$\underbrace{[(ba)^{-1}ab]^{-1} (b'a')^{-1} a'b'}_C$$

$$= (ab)^{-1} (ba) (b'a')^{-1} a'b'$$

$$= b^{-1} a^{-1} ba a'^{-1} b'^{-1} a' b'$$

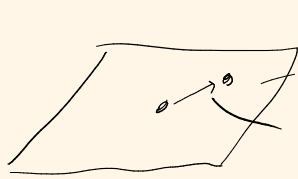
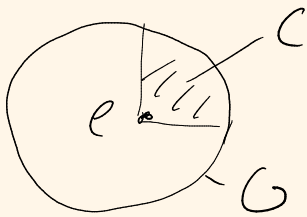
$$= b^{-1} (a^{-1} b) a$$

$$\dots \dots \dots P^{-1} Q^{-1} P Q$$

$$a' = a^{-1}$$

$$b' = a$$

simple groups are
groups with trivial subgroups



plane

vector some kind

of transformation

just one discrete subgroup

or guarantee just one
multiplication is some number

subgroup in "transformation groups" G acting on M

↓

be $s \in \text{Object}$ (being transformed by G)

$$G_s = \{g \in G \mid g \cdot s = s\} \triangleright \text{origin is a subgroup}$$

$$g, g' \in G, (g^{-1} g') s = g^{-1} \cdot (g' s) = g^{-1} \cdot s = s$$

▷ G : all motions of space, rotation and translation

fixed point $o \in M$ kept fixed $\Leftrightarrow G_o = \text{all rotations}$

$G =$ all matrices $d \times d$ be M a set of $d \times d$ matrices $\det \neq 0$
"general linear group" $GL(d)$ (anything)

$g \in G, u \in M \mapsto g u g^{-1}$ take $u = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is invariant under G
 matrices $\mapsto g u g^T = \mathbb{1}_d$ if $g g^T = \mathbb{1}_d$
 orthogonal group $O(d)$

$u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ is invariant under G $g u g^T = u$ $O(1,3)$

conjugation $g u g^{-1} = u$

if G act on space

$v \in \mathbb{R}^d$

orthogonal group
self adjoint

$g \cdot \vec{v} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$ "bilinear form" $\vec{u}, \vec{v} \mapsto \vec{u} \cdot \vec{v}$ "scalar product"

$(\vec{u}, \vec{v})_m = \vec{u}^T m \vec{v}$ bilinear form what (eg $\vec{u} = \vec{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$)

g transformation invariant $g^T m g = m$

$t^2 - x^2 - y^2 - z^2 = (\vec{t}, \vec{x})_m$

$(g \cdot \vec{u}, g \cdot \vec{v})_m = (g \vec{u})^T m (g \vec{v}) = \vec{u}^T (g^T m g) \vec{v} = \vec{u}^T m \vec{v} = (\vec{u}, \vec{v})_m$

conserved under transformation

Continuous version

another subgroup

1-parameter subgroup

$e, g, g^2, g^3, \dots, g^{-1}, g^{-2}, \dots$

rotation by axis \hat{n} , angles 0
 2π
 4π
 6π
 8π
 10π
 12π
 14π
 16π
 18π
 20π
 22π
 24π
 26π
 28π
 30π
 32π
 34π
 36π
 38π
 40π
 42π
 44π
 46π
 48π
 50π
 52π
 54π
 56π
 58π
 60π
 62π
 64π
 66π
 68π
 70π
 72π
 74π
 76π
 78π
 80π
 82π
 84π
 86π
 88π
 90π
 92π
 94π
 96π
 98π
 100π
 102π
 104π
 106π
 108π
 110π
 112π
 114π
 116π
 118π
 120π
 122π
 124π
 126π
 128π
 130π
 132π
 134π
 136π
 138π
 140π
 142π
 144π
 146π
 148π
 150π
 152π
 154π
 156π
 158π
 160π
 162π
 164π
 166π
 168π
 170π
 172π
 174π
 176π
 178π
 180π
 182π
 184π
 186π
 188π
 190π
 192π
 194π
 196π
 198π
 200π
 202π
 204π
 206π
 208π
 210π
 212π
 214π
 216π
 218π
 220π
 222π
 224π
 226π
 228π
 230π
 232π
 234π
 236π
 238π
 240π
 242π
 244π
 246π
 248π
 250π
 252π
 254π
 256π
 258π
 260π
 262π
 264π
 266π
 268π
 270π
 272π
 274π
 276π
 278π
 280π
 282π
 284π
 286π
 288π
 290π
 292π
 294π
 296π
 298π
 300π
 302π
 304π
 306π
 308π
 310π
 312π
 314π
 316π
 318π
 320π
 322π
 324π
 326π
 328π
 330π
 332π
 334π
 336π
 338π
 340π
 342π
 344π
 346π
 348π
 350π
 352π
 354π
 356π
 358π
 360π
 362π
 364π
 366π
 368π
 370π
 372π
 374π
 376π
 378π
 380π
 382π
 384π
 386π
 388π
 390π
 392π
 394π
 396π
 398π
 400π
 402π
 404π
 406π
 408π
 410π
 412π
 414π
 416π
 418π
 420π
 422π
 424π
 426π
 428π
 430π
 432π
 434π
 436π
 438π
 440π
 442π
 444π
 446π
 448π
 450π
 452π
 454π
 456π
 458π
 460π
 462π
 464π
 466π
 468π
 470π
 472π
 474π
 476π
 478π
 480π
 482π
 484π
 486π
 488π
 490π
 492π
 494π
 496π
 498π
 500π
 502π
 504π
 506π
 508π
 510π
 512π
 514π
 516π
 518π
 520π
 522π
 524π
 526π
 528π
 530π
 532π
 534π
 536π
 538π
 540π
 542π
 544π
 546π
 548π
 550π
 552π
 554π
 556π
 558π
 560π
 562π
 564π
 566π
 568π
 570π
 572π
 574π
 576π
 578π
 580π
 582π
 584π
 586π
 588π
 590π
 592π
 594π
 596π
 598π
 600π
 602π
 604π
 606π
 608π
 610π
 612π
 614π
 616π
 618π
 620π
 622π
 624π
 626π
 628π
 630π
 632π
 634π
 636π
 638π
 640π
 642π
 644π
 646π
 648π
 650π
 652π
 654π
 656π
 658π
 660π
 662π
 664π
 666π
 668π
 670π
 672π
 674π
 676π
 678π
 680π
 682π
 684π
 686π
 688π
 690π
 692π
 694π
 696π
 698π
 700π
 702π
 704π
 706π
 708π
 710π
 712π
 714π
 716π
 718π
 720π
 722π
 724π
 726π
 728π
 730π
 732π
 734π
 736π
 738π
 740π
 742π
 744π
 746π
 748π
 750π
 752π
 754π
 756π
 758π
 760π
 762π
 764π
 766π
 768π
 770π
 772π
 774π
 776π
 778π
 780π
 782π
 784π
 786π
 788π
 790π
 792π
 794π
 796π
 798π
 800π
 802π
 804π
 806π
 808π
 810π
 812π
 814π
 816π
 818π
 820π
 822π
 824π
 826π
 828π
 830π
 832π
 834π
 836π
 838π
 840π
 842π
 844π
 846π
 848π
 850π
 852π
 854π
 856π
 858π
 860π
 862π
 864π
 866π
 868π
 870π
 872π
 874π
 876π
 878π
 880π
 882π
 884π
 886π
 888π
 890π
 892π
 894π
 896π
 898π
 900π
 902π
 904π
 906π
 908π
 910π
 912π
 914π
 916π
 918π
 920π
 922π
 924π
 926π
 928π
 930π
 932π
 934π
 936π
 938π
 940π
 942π
 944π
 946π
 948π
 950π
 952π
 954π
 956π
 958π
 960π
 962π
 964π
 966π
 968π
 970π
 972π
 974π
 976π
 978π
 980π
 982π
 984π
 986π
 988π
 990π
 992π
 994π
 996π
 998π
 1000π

"Cyclic group generated by g "

$$g^4 g^m = g^{m+4} \quad g^{-4} g^m = g^{m-4}$$

$$R_n(\theta) R_n(\theta') = R_n(\theta + \theta' \text{ mod } 2\pi)$$

$$R_n(\theta) \quad \mathbb{1} = 0 = \mathbb{1}$$

has same topological structure

boosts along axis \hat{x} ,
rapidity η (or v, β, γ)

$$B_x(\eta) B_x(\eta') = B_x(\eta + \eta')$$

$$B_x(\eta) = \begin{pmatrix} \cosh \eta & \sinh \eta \\ \sinh \eta & \cosh \eta \end{pmatrix}$$

$$\beta = \frac{v}{c} = \tanh \eta$$

$$= \exp \left(\frac{\eta}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right)$$

derivative in space time
can define the infinitesimal operation
on the group

$$\begin{matrix} t, x & t', x' \\ | & | \\ 4 & 1 \\ \cdot & \cdot \\ x(0) & \xrightarrow{\quad} x(\eta) \end{matrix}$$