Continuous Symmetry Groups in Quantum Physics

Winter Term 2023/24

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Potential Examination Questions 7 Discussion: 07 February 2024 + X

Problem 7.1 – Selected Topics (10 points)

- Give three or four examples of a group "acting on something" and recall the general rules behind this construction.
- We defined a symmetry by the condition "something remains invariant". Show that this defines a sub-group.
- Explain the commutative diagram formed by symmetry transformations and time evolution.
- List the geometric transformations that preserve distances in an n-dimensional space. The name of this group? Give a few subgroups.
- Explain that the group of rotations in 3D is doubly connected.
- Justify that proper Lorentz transformations that keep one event fixed and preserve Minkowski distances in 3+1D space-time form a 6-dimensional Lie group.
- What about 2+1D space-time? [Bonus question.]
- Show that irreducible representations of an Abelian group are 1-dimensional. [Bonus question.]
- State the definition of the representation of a Lie group.
- Why do representations of a Lie algebra have the same commutators?
- Construct a situation where representations of a Lie algebra do not have the same commutators (!). [Bonus question.]
- Explain the links between rotations in 3D, the vector product, and the (orbital) angular momentum operator.
- Justify the "$\mathcal{T}^{-1}$" in the argument of a function when implementing a space-time transformation $\mathcal{T}$ in a space of functions.

- Write down an equation representing the statement "The Hermitean operator H is the generator of time translations." [Bonus question.]
- List a few Casimir operators for the following Lie algebras: SO(3), SU(2), SO(3,1), Galilei group, Poincaré group.
- Give a few examples of scalar and vectorial operators.
- Explain the transformation rules for the wave function of a spin-j particle in the relativistic context.
- Write down the Dirac and Weyl equations and explain the meaning of the symbols.
- Concepts that we could not discuss in the lecture (optional reading): Wigner-Eckardt theorem, SO(3)–SU(2) isomorphism and Bloch vector, SO(3,1)–SL(2,$\mathbb{C}$) isomorphism and Weyl spinors, (relativistic) commutation relations of Pauli-Lubanski vector, helicity for massive and massless particles and its Lorentz transformations, symmetry of Lagrange functions, rotation of multipole operators (irreducible tensor operators), representations of SU(3) (color and flavour symmetries).