Symmetries of space-time (and all the rest)

space time:
- translation
- P parity $\longrightarrow$ P parity
- boost / Galilei

C charge conjug'n

exchange of ident. part.

↓

"transformation group"

set S objects $\qquad s \longmapsto g \cdot s$

group G transformation

$$g^{-1} g \cdot s = s$$

$$(g'g) \cdot s = g' (g \cdot s)$$

$$g''(g'g) = g''(g'g)$$

QM: symmetry g "represented" by operator G

indeed any symmetry $g \longmapsto$ unitary S (Wigner theore

$|\psi\rangle \longmapsto G|\psi\rangle$

$U(t)|\psi\rangle \qquad U(t)G|\psi\rangle$

$\longrightarrow G\,U(t)|\psi\rangle \overset{?}{=}$

is symmetry if $[U(t), S$

if $U(t)$ generated by H

$[H, S] = 0 \Leftarrow [e^{-iHt/\hbar},$

eigenstates split in G-subs

unitary S with self-adj. "generator"

$$S = e^{-iG\varepsilon}$$

$G|\psi_1\rangle = \gamma_1 |\psi_1\rangle, \gamma_2$

$$0 = \langle \psi_2 | H | \psi_1 \rangle = \frac{\gamma_2 - \gamma_1}{\gamma_2 - \gamma_1} \langle \psi_2 | H | \psi_1$$

$$= \frac{1}{\gamma_2 - \gamma_1} \langle \psi_2 [G, H]$$

math structure behind all this

$\boxed{\text{"representation"}}$ $r : \mathcal{G} \ni g \longrightarrow \mathcal{U} \ni S = r(g)$

key property

$r(g')\,r(g) = r(g'g)$

Space time

Hilbert space

translation $T(3)$

$\mathcal{G}$ $\vdots$ $g \xrightarrow{\ r\ } \mathcal{U}$ unitary trafos

rot'n $SO(3)$ S

happens that

"pr

structure of class spacetime $\longrightarrow$ commutators of G (self-adj. operators) that represent symmetries

example $SO(3) \Rightarrow$

$\rightarrow SU(2) \ni M$ spi

$M = \exp\left(-i\vec{n}\cdot\vec{\sigma}\,\frac{\pi}{2}\right)$

$= \cos\left(\frac{\pi}{2}\right)\mathbb{1} - i\vec{n}$