Universität Potsdam
Institut für Informatik
Lehrstuhl Maschinelles Lernen

Models, Data, Learning Problems

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Overview

- Types of learning problems:
  - Supervised Learning (classification, regression, ordinal regression, metric learning, recommendations, sequences und structures)
  - Unsupervised Learning
  - Reinforcement-Learning (Exploration vs. Exploitation)
- Models
- Regularized empirical risk minimization
  - Loss functions,
  - Regularizer
- Evaluation

Supervised Learning: Basic Concepts

- Instance: $\mathbf{x} \in X$
  - In statistics: independent variable
  - X could be a vector space over *attributes* ($X = \mathbb{R}^m$)
  - An instance is then an assignment to the attributes.
  - $\mathbf{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix}$ feature vector
- Target variable: $y \in Y$
  - In Statistics: dependent variable
- A model maps instances to the target variable.

$$\mathbf{x} \xrightarrow{\text{Model}} y$$

Supervised Learning: Classification

- Input: Instance $\mathbf{x} \in X$.
  - e.g., a feature vector

$$\mathbf{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix}$$

- Output: Class $y \in Y$; finite set Y.
  - The class is also referred to as the target attribute.
  - y is also called the (class) label

$$\mathbf{x} \xrightarrow{\text{Classifier}} y$$

Classification: Example

- Input: Instance $\mathbf{x} \in X$
  - X : the set of all possible combinations of regiment of medication

Attribute	Instance $\mathbf{x}$
Medication #1 included?	0
	1
⋮	1
	0
	1
Medication #6 included?	0

Attribute values
Feature vector

Medication combination

- Output: $y \in Y = \{\text{toxic}, \text{ok}\}$  / 

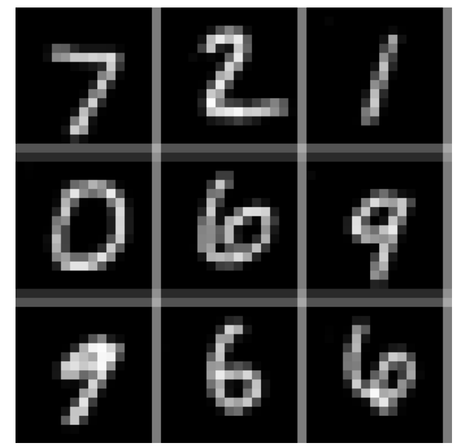
 $\longrightarrow$ **classifier** $\longrightarrow$ 

Classification: Example

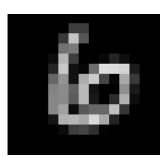
- Input: Instance $\mathbf{x} \in X$
  - X : the set of all 16×16 pixel bitmaps

Attribute	Instance $\mathbf{x}$
Gray value of pixel 1	0.1
	0.3
	0.45
$\vdots$	$\vdots$
	0.65
Gray value of pixel 256	0.87

256 pixel values



- Output: $y \in Y = \{0,1,2,3,4,5,6,7,8,9\}$: recognized digit

 $\longrightarrow$ **classifier** $\longrightarrow$ "6"

Classification: Example

- Input: Instance $\mathbf{x} \in X$
  - X : bag-of-words representation of all possible email texts

Attribute	Instance $\mathbf{x}$		Email
Word #1 occurs?	0	Aardvark	Dear Beneficiary,
	1	Beneficiary	
⋮	0	Friend	your Email address has been picked online in this years MICROSOFT CONSUMER AWARD as a Winner of One Hundred and Fifty Five Thousand Pounds Sterling…
	⋮	⋮	
	1	Sterling	
Word #m occurs?	0	Science	

$m \approx 1{,}000{,}000$

- Output: $y \in Y = \{\text{spam}, \text{ok}\}$

Dear Beneficiary,

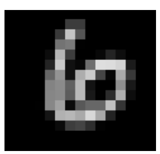
We are pleased to notify you that your Email address has been picked online in this second quarter's MICROSOFT CONSUMER AWARD (MCA) as a Winner of One Hundred and Fifty Five Thousand Pounds Sterling…

$\longrightarrow$ **classifier** $\longrightarrow$ “Spam“

Classification

- Classifier should be learned from training data.

Classifier



Classifier

"6"

Dear Beneficiary,

We are pleased to notify you that your Email address has been picked online in this second quarter's MICROSOFT CONSUMER AWARD (MCA) as a Winner of One Hundred and Fifty Five Thousand Pounds Sterling…

Classifier

“Spam“

Classifier Learning

- Input to the Learner: Training data T_n.
  - $\mathbf{X} = \begin{pmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{pmatrix}$  
  - $\mathbf{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$  
- Training Data: $T_n = \{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_n, y_n)\}$

- Output: a Model
  -  $\mapsto$ 
    $y_\theta : X \to Y$
  - for example:
    $y_{\boldsymbol{\theta}}(\mathbf{x}) = \begin{cases} \text{(sad)} & \text{if } \mathbf{x}^{\mathrm{T}}\boldsymbol{\theta} \geq 0 \\ \text{(happy)} & \textit{otherwise} \end{cases}$  

Linear classifier with parameter vector $\boldsymbol{\theta}$.

Classifier Learning

- Input to the Learner: Training data T_n.
  - $\mathbf{X} = \begin{pmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{pmatrix}$
  - $\mathbf{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$
- Training Data:
  $T_n = \{(\mathbf{x}_1, y_1), \dots, (\mathbf{x}_n, y_n)\}$

- Output: a model
  - $\mapsto$
    $y_\theta : X \to Y$
- Model classes
  - (Generalized) linear model
  - Decision tree
  - Ensemble classifier
  - …

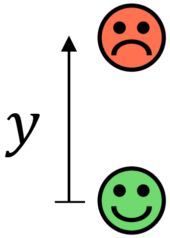
Supervised Learning: Regression

- Input: Instance $\mathbf{x} \in X$.
  - e.g., feature vector

$$\mathbf{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix}$$

How toxic is a combination?

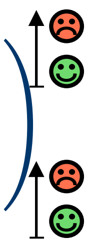
- Output: continuous (real) value, $y \in \mathbb{R}$
  - e.g., *toxicity*.



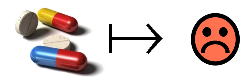
Regressor Learning

- Input to the Learner: Training data T_n.
  - $\mathbf{X} = \begin{pmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{pmatrix}$


  - $\mathbf{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$



- Training Data: $T_n = \{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_n, y_n)\}$

- Output: a model
  - 

    $y_\theta : X \to Y$
  - For example

    $y_{\boldsymbol{\theta}}(\mathbf{x}) = \mathbf{x}^{\mathrm{T}}\boldsymbol{\theta}$

Generalized linear model with parameter vector $\boldsymbol{\theta}$.

Supervised Learning: Ordinal Regression

- Input: Instance $\mathbf{x} \in X$.
- Output: discrete value $y \in Y$ like classification, but there is an ordering on the elements of Y.
- A large discrepancy between the model‘s prediction and the true value is worse than a small one.

$\longmapsto$

Satisfied with the outcome?

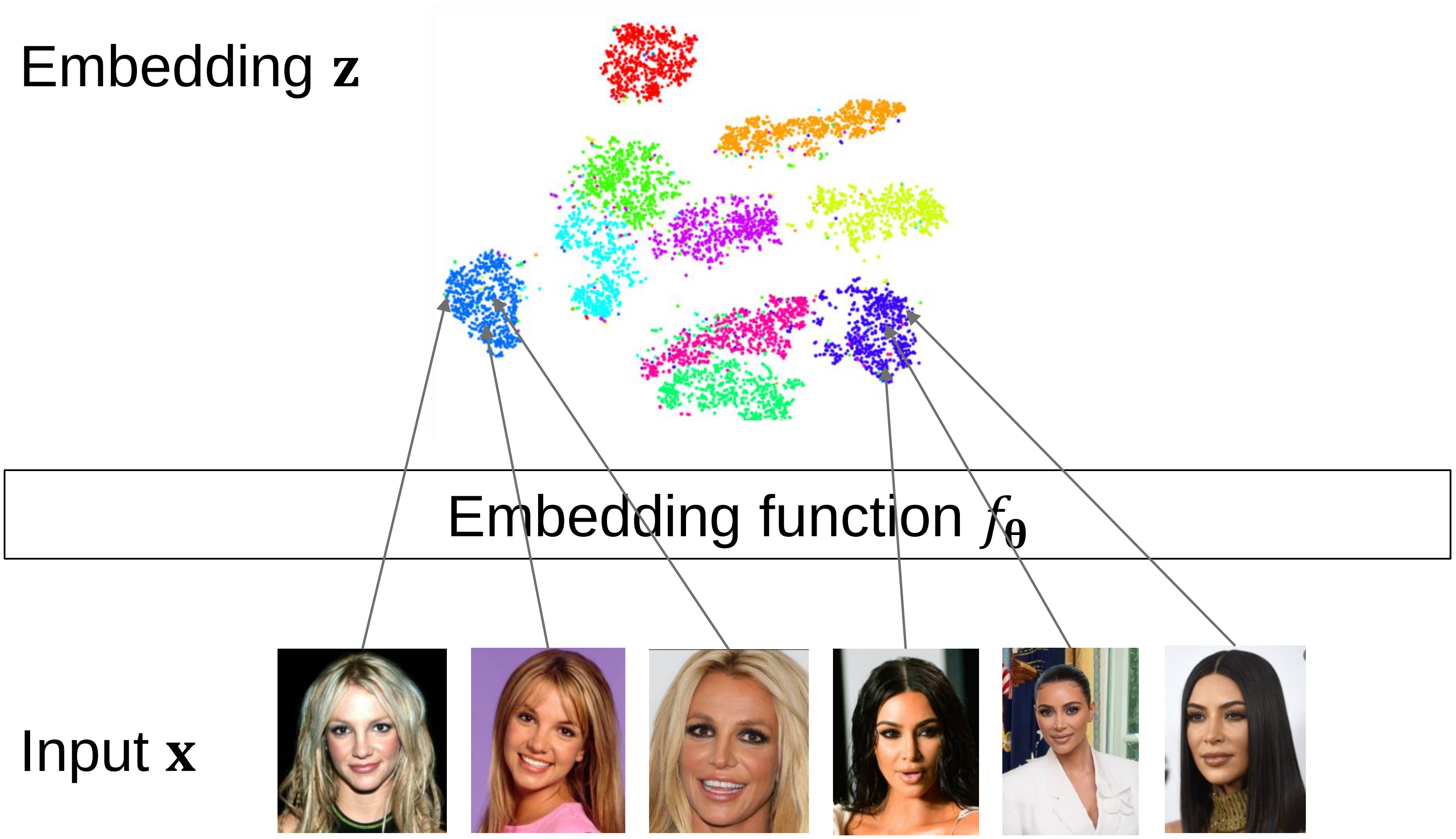
Metric Learning

- For classification problem, the classes are defined and fixed at training time.
- In some applications, one may want the model to identify previously unseen classes.
- For example, biometric identification:
  - All facial images (or fingerprint scans, or iris scans) of one individuals can be seen as one class.
  - New individuals should be recognized as such.
- Example object classification:
  - Images of new, previously unseen types of objects should be recognized as belonging to the same class.

Metric Learning

- Given: similarity metric $s(\mathbf{z}, \mathbf{z}')$.
- Input: Instance $\mathbf{x} \in X$.
- Output: embedding vector $\mathbf{z}$.
- Input to the learner:
  - Training instances $\mathbf{x}$ with classes y.
  - $T_n = \{(\mathbf{x}_1, y_1), \dots, (\mathbf{x}_n, y_n)\}$.
- Output of the learner: a model $f: \mathbf{x} \mapsto \mathbf{z}$ such that
  - All instances $\mathbf{x}, \mathbf{x}'$ with the same class y are similar: $s\big(f_{\boldsymbol{\theta}}(\mathbf{x}), f_{\boldsymbol{\theta}}(\mathbf{x}')\big) > \varepsilon$.
  - Instances $\tilde{\mathbf{x}}, \tilde{\mathbf{x}}'$ with different classes y, y' are dissimilar: $s\big(f_{\boldsymbol{\theta}}(\tilde{\mathbf{x}}), f_{\boldsymbol{\theta}}(\tilde{\mathbf{x}}')\big) < \varepsilon$.

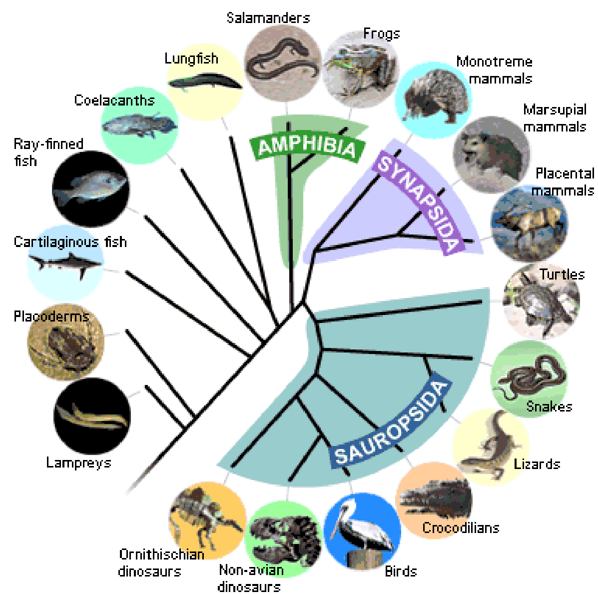
Metric Learning



Supervised Learning: Taxonomy Classification

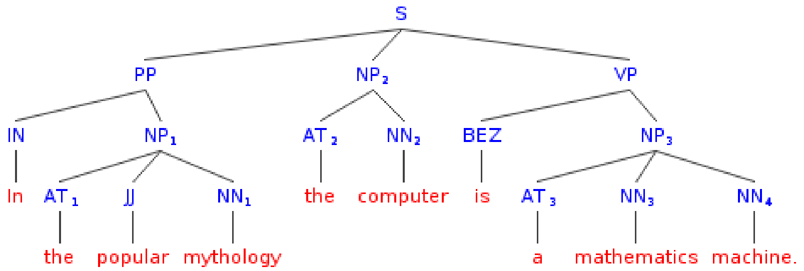
- Input: Instance $\mathbf{x} \in X$.
- Output: discrete value $y \in Y$ like classification, but there is a tree-based ordering on the elements of Y.
- The prediction is worse the farer apart the predicted and actual nodes are.

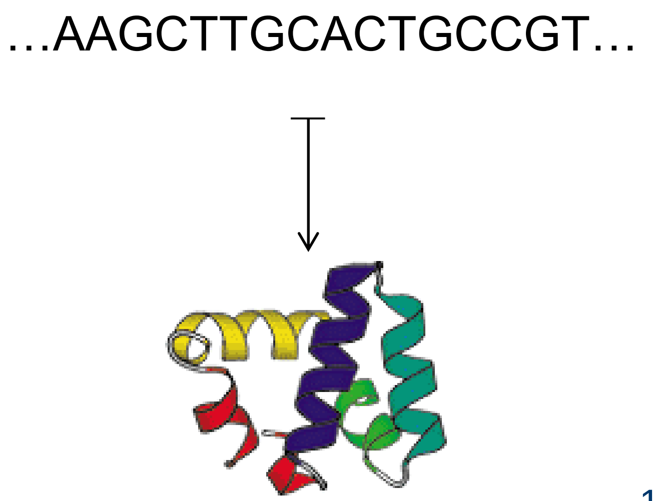
$\longmapsto$



Supervised Learning: Sequence and Structure Prediction

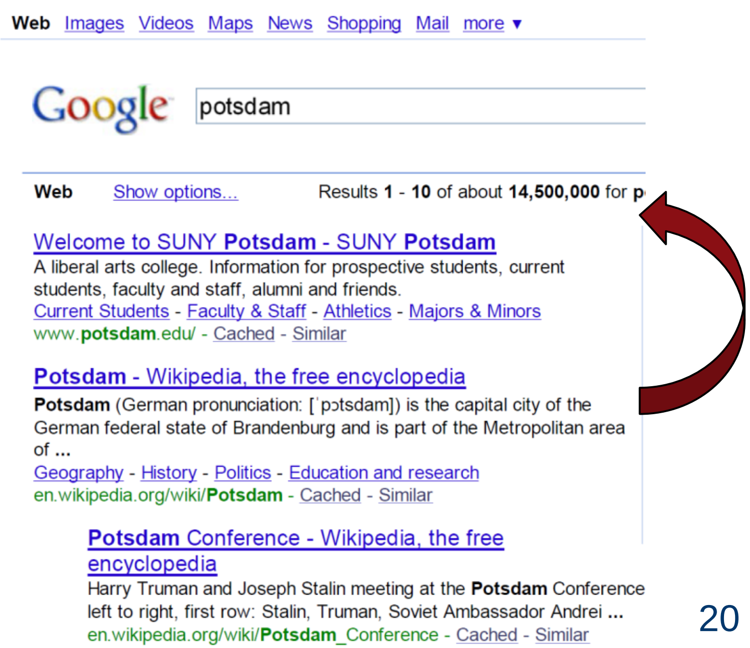
- Input: Instance $\mathbf{x} \in X$.
- Output: Sequence, Tree or Graph $y \in Y$.
- Example applications:
  - Parse natural languages
  - Protein folding





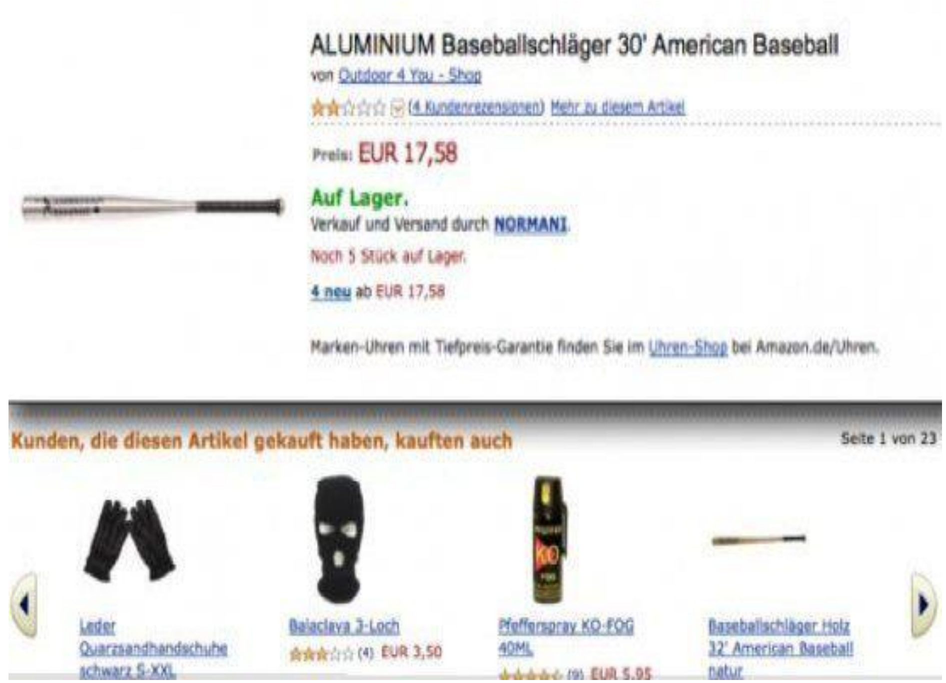
Supervised Learning: Rankings

- Input: query q and list of items $I_1, \ldots, I_n$.
- Output: a sorting of the items
- Training data: user clicks on I_j after querying with q:
  - The selected item should be ranked higher than those listed higher that were not clicked.



Supervised Learning: Recommendations

- Input: users, items, contextual information.
- Output: How much will a user like a recommendation?
- Training data: ratings, sales, page views.

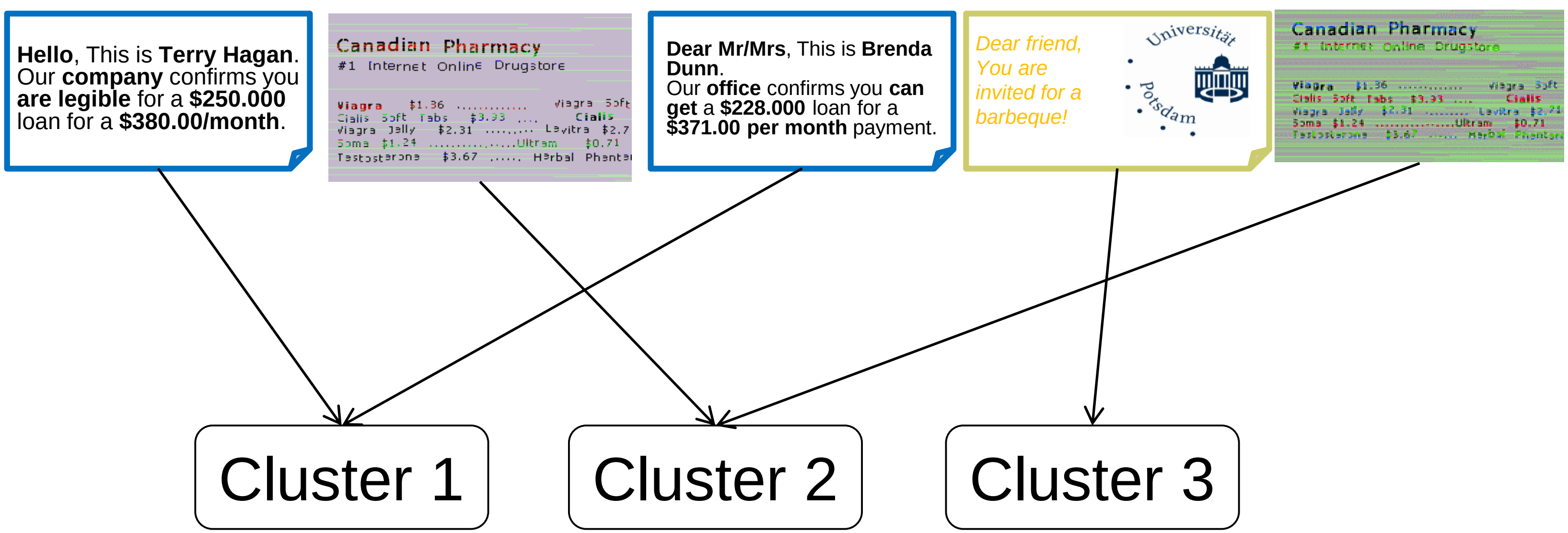


Unsupervised Learning

- Training data for unsupervised learning: Set of instances $\mathbf{x} \in X$.
- Additional assumptions about the data formation process; for example, independence of random variables.
- The goal is the detection of structure in the data:
  - For example, find the most likely grouping into clusters of instances that share certain properties, which were not directly observable in the data.

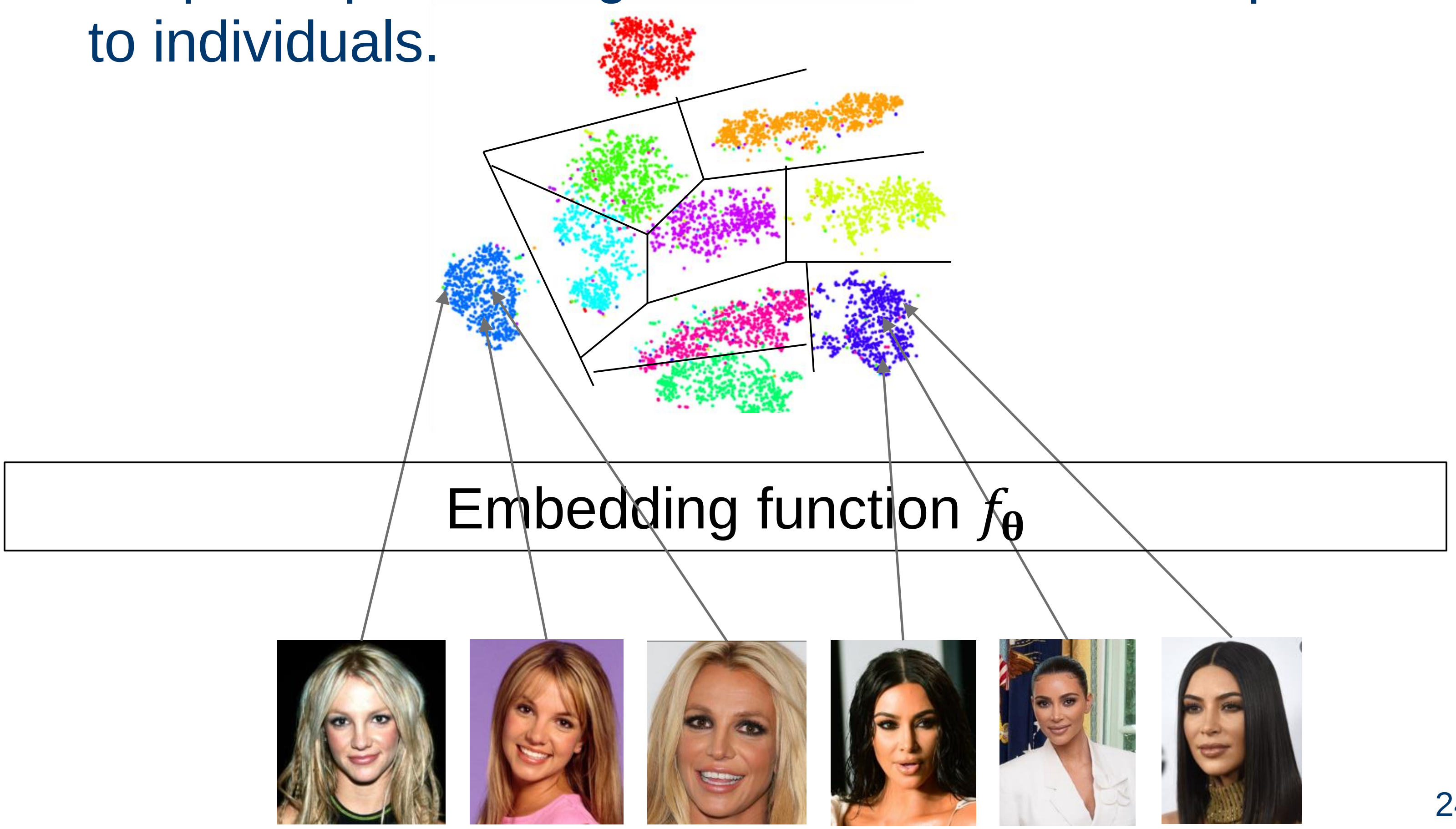
Cluster Analysis: Email Campaign Example

- Input: a stream of emails.
- Output: a partitioning into subsets that belong to the same email campaign.



Example: Clustering Embedding Space

- Input: embeddings $f(\mathbf{x})$ of images.
- Output: a partitioning into clusters that correspond to individuals.



Reinforcement Learning: Learning to Control a System

- Suppose there is a system with control parameters.
- A utility function describes the desired system behavior.
- Changes of the control parameters may have time-lagging effects.
- The Learner must experiment with the system to find a model that achieves the desired behavior (Exploration).
- At the same time, the system should be kept in the best state that is possible (Exploitation).

Reinforcement Learning: Learning to Control a System: Example

- Advertisement (Ad) placement.
- To learn which ads the user clicks, the learner must experiment.
- However, when the learner experiments with using ads other than the most popular ones, sales are lost.

Taxonomy of Learning Problems

- Supervised: Training data contain values for variable that model has to predict
  - Classification: categorial variable
  - Regression: continuous variable
  - Ordinal regression, finite, ordered set of values
  - Rankings: ordering of elements
  - Structured prediction: sequence, tree, graph, …
  - Recommendation: Item-by-user matrix

Taxonomy of Learning Problems

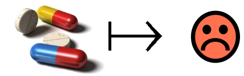
- Unsupervised: discover structural properties of data
  - Clustering
  - Unsupervised feature learning: find attributes that can be used to describe the data well
- Control / reinforcement learning: learning to control a dynamical system

Overview

- Types of learning problems:
  - Supervised Learning (Classification, Regression, Ordinal Regression, Recommendations, Sequences und Structures)
  - Unsupervised Learning
  - Reinforcement-Learning (Exploration vs. Exploitation)
- Models
- Regularized empirical risk minimization
  - Loss functions,
  - Regularizer
- Evaluation

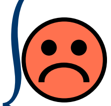
Classifier Learning

- Input to the Learner: Training data T_n.
  - $\mathbf{X} = \begin{pmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{pmatrix}$ 
  - $\mathbf{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$ 
- Training Data: $T_n = \{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_n, y_n)\}$

- Output: a classifier
  -  $y_\theta : X \to Y$
- How can a learning algorithm learn a model (classifier) from the training data?
- This is a search problem in the space of all models.

Model or Parameter Space

- Model space, parameter or hypothesis space Θ:
  - The classifier has parameters $\boldsymbol{\theta} \in \Theta$.
  - Θ is a set of models (classifiers), which are suitable for a learning method.
  - The model space is one of the degrees of freedom for maschine learning; there are many commonly used spaces.
  - Also called *Language Bias*
- Example:
  - Linear models



$$y_{\boldsymbol{\theta}}(\mathbf{x}) = \begin{cases} & \text{if } \sum_{j=1}^{m} x_j \theta_j \geq \theta_0 \\ & otherwise \end{cases}$$

Loss Function, Optimization Criterion

- Learning problems will be formulated as optimization problems.
  - The *loss function* measures the goodness-of-fit a model has to the observed training data.
  - The *regularization function* measures, whether the model is *likely* according to our prior knowledge.
  - The *optimization criterion* is a (weighted) sum of the losses for the training data and the regularizer.
  - We seek the model that minimizes the optimization criterion.

- Learning finds the overal most likely model given the training data and prior knowledge.

Loss Function

- Loss function: How bad is it if the model predicts value $y_{\boldsymbol{\theta}}(\mathbf{x}_i)$ when the true value of the target variable is y_i?

$$\ell(y_{\boldsymbol{\theta}}(\mathbf{x}_i), y_i)$$

- We average the loss over the entire training data T_n:
  - Empirical risk $\hat{R}(\boldsymbol{\theta}) = \frac{1}{n}\sum_{i=1}^{n} \ell(y_{\boldsymbol{\theta}}(\mathbf{x}_i), y_i)$
- Example: Binary classification problem with positive class (+1) and negative class (-1). False positives and false negatives are equally bad.
  - Zero-One Loss: $\ell_{0/1}(y_{\boldsymbol{\theta}}(\mathbf{x}_i), y_i) = \begin{cases} 0 & \text{if } y_{\boldsymbol{\theta}}(\mathbf{x}_i) = y_i \\ 1 & otherwise \end{cases}$

Loss Function

- Example: in diagnostic classification problems, an overlooked illness (false negative) is worse than an incorrectly diagnosed one (false positive).
  - Cost matrix

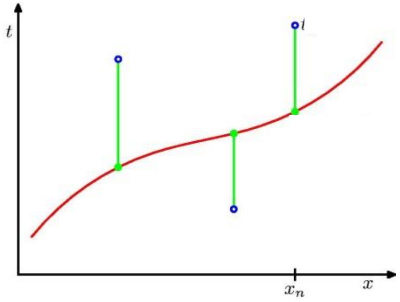
$$\ell_{c_{FP},c_{FN}}(y_{\boldsymbol{\theta}}(\mathbf{x}_i), y_i) = \begin{cases} & \\ y_{\boldsymbol{\theta}}(\mathbf{x}_i) = +1 \\ y_{\boldsymbol{\theta}}(\mathbf{x}_i) = -1 \end{cases}$$

	$y_i = +1$	$y_i = -1$
$y_{\boldsymbol{\theta}}(\mathbf{x}_i) = +1$	0	c_{FP}
$y_{\boldsymbol{\theta}}(\mathbf{x}_i) = -1$	c_{FN}	0

Loss Function

- Example of a loss function for regression: the prediction should be as close as possible to the actual value of the target attribute.
  - Quadratic error:

$$\ell_2(y_{\boldsymbol{\theta}}(\mathbf{x}_i), y_i) = (y_{\boldsymbol{\theta}}(\mathbf{x}_i) - y_i)^2$$



Loss Function

- How bad is it if the model predicts value y' when the true value of the target variable is y?
  - Loss: $\ell(y', y)$

- The selected loss function is motivated by the particular application.

Search for a Model

- Search for a Classifier of “toxic combinations“.
- Model space:

$$y_{\boldsymbol{\theta}}(\mathbf{x}) = \begin{cases} ☹ & \text{if } \sum_{j=1}^{m} x_j\theta_j \geq \theta_0 \\ ☺ & \textit{otherwise} \end{cases}$$

Training data

Medications in the combination

Sample Combinations

	x_1	x_2	x_3	x_4	x_5	x_6	y
$\mathbf{x}_1$	1	1	0	0	1	1	☹
$\mathbf{x}_2$	0	1	1	0	1	1	☹
$\mathbf{x}_3$	1	0	1	0	1	0	☺
$\mathbf{x}_4$	0	1	1	0	0	0	☺

- Approach: empirical risk should be minimal

$$\boldsymbol{\theta}^* = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \sum_{i=1}^{n} \ell_{0/1}(y_{\boldsymbol{\theta}}(\mathbf{x}_i), y_i)$$

- Is there such a model? Are there many?

Search for a Model

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- Models with 0 loss:
  - $y_{\boldsymbol{\theta}}(\mathbf{x}) =$ ☹, if $x_6 \geq 1$
  - $y_{\boldsymbol{\theta}}(\mathbf{x}) =$ ☹, if $x_2 + x_5 \geq 2$
  - $y_{\boldsymbol{\theta}}(\mathbf{x}) =$ ☹, if $2x_4 + x_6 \geq 1$
  - …

Training data

Medications in the combination

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$\mathbf{x}_1$	1	1	0	0	1	1	☹
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$\mathbf{x}_3$	1	0	1	0	1	0	☺
$\mathbf{x}_4$	0	1	1	0	0	0	☺

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$\mathbf{x}_3$	1	0	1	0	1	0	☺
$\mathbf{x}_4$	0	1	1	0	0	0	☺

- The models with an empirical risk of 0 form the *version space*.
- The version space is empty for a set of contradictory data.

Search for a Model

- Search for a Classifier of “toxic combinations“.
- Model space:

$$y_{\boldsymbol{\theta}}(\mathbf{x}) = \begin{cases} ☹ & \text{if } \sum_{j=1}^{m} x_j\theta_j \geq \theta_0 \\ ☺ & \textit{otherwise} \end{cases}$$

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Training data

Medications in the combination

Sample Combinations

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Training data

Medications in the combination

Sample Combinations

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$\mathbf{x}_1$	1	1	0	0	1	1	☹
$\mathbf{x}_2$	0	1	1	0	1	1	☹
$\mathbf{x}_3$	1	0	1	0	1	0	☺
$\mathbf{x}_4$	0	1	1	0	0	0	☺

- The models of the version space differ in their predictions of some instances, which do not appear in the training set.
- Which is the correct one?

Uncertainty

- In practice, one can never be certain whether a correct model has been found.
- Data can be contradictory (e.g. due to measurement errors)
- Many different models may achieve a small loss.
- The correct model perhaps may not even lie in the model space.

- Learning as an *optimization problem*
  - Loss function: Degree of consistency with the training data
  - Regularizer: a priori probablity of a model

Regularizer

- Loss function expresses how good the model fits the data.
- Regularisierer $\Omega(\boldsymbol{\theta})$:
  - Expresses assumptions about whether the model $\boldsymbol{\theta}$ is *a priori* probable.
  - Ω is independent from the training data.
  - The higher the regularization term is for a model, the less likely the model is.
- Often the assumptions express that few attributes should be sufficient for a suitable model.
  - Count of the non-zero weights, L_0-Regularization
  - Sum of the attribute weights, L_1-Regularization
  - Sum of the squared attribute weights, L_2-Regularization.

Regularizer

- Candidates: $(\theta_0, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6\,)$
  - $y_{\boldsymbol{\theta}}(\mathbf{x}) =$ ☹ , if $x_6 \geq 1$ $\quad \boldsymbol{\theta}_1 = (1,0,0,0,0,0,1)^{\mathrm{T}}$
  - $y_{\boldsymbol{\theta}}(\mathbf{x}) =$ ☹ , if $x_2 + x_5 \geq 2$ $\quad \boldsymbol{\theta}_2 = (2,0,1,0,0,1,0)^{\mathrm{T}}$
  - $y_{\boldsymbol{\theta}}(\mathbf{x}) =$ ☹ , if $2x_4 + x_6 \geq 1$ $\quad \boldsymbol{\theta}_3 = (1,0,0,0,2,0,1)^{\mathrm{T}}$
- Regularizer:

L_0-Regularisierung	L_1-Regularisierung	L_2-Regularisierung
$\Omega_0(\boldsymbol{\theta}_1) = 2$	$\Omega_1(\boldsymbol{\theta}_1) = 2$	$\Omega_2(\boldsymbol{\theta}_1) = 2$
$\Omega_0(\boldsymbol{\theta}_2) = 3$	$\Omega_1(\boldsymbol{\theta}_2) = 4$	$\Omega_2(\boldsymbol{\theta}_2) = 6$
$\Omega_0(\boldsymbol{\theta}_3) = 3$	$\Omega_1(\boldsymbol{\theta}_3) = 4$	$\Omega_2(\boldsymbol{\theta}_3) = 6$

Optimization Criterion

- Regularized empirical risk: Trade-off between average loss and regularizer

$$\frac{1}{n}\sum_{i=1}^{n}\ell(y_{\boldsymbol{\theta}}(\mathbf{x}_i), y_i) + \lambda\Omega(\boldsymbol{\theta})$$

- The parameter $\lambda > 0$ controls the trade-off between loss and the regularizer.

Optimization Problem

- Is there a reason to use this optimization criterion (a *regularized* empirical risk)?
- There are several justifications and derivations:
  - Most probable (a posteriori) model (*MAP-Model*).
  - One can obtain a smaller upper bound for the error on future data depending on $|\boldsymbol{\theta}|$ (*SRM*).
  - Learning without regularization is an *ill-posed* problem; there is no unique solution or it is strongly influenced by minimal changes of the data.

Regularized Empirical Risk Minimization

- Search for a Classifier of “toxic combinations“.

Training data

Medications in the combination

Sample Combinations	x_1	x_2	x_3	x_4	x_5	x_6	y
$\mathbf{x}_1$	1	1	0	0	1	1	☹
$\mathbf{x}_2$	0	1	1	0	1	1	☹
$\mathbf{x}_3$	1	0	1	0	1	0	☺
$\mathbf{x}_4$	0	1	1	0	0	0	☺

- Model space:

$$y_{\boldsymbol{\theta}}(\mathbf{x}) = \begin{cases} ☹ & \text{if } \sum_{j=1}^{m} x_j \theta_j \geq \theta_0 \\ ☺ & \textit{otherwise} \end{cases}$$

- Regularized empirical risk:

$$\frac{1}{n} \sum_{i=1}^{n} \ell_{0/1}(y_{\boldsymbol{\theta}}(\mathbf{x}_i), y_i) + 0.1\Omega_0(\boldsymbol{\theta})$$

- Model with minimal regularized empirical risk
  - $y_{\boldsymbol{\theta}}(\mathbf{x}) = \begin{cases} ☹ & \text{if } x_6 \geq 1 \\ ☺ & \textit{otherwise} \end{cases}$

Evaluation of Models

- How good will a model function in the future?
- Future instances will be drawn according to an (unknown) probability distribution $p(\mathbf{x}, y)$.
- Risk: expected loss under distribution $p(\mathbf{x}, y)$.

$$R(\boldsymbol{\theta}) = \sum_{y} \int \ell\Big((y_{\boldsymbol{\theta}}(\mathbf{x}), y\,)\Big) p(\mathbf{x}, y) d\mathbf{x}$$

- Is the empirical risk on the training data a useful estimator for the risk?

Evaluation of Models

- How good will a model function in the future?
- Future instances will be drawn according to an (unknown) probability distribution $p(\mathbf{x}, y)$.
- Risk: expected loss under distribution $p(\mathbf{x}, y)$.
- Is the empirical risk on the training data a useful estimator for the risk?
  - Problem: All models in the version space have an empirical risk of 0 on the training data.
  - A classifier can achieve 0 empirical risk on the training data by simply storing each training instance in a table and reproducing the stored label when queried.

Evaluation of Models

- How good will a model function in the future?
- Future instances will be drawn according to an (unknown) probability distribution $p(\mathbf{x}, y)$.
- Risk: expected loss under distribution $p(\mathbf{x}, y)$.
- Empirical risk on the training data is an extremely optimistic estimator for the risk.
- Risk is evaluated using instances that were not used for training.
  - Training and Test datasets.
  - N-fold cross validation.

Optimization Problem

- How should λ be set?
- Divide available data into training and test data.
- Iterate over values of λ
  - Train on training data to find a model
  - Evaluate it on the test data
- Choose the value of λ giving minimal loss
- Train with all data

Data, Models, & Learning Problems

- Supervised Learning: Find the function most likely to have generated the training data.
- Loss function: Measures the agreement between the model's predictions and the values of the target variable in the training data.
- Regularizer: Measures the agreement to prior knowledge.
- Unsupervised Learning: with no target variable, discover structure in the data; e.g., by dividing the instances into clusters with common properties.
- Reinforcement Learning: Control of processes.