lecture 2024-01-18

binary stars

binary stars simulation in different cases

$$Y_e = \frac{n_e}{n_p + n_n}$$

electron fractions

$Y_e \approx 0.1$ Neutron stars

$Y_e \approx 0.5$

equation of state (EOS) using tables of pressure and entalpie

$$P, \varepsilon \ldots (\rho, T, Y_e)$$

Nucleosynthesys

$$Y_e \leq 0.25$$

Kilonova: merge of two neutron stars
the radiation magnitude is decreasing

$$\nu_e + n \leftrightarrow p + e$$

$$\bar{\nu}_e + p \leftrightarrow n + e^+$$

changing of the particles in the star

conservation law

$$\nabla_\mu (\underbrace{\rho Y_e} u^\mu) = S$$

$$n_b m_b \frac{n_p}{n_b} = \rho_p$$

conservation of binary density

rest mass density

a source term in the conservation law

include neutrino interaction

$$\nabla_\mu T^{\mu\nu} = -\Psi^\nu$$

leakage: neutrino cooling

$$p + e^- \rightarrow n + \nu_e$$

$$n + e^+ \rightarrow p + \bar{\nu}_e$$

ignoring the reverse interaction

$$\nabla_\mu (\rho Y_e u^\mu) = \rho R_{eff}$$

$\nu$ neutrino emission $/ n_b$ $/ J$

$$\Psi^\nu \rightarrow \Psi^\nu_{eff}$$

neutrinos in thermal equilibrium

$$n_{\nu_i}(\varepsilon, T, \mu_{\nu_i}) = \frac{4\pi}{(hc)^2}\frac{\varepsilon^2}{1+e^{\frac{\varepsilon-\mu}{T}}}$$

$\nu_i \in \{\nu_e, \bar{\nu}_e\}$

neutrino emission isotropic

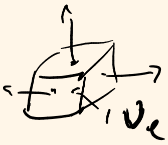
energy of neutrinos

$$\Psi^{\nu}_{eff} = n_b Q_{eff} u^{\nu}$$

$$Q_{eff} = \sum_{\nu_i} Q_{eff}$$

$$R_{eff} = R_{eff}(\bar{\nu}_e) - R_{eff}(\nu_e)$$

free streaming:



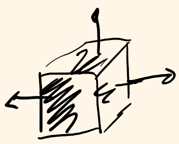
$\nu_e$ emission per volume element

$$Q_{eff,\nu_i} = Q^F_{\nu_i} = \sum_r Q_r$$

emission rates (from cross sections)

$$R_{eff} = R^F_{eff} = \sum_r R^{\nu_i}_{eff}$$

diffusive regime
very hgh density



mean free streaming path $\lambda$

$$\lambda^{-1} = n_p \sigma_\nu(p) + n_n \sigma_\nu(n)$$

cross section

$\sigma_\nu \sim \varepsilon^2$ $\quad\quad$ $\varepsilon$ = $\nu$ energy

$$\lambda \sim \varepsilon^{-2}$$

optical depth $\tau$ opacity of medium

$$\tau(\varepsilon) = \int_s^{\infty} \frac{ds}{\lambda(\varepsilon)} = \int_s^{\infty} \varepsilon^2 \Big(\underbrace{\frac{1}{\varepsilon^2 \lambda(\varepsilon)}}_{\xi}\Big) ds = \varepsilon^2 \overbrace{\int_s^{\infty} \xi \, ds}^{\chi}$$

energy independent mean free path

calculation by Rossvvog 2009

$$Q_{eff} = Q_{eff}^{D} = \frac{4\pi}{(hc)^3} \frac{\xi}{3\chi^2} T^2 F_1\left(\frac{\mu}{T}\right)$$

emission
number rate of escape $R_{eff}$

$$R_{eff} = R_{eff}^{D} = \frac{4\pi}{(hc)^3} \frac{\xi}{3\chi^2} T F_0\left(\frac{\mu}{T}\right)$$

$$F_k(\eta) = \int_0^{+\infty} \frac{x^k}{1 + e^{x-\eta}} dx$$

$$Q_{eff} = \frac{Q^F Q^D}{Q^F + Q^D} \qquad R_{eff} = \frac{R^F R^D}{R^F + R^D}$$

free streaming $\chi \ll 1$ $Q^D \gg Q^F$

diffusion

$$Q_{eff} = Q^F$$

thick limit: $\chi \gg 1$ $Q^D \ll Q^F$

$$Q_{eff} = Q^D$$

no neutrino heating:

no causality

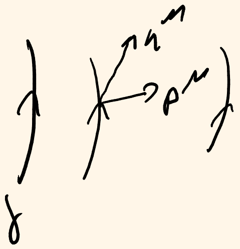
arbitrary $$Q_{ext} = A(\tau)\, Q^{F} + B(\tau)\, Q^{D}$$

Transport of neutrinos:

$$\int f_\nu (x^\mu, p^\mu)\, dx^\mu\, dp^\mu = N$$

momentum

general relativistic Bolztmann equation:



$$p = \frac{d}{d\varepsilon}$$

$$dl = -p^\mu u_\mu d\xi$$

length of line

$$\frac{d f(x^\mu, p^\mu)}{dl} = S[f](x^\mu, p^\mu)$$

linear tion of neutrinos

exact solutions are possible

$$p^\mu p_\mu = 0$$

$$p^0 \nu_0 = 0$$

$$\frac{d f(x^\mu, p^i)}{dl} = S[f](x^\mu, p^\mu)$$

$\Rightarrow$ $3 + 3 + 1$

place, momentum, time

order of momentum here the first order of momentum

M1:

$$\mu^{\alpha_1 \ldots \alpha_k}(x^\beta) := \int_\nu \int_\Omega \nu^3 \frac{f(x^\mu, p^i)}{\nu^k} p^{\alpha_1} \cdots p^{\alpha_n}\, d\nu\, d\Omega$$

$$p^\alpha = \nu (u^\alpha + l^\alpha) \qquad l^\mu u_\mu = 0$$

frequency nr with $E = h\nu$ $\rightarrow$ $E \sim \nu$ so energy second order momentum

$$M^{\alpha\beta} = J u^\alpha u^\beta + H^\alpha u^\beta + H^\beta u^\alpha + S^{\alpha\beta}$$

$$J = \int_\nu \nu^3 \int_\Omega f(\nu, \Omega, x^\mu)\, d\nu\, d\Omega$$

Energy

$$H^\alpha = \int_\nu \nu^3 \int_\Omega f(\nu, \Omega, x^\mu)\, \ell^\alpha\, d\nu\, d\Omega$$

momentum

$$S^{\alpha\beta} = \int_\nu \nu^3 \int_\Omega f(\nu, \Omega, x^\mu)\, \ell^\alpha \ell^\beta\, d\nu\, d\Omega$$

$$M^{\alpha\beta} = T_\nu^{\alpha\beta}$$

frequency no 4-index

$$T^{\alpha\beta} = E n^\alpha n^\beta + F^\alpha n^\beta + F^\beta n^\alpha + P^{\alpha\beta}$$

labour frame Energy momentum Stress energy tensor

$$(E, F^\alpha, P^{\alpha\beta}) \to (J, H^\alpha, S^{\alpha\beta})$$

$$\frac{df}{dl} = S[f]$$

$$\nabla_\beta M^{\alpha_1 \dots \alpha_k \beta} - (k-1) M^{\alpha_k \beta\delta} \nabla_\delta u_\beta = S^{\alpha_1 \dots \alpha_k}$$

$k = 1$ $\Rightarrow 0$

$$\nabla_\beta M^{\beta\alpha} = S^\alpha$$

Shibata 2011

$$\partial_t(\sqrt{\gamma}E) + \partial_j[\underbrace{\sqrt{\gamma}(\alpha F^j - \beta^j E)}_{\text{flux}}] = \alpha\sqrt{\gamma}[P^{ij}K_{ij} - F^j\partial_j \ln\alpha - S^\alpha n_\alpha]$$

$$\partial_t(\sqrt{\gamma}F_i) + \partial_j(\sqrt{\gamma}(\alpha P^j_i - \beta^j F_i)) = \sqrt{\gamma}[-E\partial_i\alpha + F_k\partial_i\beta^k + \frac{\alpha}{2}P^{jk}\partial_i\gamma_{jk} + \alpha S^\alpha\gamma_{i\alpha}]$$

$$S^\alpha = \eta u^\alpha - k_a J u^\alpha - (k_a + k_s)H^\alpha$$

$$\nabla_\alpha M^{\alpha\beta} = \nabla_\alpha T^{\alpha\beta}_\nu = S^\beta$$

$$\nabla_\alpha T^{\alpha\beta}_{\cancel{tot}} = \nabla_\alpha(T^{\alpha\beta}_{fluid} + T^{\alpha\beta}_\nu) = 0$$

$$\nabla_\alpha T^{\alpha\beta}_{fluid} = -\underbrace{\nabla_\alpha T^{\alpha\beta}_\nu}_{S^\beta} = -S^\beta$$

$$\nabla_\alpha(\rho Y_e u^\alpha) = m_b R$$

lumnosity emission rate from Neutrinos

$$R = \sum_{\nu_i} \mathrm{sign}(\nu_i)\frac{\eta_{\nu_i} - \cancel{k}\,\kappa_{,\nu_i} J_{\nu_i}}{\langle\epsilon_{\nu_i}\rangle}$$

M1 scheme

1 grey: $k_a$, $k_s(\epsilon)$ opacity   $f_\nu(\epsilon)$   $J = \int_\epsilon$

$k_{a,s} \sim \sigma \sim \epsilon^2$

thermal equilibrium, but Neutrinos not the most of the time in this state

2. closure

$\partial_t E, \partial_t F_i \qquad (P^{ij})$

- free streaming $\quad P^{ij} = E \frac{F^i F^j}{F^2}$

$E, F_x$

$$\partial_t E + \partial_x E = 0$$

$$\partial_t F_x + \partial_x F_x = 0$$

$E = |F|$

$$(E, F^i, P^{ij}) \longleftrightarrow (J, H^\alpha, S^{\alpha\beta})$$

$$S^{\alpha\beta} = \frac{1}{3} J h^{\alpha\beta} \qquad h^{\alpha\beta} = g^{\alpha\beta} + u^\alpha u^\beta$$

$\downarrow$ $P^{ij}$ projection metric