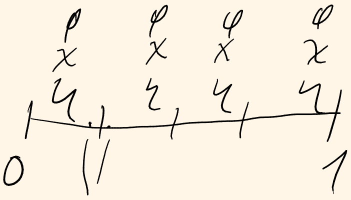
lecture 2023-11-23

Implementation for mpi



communicate the boundary points

boundary corrections

2.1 Reminder of black hole simulations

2.1.1. general relativity

4d dimensions proper seperation: $ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$ $g_{\mu\nu}$ signature $= (-+++)$

$ds^2 < 0$ timelike seperated
$ds^2 = 0$ light –||–
$ds^2 > 0$ space like seperated

Einstein field equation: $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$

$R, R_{\mu\nu}(g_{\mu\nu}, \partial g_{\mu\nu}, \partial\partial g_{\mu\nu})$ $T_{\mu\nu}$ stress energy momentum tensor

in vacuum $T_{\mu\nu} = 0$

perfect fluid $T_{\mu\nu} = (\underbrace{\overset{\text{density}}{\rho} + \underset{\text{pressure}}{p}})\, \underbrace{u_\mu u_\nu}_{\text{flow velocity}} + P g_{\mu\nu}$

$\mu,\nu = 0,1,2,3$ $i,j = 1,2,3$

T^{00} energy density
T^{i0} momentum density
T^{ij} flux of momentum i in direction j

four velocity $u^\mu = \frac{dx}{d\tau}$ $u^\mu u_\mu = -1$, is fixed
rest frame $u^\mu = (1,0,0,0)$
four-momentum $p^\mu = m u^\mu$

n-particles: number flux 4-vector $N^\mu = n u^\mu$

density $\rho = n m$

$T^{00}_{rest} = \rho$ all other $T^{\mu 0}_{rest} = 0$ $\qquad T^{\mu\nu} = \rho u^\mu u^\nu$

perfect fluid not shear force
pushing force

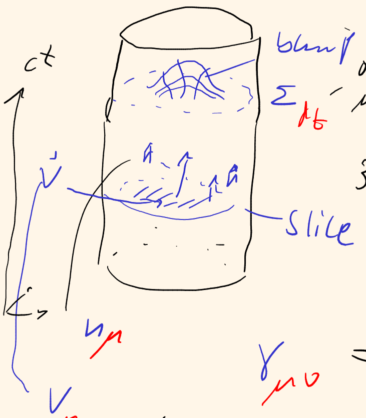
$$T^{\mu\nu}_{rest} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

$$\eta^{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

2.1.2. 3+1 decomposition

$R = R_{\mu\nu} g^{\mu\nu}$

$R_{\mu\nu} = R^\rho{}_{\mu\rho\nu}$



Σ_{dt} - n-Mannigfold spacetime

3D Zylinder

$g_{\mu\nu}$

$\Sigma_0, \gamma_{\mu\nu}$ $\qquad \gamma_\mu{}^\nu = \delta_\mu^\nu + n^\nu n_\mu$

$\gamma_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu \qquad n^\mu n_\mu = -1$

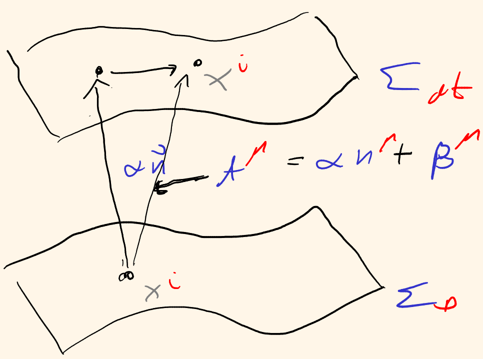
V_μ spatial vector only in Σ_0 $\quad$ V spacelike (in Σ_0) $\quad n^\mu V_\mu = 0$

$V^\mu = \delta^\mu_\nu V^\nu = \gamma^\mu_\nu V^\nu + n^\mu n_\nu V^\nu$

evolving in time of the normal vector you need the extrinsic curvature $K_{\mu\nu}$

$$K_{\mu\nu} = \gamma^\rho_\mu \gamma^\eta_\nu \nabla_\rho n_\eta = -\frac{1}{2}\alpha \quad \gamma_{\mu\nu}$$

move from one slice to the other slice



you can choose in every slice new coordinates

$$g_{\mu\nu} = \left(\begin{array}{c|c} -\alpha^2 + \beta_i\beta^i & \beta_i \\ \hline \beta_i & \gamma_{ij} \end{array}\right)$$

2.1.3. evolution equations

from $K_{\mu\nu} = -\frac{1}{2}\alpha_n \gamma_{\mu\nu}$ we get $\alpha_n \gamma_{ij} = -2\alpha K_{ij} + D_i \beta_j + D_j \beta_i$

derivative

$G_{\mu\nu} = 8\pi T_{\mu\nu}$ contract with $\gamma^\mu_i \gamma^\nu_j$

$$\partial_t K_{ij} = \beta^\nu \partial_n K_{ij} + K_{ni} \partial_j \beta^n + K_{nj} \partial_i \beta^n - D_i D_j \alpha + \alpha \left({}^{(3)}R_{ij} + K K_{ij} - 2 K_{in} K^n_j \right) +$$

$$+ 4\pi\alpha \left(\gamma_{ij} (S - \rho) - 2 S_{ij} \right)$$

$S_{ij} = T_{nr} \gamma^n_i \gamma^r_j$ $\quad S = S_{ij} \gamma^{ij}$

$\rho = n^a n^b T_{ab}$

matter in your slice

2.1.4. constraint equations

$$\left(G_{ab} = 8\pi T_{ab} \right) n^a n^b = \mathcal{H} = {}^{(3)}R + K^2 - K_{\mu\nu} K^{\mu\nu} - 16\pi\rho = 0$$

Hamiltonian constraint conservation of energy

$$\left(G_{ab} = 8\pi T_{ab} \right) n^a \gamma^b = \mathcal{M}^i = D_j \left(K^{ij} - \gamma^{ij} K \right) - 8\pi j^i = 0$$

$j^i = -T_{ab} n^a \gamma^{ai}$

momentum constraint