Quantum Gravity

https://pirsa.org/20070001 talk he has given
Klaus Kiefer: Quantum Gravity (2004) book

Disclaimer: we do not have a theory of Quantum Gravity (QG)!
No consens about what it would be

Foundamental constants

G Newtonian gravity
c special relativity } GR
$\hbar$ Quantum theory

} Quantum field theory (QFT) } expectation QG
Bogomolstein ...

Planck units (1899):
at which scale a theory comes relevant

$$L_P = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35}\,m \quad \text{Planck length}$$

$$M_P = \sqrt{\frac{\hbar c}{G}} \sim 10^{-8}\,kg \sim 10^{19}\,GeV = 10^{16}\,\underbrace{TeV}_{LHC} \quad \text{Planck mass}$$

$$t_P = \sqrt{\frac{\hbar G}{c^5}} \sim 10^{-43}\,s \quad \text{Planck time}$$

1 TeV - energy can be measured
=> no direct tests of QG!

$$T_P = \frac{M_P c^2}{k_B} \sim 10^{32}\,{}^\circ K \quad \text{Planck temperature}$$

What is the rpoblem? Why should we do it?

$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\substack{\text{space-time}\\ \text{geometry}\\ \text{classical}}} = \frac{8\pi G}{c^4} \underbrace{T_{\mu\nu}}_{\substack{\text{matter}\\ \text{quantum}}} \leftarrow \langle \Psi | T_{\mu\nu} | \Psi \rangle \,?$$

conceptional issue

both GR and QFT are incomplete (inconsistent?)

GR:
- singularity theorems:
spacetime singularities unavoidable?

collapse of atoms
decay
Quantum mechanics solved this problem

QFT:
- UV divergences → renormalization in perturbation
Standard model of particle physics
may not exists as mathematical theory!

pertubativly quantized GR is non-renormalizable

$$g_{\mu\nu} = \eta_{\mu\nu} + \underset{\ll 1}{\kappa}\, h_{\mu\nu} \leftarrow \text{spin 2 field}$$

$$\frac{1}{\kappa^2}\int \sqrt{g}\, R = \underset{\text{Einstein Hilbert action}}{S_{EH}} = \int \{ \partial h\, \partial h + \underbrace{\kappa h \partial h \partial h + \kappa^2 h^2 \partial h \partial h + \dots}_{\text{complicated !}} \}$$

one loop calculation very complicated

two loops: need counter term $\Gamma^{(2)}_{div} = \frac{1}{\epsilon}\,\frac{209}{2880}\int R_{\mu\nu\rho\sigma} R^{\rho\sigma\lambda\tau} R_{\lambda\tau}{}^{\mu\nu}$

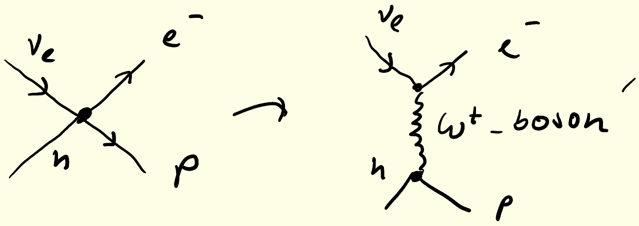
ε- dimensional regularisation $\varepsilon = D - 4$

not predictive ! you have to fix an infinite amount of parameters

(1) particle physics strategy

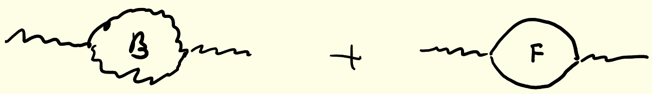
amend Einstein's theory

famous example: β - decay



renormalizable theory
⇒ Standard model of particle physics

eliminate divergences by adding fermions in calculations

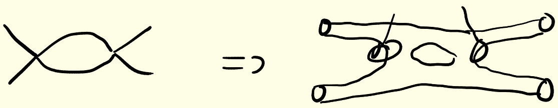


⇒ supersymmetry !

⇒ supergravity = supersymmetric GR

there is a limit for bringing new symmetries
-> most "symmetric" extension of GR N=8 Supergravity
impossible to ask, if it is possible to eliminate all infinities
finit up to $L \leq 5$ loops point to rotation of points

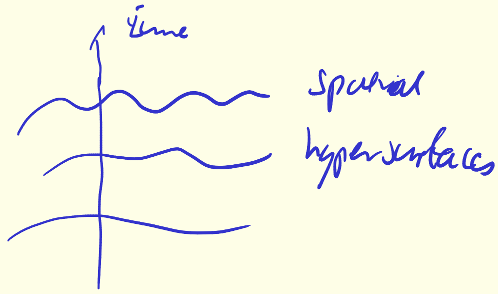
superstring theory



(2) Canonical quantization strategy

keep Einstein and quantize non-pertubatively
go to phase space

$q, p \to$ operators $q, p = i\hbar \partial_q$



broke spacetime covariance

$g_{\mu\nu} = g_{\mu\nu}(t, \vec{x})$ $\quad$ g_{00}, g_{0m} $\quad$ $m = 1, 2, 3$ no time dependency

Lagrange multiplier fields $\to$ constraints

$$"q" = g_{mn}(\vec{x}) \qquad \pi^{mn}(\vec{x}) = \frac{\delta S_{EH}}{\delta \dot{g}_{mn}}$$

Hamiltonian constraint from g_{00},

$$\mathcal{H}(\vec{x}) = G_{mn,pq}\, \pi^{mn}\pi^{pq} - \sqrt{g^{(3)}}\, R^{(3)}$$

for each point a constraint $\Rightarrow$ infinit constraint

$$" \frac{1}{\sqrt{g}}(2 g_{mp} g_{qn} - g_{mn} g_{pq})$$

$$\pi^{mn}(\vec{x}) = -i\hbar \frac{\delta}{\delta g_{mn}(\vec{x})}$$

from that you could get Schrödinger equation

$$\forall \vec{x} \quad \mathcal{H}(\vec{x})\, \Psi(g_{mn}) = 0$$

Wheeler - DeWitt equation (1963)

timeless equation (like Schrödinger equation but without time dependency)

wavefunction of the universe

notion of time, problem of time in Quantum Gravity

loop quantum theory changing variables here, to tackle the problem

metric is flucates