2.) $x = r\cos\varphi \qquad dx = dr\cdot\cos\varphi - r\sin\varphi\, d\varphi = dr\,c - rs\,d\varphi$

$y = r\sin\varphi \qquad dy = dr\sin\varphi + r\cos\varphi\, d\varphi = dr\,s + rc\,d\varphi$

$$ds^2 = dx^2 + dy^2 = (dr\cdot c - r\,s\,d\varphi)^2 + (dr\,s + r\,c\,d\varphi)^2$$

$$= dr^2 sc + dr\,d\varphi\cdot c^2 r - rs^2 dr\,d\varphi - r^2 cs\,d\varphi^2$$

$$= dr^2 sc + dr\,d\varphi(c^2 r - rs^2) - r^2 cs\,d\varphi^2$$

$$\bar g_{\mu\nu} = \frac{\partial x^\alpha}{\partial \bar x^\mu}\frac{\partial x^\beta}{\partial \bar x^\nu} g_{\mu\nu} \qquad \alpha = 0,1 \quad x\,y \qquad x = x, y \quad \bar x = r, \varphi$$

$$\bar g_{\mu\nu} = \mathrm{diag}(1, r) \qquad g_{\mu\nu} = \mathrm{diag}(1,1)$$

3) $D_\mu A_\lambda = \partial_\mu A_\lambda - \Gamma^\rho_{\mu\lambda} A_\rho \qquad \frac{\partial x^\nu}{\partial \bar x^\mu} b_\nu = b_\mu$

$$\frac{\partial A_\lambda}{\partial x^\mu} = \frac{\partial}{\partial x^\mu}\frac{\partial x^\nu}{\partial \bar x^\lambda}\bar A_\nu = \frac{\partial \bar x^\rho}{\partial x^\mu}\frac{\partial}{\partial \bar x^\rho}\left(\frac{\partial x^\nu}{\partial \bar x^\lambda}\bar A_\nu\right)$$

$$= \frac{\partial \bar x^\rho}{\partial x^\mu}\left(\frac{\partial}{\partial \bar x^\rho}\frac{\partial x^\nu}{\partial \bar x^\lambda}\bar A_\nu + \frac{\partial x^\nu}{\partial \bar x^\lambda}\frac{\partial}{\partial \bar x^\rho}\bar A_\nu\right)$$

$$= \frac{\partial \bar x^\rho}{\partial x^\mu}\frac{\partial^2 x^\nu}{\partial \bar x^\rho \partial \bar x^\lambda}\bar A_\nu + \frac{\partial \bar x^\rho}{\partial x^\mu}\frac{\partial x^\nu}{\partial \bar x^\lambda}\frac{\partial \bar A_\nu}{\partial \bar x^\rho}$$

$$\frac{\partial \bar A_\lambda}{\partial x^\mu} - \frac{\partial \bar x^\rho}{\partial x^\mu}\frac{\partial^2 x^\nu}{\partial \bar x^\rho \partial \bar x^\lambda}\bar A_\nu = \frac{\partial \bar x^\rho}{\partial x^\mu}\frac{\partial x^\nu}{\partial \bar x^\lambda}\frac{\partial \bar A_\nu}{\partial \bar x^\rho}$$

4.)

$$\nabla_\alpha g_{\mu\nu} = 0 \qquad \nabla_\nu T_{\lambda\mu} = \partial_\nu T_{\lambda\mu} - \Gamma^{\alpha}_{\lambda\nu} T_{\mu\alpha} - \Gamma^{\alpha}_{\mu\nu} T_{\lambda\alpha}$$

$$g_{\mu\alpha} g^{\mu\beta} = \delta^{\beta}_{\alpha}$$

$$g_{\mu\nu} = g_{\nu\mu}$$

$$\nabla_\alpha g_{\mu\nu} = \partial_\alpha g_{\mu\nu} - \Gamma^{\beta}_{\nu\alpha} g_{\mu\beta} - \Gamma^{\beta}_{\mu\alpha} g_{\nu\beta}$$

$$\Gamma^{\alpha}_{\mu\beta} = \frac{1}{2} g^{\alpha\nu}(\partial_\beta g_{\mu\nu} + \partial_\mu g_{\nu\beta} - \partial_\nu g_{\beta\mu})$$

$$\Gamma^{\beta}_{\nu\alpha} = \frac{1}{2} g^{\beta\sigma}(\partial_\alpha g_{\nu\sigma} + \partial_\nu g_{\sigma\alpha} - \partial_\sigma g_{\alpha\nu})$$

$$\Gamma^{\beta}_{\mu\alpha} = \frac{1}{2} g^{\beta\sigma}(\partial_\alpha g_{\mu\sigma} + \partial_\mu g_{\sigma\alpha} - \partial_\sigma g_{\alpha\mu})$$

$$\frac{1}{2} g^{\beta\sigma}(\partial_\alpha g_{\nu\sigma} + \partial_\nu g_{\sigma\alpha} - \partial_\sigma g_{\alpha\nu}) g_{\mu\beta}$$

$$\frac{1}{2} g^{\beta\sigma}(\partial_\alpha g_{\mu\sigma} + \partial_\mu g_{\sigma\alpha} - \partial_\sigma g_{\alpha\mu}) g_{\nu\beta}$$

$$\frac{1}{2} g^{\sigma}_{\mu}(\partial_\alpha g_{\nu\sigma} + \partial_\nu g_{\sigma\alpha} - \partial_\sigma g_{\alpha\nu})$$

$$\frac{1}{2} \delta^{\sigma}_{\nu}(\partial_\alpha g_{\mu\sigma} + \partial_\mu g_{\sigma\alpha} - \partial_\sigma g_{\alpha\mu})$$

$$g^{\beta\sigma} g_{\mu\beta} \partial_\alpha g_{\nu\sigma} + g^{\beta\sigma} g_{\mu\beta} \partial_\nu g_{\sigma\alpha} - g^{\beta\sigma} g_{\mu\beta} \partial_\sigma g_{\alpha\nu}$$

$$\underbrace{\delta^{\sigma}_{\mu} \partial_\alpha g_{\nu\sigma}}_{\partial_\alpha g_{\nu\mu}} + \underbrace{\delta^{\sigma}_{\beta} \partial_\nu g_{\sigma\alpha}}_{\partial_\nu g_{\beta\alpha}} - \partial_\mu g_{\alpha\nu}$$

$$\nabla_\alpha g_{\mu\nu} = 0$$

$$\nabla_\alpha g_{\nu\mu} = 0$$

$$\nabla_\alpha g_{\nu\mu} = \partial_\nu g_{\nu\mu} - \Gamma^{\beta}_{\nu\alpha} g_{\mu\beta} - \Gamma^{\beta}_{\mu\alpha} g_{\nu\beta}$$

$$\frac{1}{2} g^{\beta\sigma} (\partial_\alpha g_{\nu\sigma} + \partial_\nu g_{\sigma\alpha} - \partial_\sigma g_{\alpha\nu}) g_{\mu\beta}$$

$$\frac{1}{2}(\delta^{\sigma}_{\mu} \partial_\alpha g_{\nu\sigma} + \delta^{\sigma}_{\mu} \partial_\nu g_{\sigma\alpha} - \partial_\mu g_{\alpha\nu})$$

$$\frac{1}{2}(\underline{\partial_\alpha g_{\nu\mu}} + \underline{\partial_\nu g_{\mu\alpha}} - \underline{\partial_\mu g_{\alpha\nu}})$$

$$\frac{1}{2} g^{\beta\sigma} (\partial_\alpha g_{\mu\sigma} + \partial_\mu g_{\sigma\alpha} - \partial_\sigma g_{\alpha\mu}) g_{\nu\beta}$$

$$\frac{1}{2}(\delta^{\sigma}_{\nu} \partial_\alpha g_{\mu\sigma} + \delta^{\sigma}_{\nu} \partial_\mu g_{\sigma\alpha} - \underline{\partial_\nu g_{\alpha\mu}})$$

$$\frac{1}{2}(\underline{\partial_\alpha g_{\mu\nu}} + \underline{\partial_\mu g_{\nu\alpha}} - \underline{\partial_\nu g_{\alpha\mu}})$$

$$\partial_\alpha g_{\mu\nu} - 2 \cdot \frac{1}{2} \partial_\alpha g_{\mu\nu} = \underline{\underline{0}}$$

5.) $T^{\alpha}{}_{\beta\gamma}$

$$\bar{T}^{\alpha}{}_{\beta\gamma} = \frac{\partial \bar{x}}{\partial x}\frac{\partial x}{\partial \bar{x}}\frac{\partial x}{\partial \bar{x}} T$$

$$\delta$$

$$T^{\alpha}{}_{\alpha\gamma} = T_{\gamma} \qquad \bar{T}^{\beta}{}_{\beta\gamma} = \frac{\partial x^{\sigma}}{\partial \bar{x}^{\gamma}} T^{\rho}{}_{\rho\sigma}$$

$$T^{\alpha}{}_{\beta\gamma} = g^{\beta}_{\alpha} T^{\alpha}{}_{\beta\gamma} = T^{\beta}{}_{\beta\gamma} = T_{\gamma}$$

6.) $T^{r} U_{ir}$ for T^{r}, U_{ir} are tensors

$$\bar{T}^{r}\bar{U}_{ir} = \frac{\partial \bar{x}^{r}}{\partial x_{\gamma}} T^{\gamma} \cdot \frac{\partial x^{\alpha}}{\partial \bar{x}_{i}}\frac{\partial x^{\beta}}{\partial \bar{x}_{r}} U_{\alpha\beta} = \frac{\partial \bar{x}^{r}}{\partial x_{\gamma}}\frac{\partial x^{\alpha}}{\partial \bar{x}^{i}}\frac{\partial x^{\beta}}{\partial \bar{x}_{r}} T^{\gamma} U_{\alpha\beta}$$

$$= \frac{\partial x^{\alpha}}{\partial x^{\gamma}}\frac{\partial x^{\beta}}{\partial \bar{x}^{i}} T^{\gamma} U_{\alpha\beta}$$

$$\delta^{\alpha}_{\gamma}$$

$$= \frac{\partial x^{\beta}}{\partial \bar{x}^{i}} T^{\alpha} U_{\alpha\beta}$$

$$\bar{B}_{i} = \frac{\partial x^{j}}{\partial \bar{x}^{i}} B_{j}$$

$$= \frac{\partial x^{\beta}}{\partial \bar{x}^{i}} B_{\beta}$$