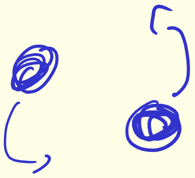
Classical Gravity from Quantum Field Theory

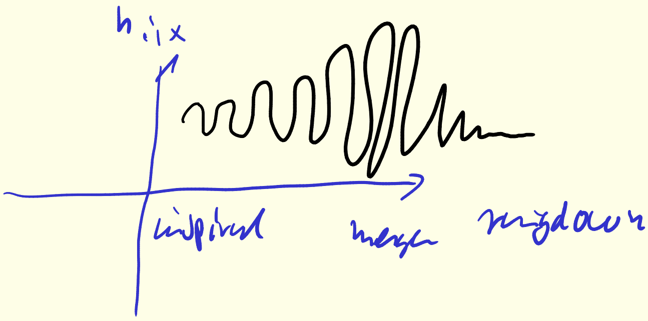
based on Rafael Porto 1601.04914 Michele Levi 1807.01699

1 Intro Binary Inspiral

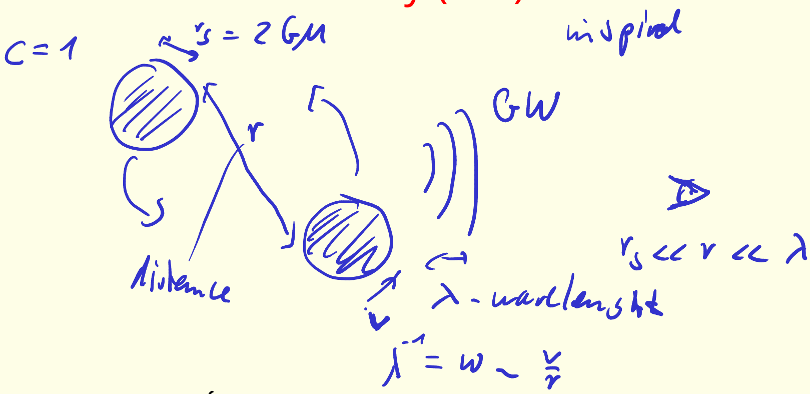


$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Fo solving use
1. numerical relativity(NR) - good for merger
2. Pertubation theory - good for inspiral



2. effective field theory (EFT)



EFT Tower

internal zone: finite-size effects
near zone: orbital scale
far zone: GW scale

$$e^{iW[x_a^\mu, h^{far}_{\mu\nu}]} = \int D h^{near}_{\mu\nu} \exp\left(i S_{tot}[g_{\mu\nu}, x_1^\mu(\tau_1), x_2^\mu(\tau_2)]\right)$$

2.1. Post-Newtonian (PN) Regime

Newtonian = OPN
... = IPN

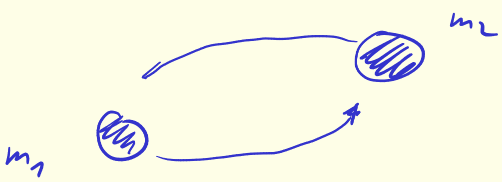
$$\epsilon = \frac{GM}{r} \sim v^2 \quad \text{virial theorem}$$

Parameter

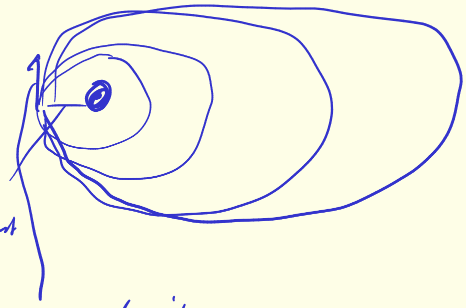
relvant to bound, circular orbits
e.g. 2-ody Hamiltonian
expanding in Epsilon to higher order in Hamiltonian

2.2. Post-Minkowskian (PM) Regime

$$\epsilon_{PM} = \frac{GM}{r} \qquad g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G_N}\, h_{\mu\nu}$$

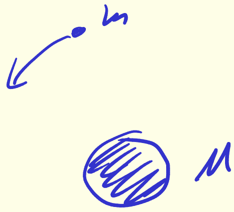


* scattering process
* weak fields, fast velocities
* infinitly igh PN orders
* 2-body Hamiltonian is universal



Gravitational self-force (GSF)

$$\varepsilon_{GSF} = \frac{m}{M} \qquad m \ll M$$



reduced to a one body problem
solved with geodesic equation
probe limit

ultimately: incorporate different regimes, move complete picture of dynamics

3. Single-particle EFT

action for point particle $\quad S_{pp} = -m\int ds = -m\int\sqrt{g_{\mu\nu}\,dx^\mu dx^\nu}$

effective field theory framework $\quad$ particle proper time

$$= -m\int d\tau\,\sqrt{g_{\mu\nu}\,\dot x^\mu \dot x^\nu} \qquad \dot x^\mu = \frac{dx^\mu}{d\tau}$$

$$S_{pp} = -\frac{m}{2}\int d\tau\, g_{\mu\nu}\,\dot x^\mu \dot x^\nu \qquad \text{Polyakov form}$$

$$g_{\mu\nu}(x^\sigma)$$

$$\Rightarrow \frac{d}{d\tau}\left(\frac{\partial L}{\partial \dot x^\mu}\right) = \frac{\partial L}{\partial x^\mu} \qquad \frac{\partial L}{\partial x^\mu} = \frac{\partial g_{\nu\sigma}}{\partial x^\mu}\dot x^\nu \dot x^\sigma$$

$$\frac{\partial L}{\partial \dot x^\mu} = 2 g_{\mu\nu}\dot x^\nu \Rightarrow \frac{d}{d\tau}\left(\frac{\partial L}{\partial \dot x^\mu}\right) = 2 g_{\mu\nu}\ddot x^\nu + 2\,\frac{\partial g_{\mu\nu}}{\partial x^\rho}\dot x^\nu \dot x^\rho$$

$$\ddot x^\mu(\tau) = -\Gamma^\mu_{\nu\rho}\,\dot x^\nu \dot x^\rho$$

$$\Gamma^\mu_{\nu\rho} = \frac{1}{2} g^{\mu\sigma}\left(\frac{\partial g_{\sigma\nu}}{\partial x^\rho} + \frac{\partial g_{\sigma\rho}}{\partial x^\nu} - \frac{\partial g_{\nu\rho}}{\partial x^\sigma}\right)$$

$$S = m\int d\tau \left[\frac{1}{2} g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu + C_{E^2} E_{\mu\nu} E^{\mu\nu} + C_{B^2} B_{\mu\nu} B^{\mu\nu} + \dots \right]$$

$R_{\mu\nu} = 0 \quad R, R_{\mu\nu}, R_{\mu\nu\rho\sigma}$ $\quad E_{\mu\nu} = R_{\mu\alpha\nu\beta}\dot{x}^\alpha \dot{x}^\beta \quad [E_{\mu\nu}] = 2$ mass dimension

$R = g^{\mu\nu}R_{\mu\nu}$ $\quad B_{\mu\nu} = \epsilon_{\alpha\beta\gamma\mu} R^{\alpha\beta}{}_{\delta\nu}\dot{x}^\gamma \dot{x}^\delta$

$C_{E^2} \sim r_s^4 \quad C_{B^2} \sim r_s^4$

"Love Numbers"

$$S_{tot}[g_{\mu\nu}, x_1^\mu(\tau_1), x_2^\mu(\tau_2)] = S_{EH}[g_{\mu\nu}] + S_{pp}[g_{\mu\nu}, x_1^\mu(\tau_1)] + $$
$$+ S_{pp}[g_{\mu\nu}, x_2^\mu(\tau_2)]$$

$$S_{EH}[g_{\mu\nu}] = -\frac{1}{16\pi G}\int d^4x \sqrt{-\det g_{\mu\nu}}\, R[g_{\mu\nu}] \Rightarrow G_{\mu\nu} = 0$$

$$\frac{\delta S_{tot}}{\delta g_{\mu\nu}} = 0 \quad \Rightarrow G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$T_{\mu\nu}(x) = \sum_{a=1,2} m_a \int d\tau_a\, \dot{x}_\mu \dot{x}_\nu\, \delta^4(x - x_a(\tau))$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \qquad \kappa = \sqrt{32\pi G}$$

Taylor expansion κ $\quad \Box h_{\mu\nu} = -\frac{\kappa}{2} P_{\mu\nu;\rho\sigma} T^{\rho\sigma} + O(\kappa^2)$

$$\Box = -\partial_t^2 + \partial_x^2 + \partial_y^2 + \partial_z^2$$

$$P_{\mu\nu;\rho\sigma} = \frac{1}{2}(\eta_{\mu\rho}\eta_{\sigma\nu} + \eta_{\mu\sigma}\eta_{\rho\nu} - \eta_{\mu\nu}\eta_{\rho\sigma})$$

$$\int d^4x\, e^{i\vec{k}\cdot\vec{x}}\, \Box h_{\mu\nu}(x) = -\frac{\kappa}{2} P_{\mu\nu;\rho\sigma}\int d^4x\, e^{i\vec{k}\cdot\vec{x}}\, T^{\rho\sigma}(x)$$

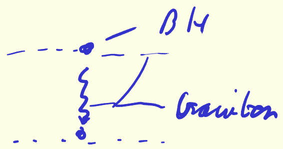
$$= \int d^4x\, (-k^2)\, e^{i\vec{k}\cdot\vec{x}} h_{\mu\nu}(x) \qquad \tilde{h}_{\mu\nu}(k) \equiv \int d^4x\, e^{i\vec{k}\cdot\vec{x}} h_{\mu\nu}(x)$$

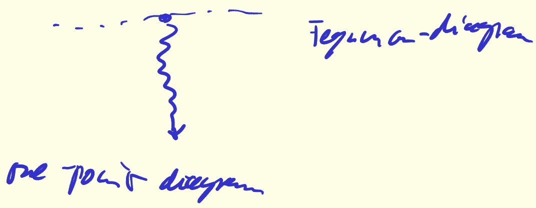
$$\tilde{T}_{\mu\nu}(k) = \int d^4x\, e^{i\vec{k}\cdot\vec{x}}\, T_{\mu\nu}(x)$$

$$\tilde{h}_{\mu\nu}(k) = \frac{\kappa}{2}\frac{P_{\mu\nu;\rho\sigma}}{k^2} T^{\rho\sigma}(k)$$

$$h_{\mu\nu}(x) = \int \frac{d^4k}{(2\pi)^4} e^{-i\vec{k}\cdot\vec{x}}\, \tilde{h}_{\mu\nu}(k)$$

$$\bar{T}_{\mu\nu}(k) = \sum_{a=1,2} m_a \int d\tau_a \, e^{i\vec{k}\cdot\vec{x}_a(\tau_a)} \cdot \dot{x}^\mu_a \dot{x}^\nu_a$$





5. Non-relativitistc General Relativity (NRGR)

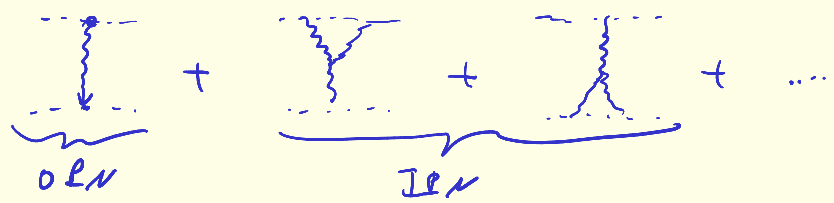
$$g_{\mu\nu} = \eta_{\mu\nu} + h^{\text{internal}}_{\mu\nu} + h^{\text{near}}_{\mu\nu} + h^{\text{far}}_{\mu\nu}$$

here only near zone $\to 0$ and far zone

$i = x, y, z$

$$\left.\begin{array}{ll} \partial_t h^{\text{near}}_{\mu\nu} \sim \frac{v}{r} h^{\text{near}}_{\mu\nu} & \partial_i h^{\text{near}}_{\mu\nu} \sim \frac{1}{r} h^{\text{near}}_{\mu\nu} \\ \partial_t h^{\text{far}}_{\mu\nu} \sim \frac{v}{r} h^{\text{far}}_{\mu\nu} & \partial_i h^{\text{far}}_{\mu\nu} \sim \frac{v}{r} h^{\text{far}}_{\mu\nu} \sim \frac{1}{\lambda} h^{\text{far}}_{\mu\nu} \end{array}\right\} \begin{array}{l} p^{\text{near}}_\mu \sim \left(\frac{v}{r}, \frac{1}{r}\right) \\ p^{\text{far}}_\mu \sim \left(\frac{v}{r}, \frac{v}{r}\right) \end{array}$$

$e^{iW[\vec{x}_a]} =$



Feynman-diagram

$$W = \int dt \left[L^{0PN} + L^{1PN} + \dots \right]$$

Zero post Newtonian Order Lagrangian $\quad L^{0PN} = \overbrace{\frac{1}{2} m_1 \vec{v}_1^{\,2} + \frac{1}{2} m_2 \vec{v}_2^{\,2}}^{T} + \overbrace{\frac{G m_1 m_2}{r}}^{V} \qquad r = |\vec{x}_1 - \vec{x}_2|$

Newtonian physics

first -||-

$$L^{1PN} = \frac{1}{8} m_1 \vec{v}_1^{\,4} + \frac{1}{8} m_2 \vec{v}_2^{\,4} - \frac{G^2 m_1 m_2 (m_1 + m_2)}{2r^2} + \frac{G m_1 m_2}{2r} \left[3(\vec{v}_1^{\,2} + \vec{v}_2^{\,2}) - 7 \vec{v}_1 \cdot \vec{v}_2 - \frac{(\vec{v}_1 \cdot \vec{r})(\vec{v}_2 \cdot \vec{r})}{r^2} \right]$$

Einstein-Infield-Hoffmann Lagrangian

$$\varepsilon = \frac{G m}{r} \sim v^2$$

It is done to the fourth order 4PN order = state-of-the-art

5PN is not yet fully done

combination of concepts in particle physics and GR