lecture-2024-03-01

Intro to Cosmology

just one object, it can not created again and we can measure the changes

1. Cosmological Principle

100 million lightyears

distribution along filaments, galaxies are distributed a long filaments, with voids in between

on larger scales $\gtrsim 300$ million light-years
the universe looks similar in all directions

Cosmological principle (CP): the universe spationally isotropic and homogeneous on large scales.
(observed fact, not a theorem)

- needs an explanation
- if it were not true, we would know little about comosology

speed of light finite $3\cdot 10^8 \frac{m}{s}$

- so we see only the past of distant regions
- if the comology principle is true, then this will be similar to our own past
- the breaks down on small scales, it might break down

on scales larger then currently observable universe

- CP treats space and time diffenrently

there is a prefered frame in the universe, where the CP true
it does not imply that the universe is static or that galaxies are at rest w.r.t. each other
universe is expanding, preferred frame is determined by this expansion

2. Robertson-Walker metrics (RW)

use GR to describe the evolution of universe, with the CP there are only 3 possible metrics for the universe
it is an approximation (0 to 1. order)
at first assume exact spatial isotropy and homogeneity
very restrictive -> allows for 3 distinct spatial geometries

Curvature

0	positive	negative
flat	sphere	hyperbolic
$\mathbb{R}^3$,	S^3,	H^3

all have constant curvature

metric on sphere S^3
embed S^3 into $\mathbb{R}^3$

$$x^2+y^2+z^2+w^2=1$$

differentiate

$$2x\,dx+2y\,dy+2z\,dz+2w\,dw=0$$

$$w=\pm\sqrt{1-x^2-y^2-z^2}$$

$$dw=\pm\frac{x\,dx+y\,dy+z\,dz}{\sqrt{1-x^2-y^2-z^2}}$$

metric on $\mathbb{R}^4$ $\quad dl^2 = dx^2+dy^2+dz^2+dw^2$

metric S^3 dw in ds^2

$$dl^2 = dx^2 + dy^2 + dz^2 + \frac{(x\,dx + y\,dy + z\,dz)}{1 - x^2 - y^2 - z^2}$$

by analogy for H^3 $x^2 + y^2 + z^2 - w^2 = -1$

$$dl^2 = dx^2 + dy^2 + dz^2 + \frac{k(x\,dx + y\,dy + z\,dz)^2}{1 - k(x^2 + y^2 + z^2)}$$

$k = 1$ S^3

$k = 0$ R^3

$k = -1$ H^3

in polar coordinates

spatial metric

$$dl^2 = \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 d\varphi^2)$$

Can allow for an arbitary size by multiplying with the (time-dependent) factor $a(t)$
scalar factor, end up with 4d Robertson Walker metric

$c = 1$ $8\pi G = 1$

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta + \sin^2\theta\, d\varphi^2) \right]$$

at fixed t, θ, φ distance between $r = 0$ and $r = R$ fixed spatial coordinate $\rightarrow$ comoving frame

is $\rightarrow dt = 0 = d\varphi = d\theta$

$$\int_{r=0}^{r=R} ds = a(t) \int_{r=0}^{r=R} \frac{dr}{\sqrt{1 - kr^2}} = \begin{cases} \operatorname{arcsinh}(R) & k = -1 \\ R & k = 0 \\ \arcsin(R) & k = 1 \end{cases}$$

to calculate proper-time interval would calculate

$$\int \sqrt{-ds^2} = \Delta\tau$$

redshift

shift of spectral lines due to the scalar factor $a(t)$
propagation of light rays $ds^2 = 0$ the radius is aligned to the light direction

choose $\theta, \varphi = \text{const}$ obtain light comes to us

$$dt = -a(t) \frac{dr}{\sqrt{1 - kr^2}}$$

light observed at r=0, emitted at r=R

integrate $\displaystyle\int_{t_0}^{t_R} \frac{dt}{a(t)} = \int_0^R \frac{dr}{\sqrt{1-kr^2}}$

minus sign is gone, because the differentials behave couterwise, time increase, spatial decrease

independent of t => differential $\dfrac{\delta t_0}{a(t_0)} = \dfrac{\delta t_R}{a(t_R)}$)

frequency of light $\nu \sim \dfrac{1}{\delta t}$ $\qquad \nu_0 = \nu_R \dfrac{a(t_R)}{a(t_0)}$

alternativly wavelenght $\lambda_0 = \lambda_R \dfrac{a(t_0)}{a(t_R)} = \lambda_R (1 + \underbrace{z}_{\text{not a coordinate}})$ — redshift

follows expansion or contraction
changing in frequency is due to the changing in spacetime, but also due to gravity
you can not distinct between them in general

if the wavelength is lengthened -> redshift
shorted -> blueshift

for nearby sources expand $\quad a(t) = a(t_0)\left[1 + \underbrace{H_0}_{\text{Hubble constant}}(t - t_0) + \dots\right]$

subscript 0 : for today $\qquad H_0 = \dfrac{\dot a(t_0)}{a(t_0)}$ rate of changing

hence $\quad z = H_0 \overbrace{(t - t_0)}^{\text{distance } c=1} + \dots$

$\quad = H_0 \cdot d$

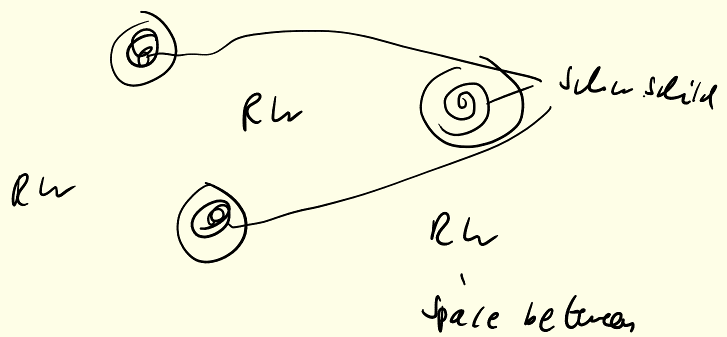
$z = H_0 \cdot d \qquad$ Hubble's law

velocity on top of the hubble flow = "peculiar velocity"

observation show $\quad H_0 \simeq 70 \frac{km}{s\,Mpc} \qquad 1pc \simeq 3{,}26$ light-years

Hubble's law gives us the velocity of spacegrowing between big objects and very high distances

metric in universe to 2nd order approximation as Swiss cheese



the crucial part is the distant to measure

Hubble tension

the universe is expanding

distances with standard candles. Supernova, at some mass limit, stars exploding, for that you now the mass and the luminosity. It is very bright and you can measure the distance of galaxies.

variable stars (Cepheids)
type Ia supernovae (white dwarf star that accrete too much matter)

3. Friedmann equations

measuring the distance with CMB (Cosmic microwave background)

$G_{\mu\nu} = T_{\mu\nu}$ – must also impose
- isotropy & homogeneity

RW metric

$\mu = 0,1,2,3$

$i = 1,2,3$

$T_{ij} = g_{ij}\, p(t)$ – pressure

RW metric

$T_{0i} = 0$ like a vector so because of isotropy, no direction preferred

$T_{00} = \rho(t)$ – energy density

=> perfect fluid, average distribution of matter

equation of state $p = w \rho$

parameter

We will assume $w = \text{const}$ for each matter component

GR: Bianchi-identity $D^{\mu} G_{\mu\nu} = 0$

likewise $D^{\mu} T_{\mu\nu} = 0$ covariantly conserved

$\nu = i$ trivial

$\nu = 0$ $\dot{\rho} + 3H(\rho + p) = 0$ equation of continuity

$H = \frac{\dot{a}}{a}$

- energy is changing because of the expansion of the universe, it is not conserved in general $H \neq 0$

$p = w\rho \rightarrow \dot{\rho} + 3H(1+w)\rho = 0$

$\rho \sim \frac{1}{a^{3(1+w)}}$

examples: $w = 0$ pressure free matter, like humans

$\rho \sim \frac{1}{a^3} \sim \frac{1}{\text{volume}}$ standard baryonic matter + dark matter

(or maybe only weak) does not interact with light

- $w = \frac{1}{3}$ radiation / relativistic matter

$$\rho \sim \frac{1}{a^4} = \frac{1}{a^3} \cdot \frac{1}{a}$$

redshift of expansion

diluting number of photons

- $w = -1$ vacuum energy, dark energy, cosmological constant

$$\rho = const = \Lambda$$

scalar fields (Higgs) they have w(t)

now plugin into the Einstein equations $G_{\mu\nu} = T_{\mu\nu}$

$$H^2 + \frac{k}{a^2} = \frac{1}{3}\rho$$ Friedman equation $H(t) = \frac{\dot{a}(t)}{a(t)}$

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p) = -\frac{1}{6}\rho(1+3w)$$ acceleration equation

with $k=0$
examples: $w = 0$ $a(t) = a_0(t-t_0)^{2/3}$

$w = \frac{1}{3}$ $a(t) = a_0(t-t_0)^{1/2}$

$w = -1$ $a(t) = a_0 e^{Ht}$ $H = \sqrt{\frac{\Lambda}{3}}$ acceleration solution

In our universe all three matter types(plus others) are present

$$3H^2 = \frac{\rho_{r,0}}{a^4} + \frac{\rho_{m,0}}{a^3} - \frac{3k}{a^2} + \Lambda$$ convention $a(t_0) = 1$ today

$\rho_{r,0}$ current energy density in radiation

$$\left(\frac{H}{H_0}\right)^2 = \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda$$

spatial curvature

today $\Omega_r + \Omega_m + \Omega_k + \Omega_\Lambda = 1$

Ω current fractional energy densities

remarks:
- even k=0 universe is curved (in 4d)
- critical density $\rho_{crit} = 3H^2$

if

$\rho_r + \rho_m + \rho_\Lambda = \rho_{crit}$ → $k = 0$ universe flat

if more → $k = +1$ positive curvature

if less → $k = -1$ negative curvature

current critical density $\rho_{crit,0} = 3H_0^2 = \frac{3}{8\pi G}\left(70 \frac{km}{s\,Mpc}\right)^2 = 10^{-26} \frac{kg}{m^3}$

average density

- Friedmann equations describe both expanding and contracting universes
- no static universe is possible (unstable state)
spacetime is evolving, the universe has
a history
- looking for matter and radiation solutions,
there is a time when $a \to 0$

from stringtheory it could be that
the vacuum energy could be go negative
-> recolapse of the universe

generally $R \to \infty$ curvature singularity "big bang"

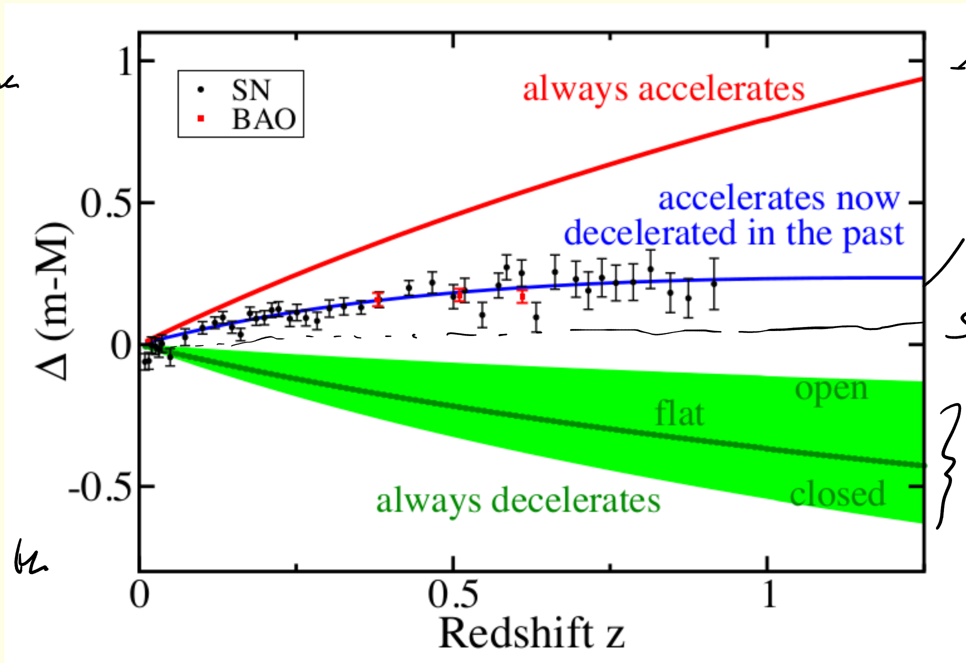
theory breaks down at this point -> to solve we will need quantum gravity

Expansion history

arXiv:1709.01091

distance

change in brightness of type Ia supernovae



brightness

Λ only $\Omega_m = \Omega_r = \Omega_k = 0$

$\Omega_m \approx 0,3$ $\Omega_\Lambda \approx 0,7$

Ω_r small $\Omega_k = 0$

$\Omega_m = 1$

Ω_m and Ω_k only

for different matter content, measured the brightness

that is how dark matter was discovered

of which only 20% in ordinary matter

$$\frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda$$

today $\quad 10^{-5} \quad 0,3 \quad 0 \quad 0,7$

back in time: now: vacuum dominated epoch

transition at $\frac{\Omega_m}{a^3} = \Omega_\Lambda$ to matter dominate

→ time of radiation-matter quality $\frac{\Omega_r}{a^4} = \frac{\Omega_m}{a^3}$

domination $\quad$ Years

radiation $\quad z \geq 5000 \quad$ 390000 years after Big Bang

↓ limit $\quad z = 1090 \quad$ emission of CMB $\quad$ universe becomes transparent

matter $\quad$ until 5 billion years ago

↓

vacuum energy $\quad$ since then

calculate age via: $H = \frac{\dot{a}}{a} = \frac{da}{dt}\frac{1}{a}$

$$\int dt = \int_0^{a=1} \frac{da}{aH} \quad \text{age of universe}$$

in the past

standard ruler for measering the curvature