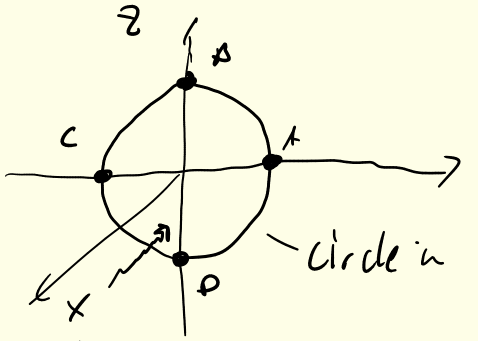
2024-03-04 tutorial



circle in z-y-plane

wave propagation in -x-direction

$$\delta x^1 = \delta x_0^1$$

$$\delta x^2 = \delta x_0^2 + \tfrac{1}{2}\left(h^{TT}_{22}\,\delta x_0^2 + h^{TT}_{23}\,\delta x_0^3\right)$$

$$\delta x^3 = \delta x_0^3 + \tfrac{1}{2}\left(h^{TT}_{23}\,\delta x_0^2 + h^{TT}_{33}\,\delta x_0^3\right)$$

$$h_{yy} = -h_{zz} = 2\,\mathrm{Re}\left\{A_+ e^{i\omega(t-\frac{x}{c})}\right\}$$

$$= 2A_+ \cos(\omega(t-\tfrac{x}{c}))$$

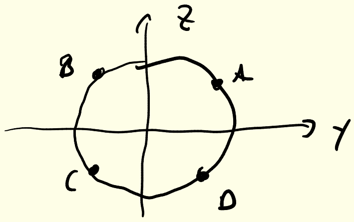
$$h_{yz} = h_{zy} = 2A_\times \cos(\omega(t-\tfrac{x}{c}))$$

① $A_+ = \frac{1}{2}$ plus polarized

$A_\times = 0$

② $A_+ = 0$ cross polariz.

$A_\times = \frac{1}{2}$



$$\delta x^2 = \delta x_0^2 + \tfrac{1}{2}\left(h^{TT}_{22}\,\delta x_0^2 + h^{TT}_{23}\,\delta x_0^3\right)$$

$$\delta x^3 = \delta x_0^3 + \tfrac{1}{2}\left(h^{TT}_{23}\,\delta x_0^2 + h^{TT}_{33}\,\delta x_0^3\right)$$

$$\delta x^2 = \delta x_0^2 + \tfrac{1}{2}\left(2A_+ c\,\delta x_0^2 + 2A_\times c\,\delta x_0^3\right) \qquad A_\times = 0$$

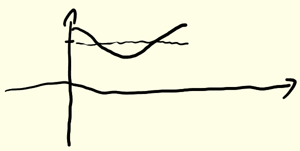
$$= \delta x_0^2(1 + A_+ c) + A_\times c\,\delta x_0^3 = \delta x_0^2(1 + A_+ c)$$

$$\delta x^3 = \delta x_0^3(1 - A_+ c) + A_\times c\,\delta x_0^2 \qquad A_\times = 0$$

$A = 1/2$ $\qquad = \delta x_0^3(1 - A_+ c)$ $\qquad c(\frac{\pi}{2}) = 0$

$$\delta x^2 = \delta x_0^2\left(1 + \tfrac{1}{2}c\right)$$

$$\delta x^3 = \delta x_0^3\left(1 - \tfrac{1}{2}c\right)$$



$\frac{2\pi}{T}(t - \frac{x}{c}) = \frac{\pi}{2}$

$t = 0$

$-\frac{2\pi}{T}\frac{x}{c} = \frac{\pi}{2}$

$T = -\frac{4x}{c}$

$\omega(-\frac{x}{c}) = \frac{\pi}{2}$

$\omega = -\frac{\pi c}{2x}$

② $A_+ = 0 \quad A_\times = \frac{1}{2}$

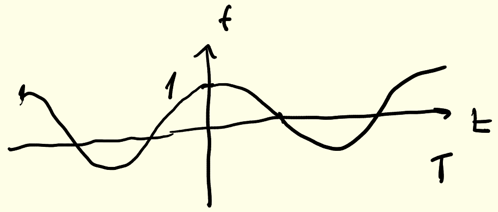
$$h_{yy} = -h_{zz} = 2\mathrm{Re}\{A_+ e^{i\omega(t-\frac{x}{c})}\}$$
$$= 2A_+ \cos(\omega(t-\tfrac{x}{c}))$$
$$h_{yz} = h_{zy} = 2A_\times \cos(\omega(t-\tfrac{x}{c}))$$

$$\delta x^2 = \delta x_0^2 + \tfrac{1}{2}(h_{22}^{TT}\delta x_0^2 + h_{23}^{TT}\delta x_0^3)$$
$$\delta x^3 = \delta x_0^3 + \tfrac{1}{2}(h_{23}^{TT}\delta x_0^2 + h_{33}^{TT}\delta x_0^3)$$

$$\delta x^2 = \delta x_0^2(1 + A_+ c) + A_\times c\, \delta x_0^3 \qquad A_+ = 0 \quad A_\times = \tfrac{1}{2}$$
$$\delta x^3 = \delta x_1^3(1 - A_+ c) + A_\times c\, \delta x_0^2$$

$$\delta x^2 = \delta x_0^2 + A_\times c\, \delta x_0^3 = \delta x_0^2 + \tfrac{1}{2} c\, \delta x_0^3$$
$$\delta x^3 = \delta x_0^3 + A_\times c\, \delta x_0^2 = \delta x_0^3 + \tfrac{1}{2} c\, \delta x_0^2$$

$\omega(t - \frac{x}{c}) = 1 \quad t = 0$



$$-\omega\frac{x}{c} = -\frac{2\pi}{T}\frac{x}{c} = 1$$
$$T = \frac{-2\pi x}{c}$$

solution:

$\delta x^1 = \delta x^1_0$

$\delta x^2 = \delta x^2_0 + \frac{1}{2} h_{yy} \delta x_0^2$

$\delta x^3 = \delta x^3_0 - \frac{1}{2} h_{yy} \delta x$

$A_x = 0$

$A_+ = \frac{1}{2}$

$\delta x^2 = \delta x_0^2 \left(1 + \frac{1}{2} A_+ \cos\left(\omega(t - \frac{x}{c}\right)\right)$

$\delta x^3 = \delta x^3 \left(1 - \frac{1}{2} A_+ \cos(\omega(t - \frac{x}{c})\right)$

$t=0$ $\qquad \frac{\pi}{2}$

$A(0, r, 0) \quad C(0, -r, 0)$

$B(0, 0, r) \quad D(0, 0, -r)$

π

$A(0, \frac{3}{2}r, 0) \quad C(0, -\frac{3r}{2}, 0)$

$B(0, 0, \frac{r}{2}) \quad D(0, 0, -\frac{r}{2})$

$\frac{3\pi}{2}$

–||– $\quad$ like $\frac{\pi}{2}$

2π

$A(0, \frac{r}{2}, 0) \quad C(0, -\frac{r}{2}, 0)$

$B(0, 0, \frac{3r}{2}) \quad D(0, 0, -\frac{3r}{2})$

in cross polerisation, both arms are streched in the same way.
You need both plus and cross waves to detect and do not have blind spots.
So you need a network of many detectors around the world.