Gravitational wave data analysis: an introduction

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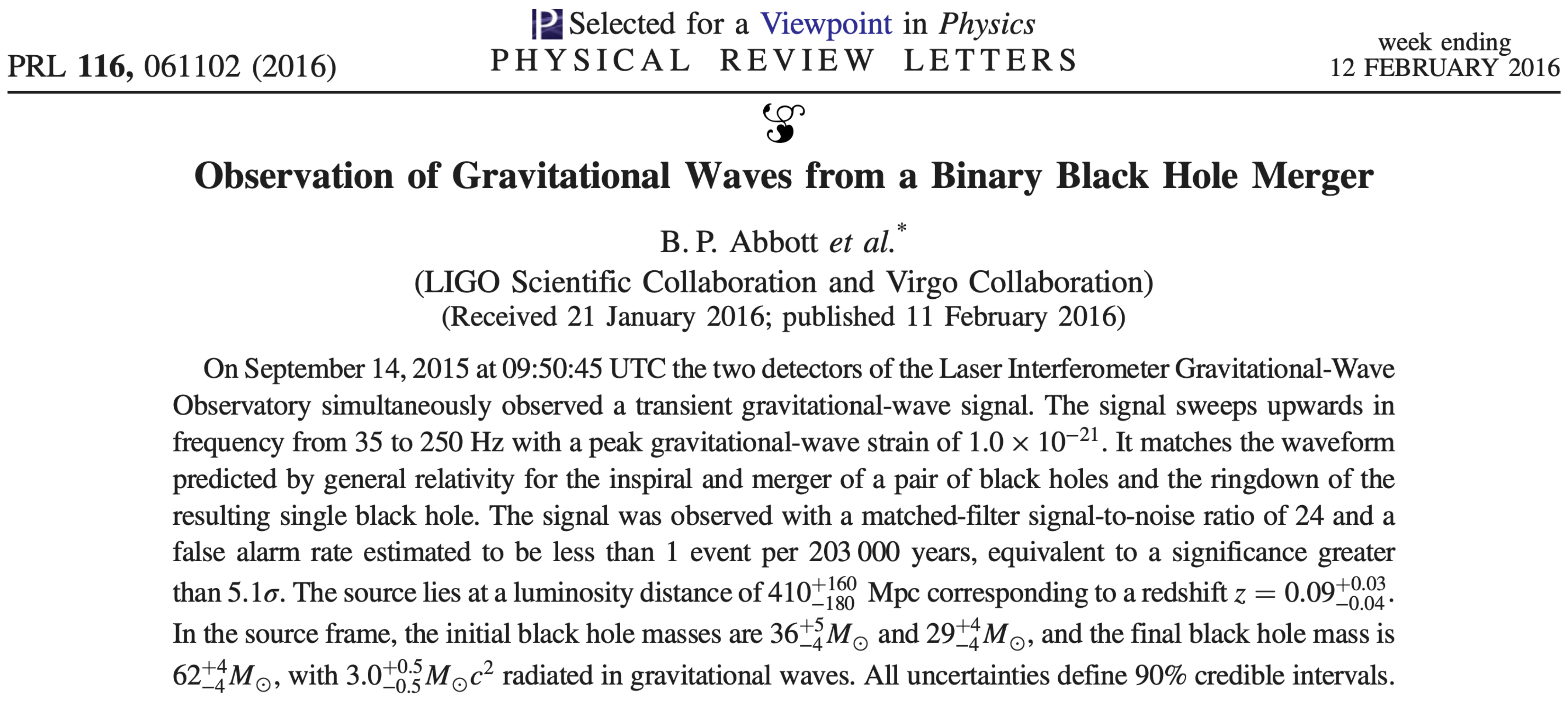
Jürgen Ehlers Spring School on Gravitational Physics

Index

1. Introduction

Gravitational wave astronomy

First direct observation of GWs from a BBH system on September 14, 2015: GW150914



Selected for a Viewpoint in *Physics*

PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

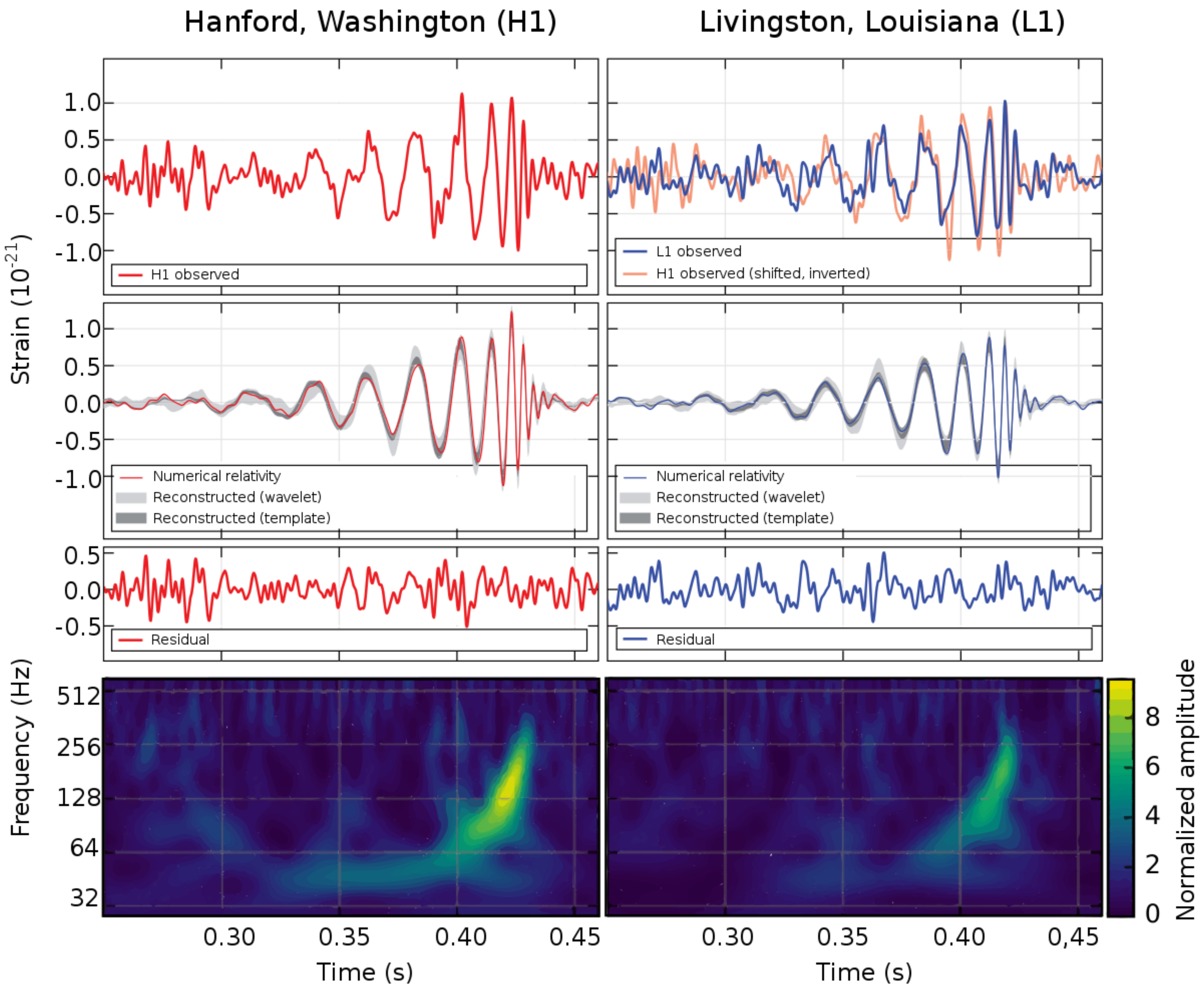
Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*[*]

(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 21 January 2016; published 11 February 2016)

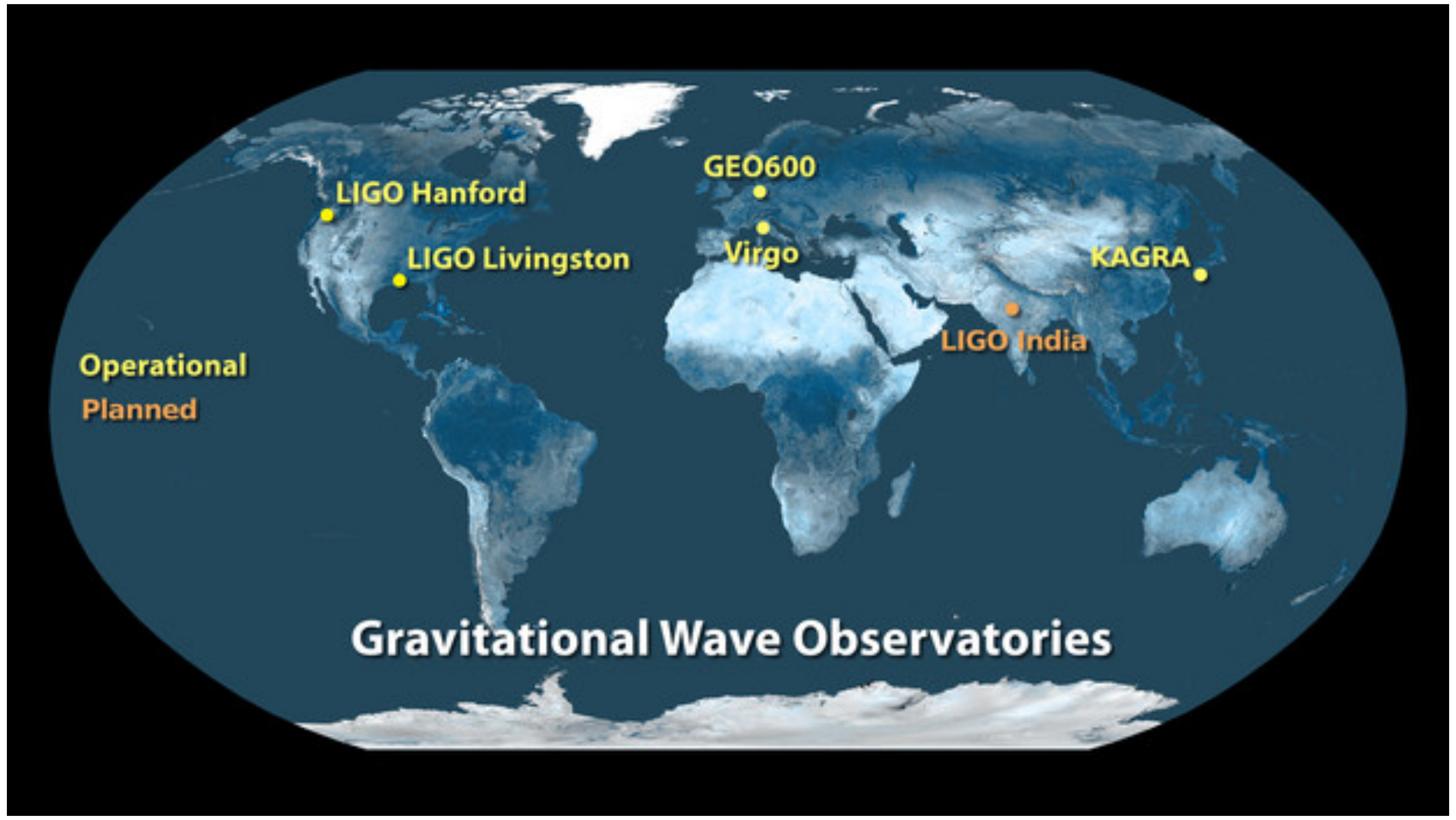
On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_\odot$ and $29^{+4}_{-4} M_\odot$, and the final black hole mass is $62^{+4}_{-4} M_\odot$, with $3.0^{+0.5}_{-0.5} M_\odot c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals.

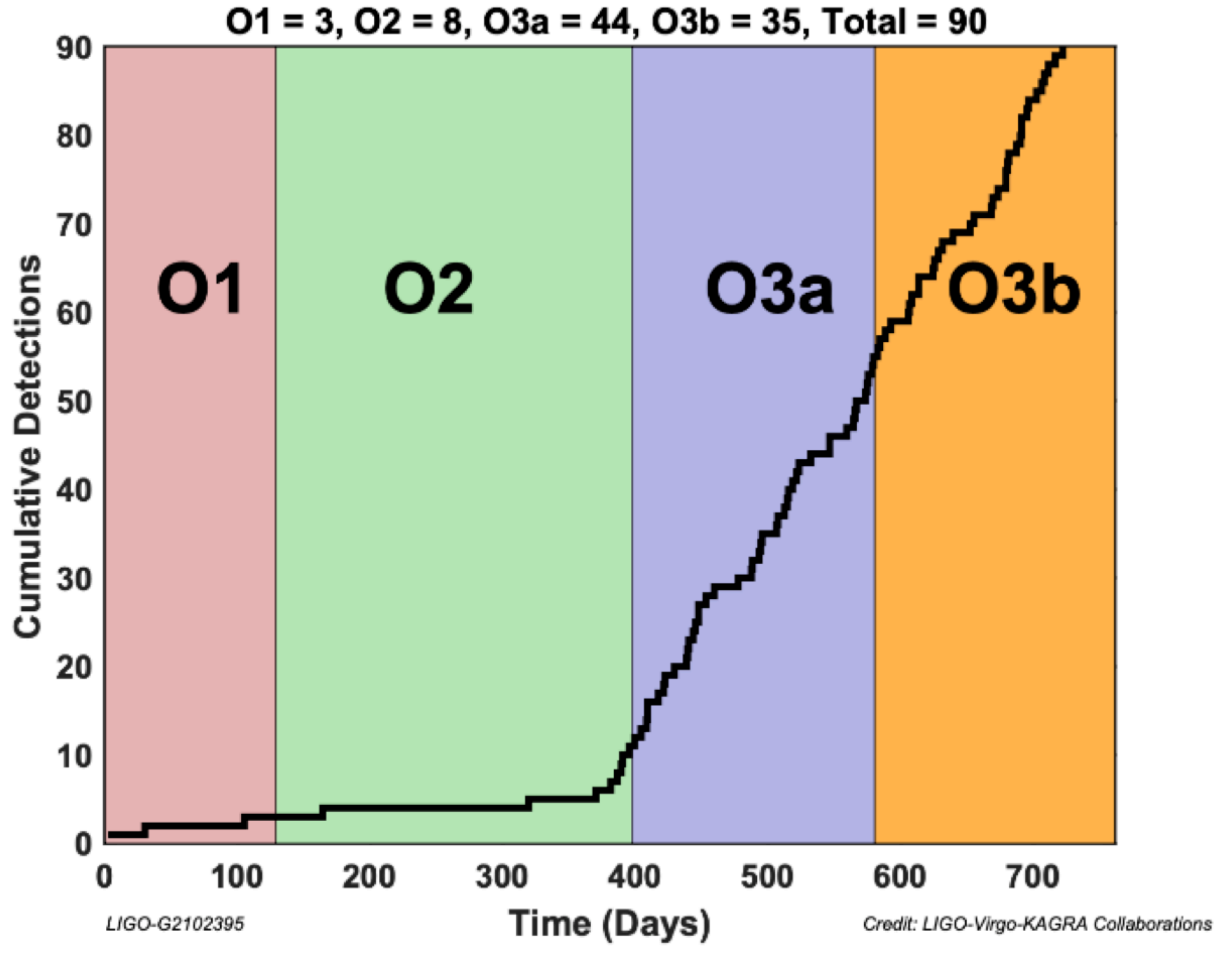


Gravitational wave astronomy

90 GW signals compatible with CBC systems detected so far by LIGO-Virgo-KAGRA Collaboration.

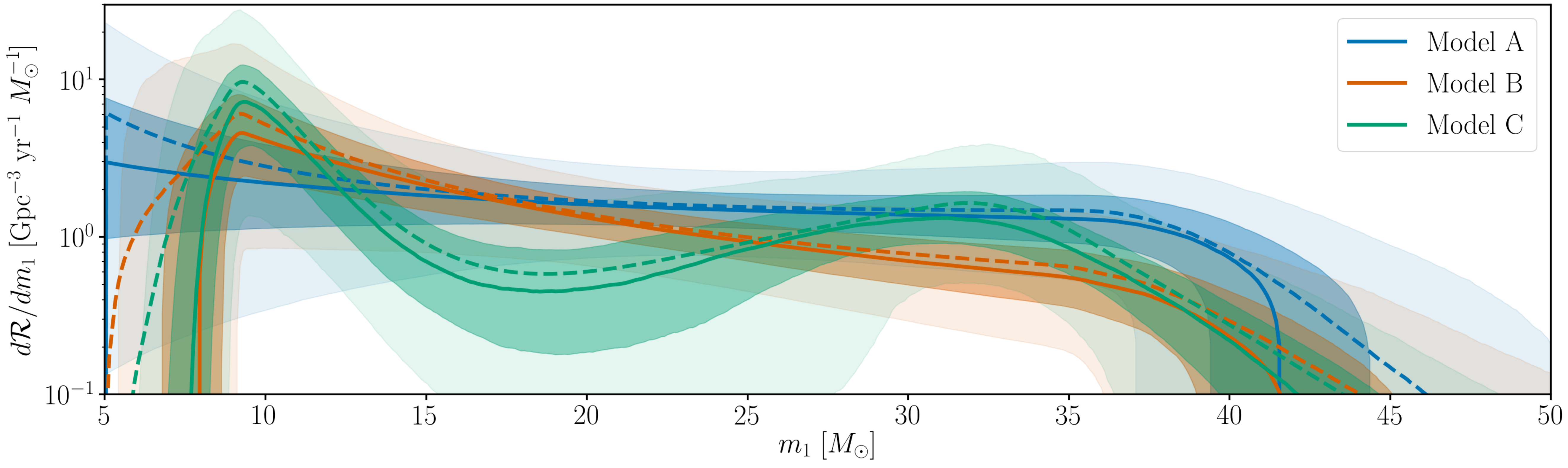
Increased rate of detections with improved sensitivity.



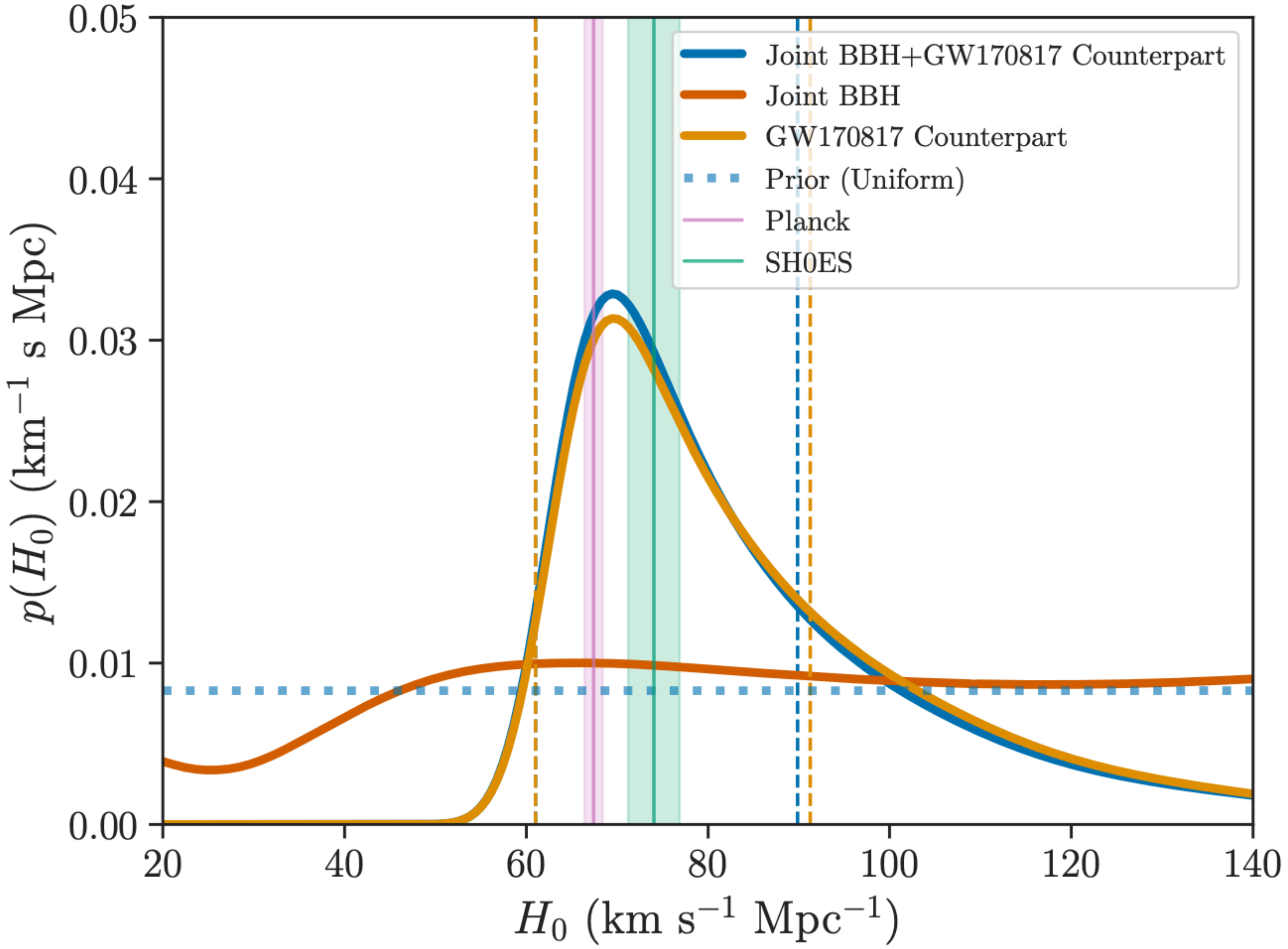


Gravitational wave astronomy

GW detections and analysis enable lots of interesting science: **astrophysical population models, cosmology, tests of General Relativity, matter in extreme states, ….**



Astrophysical population of BHs. (Figure: rate of black holes of a given mass which participate in mergers over a given volume of space, from GWTC, LVK+2019)



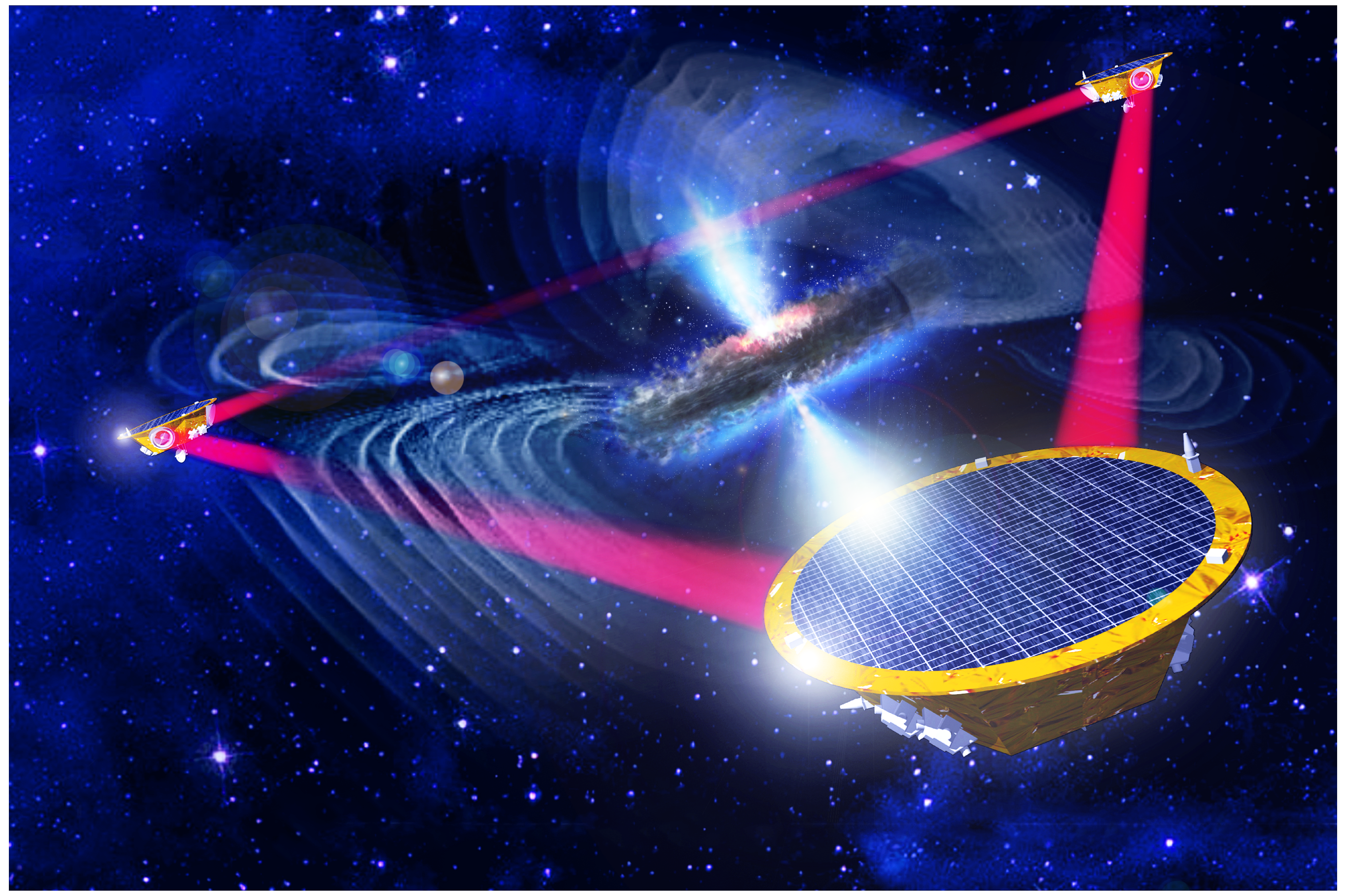
Hubble constant inference. (Figure: inference of the Hubble constant using GWs + BNS electromagnetic counterpart, from Abbott+2020)

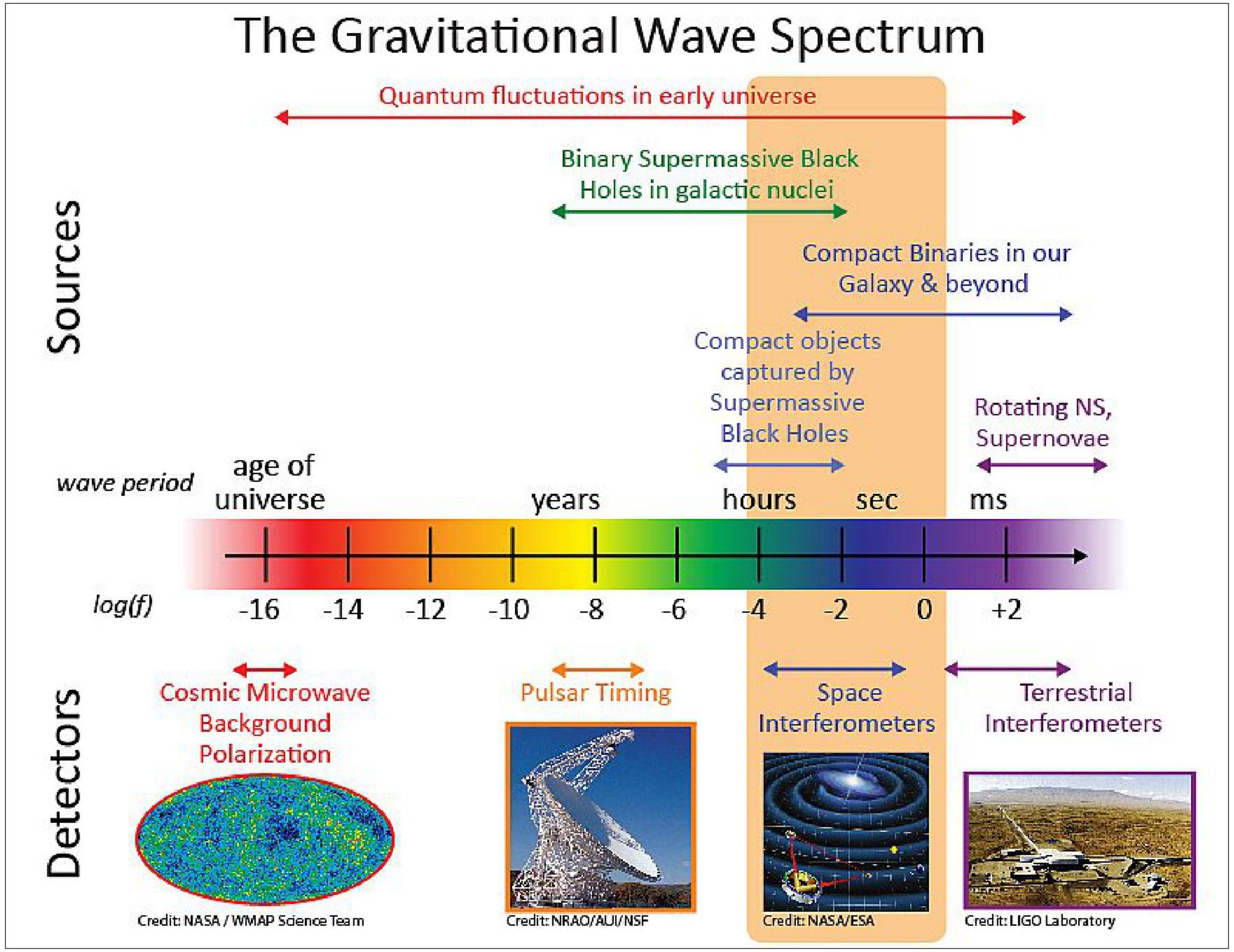
Future detectors

Next generation ground-based observatories: Einstein Telescope, Cosmic Explorer

Spaced-based detectors: LISA

Thousands of signals expected, some very loud





Interferometric detectors

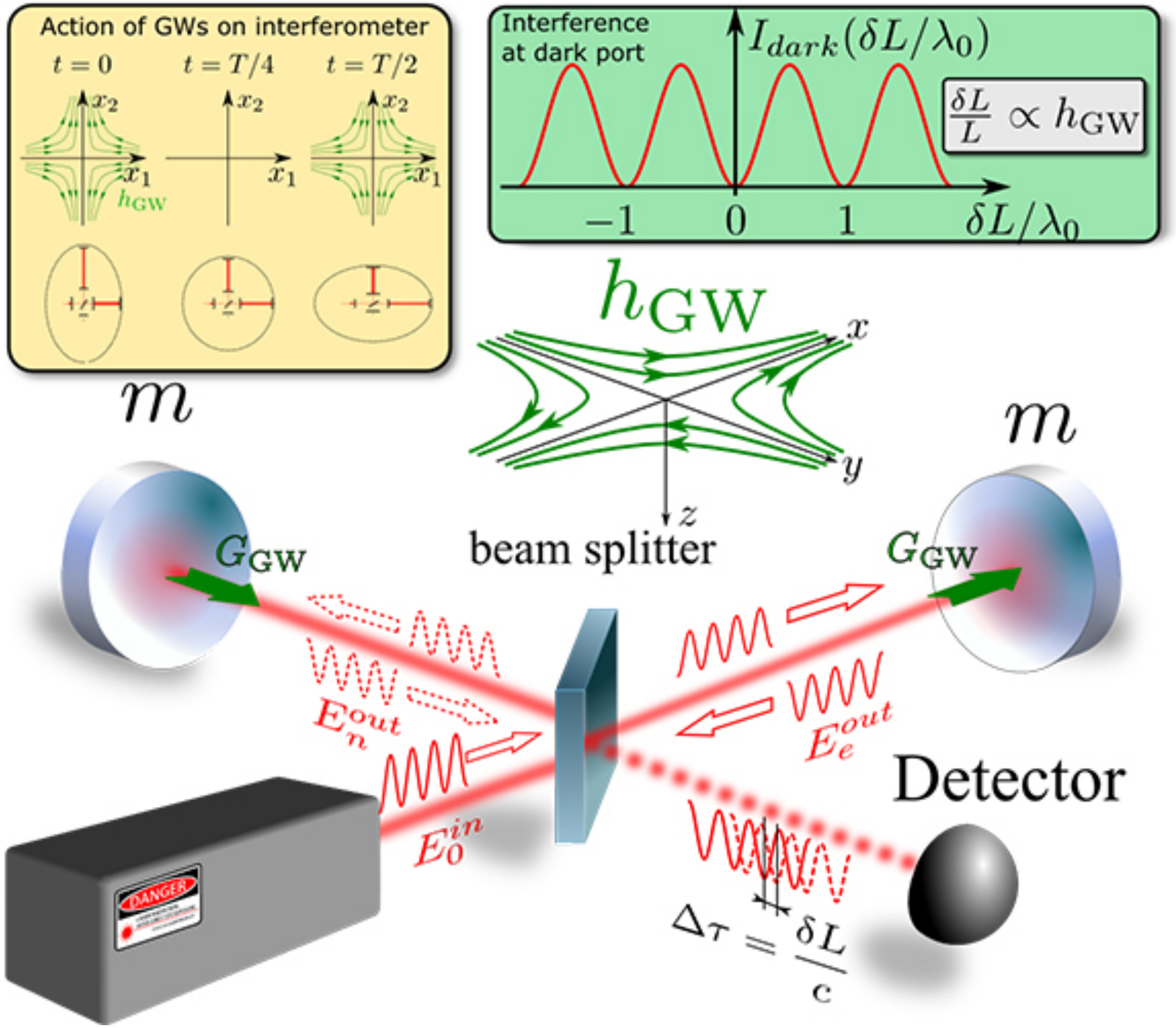
Current ground-based GW observatories:

Michelson-like interferometers.

Measure the relative change in the length of the the arms:

$$d(t) = \frac{\Delta L}{L}$$

GWs modify the spacetime, therefore produce a modification of the proper length travelled by the laser.



Gravitational wave strain

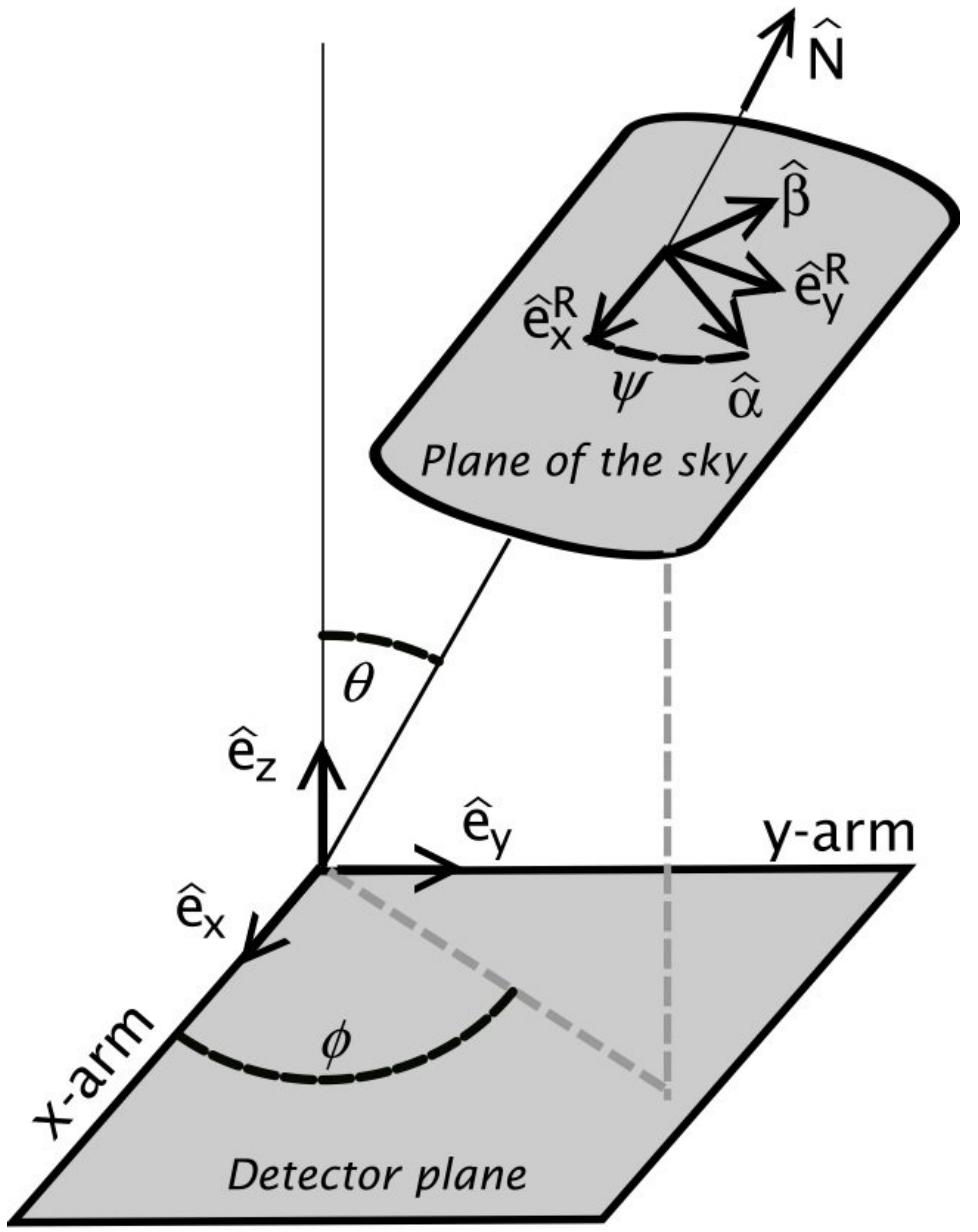
The **response of a detector** to a gravitational wave is a **linear combination of the polarizations**

$$h_{ij}(t) = \begin{pmatrix} h_+(t) & h_\times(t) & 0 \\ h_\times(t) & -h_+(t) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Detector response:

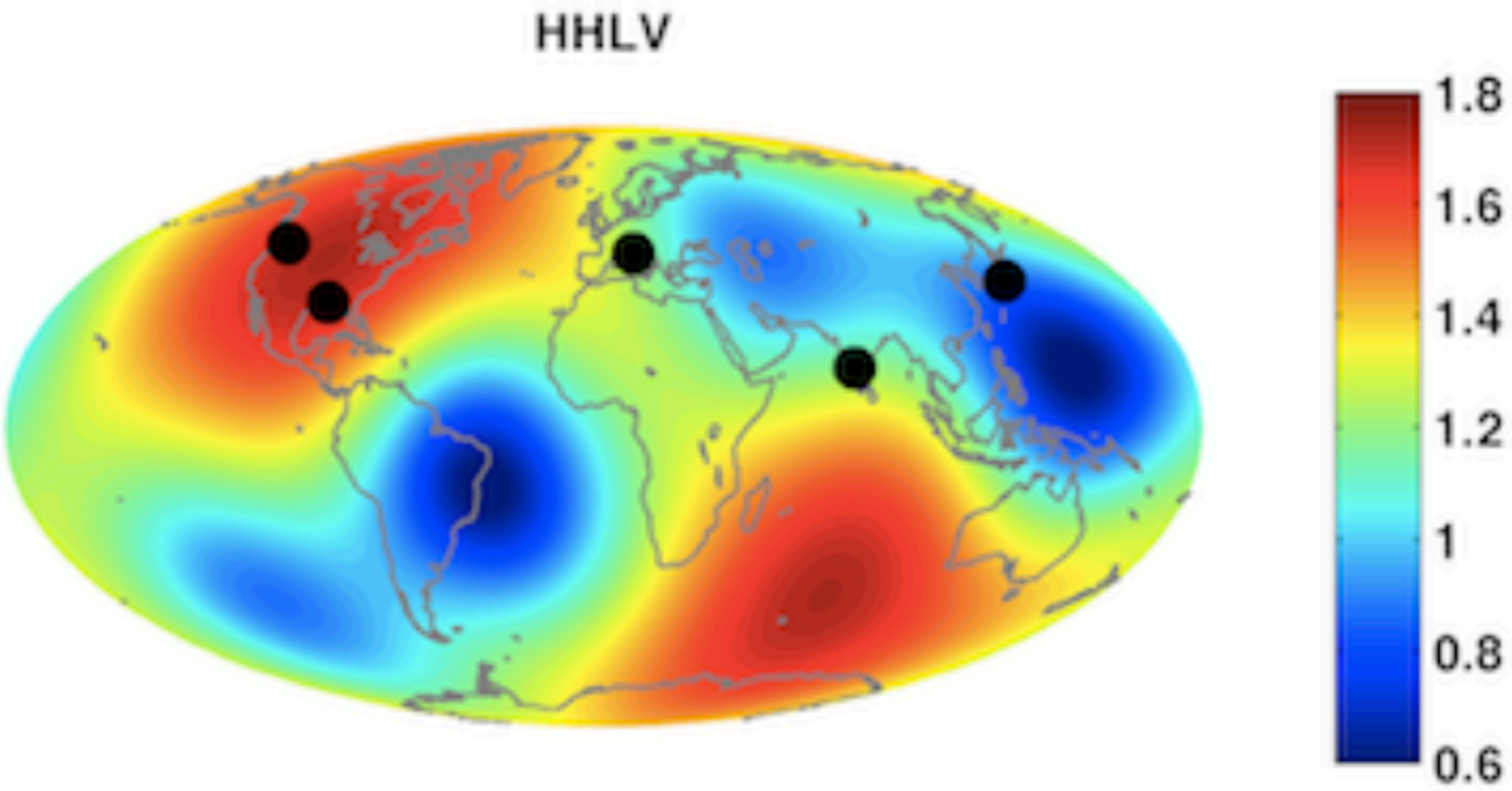
$$h(t) = D^{ij} h_{ij}(t) = F_+(\theta, \phi, \psi) h_+(t) + F_\times(\theta, \phi, \psi) h_\times(t)$$

D^{ij} **is a tensor that depends on the geometry of the detector.** h_{ij} has to be expressed in the same frame, which implies **dependence on the sky position** (θ, ϕ) and the **wave polarization** ψ

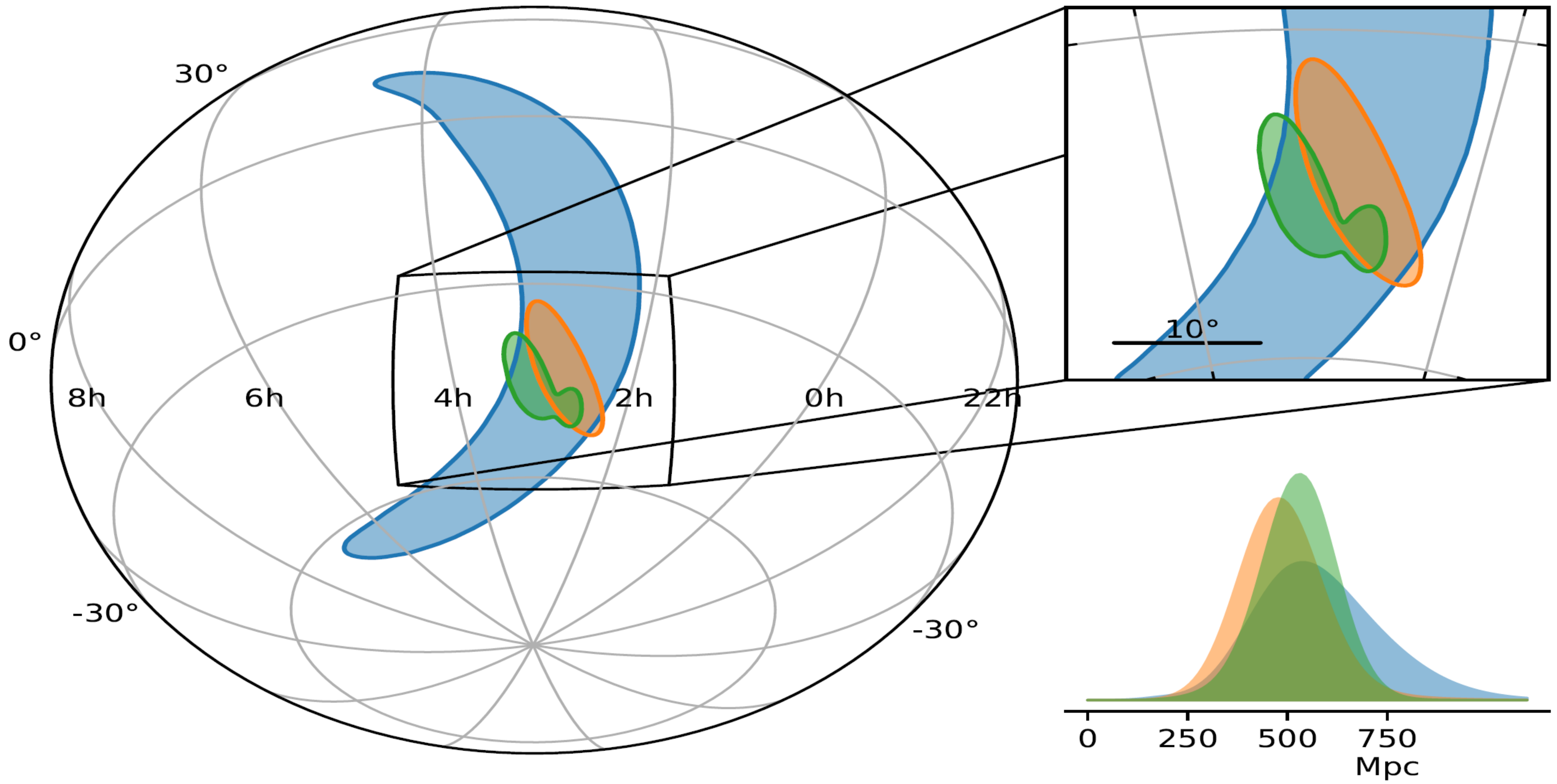


Relative orientation of the GW and detector frames. From Schutz 2011.

Sky sensitivity



Combined antenna patterns for HLV network.
From Indigo.



Inferred sky location of GW170814 using two LIGO detectors (blue) and adding the Virgo detector (orange and green). From Abbott+2017

Detector data

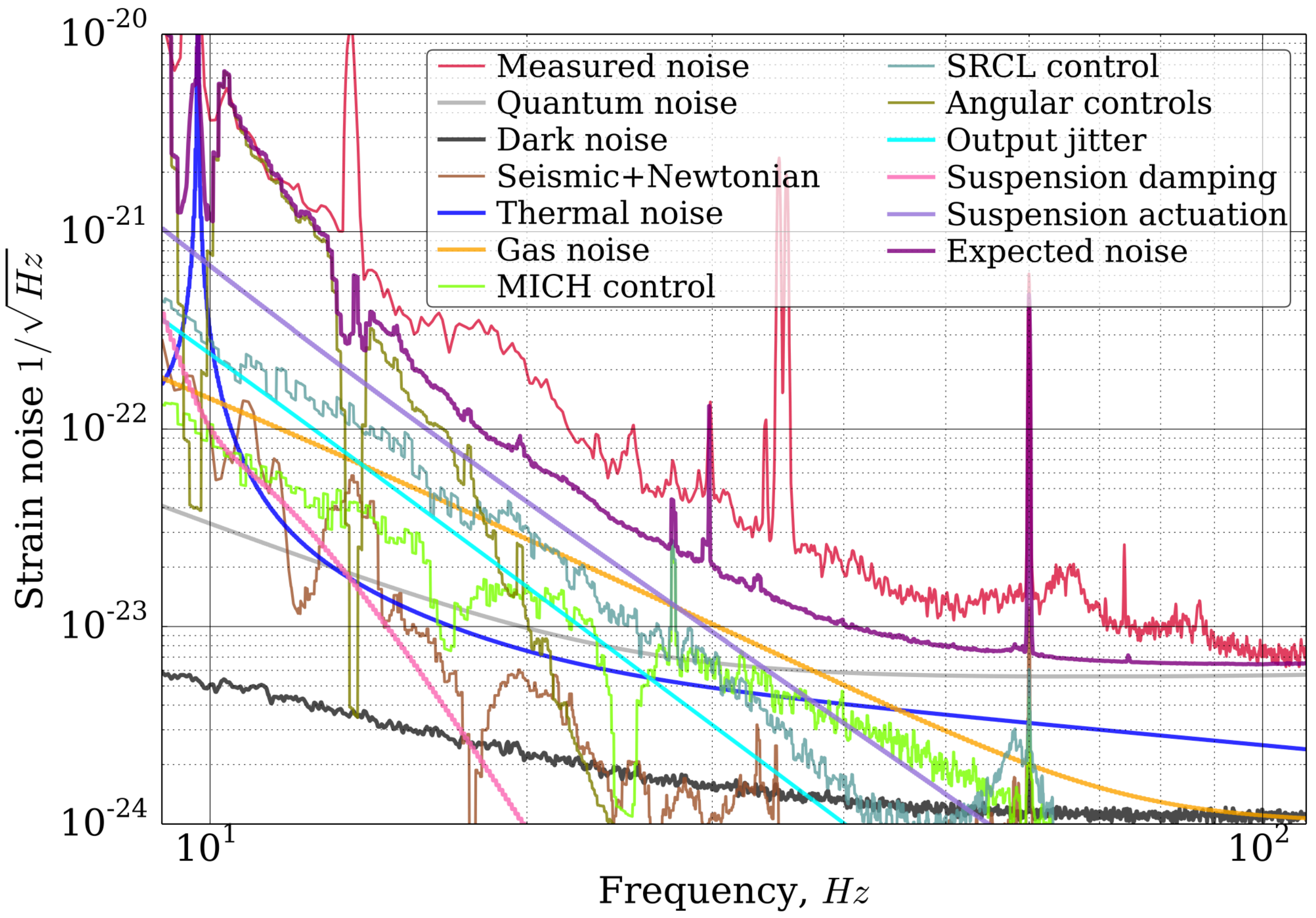
Detector strain for GW signals really small!

$$h \sim 10^{-21} \left(\frac{\mathcal{M}_c}{1.2\, M_\odot} \right)^{5/3} \left(\frac{f}{100\, \textbf{Hz}} \right)^{2/3} \left(\frac{D}{100\, \textbf{Mpc}} \right)^{-1}$$

GW detectors are designed to be among the most accurate measurement devices.

But this implies that they will be sensitive to many other terrestrial sources … **noise!**

$$d(t) = h(t) + n(t)$$



Credit: Martynov+ 2016

Detector data

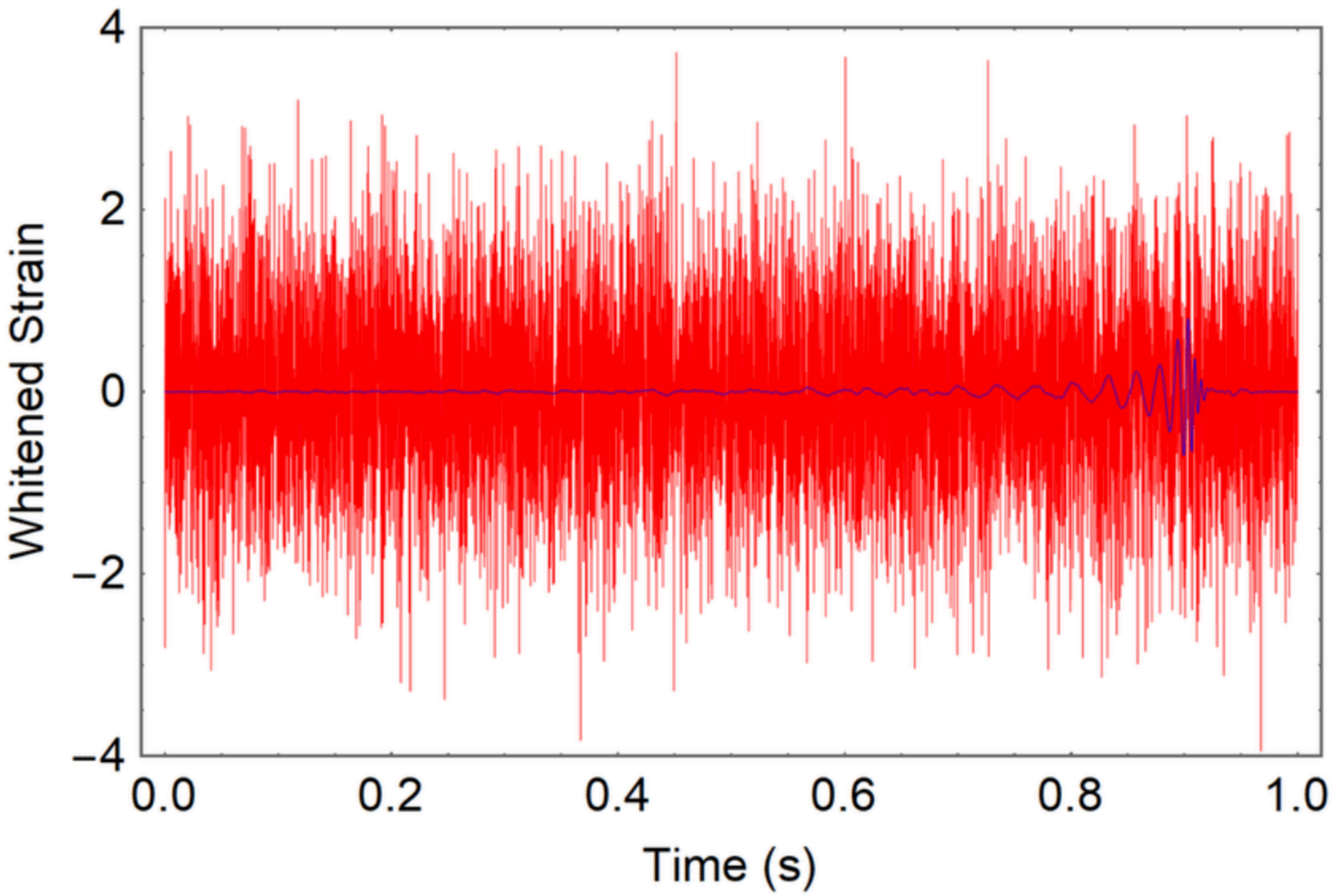
The detector **strain data** $d(t)$ measured by the detector will be a combination of **instrumental noise** $n(t)$ and possibly an **astrophysical GW signal** $h(t)$:

$$d(t) = n(t) + h(t)$$

Naively we could think that GW are detectable if $h(t) > n(t)$ ...

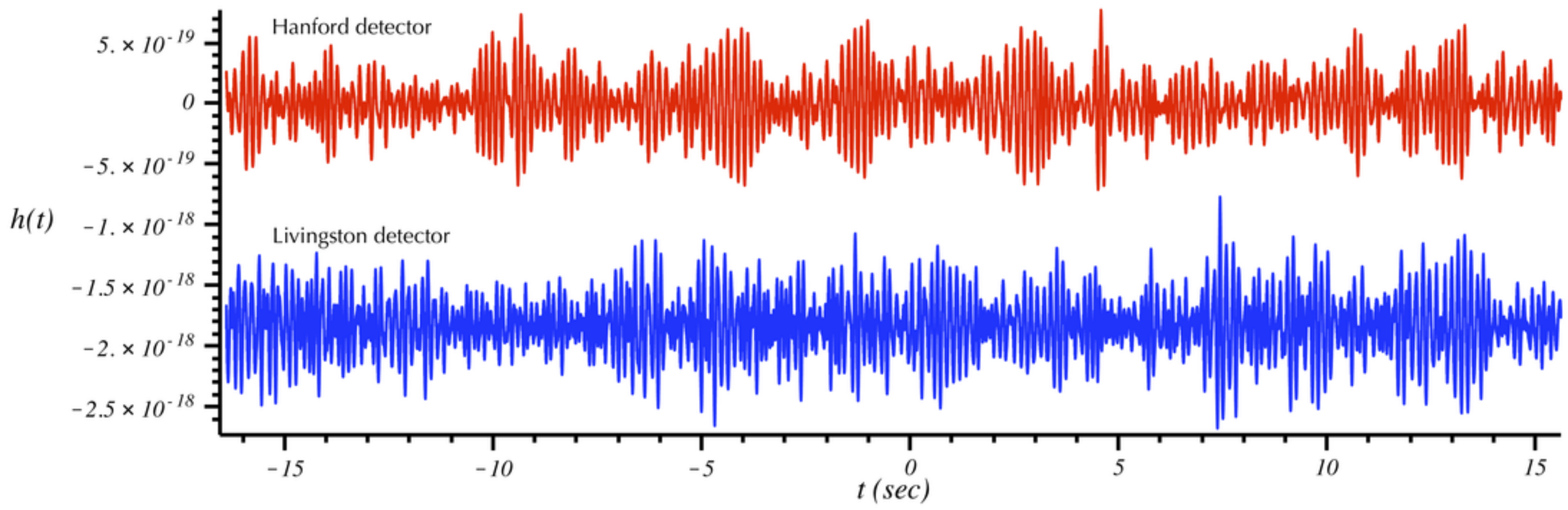
but $n(t) \geq h(t)$!

How can we extract the signals from the louder noise?



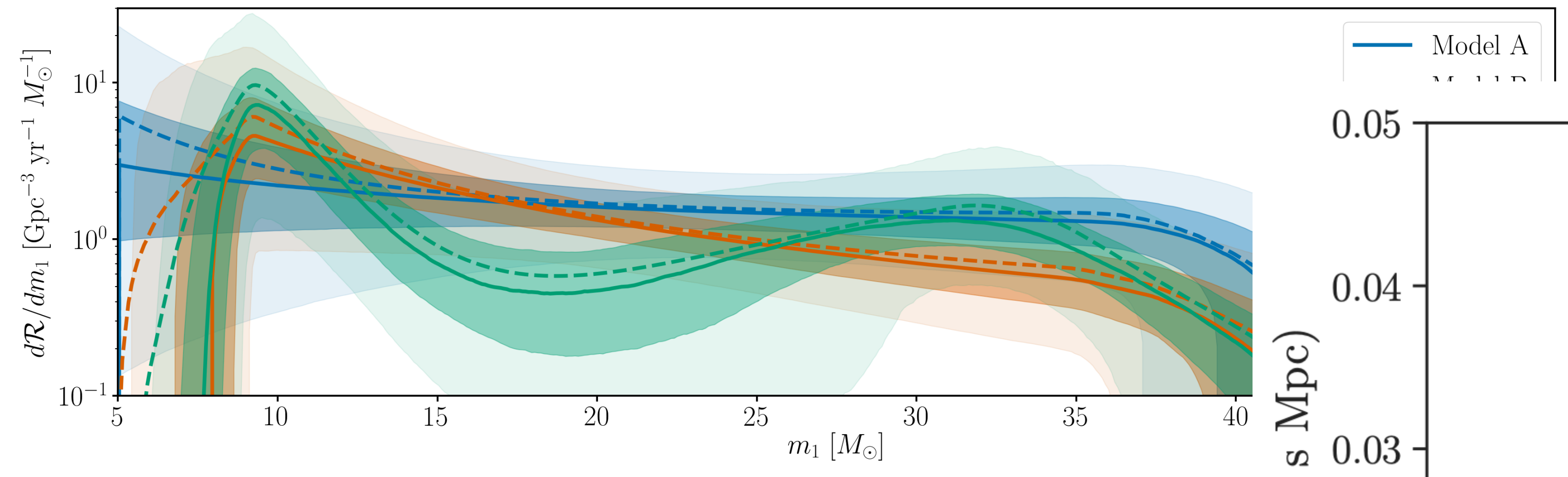
Sample signal injected into real whitened noise (the raw data is much worse ...). From George+2017

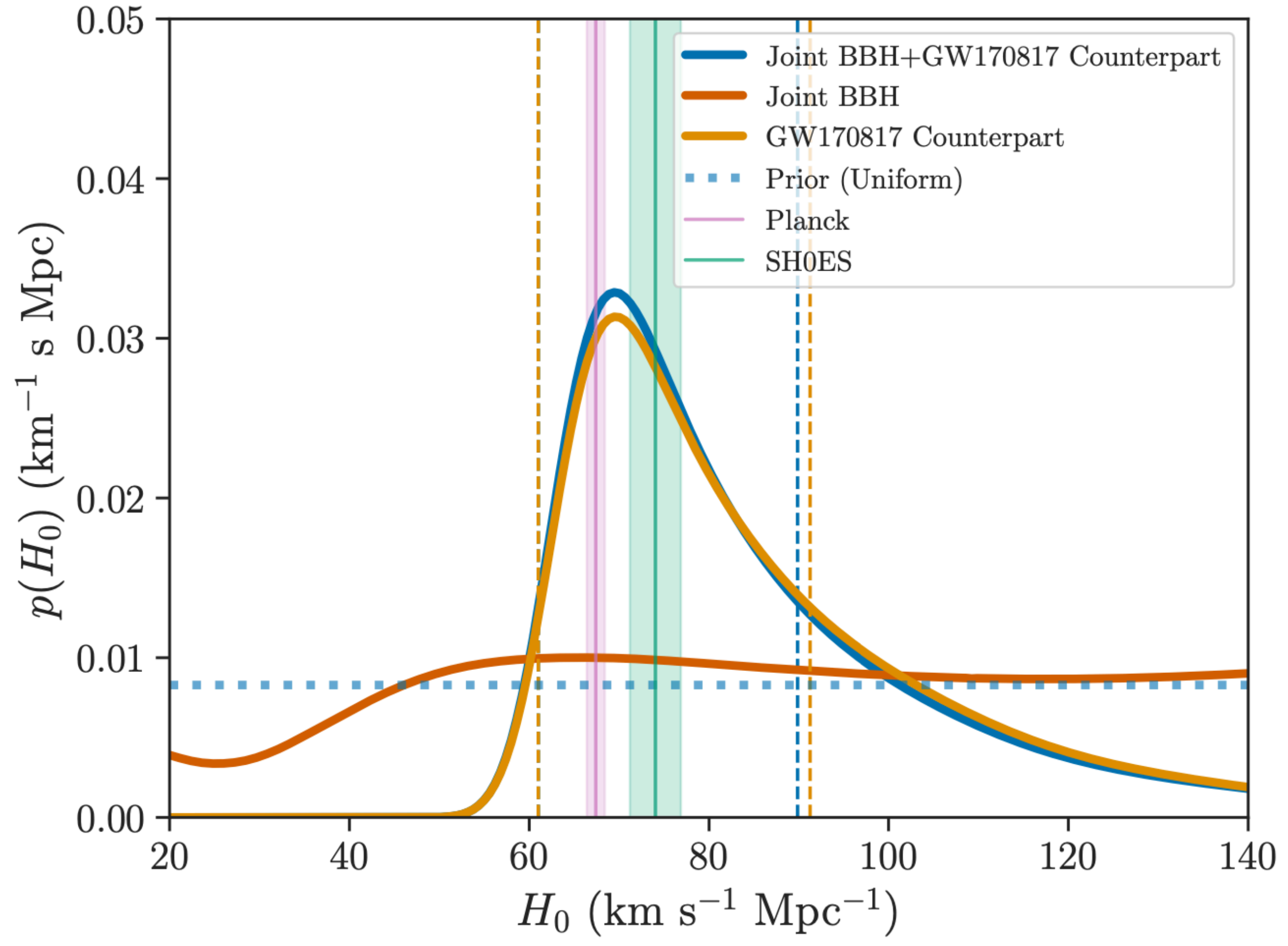
Gravitational wave data analysis

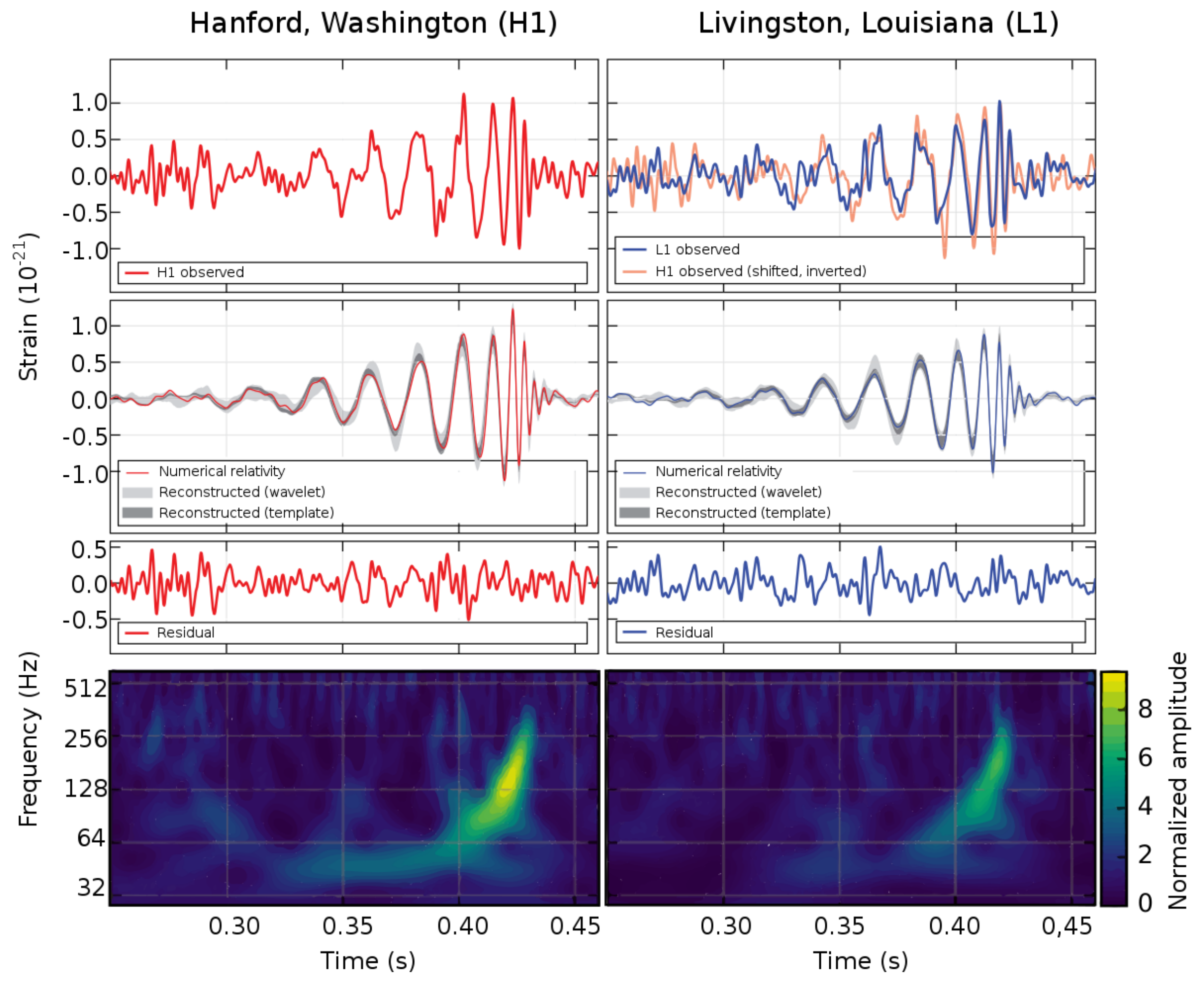


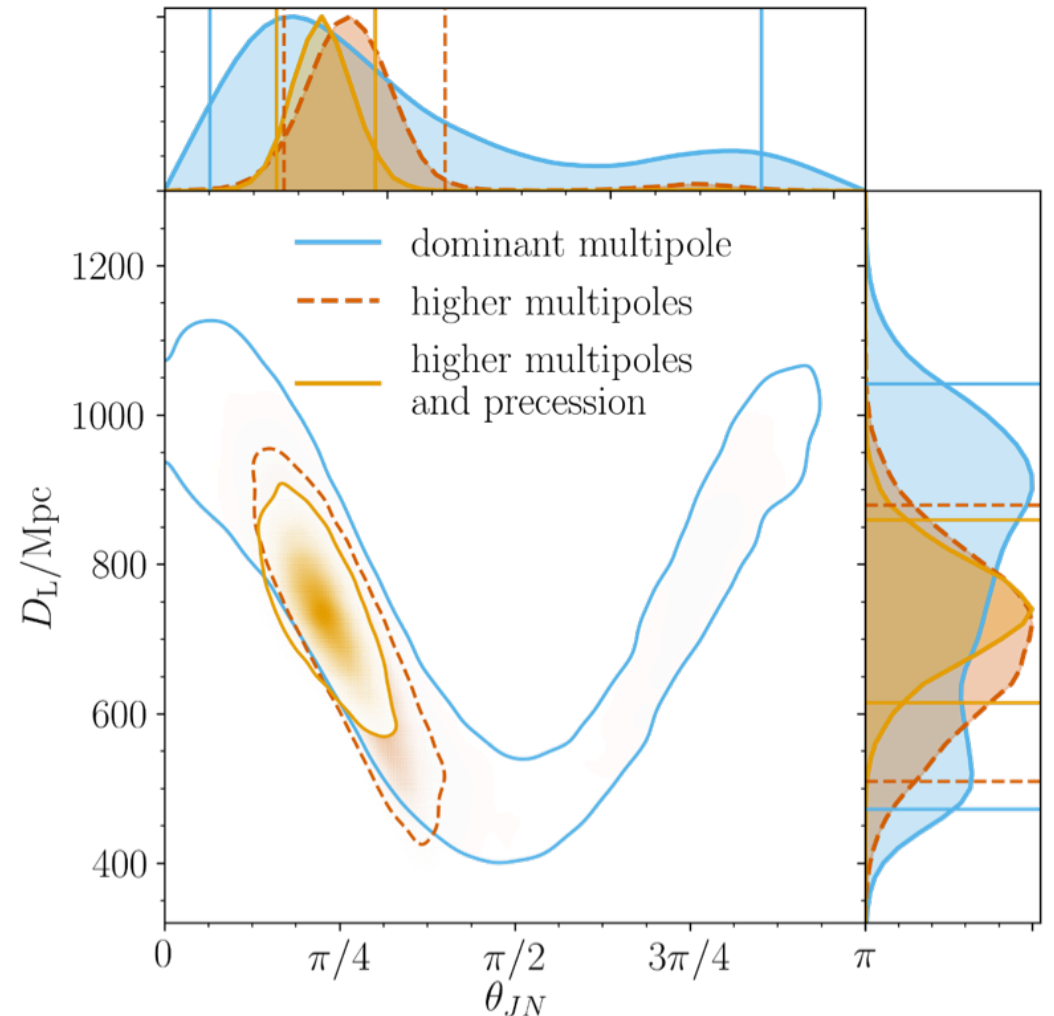
From noise ...

... to Science









2. Modelling the instrumental noise

Random variables

Random variables X are quantities that are not fixed, but can take new values each time they are observed (a ***realisation*** x). For example, the outcome of an experiment.

Random variables can be **discrete** or **continuous**, and the distribution of many realisations will follow a **probability distribution:**

$P(X = x)$ **probability** of X taking the **discrete** value x , $\sum_{\text{all x}} P(X = x) = 1$

$p_X(x)dx$ **probability** of X taking a **continuous** value x between x and $x + dx$, $\int_{x \in \mathcal{X}} p_X(x)dx = 1$

Expected values $\langle \cdot \rangle$ can be computed for **any function of the random variable:**

E.g, mean $\mu_X = \langle x \rangle = \int_{x \in \mathcal{X}} xp(x)dx$ **, variance** $\sigma^2 = \langle (x - \mu)^2 \rangle$ **, …**

Random variables

Usually, for a collection of experimental data, **we don't know the underlying probability distribution.**

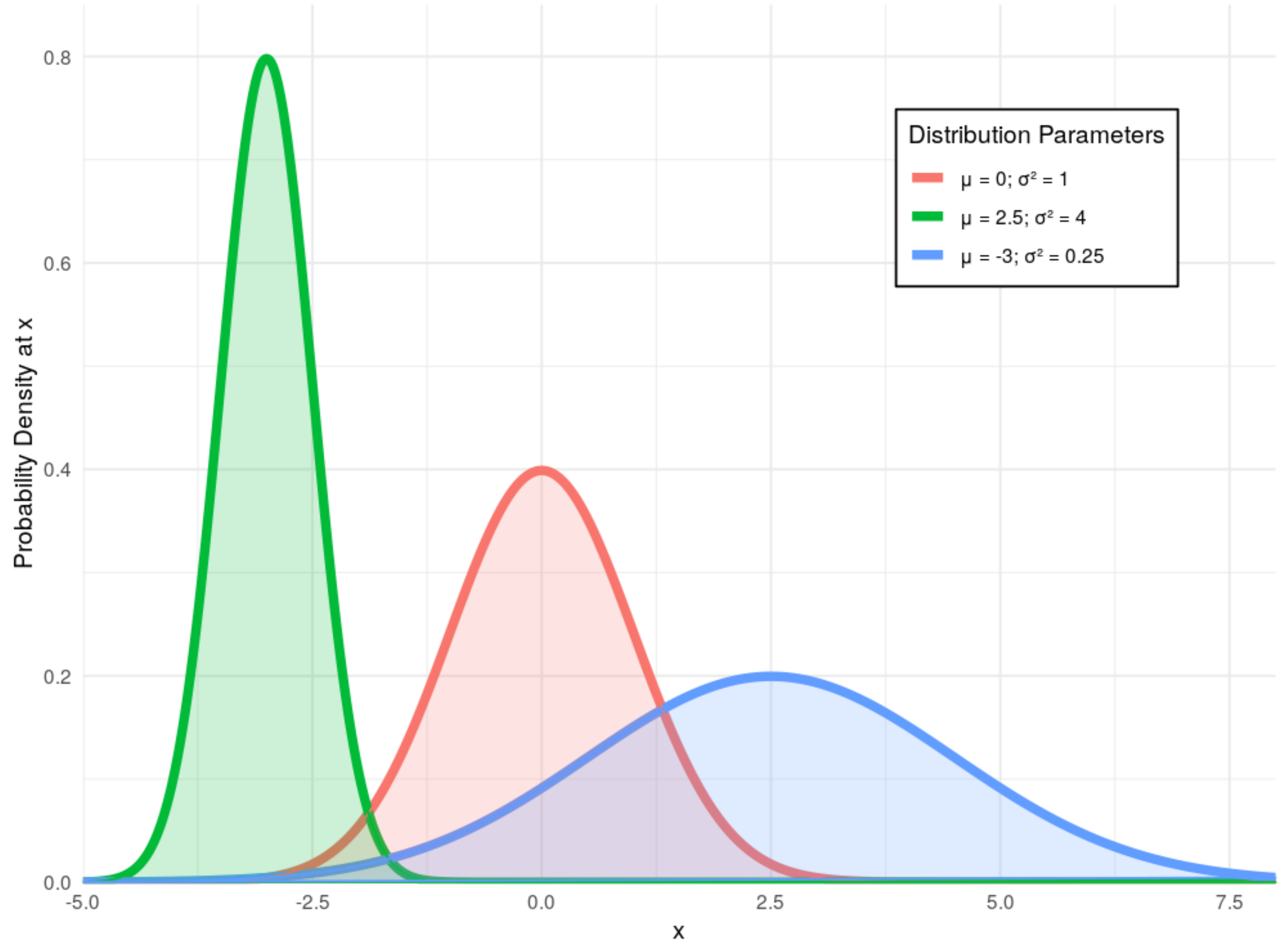
We can **estimate** the **expected values** from a large enough sample of the data **averaging over the ensemble**:

$$\mu_x = \langle x \rangle = \lim_{N\to\infty} \frac{1}{N} \sum_i^N x_i$$

$$\sigma_x^2 = \langle |x - \langle x \rangle|^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$

Gaussian variables follow a **Gaussian distribution:**

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$



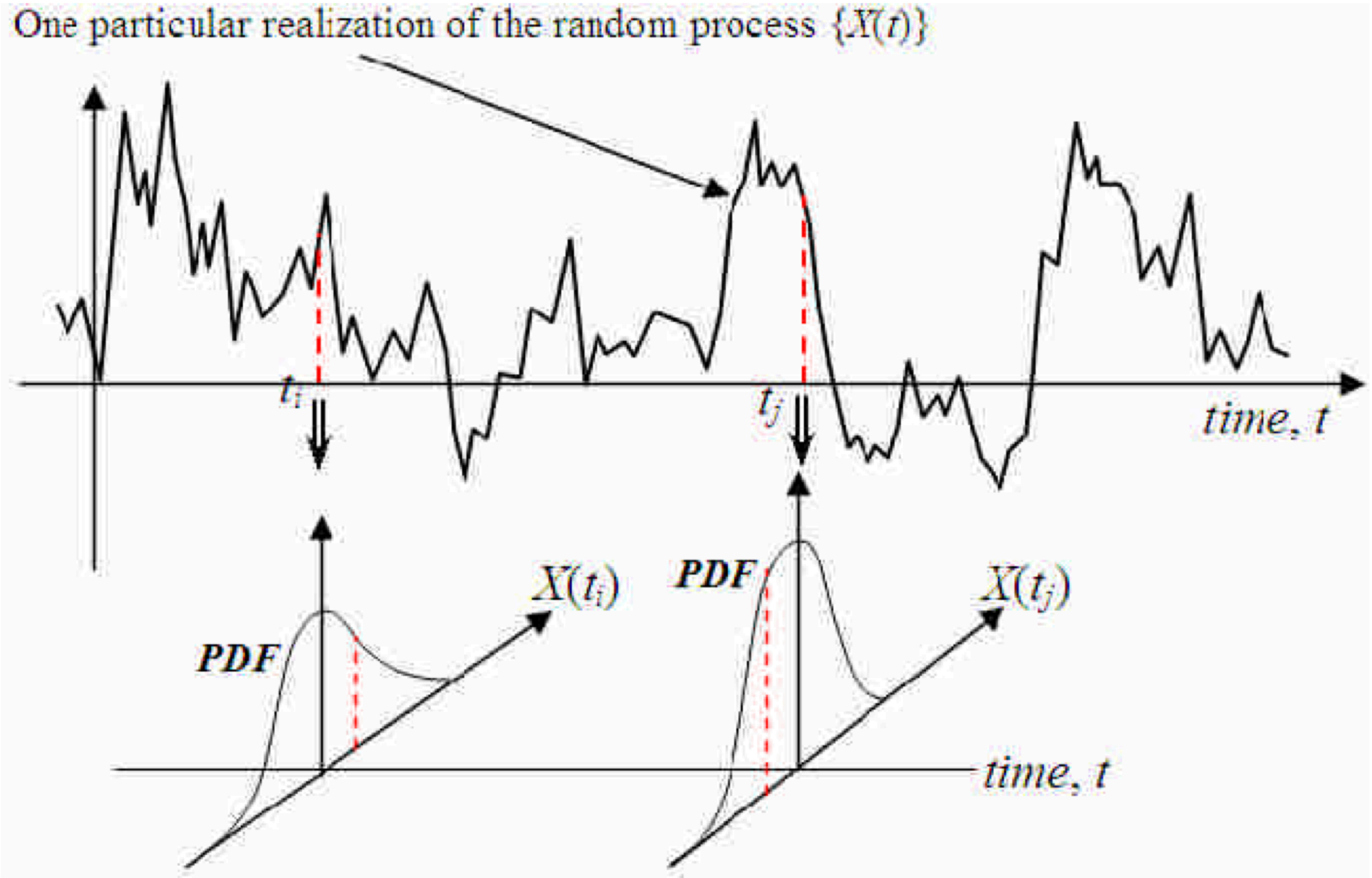
Random processes

Noise is a stochastic (random) process: we cannot determine in advance its exact value, but we can describe it by its statistical properties.

A **random process** is a set of random variables $\{X_t\}_{t \in T}$ usually describing the stochastic evolution of some system.

A **realisation** of the process $X(t)$ is a draw from the joint probability distribution of the set of random variables $\{X_t\}_{t \in T}$:

$$p(X_{t_N}; X_{t_{N-1}}; \ldots; X_{t_1}) dX_{t_N} dX_{t_{N-1}} \ldots dX_{t_1}$$



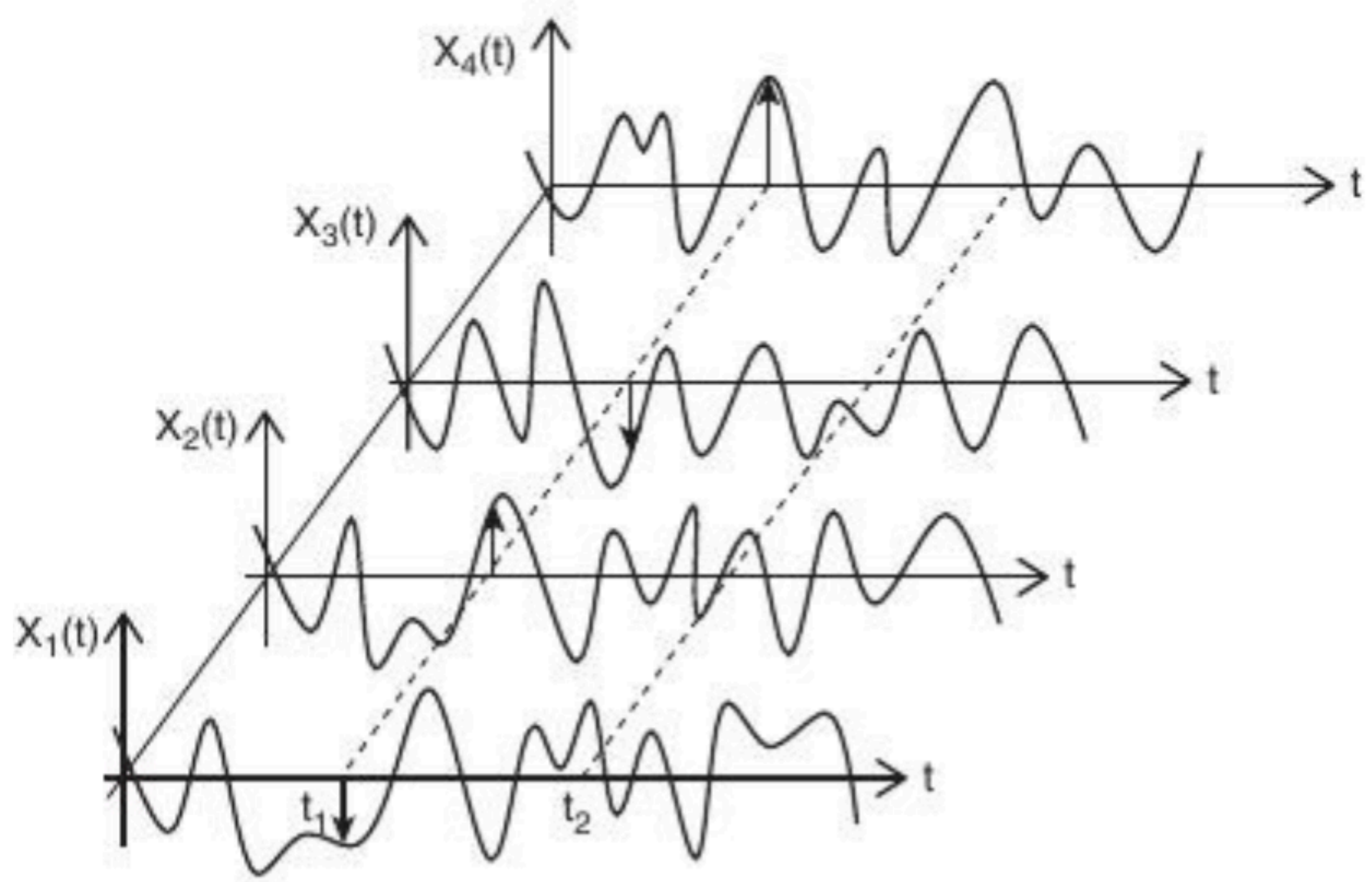
Random processes

We can estimate the **expected values** for the process considering an **ensemble of realisations**

$$\{X_1(t), X_2(t), \ldots, X_M(t)\}$$

$$\mu(t) = \langle X(t) \rangle = \lim_{M\to\infty} \frac{1}{M} \sum_{j=1}^{M} X_j(t)$$

$$\begin{aligned}\sigma^2(t) &= \langle |X(t) - \mu(t)|^2 \rangle \\ &= \lim_{M\to\infty} \frac{1}{M-1} \sum_{j=1}^{M} |X(t_j) - \mu(t_j)|^2\end{aligned}$$



Notice that in general the **expected values will be functions of time**.

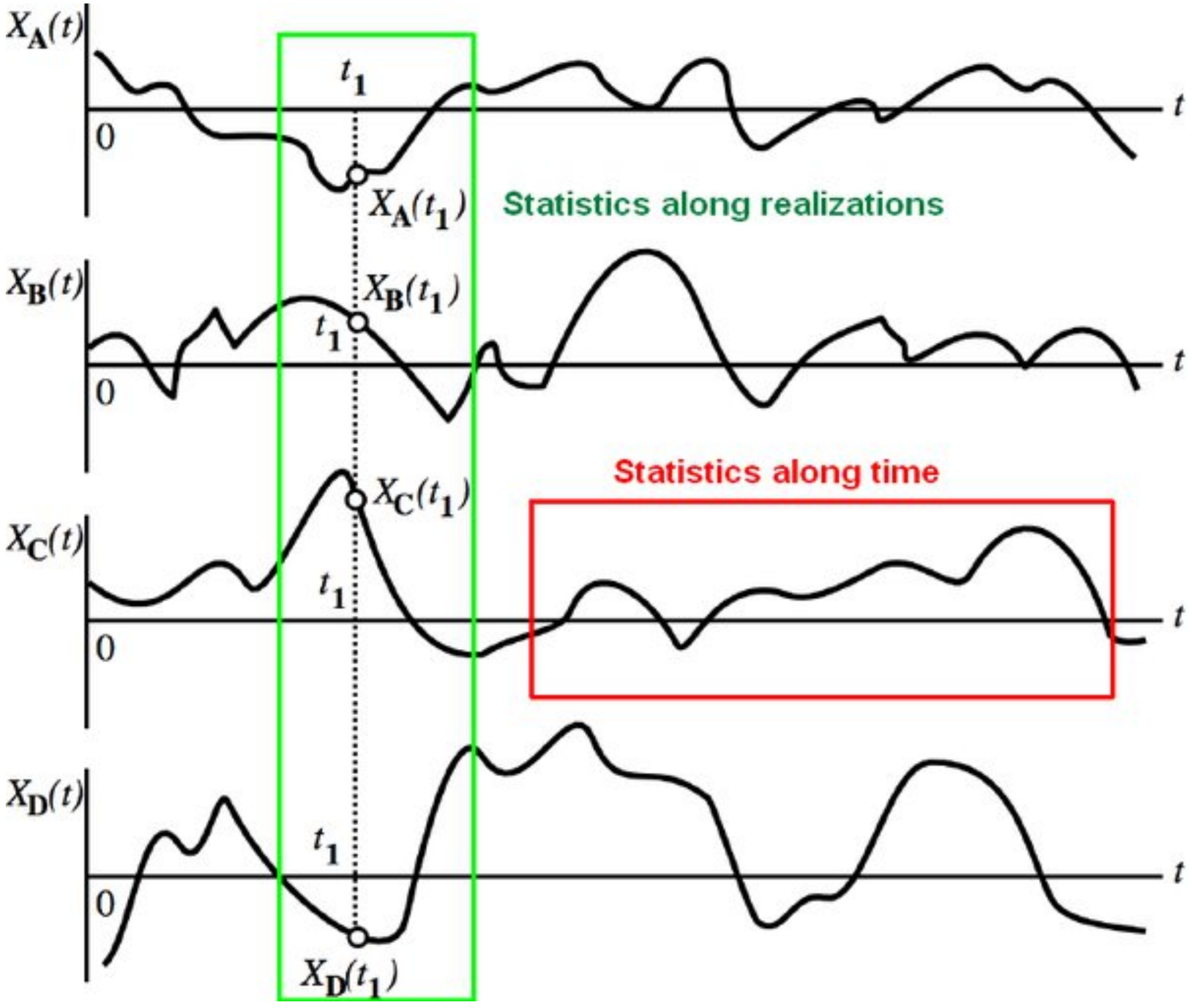
Stationarity

In our case, for a detector we **have only 1 noise realisation** $n(t)$**. How can we compute the statistical properties of the underlying process?**

Let's assume the process is **ergodic** (which implies that it is also **stationary**): the ensemble average is equal to the time average.

$$\hat{\mu} = \langle X(t) \rangle = \lim_{M\to\infty} \frac{1}{M} \sum_{j=1}^{M} X(t_j)$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} X(t)dt$$



This implies that the **statistical properties of the process do not change with time.**

Variance of the noise

We can shift the noise dataset to have a new dataset with zero-mean:

$n(t) \to n(t) - \hat{\mu}$.

Therefore we consider from now on that we work with a dataset with $\hat{\mu} = 0$.

We can assume noise is a **ergodic** process, its variance correspond to the **power** of the realisation:

$$\sigma^2 = \langle n(t)^2 \rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} n(t)^2 dt \equiv P_n$$

which is the zero-lag $\tau = 0$ value of **auto-correlation function** $R(\tau)$:

$$R(\tau) \equiv \langle n(t)n(t+\tau) \rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} n(t)n(t+\tau)dt$$

Variance of the noise

It is more useful to consider the **frequency dependence** of the noise.

Defining a **windowed noise segment** $n_T(t)$:

$$n_T(t) = \begin{cases} n(t) & -T/2 \leq t \leq T/2 \\ 0 & \text{otherwise} \end{cases}$$

And the **Fourier transform**:

$$\tilde{n}_T(f) = \int_{-\infty}^{\infty} dt n_T(t) \exp(-i2\pi f t)$$

Then:

$$\begin{aligned}
\langle n(t)^2 \rangle &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} n(t)^2 dt \\
&= \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} n_T(t)^2 dt \\
&= \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} |\tilde{n}_T(f)|^2 df \\
&= \lim_{T\to\infty} \frac{2}{T} \int_{0}^{\infty} |\tilde{n}_T(f)|^2 df \equiv \int_0^{\infty} S_n(f) df
\end{aligned}$$

Using Parseval's theorem

Using that for real $n(t)$ **->** $\tilde{n}(-f) = \tilde{n}^*(f)$

Power Spectral density

Therefore, we can define the **power spectral density** as:

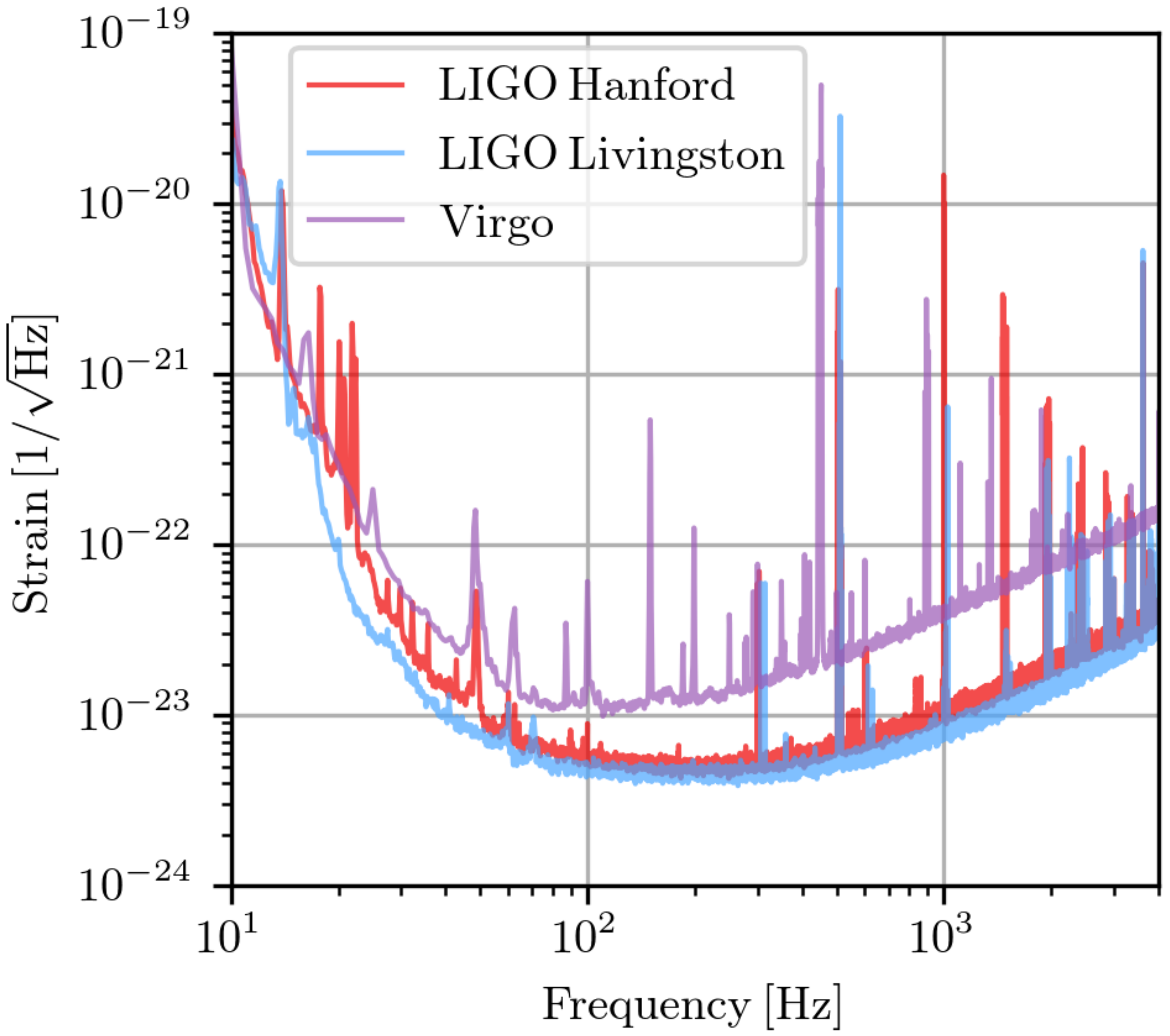
$$S_n(f) = |\tilde{n}(f)|^2 = \lim_{T\to\infty} \frac{1}{T} \left| \int_{-T/2}^{T/2} n(t) e^{-i2\pi f t} dt \right|^2$$

Using the same derivation with the auto-correlation function $R(\tau) = \langle n(t) n(t+\tau) \rangle$, it can be proven that:

$$R(\tau) = \int_{-\infty}^{\infty} (1/2) S_n(f) e^{-i2\pi f \tau} df$$

$R(\tau)$ and $S_n(f)/2$ are **Fourier conjugates**, therefore:

$$(1/2) S_n(f) = \int_{-\infty}^{\infty} d\tau R(\tau) e^{i2\pi\tau f}$$



Noise correlation

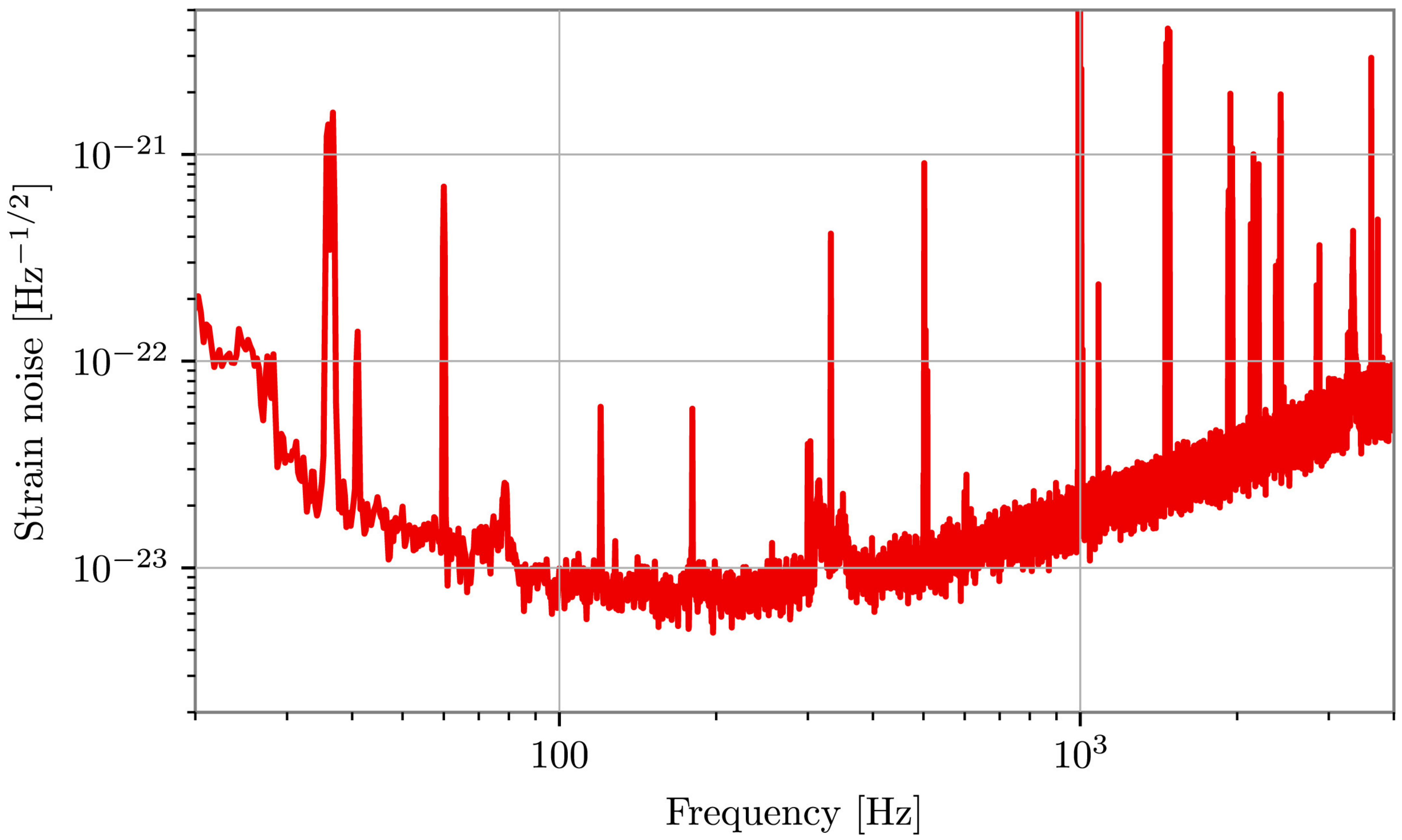
Another important features that follows from **stationarity** is that **noise is uncorrelated** in the **Fourier domain:**

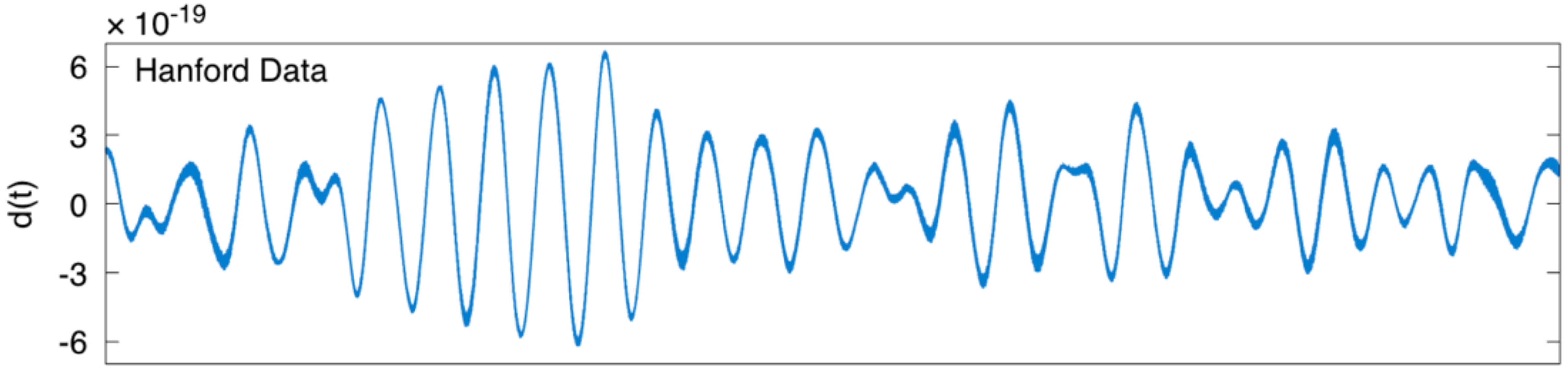
$$\langle \tilde{n}^*(f)\tilde{n}(f)\rangle = \frac{1}{2}\delta(f-f')S_n(f)$$

(you will prove this afternoon)

But in time-domain it is **correlated** (auto-correlated):

$$\langle n(t)n(t+\tau)\rangle = R(\tau) = \int_{-\infty}^{\infty}(1/2)S_n(f)e^{-i2\pi f\tau}df$$





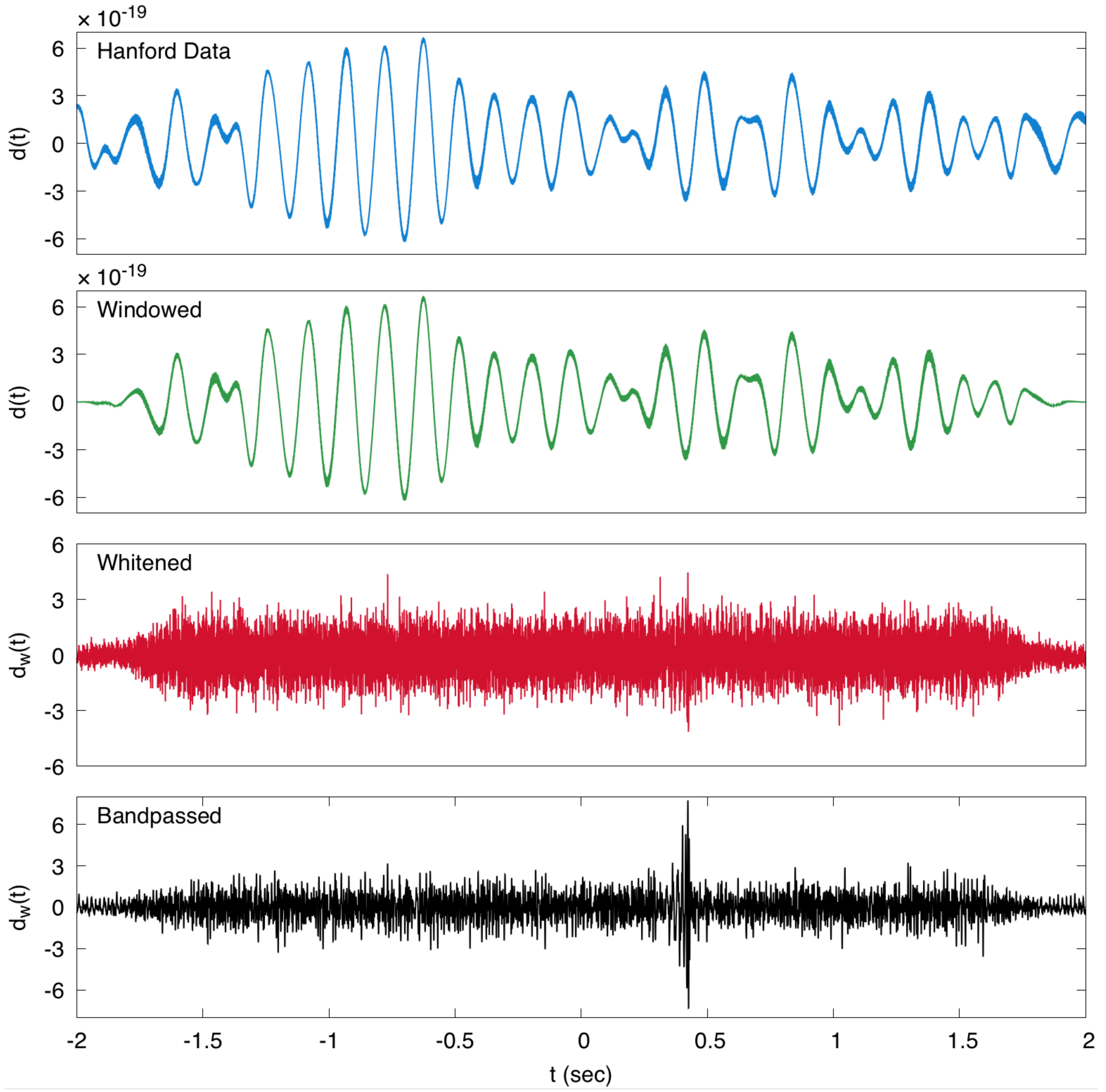
Whitening

In general the **power spectral density** $S_n(f)$ is a function of the frequency f, and we say that the noise is **colored.**

If $S_n(f) = \text{constant}$, then the noise at different time samples is uncorrelated, since $R(\tau) \propto \delta(\tau)$. We call this **white noise.**

We can **whiten** colored noise to have a new dataset with a constant PSD:

$$n_w(t) \equiv \int_{-\infty}^{\infty} df \frac{\tilde{n}(f)}{\sqrt{S_n(f)}} e^{i2\pi ft}$$



Gaussian noise

A useful assumption is that the instrumental noise in the detector follows approximately a **gaussian distribution:**

$$p(n(t)) \propto \exp\left[-\frac{n(t)^2}{2\sigma_n^2}\right]$$

Central limit theorem: a sum of a large number of independent random variables will tend to a Gaussian distribution, irrespective of their individual distributions.

Gaussian distributions are characterized only by the **mean** (which we can always set to 0) and the **variance** σ^2 (which we now know how to compute). The **probability density** of having a particular noise realisation $n_0(t)$ is given by:

$$p(n_0) \propto \exp\left[-\frac{1}{2}\int_{-\infty}^{\infty} df \frac{|\tilde{n}_0(f)|^2}{(1/2)S_n(f)}\right] \propto \exp\left[-\frac{1}{2}4\int_{0}^{\infty} df \frac{|\tilde{n}_0(f)|^2}{S_n(f)}\right]$$

Gaussian noise

It is useful to define a **noise-weighted inner product** between two **real** functions $a(t)$ and $b(t)$:

$$(a\,|\,b) \equiv \mathrm{Re}\int_{-\infty}^{\infty} df\,\frac{\tilde{a}(f)\tilde{b}(f)^*}{(1/2)S_n(f)} = 4\mathrm{Re}\int_{0}^{\infty} df\,\frac{\tilde{a}(f)\tilde{b}(f)^*}{S_n(f)}$$

Notice that since $S_n(f) > 0$, this inner product is **positive-definite**:

$$(a\,|\,a) = 4\mathrm{Re}\int_{0}^{\infty} df\,\frac{|\,\tilde{a}(f)\,|^2}{S_n(f)} \geq 0$$

In terms of this inner product, the probability density for a particular noise realisation is:

$$p(n(t) = n_0) \propto e^{-(n_0|n_0)/2}$$

Summary

Instrumental noise can be approximated as a **stationary Gaussian process.**

From **stationarity**, we can relate its **statistical variance** to the **power spectral density**.

From **Gaussianity**, we can **completely model** the noise using this variance.

$$p(n(t) = n_0) \propto e^{-(n_0|n_0)/2}$$

This will be very useful in the next sections.

3. Matched filtering and detection statistics

Signal hypothesis

What is the probability of having a given GW signal $h(t)$ in the detector data?

- Assume we **know the noise** (we assume it is **gaussian, stationary** and therefore we can fully characterise it with its variance σ_n^2, that we can compute)
- Assume we **know the signal** $h(t)$ we are looking for

Null hypothesis: $\mathscr{H}_0$: $d(t) = n(t)$ no signal in the detector data

Signal hypothesis: $\mathscr{H}_1$: $d(t) = n(t) + h(t)$ signal in the detector data

We can compute the **likelihood ratio** of the signal hypothesis as:

$$\Lambda(\mathscr{H}_1, d) = \frac{p(d, \mathscr{H}_1)}{p(d, \mathscr{H}_0)}$$

probability of data being described by signal hypothesis

probability of data being described by null hypothesis

Likelihood ratio

$$\Lambda(\mathscr{H}_1, d) = \frac{p(d, \mathscr{H}_1)}{p(d, \mathscr{H}_0)}$$ how **likely** is the signal hypothesis versus the null hypothesis.

We can compute this using the **model of the noise**:

$$p(d, \mathscr{H}_0) = p_n[n(t) = d(t)] \propto e^{-(d|d)/2}$$

probability of data being described by null hypothesis

$$p(d, \mathscr{H}_1) = p_n[n(t) = d(t) - h(t)] \propto e^{-(d-h|d-h)/2}$$

probability of data being described by signal hypothesis

Therefore:

$$\Lambda(\mathscr{H}_1, d) = \frac{e^{-(d-h|d-h)/2}}{e^{-(d|d)/2}} = e^{(d|h)}e^{-(h|h)/2}$$

Likelihood ratio of the signal hypothesis, i.e, how likely is that the signal $h(t)$ is in the data

Matched filter

$$\Lambda(\mathscr{H}_1, d) = e^{(d|h)} e^{-(h|h)/2}$$

The **likelihood ratio** of the **signal hypothesis** increases monotonically with

$$\hat{s} \equiv (d\,|\,h) = 4\mathrm{Re} \int_0^\infty df \, \frac{\tilde{d}(f)\tilde{h}(f)^*}{S_n(f)} \,,$$

which is called the **matched filter** of the data, and it is an **optimal detection statistic for gaussian stationary noise.**

We can **establish a threshold value** $\hat{s}_{\mathrm{threshold}}$ **and above that value we accept the signal hypothesis.**

Matched filter

$\hat{s} \equiv (d\,|\,h)$ is called **matched filter** because it is equivalent to **filter** (convoluting) the detector data with a linear filter $K(t)$:

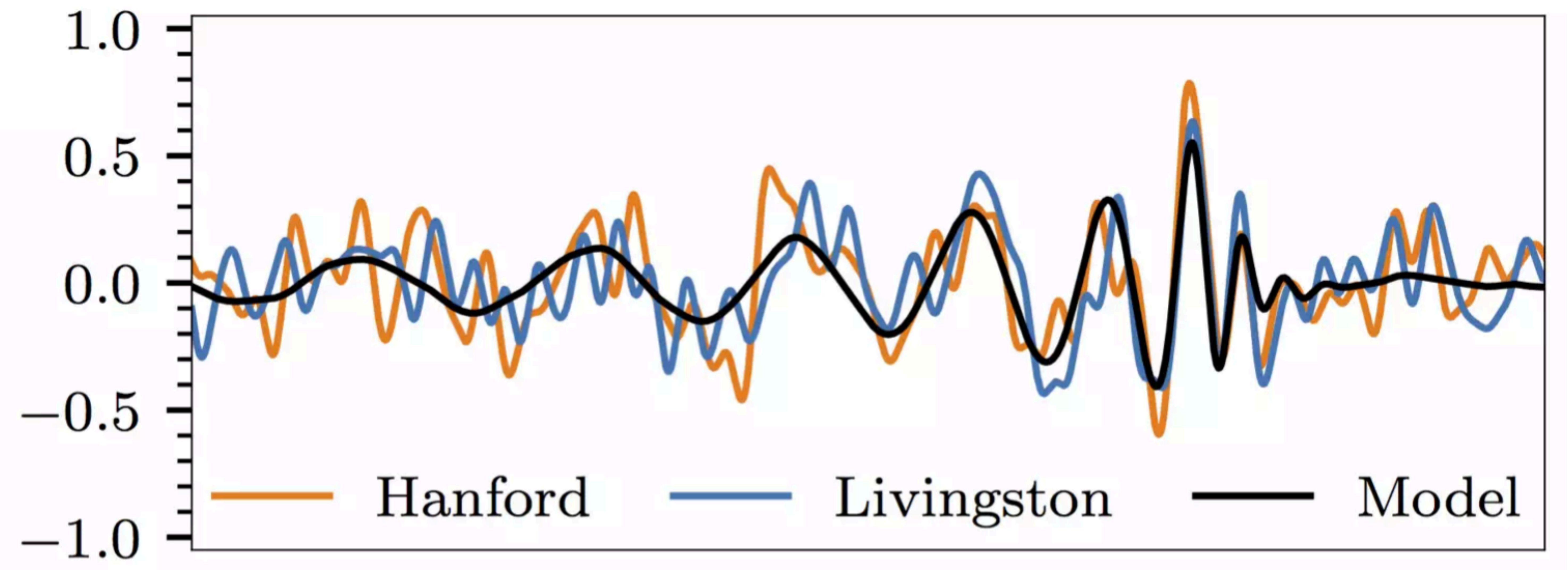
$$\hat{s} = \int_{-\infty}^{\infty} dt d(t) K(t) = \int_{-\infty}^{\infty} df \tilde{d}(f) \tilde{K}^*(f)$$

such that $\tilde{K}(f) \propto \tilde{h}(f)/S_n(f)$, so the filter is proportional to the noise-weighted signal, the **filter matches the signal**. (you will prove it this afternoon)

Matched filter

We can also interpret the matched filter as the **convolution** (filtering) of the **whitened data** $d_w(t)$ with the **whitened expected signal** $h_w(t)$ **:**

$$\hat{s} = (d\,|\,h) = 4\mathrm{Re}\int_0^\infty df\,\frac{\tilde{d}(f)\tilde{h}(f)^*}{S_n(f)} = \int_{-\infty}^{\infty} df\,\frac{\tilde{d}(f)}{\sqrt{S_n(f)/2}}\frac{\tilde{h}^*(f)}{\sqrt{S_n(f)/2}} = \int_{-\infty}^{\infty} dt d_w(t) h_w(t)$$



Signal-to-noise ratio

The **matched filter $\hat{s}$ is a random Gaussian variable** (since it depends on the detector data and hence the noise, which we have assumed is Gaussian).

When **no signal** is present, its expected value is 0:

$$\langle \hat{s} \rangle_{h=0} = (\langle d \rangle_{h=0} \,|\, h) = 0$$

Notice the abuse of notation, we are saying that the true signal is 0, but we are filtering with the signal we expect to find, which is not 0.

and its variance is:

$$\sigma_{\hat{s}}^2 \equiv \langle \hat{s}^2 \rangle_{h=0} = \langle (d\,|\,h)(d\,|\,h) \rangle_{h=0} = (\langle d \rangle_{h=0} \,|\, \langle d \rangle_{h=0}) + 2(\langle d \rangle_{h=0} \,|\, h) + (h\,|\,h) = (h\,|\,h)$$

It is useful to construct a new detection statistics with **normalised variance:**
the matched filter signal-to-noise ratio:

$$\rho = \hat{s}/\sigma_{\hat{s}} = (d\,|\,h)/(h\,|\,h)^{1/2}$$

Signal-to-noise ratio

The **signal-to-noise ratio (SNR)** ρ is a random variable with unit variance:

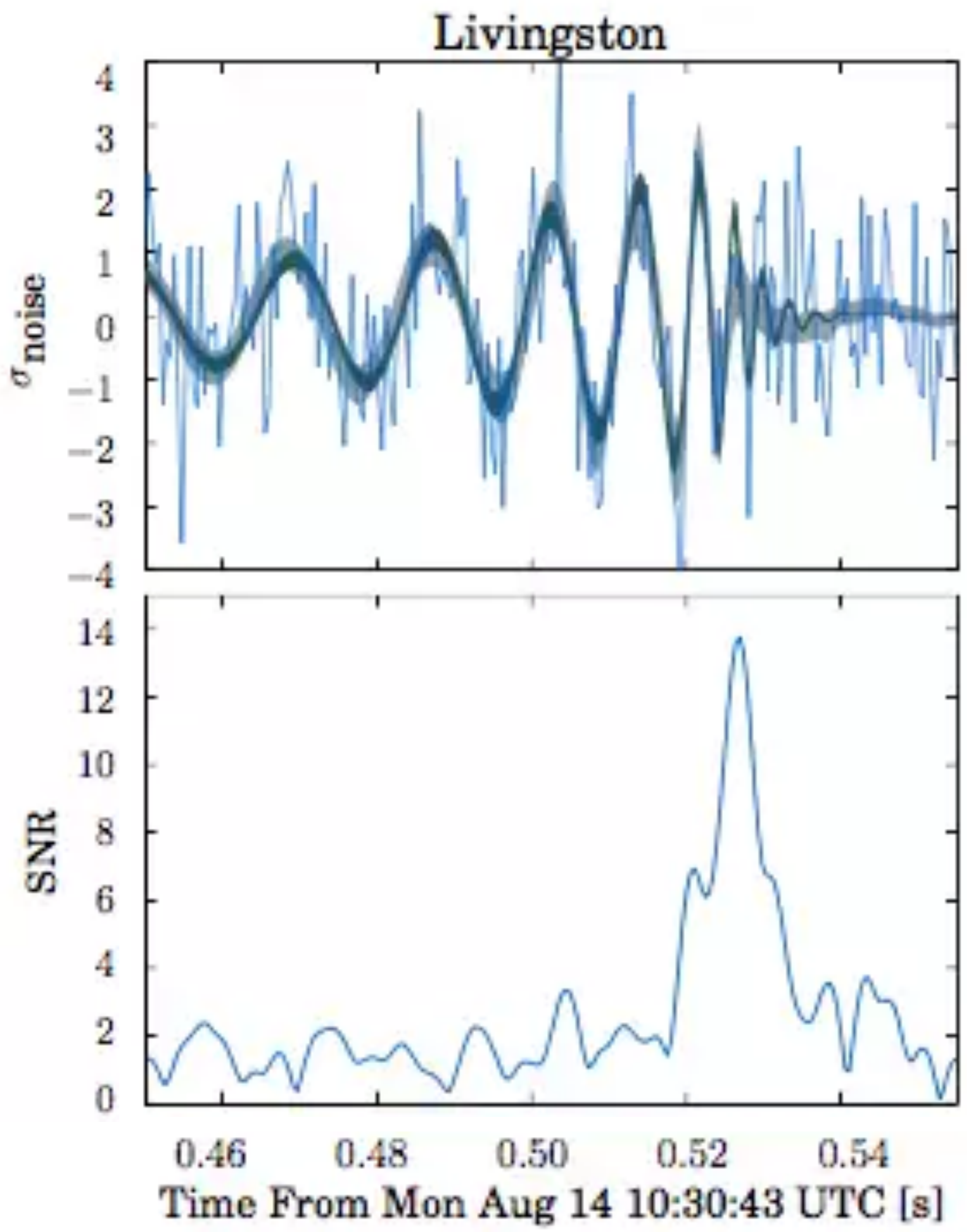
$$\sigma_\rho^2 \equiv \langle \rho^2 \rangle_{h=0} = 1$$

Its expected value in the presence of the signal $h(t)$ is:

$$\rho_{\rm opt} \equiv \langle \rho \rangle_{h=h} = (\langle d \rangle_{h=h} \,|\, h)/(h\,|\,h)^{1/2} = (h\,|\,h)^{1/2}$$

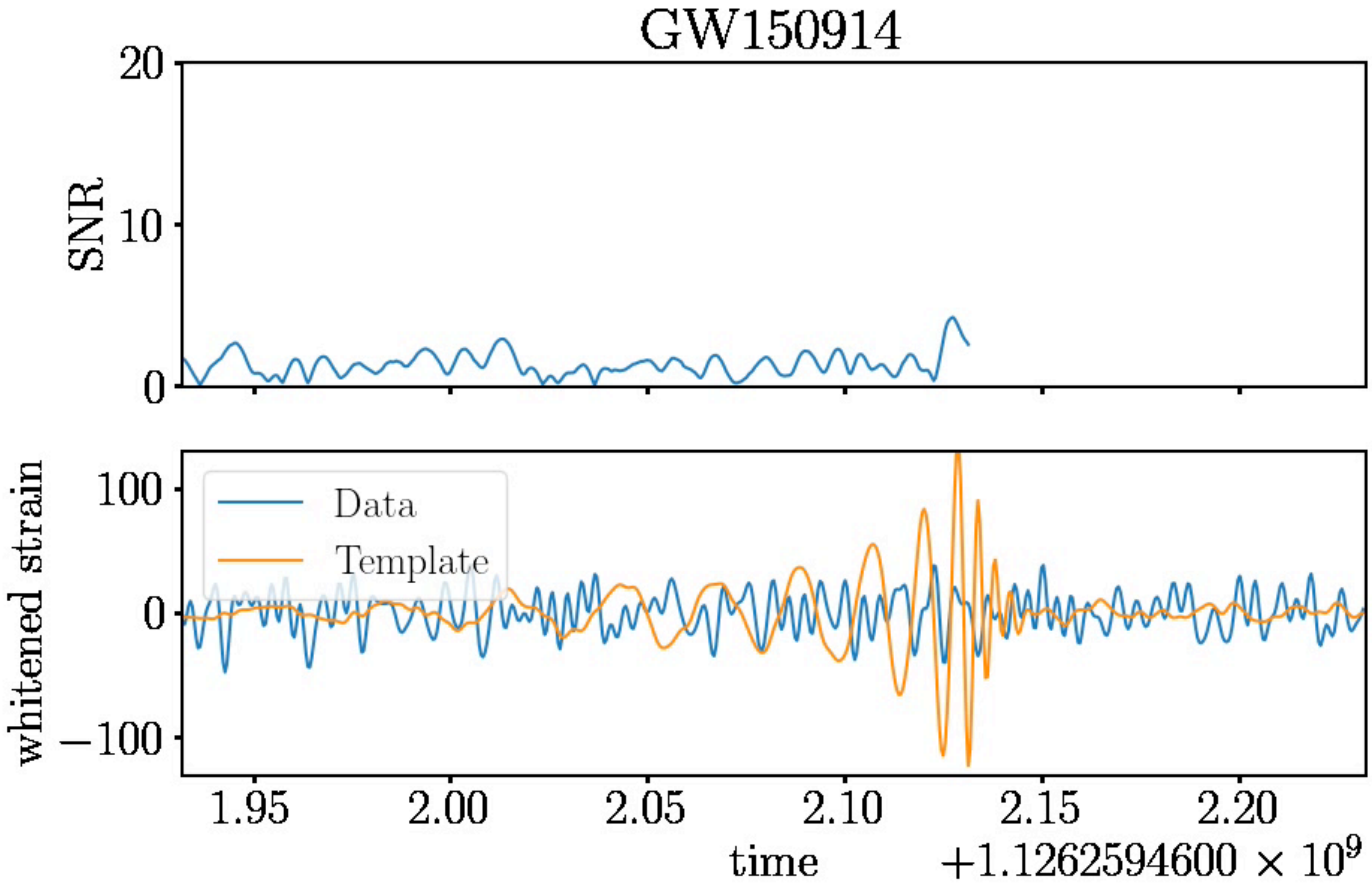
this expected value is usually called **optimal SNR** for the signal $h(t)$

GW170814



From Abbott et al. 2018

Signal-to-noise ratio



Credit: Yifan Wang

Horizon distance

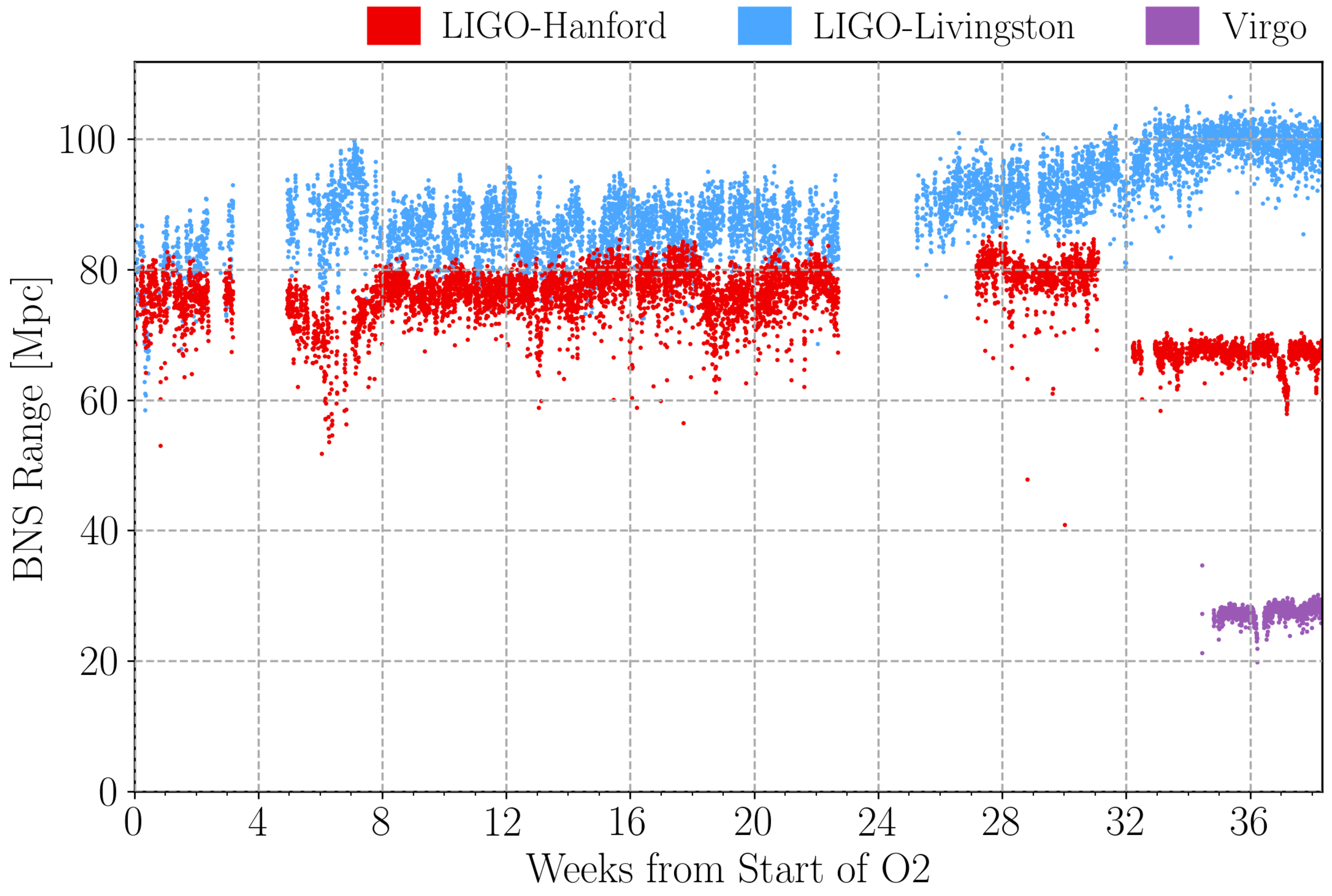
The **optimal SNR** $\rho_{\rm opt} = (h\,|\,h)^{1/2}$ can be employed to see how sensitive is a detector for a particular signal.

Equivalently, using a **threshold SNR** $\rho_{\rm thres}$ for detection, we can estimate **how far** we can detect a particular signal. This is called **horizon distance** and it is a practical quantity to quantify the sensitivity of a detector.

We have seen that $h(t) \propto 1/D$, therefore $\rho_{\rm opt} = (h\,|\,h)^{1/2} \propto 1/D$. We want to see at which distance $D_{\rm thres}$:

$$\frac{D}{D_{\rm thres}}\rho_{\rm opt} = \rho_{\rm thres} \rightarrow D_{\rm thres} = \frac{\rho_{\rm opt}/D}{\rho_{\rm thres}}$$

From GWTC-1, Abbott+2019



Typically the **BNS range** is reported, the distance at which we can observe a $1.4M_\odot + 1.4M_\odot$ system (averaged over sky location and assuming optimal inclination) with a **threshold SNR** of 8

Summary

We can consider detecting a GW signal in terms hypothesis testing.

Since we have a **model of the noise**, we can compute the **likelihood** of the data being described by **only the noise**, or **noise plus a signal.**

The **matched-filter SNR** is a useful **detector statistic** (we can consider a signal is present if the SNR is greater than a threshold).

4. Searching for GW signals

Maximum likelihood estimator

In practice, **we do not know the possible signal** at the detector.

We want to check **if there is a signal of a particular kind**, for example a CBC signal: $h(t; \lambda_{\text{true}})$, that can be parameterised by some parameters λ_{true} (masses, spins, inclination of the source, sky position, arrival time, …)

We **can model CBC signals** and produce **template signals** $\hat{h}(t; \lambda)$.

For each template, we have the **template hypothesis** $\mathcal{H}_\lambda$ of the data being $d(t) = n(t) + \hat{h}(t; \lambda)$, with likelihood $p(d, \mathcal{H}_\lambda)$.

But we want to check the **signal hypothesis** $\mathcal{H}_1$**,** we want to know **if a signal is present in the data,** and in general $p(d, \mathcal{H}_\lambda) \neq p(d, \mathcal{H}_1)$.

Maximum likelihood estimator

If there is a **strong signal** in the data, and we have **covered sufficiently the parameter space** with **templates**, the **likelihood** $p(d, \mathscr{H}_\lambda)$ will be strongly peaked with a **maximum** at $\lambda_{\max}$close to the true parameters λ_{true}.

In this case, we can approximate:

$$\Lambda(\mathscr{H}_1, d) \approx \Lambda(\mathscr{H}_{\lambda_{\max}}, d)$$

and use $\Lambda(\mathscr{H}_{\lambda_{\max}}, d)$ as the detection statistics.

This is called the **maximum likelihood estimator.**

Some parameter can be maximised analytically (we will see it for the amplitude and the arrival time in the **exercises** this afternoon). For the others, we have to construct **template banks** to find the maximum likelihood estimator.

Template banks

Let us split the **parameters** $\boldsymbol{\lambda}$ into parameters $\boldsymbol{\theta}$ we can maximise analytically (**extrinsic parameters**) and parameters $\boldsymbol{\lambda}_{\rm template}$ that we have to fill with templates for obtaining the maximum likelihood estimator (**intrinsic parameters**).

How can we quantify if we **cover well** the parameter space $\boldsymbol{\lambda}_{\rm template}$ with templates?

Consider **normalized templates** $g_i(t; \boldsymbol{\lambda}_{\rm template}) \rightarrow (g_i \,|\, g_i) = 1$ (you'll see this afternoon that the amplitude can be maximised analytically)

The **inner product** $(g_i \,|\, g_j)$ is a quantity between 0 and 1 (since the templates are normalized), quantifying **how similar** the g_i and g_j templates are.

We **maximise it analytically** over $\boldsymbol{\theta}$ to see **how close** g_i **and** g_j **in the parameter space** $\boldsymbol{\lambda}_{\rm template}$**:**

$O(g_i, g_j) \equiv \max_{\boldsymbol{\theta}} (g_i \,|\, g_j)$ This is called the **overlap** (notice that $O(g_i, g_j) \geq (g_i \,|\, g_j)$)

Template banks

Consider the **SNR of the signal** $h_{\mathrm{true}} \equiv h(t; \boldsymbol{\lambda}_{\mathrm{true}})$ **with a template** $g_i \equiv g(t; \boldsymbol{\lambda}_i)$ **maximized analytically over** $\boldsymbol{\theta}$**:**

$$\max_{\boldsymbol{\theta}} \rho_i = \max_{\boldsymbol{\theta}} (h_{\mathrm{true}} | g_i) = ||h_{\mathrm{true}}|| \max_{\boldsymbol{\theta}} (g_{\mathrm{true}} | g_i) = \rho_{\mathrm{opt}} \max_{\boldsymbol{\theta}} (g_{\mathrm{true}} | g_i) = \rho_{\mathrm{opt}} O(g_{\mathrm{true}} | g_i) \leq \rho_{\mathrm{opt}}$$

The **relative SNR loss** due to the **mismatch** of the template and the signal is:

$$\Delta_{\mathrm{rel}} \rho \equiv \frac{\rho_{\mathrm{opt}} - \rho_i}{\rho_{\mathrm{opt}}} = 1 - O(g_{\mathrm{true}}, g_i)$$

From $\rho_{\mathrm{opt}} \propto 1/D \rightarrow \Delta_{\mathrm{rel}} D_{\mathrm{horizon}} = \Delta_{\mathrm{rel}} \rho$,

and we lose a horizon volume $\Delta_{\mathrm{rel}} V_{\mathrm{horizon}} = 1 - O(g_{\mathrm{true}}, g_i)^3$

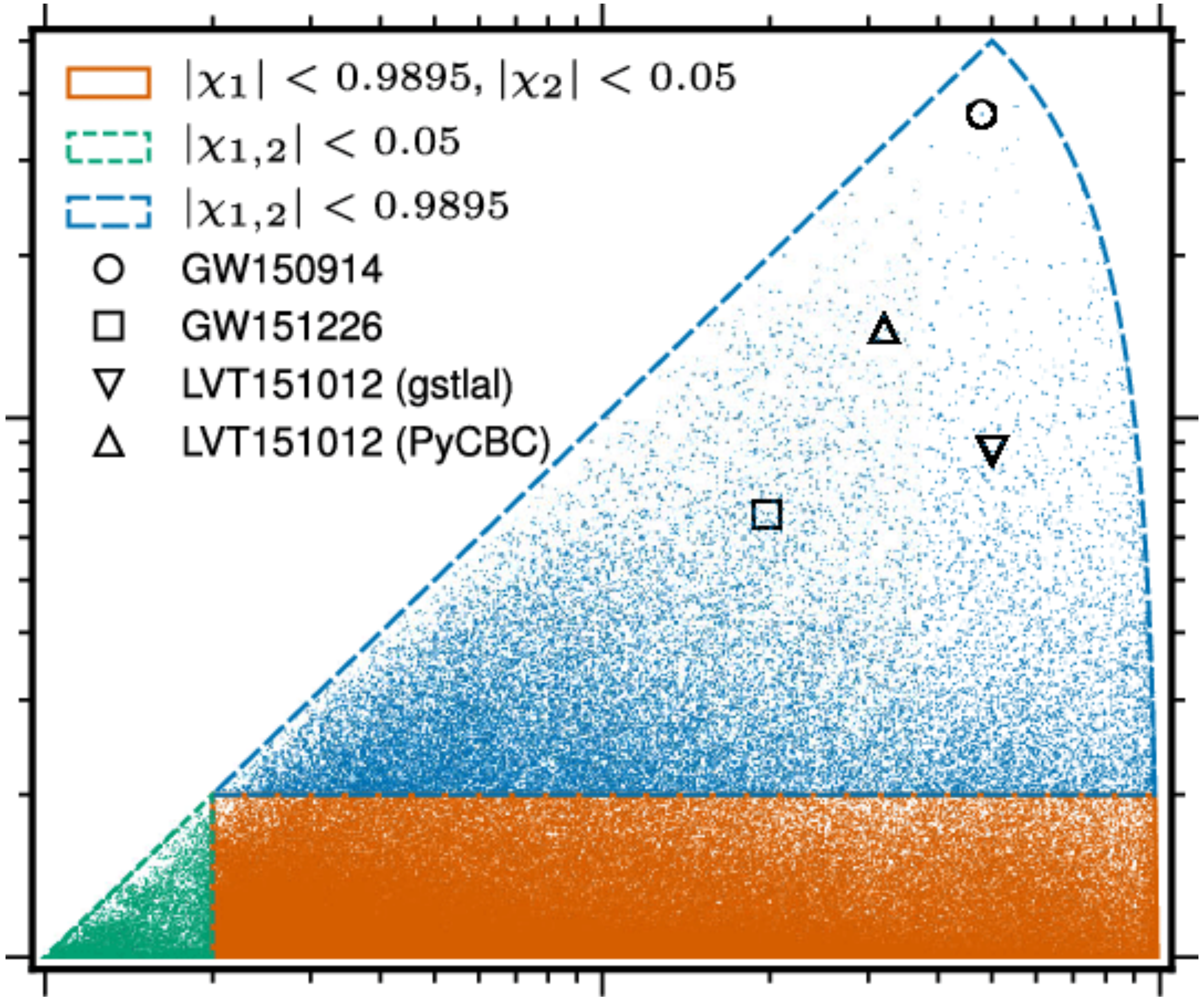
Template banks

A common criterion is to **require** that we **do not lose more than a 10% of observable volume.**

This translates into $O(g_{\rm true}, g_i) \geq (0.9)^{1/3} \approx 0.97$ for all g_i.

Therefore, the **template bank** is constructed such that for each template g_i, the closest template satisfies $O(g_{i,\rm closest} \,|\, g_i) \geq 0.97 \,\forall g_i$.

Typically this is done with **stochastic placing** techniques.

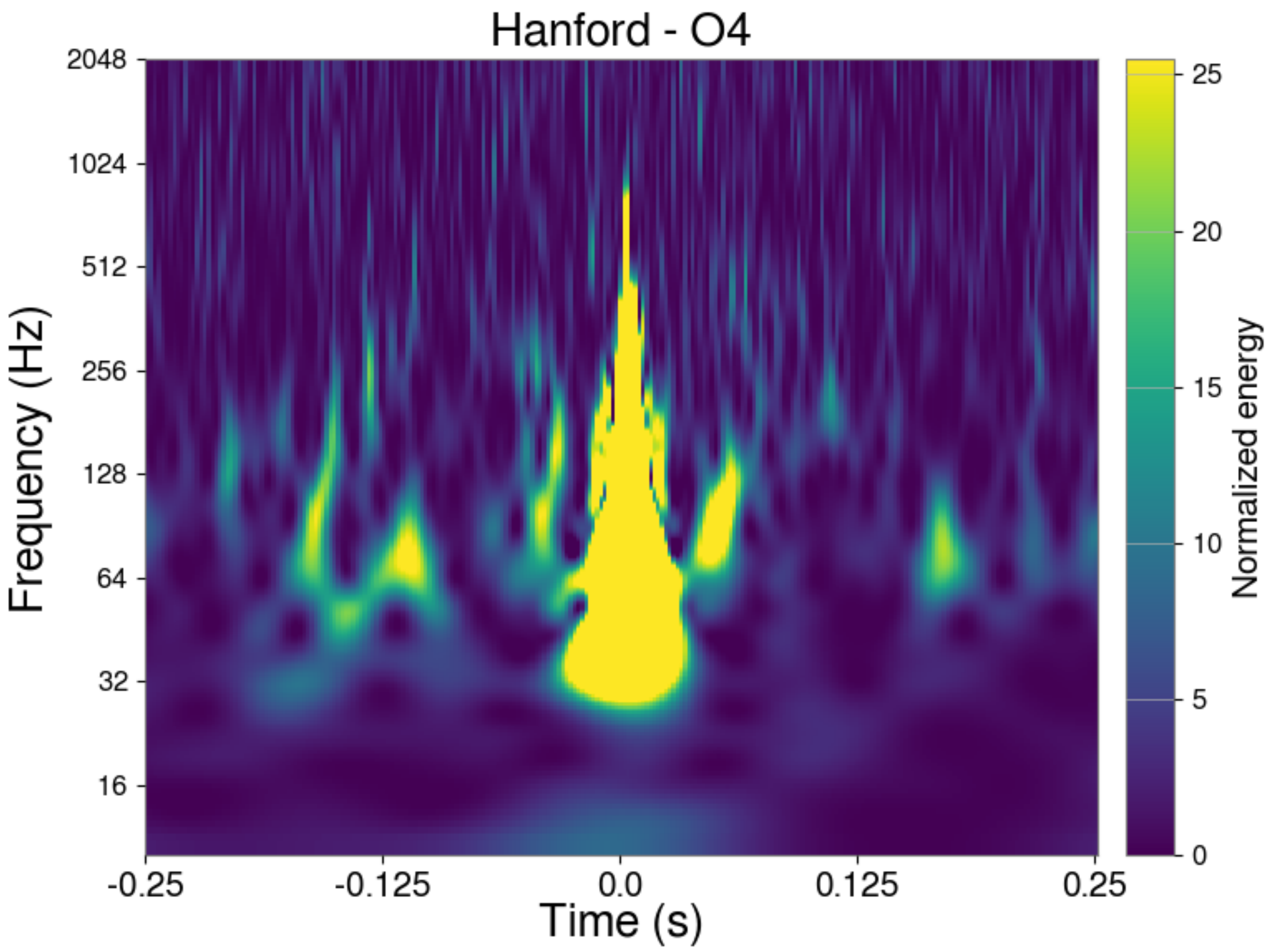


Follow-up template bank triggers

Once a **template bank** is constructed, the detector data is **matched filtered** with the template bank, and **triggers** with $\rho > \rho_{\text{thres}}$ are stored.

We have **assumed** that the **noise is gaussian and stationary**, but in practice this is just an **approximation**.

There can be **non-gaussianities** and **non-stationary glitches** that usually have **sufficient power to trigger the detection statistic**.



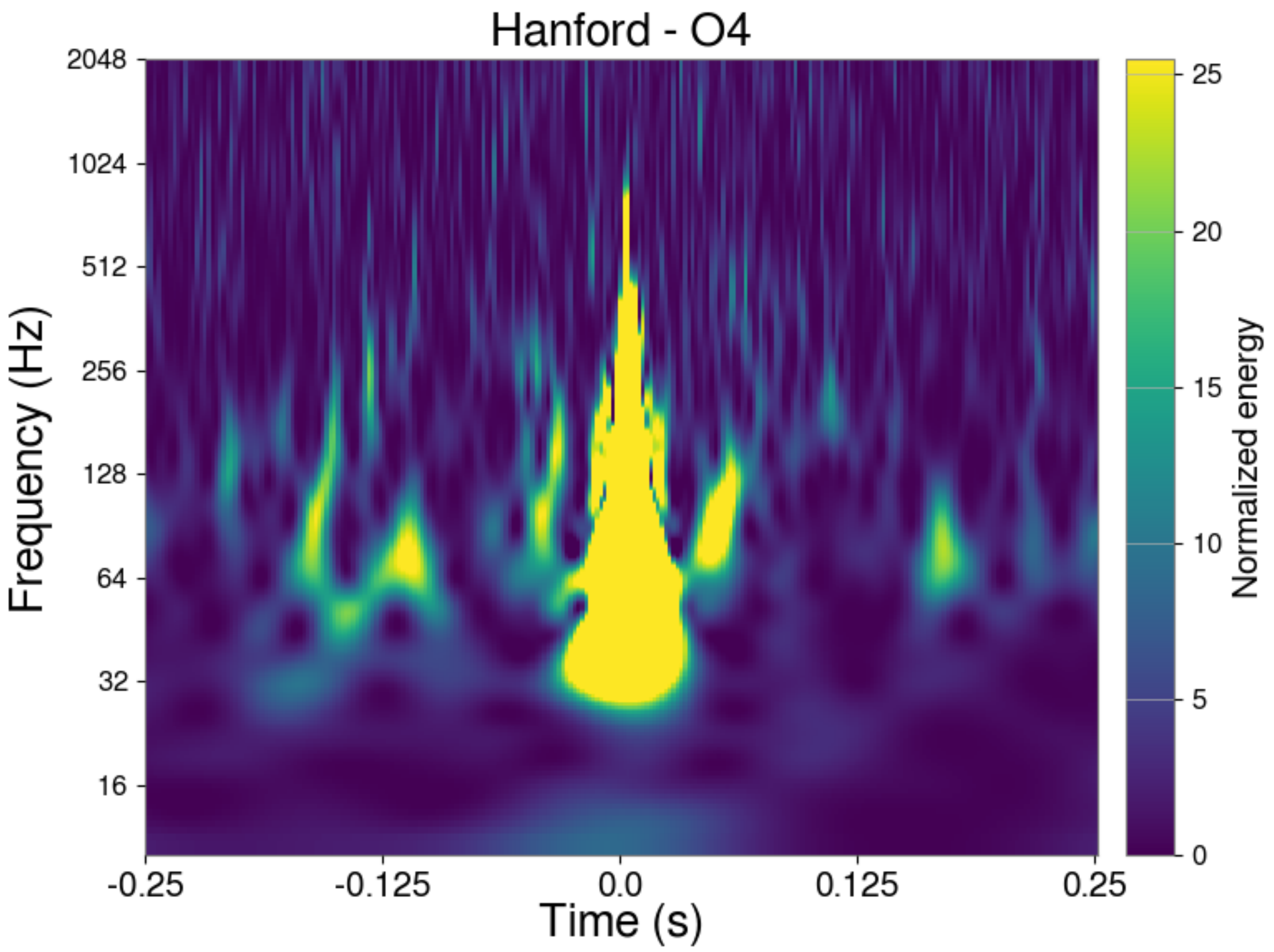
Follow-up template bank triggers

There are several **additional steps** that can be done to mitigate **false detections**:

- **Waveform consistency tests** (χ^2-test)

- **Veto problematic data, coincidence test between several detectors**

- **Estimate False Alarm Probability (FAR) from the background.**



Waveform consistency: χ^2-test

A typical **follow-up test** performed on the triggers is the χ^2**-test**.

The motivation behind is that **if the trigger is a real GW signal**, after subtracting the **MLE template**, the **residual** $d(t) - \hat{h}(t; \lambda_{\rm max})$ **should follow a Gaussian distribution** (the residual should be Gaussian noise):

$$p[d(t) - \hat{h}(t)] \propto e^{-(d-\hat{h}|d-\hat{h})/2} \to p[\tilde{d}(f) - \tilde{\hat{h}}(f)] \propto \exp\left[\frac{|\tilde{d}(f) - \tilde{\hat{h}}(f)|^2}{S_n(f)}\right]$$

Waveform consistency: χ^2-test

$$p[d(t) - \hat{h}(t)] \propto e^{-(d-\hat{h}|d-\hat{h})/2} \rightarrow p[\tilde{d}(f) - \tilde{\hat{h}}(f)] \propto \exp\left[\frac{|\tilde{d}(f) - \tilde{\hat{h}}(f)|^2}{S_n(f)}\right]$$

The quantity $\chi^2 = \sum_{i=1}^{N} \frac{|\tilde{d}(f_i) - \tilde{\hat{h}}(f_i)|^2}{S_n(f_i)}$ is the sum of N Gaussian variables with 0 mean and unitary variance, and hence it follows a χ^2 distribution.

For a true GW detected, it will be close to N, and **for glitches much larger.**

Re-weighted SNR:

$\hat{\rho} = \frac{\rho}{[1 + (\chi^2/N)^3]^{1/6}} \rightarrow \hat{\rho} \approx \rho$ **for real GW,** $\hat{\rho} \ll \rho$ **for glitches**

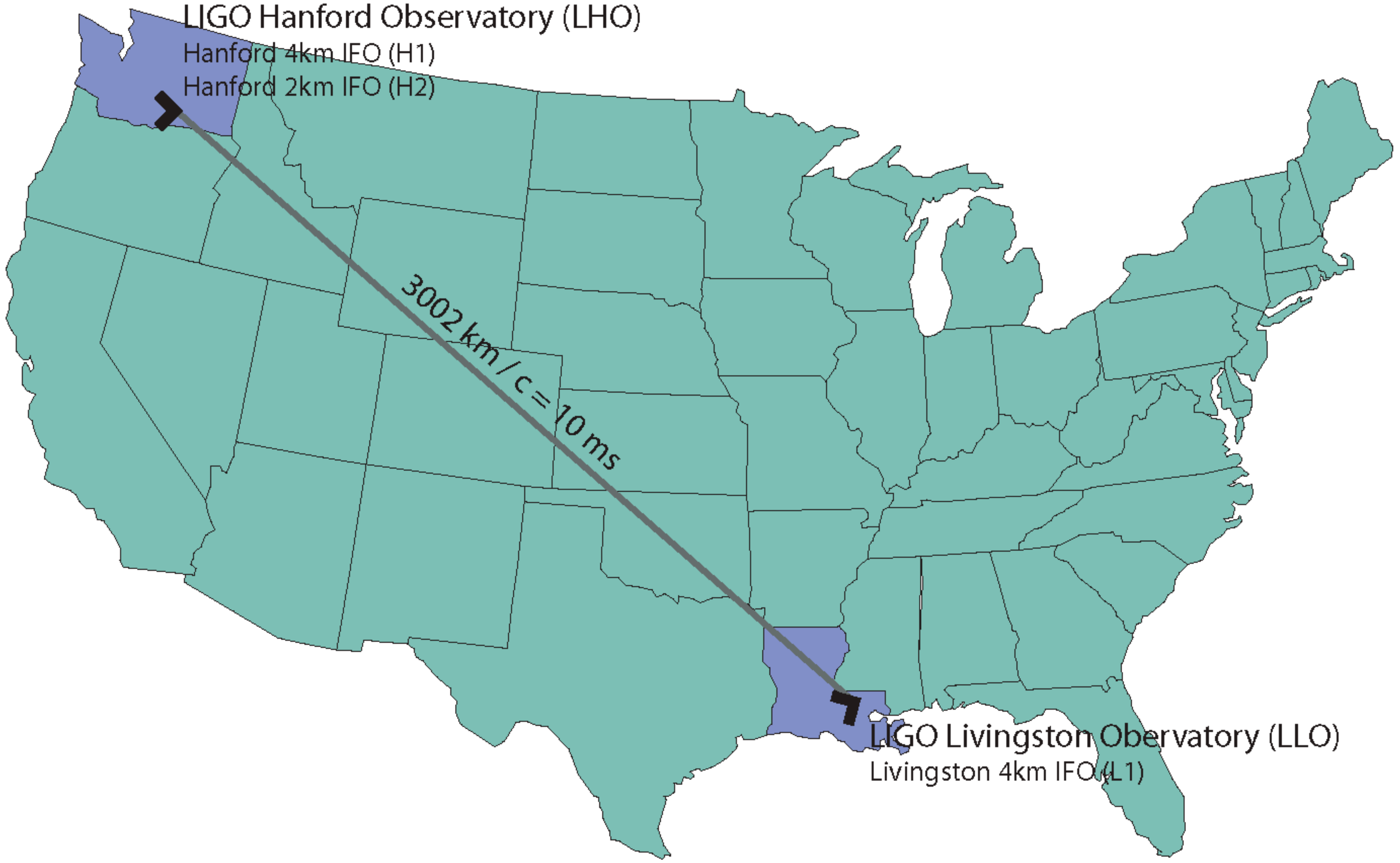
Coincidence between detectors

If **more than one detector is observing**, a powerful test is to check **if the trigger is consistent in the several detectors.**

GW travel at the speed of light, therefore the **time delay between different detector triggers for the same GW signal cannot be greater than:**

$$t_{\mathrm{det,i}} - t_{\mathrm{det,j}} \leq |\boldsymbol{r}_{\mathrm{det,i}} - \boldsymbol{r}_{\mathrm{det,j}}|/c$$

Triggers that have a time delay greater than this time cannot correspond to the same GW signal.



Coincidence between detectors

Also, the **intrinsic parameters of the triggers** in the several detectors have to **very similar** if a real GW signal has triggered them:

$\lambda_{det,i} \approx \lambda_{det,j} \forall \; det_i, det_j$ (notice that some **extrinsic parameters** can differ, as the phase in each detector).

Nowadays, **single-detector triggers** are still followed-up and can be candidate detections if their significance is enough, but the **coincidence test is very powerful** discarding false triggers, and this is one of the reasons for constructing a **network of several detectors**.

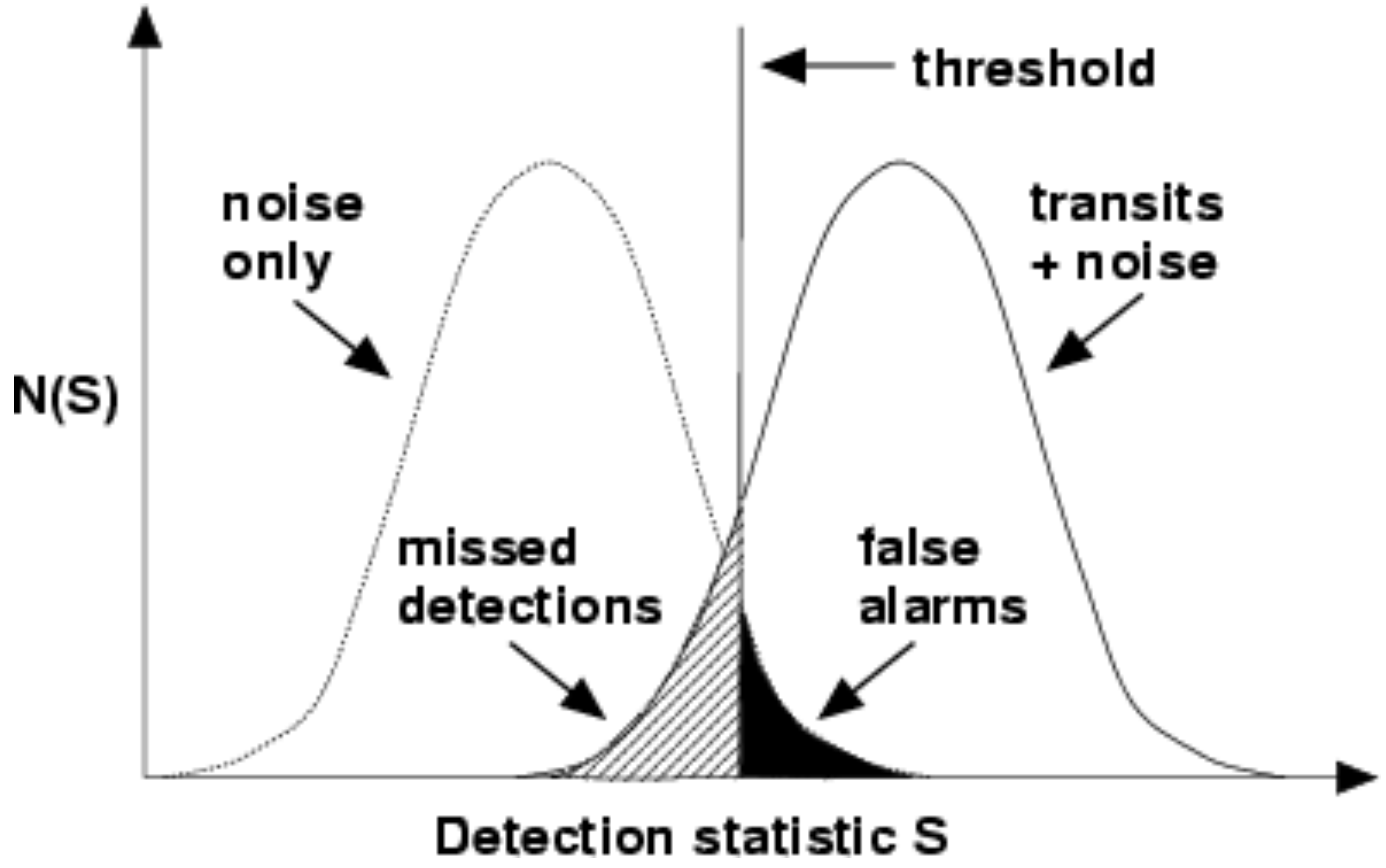
Background and False Alarm Rate

To assess the significance of a candidate detection, we want to estimate how often the background noise can mimic the given candidate.

False alarm probability: probability of having a trigger with $\hat{\rho} > \hat{\rho}_c$ (SNR of the candidate) for a non-GW trigger (so the null hypothesis $\mathscr{H}_0$ being true)

$$\mathrm{FAP}(\rho_c) = \int_{\rho_c}^{\infty} p(\hat{\rho}, \mathscr{H}_0) d\rho$$

Usually it cannot be computed analytically, but it can be estimated from the **background.**



Background and False Alarm Rate

GW-free background: It can be constructed time-shifting the data of several detectors such that $\Delta t > |\boldsymbol{r}_{\mathrm{det,i}} - \boldsymbol{r}_{\mathrm{det,j}}|/c$.

This process is usually called **time sliding.**

Therefore, **coincident triggers in the time shifted data cannot correspond to real GW events.**

Background and False Alarm Rate

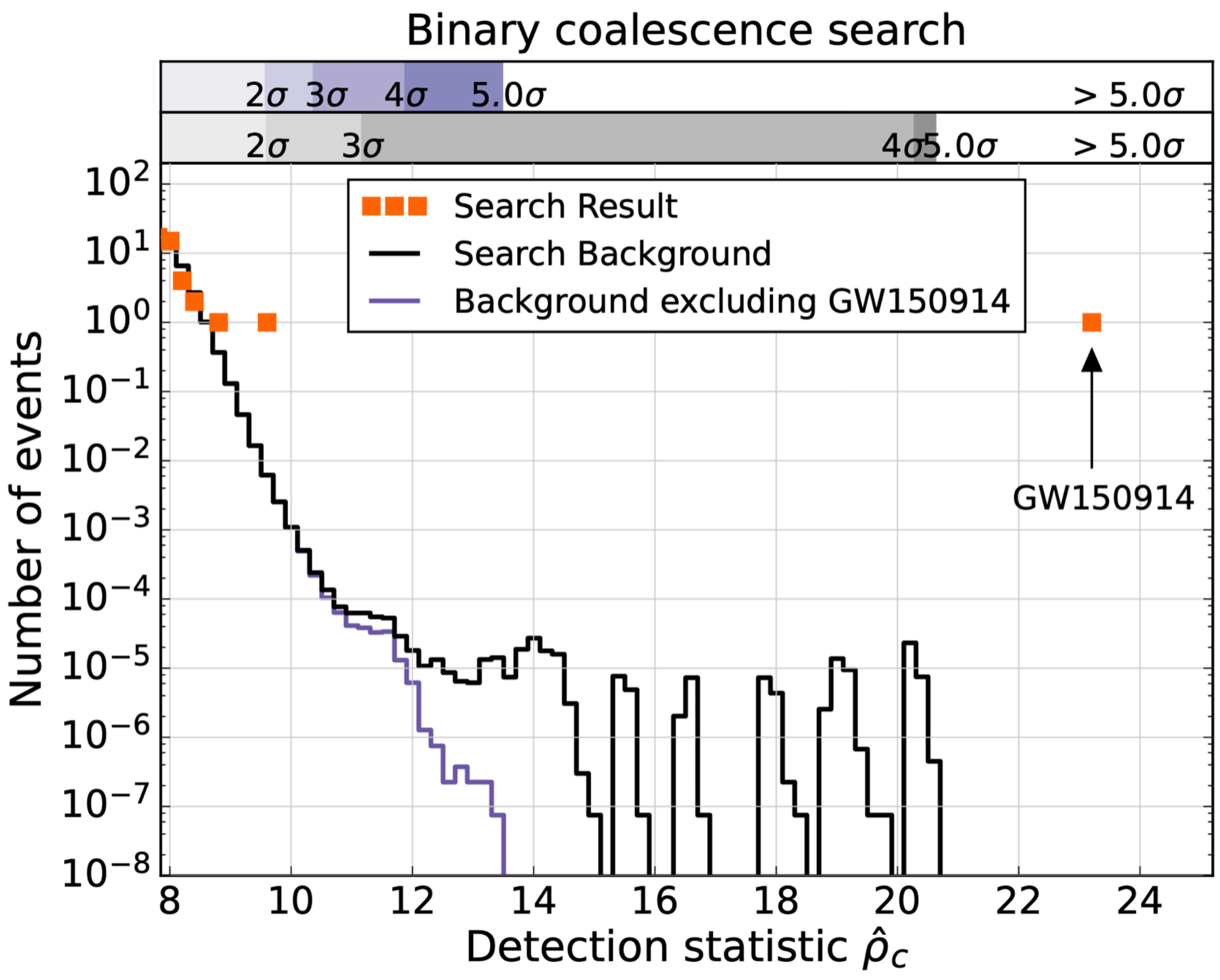
The **template bank** is **matched filtered repeatedly with different time-shifts applied to the data.**

Background triggers are collected.

FAP: fraction of **background triggers** with $\hat{\rho} > \hat{\rho}_{\text{candidate}}$.

FAR (False alarm rate): number of false alarms per year.

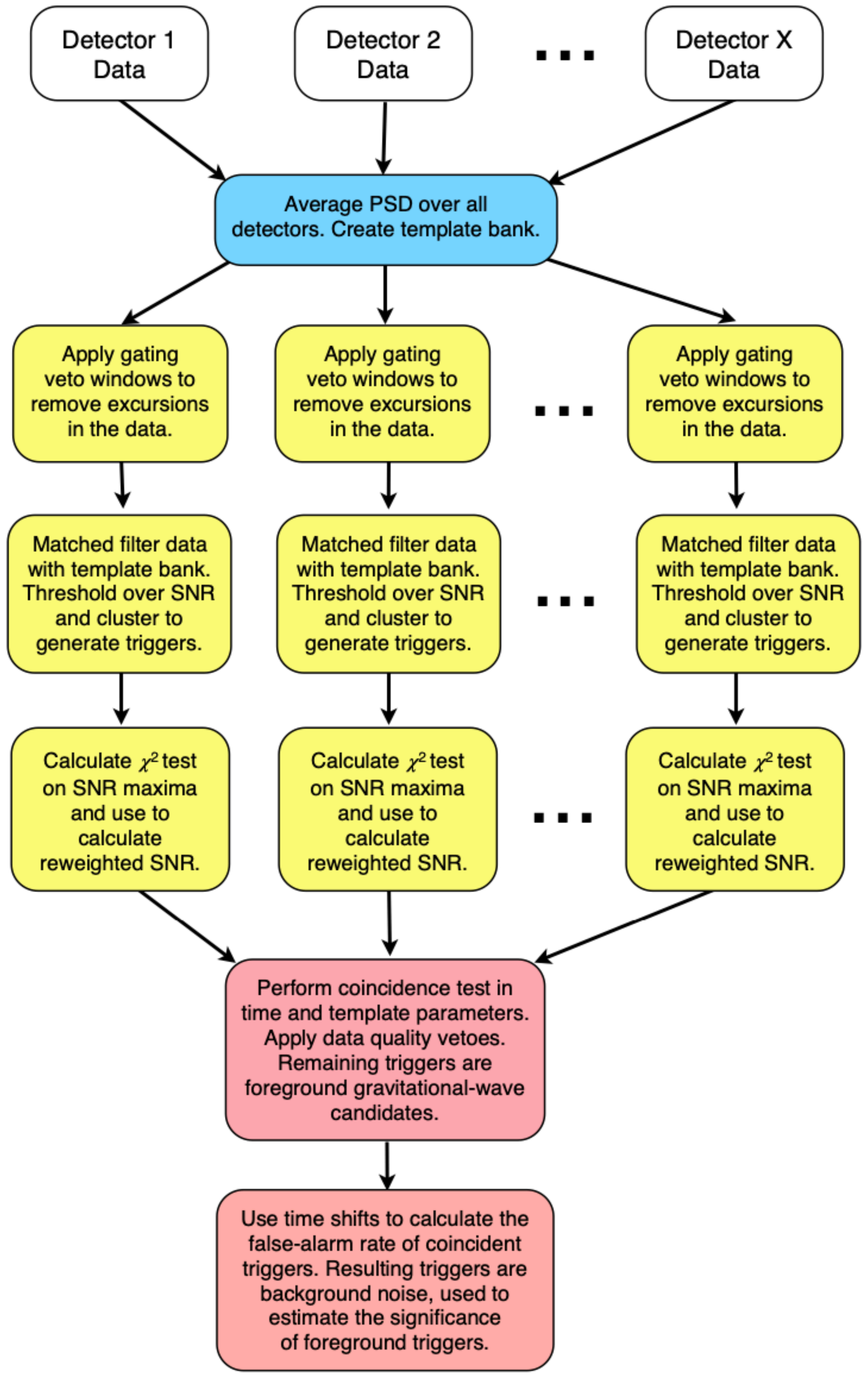
Detection can be claimed if the FAP or FAR is lower than some threshold value, for example FAR=1/100 years.



Summary

Steps for searching CBC GW events:

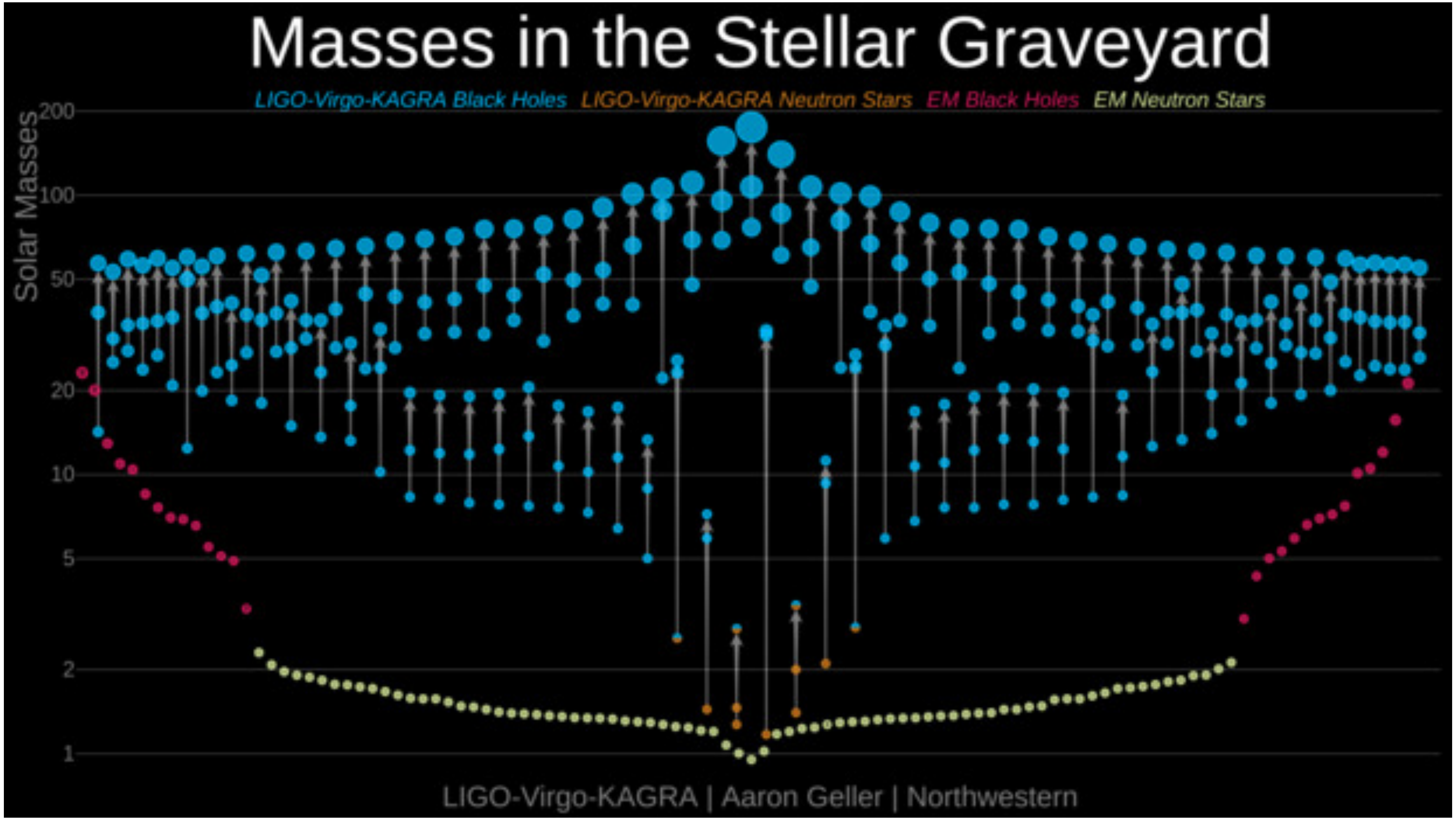
- Model noise (compute PSD)
- Create template bank
- Matched filter detector data with template bank.
- Collect triggers.
- Waveform consistency: re-weighted SNR from χ^2-test
- Additional checks: data quality, coincidence between detectors.
- Compute background and collect noise events.
- Assess significance based on FAP or FAR.



Summary

LVK matched-filter search pipelines: PyCBC, gstLAL

90 confident detections so far!



5. Parameter estimation

Conditional probabilities

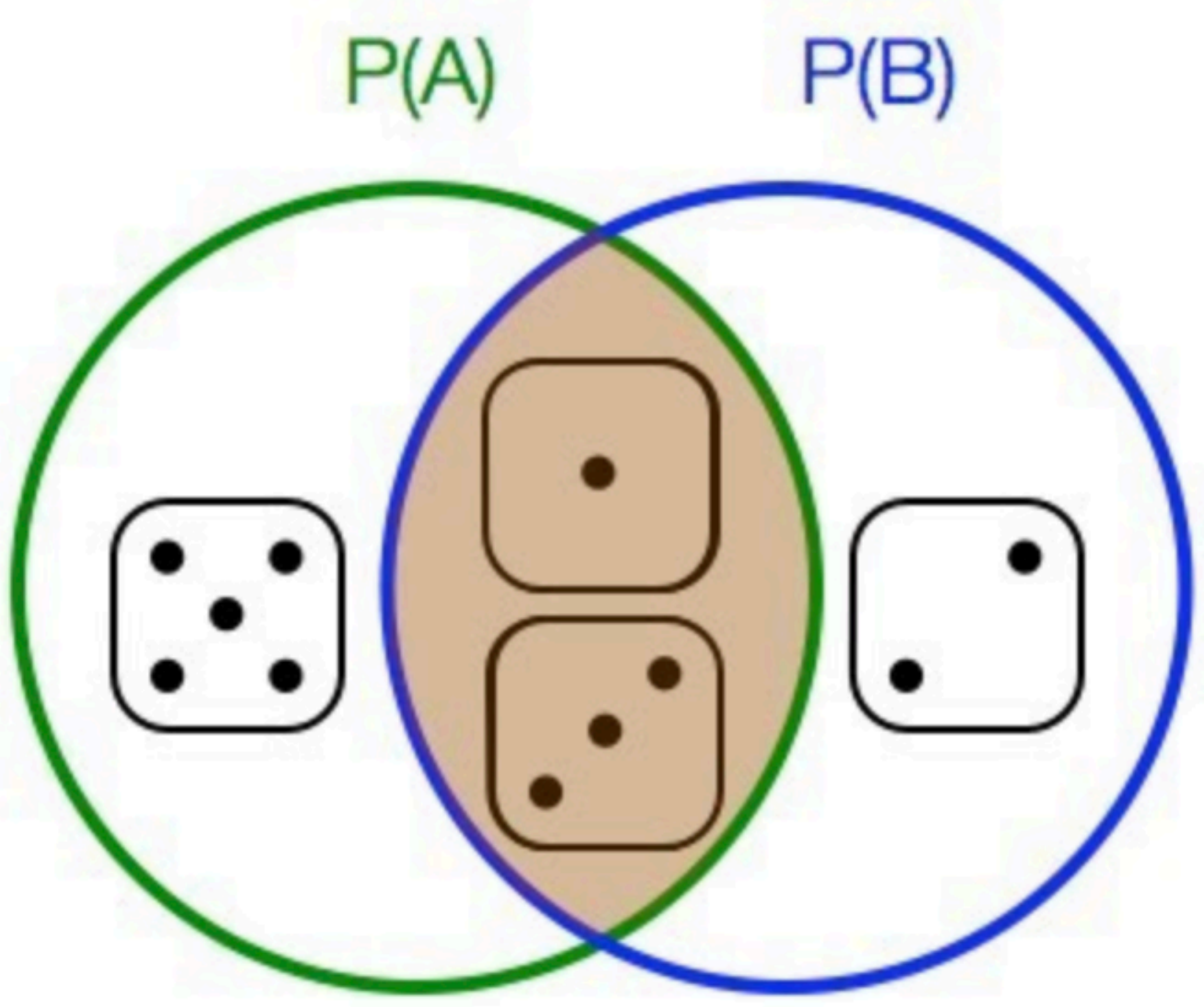
The **conditional probability** of an event **A** that depends on some event **B** is:

$$P(A, B) = \frac{P(A \cap B)}{P(B)} \,,$$

where $P(A \cap B)$ is the probability of both A and B occur.

Notice that **if A is independent of B**, then

$$P(A, B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A)$$



Example: what is the probability of rolling a dice and getting an odd number (P(A)) given that the number is less that 4 (P(B))?

P(A) = 3/6=1/2
P(B) = 3/6=1/2
P(A∩B) = 2/6=1/3
P(A,B) = (1/3)/(1/2) =2/3

Bayes rule

Similarly, the **conditional probability** of the event B given the event A**:**

$$P(B,A) = \frac{P(A \cap B)}{P(A)}$$

We can see that: $P(B,A)P(A) = P(A \cap B) = P(A,B)P(B)$

And therefore: $P(B,A) = \dfrac{P(A,B)}{P(A)}P(B)$, this is the famous **Bayes rule.**

It is a simple **but very powerful** mathematical identity.

Parameter probability distribution

For a **detected GW signal**, we want to know the **parameters** describing it: the physical **parameters of the source**, the **sky location**, the **distance** of the source, the **orientation** of the source, …

Assuming that the signal is described by a **model**, for example, a semi-analytical model for CBC signals $\hat{h}(t; \boldsymbol{\lambda})$, we can ask:

What is the **probability distribution** $p[\hat{h}(\boldsymbol{\lambda}), d(t)]$ of the model $\hat{h}(t; \boldsymbol{\lambda})$ with **parameters** $\boldsymbol{\lambda}$ describing the **data** $d(t)$?

We can use **Bayes rule** to compute this:

$$p[\hat{h}(\boldsymbol{\lambda}), d(t)] = \frac{p[d, \hat{h}(\boldsymbol{\lambda})]}{p(d)} p[\hat{h}(\boldsymbol{\lambda})]$$

Parameter probability distribution

$$p[\hat{h}(\boldsymbol{\lambda}), d(t)] = \frac{p[d, \hat{h}(\boldsymbol{\lambda})]}{p(d)} p[\hat{h}(\boldsymbol{\lambda})]$$

Assuming a given model $\hat{h}$ for the signal, this is a statement about the probability of the parameters $\boldsymbol{\lambda}$ taking certain values $p(\boldsymbol{\lambda}, d)$.

$p(\boldsymbol{\lambda}, d)$ is the **posterior probability distribution** of the parameters $\boldsymbol{\lambda}$.

$p[\hat{h}(\boldsymbol{\lambda}), d(t)] \equiv \mathscr{L}(d, \boldsymbol{\lambda})$ is the **likelihood of the data being described by** $\boldsymbol{\lambda}$ (we have already seen this likelihood!)

$p[\hat{h}(\boldsymbol{\lambda})] \equiv \pi(\boldsymbol{\lambda})$ is the **prior probability distribution** of the parameters $\boldsymbol{\lambda}$ (our knowledge of the parameters before the observation of the data)

$p(d) \equiv Z(d)$ is **the evidence** of the data being observed (assuming the model $\hat{h}$)

Likelihood function

$$p(\boldsymbol{\lambda}, d) = \frac{\mathscr{L}(d, \boldsymbol{\lambda})}{Z(d)} \pi(\boldsymbol{\lambda})$$

Our prior knowledge of the parameters gets updated with the observation of the data, assuming a model for the signal.

Assuming **stationary Gaussian noise** $n(t)$and assuming the data being described by:

$$d(t) = n(t) + \hat{h}(t; \boldsymbol{\lambda})$$

the **likelihood** is familiar to us:

$$\mathscr{L}(d, \boldsymbol{\lambda}) = \frac{1}{2\pi\sigma_n^2} \exp[-(d - \hat{h}(\boldsymbol{\lambda}) \,|\, d - \hat{h}(\boldsymbol{\lambda}))]$$

Prior probability

The **prior probability** reflects our knowledge or assumptions about the distribution of the parameters.

For a **precessing binary system** in **quasi-circular orbits,** we need 15 parameters to fully describe the signal:

- Intrinsic parameters: m_1, m_2, χ_1, χ_2
- Source orientation: ι, ϕ_{ref}
- Source location: α, δ, D_L
- Phase and time at detector: ψ, t_c

Priors can be **uninformative** (for example, uniform distribution) or reflect some motivated knowledge (for example, the expected astrophysical distribution for the masses)

Evidence and model selection

The **evidence of the data being described by the model** is a normalisation constant:

$$Z(d) \equiv \int d\lambda \mathscr{L}(d, \lambda)\pi(\lambda)$$

Useful for model comparison:

$Z_S \equiv \int d\lambda \mathscr{L}(d, \lambda)\pi(\lambda)$ Evidence of $d(t) = n(t) + \hat{h}(t)$.

$Z_N = \mathscr{L}(d,0)$ Evidence of $d(t) = n(t)$ (**noise evidence**)

$\mathrm{BF}_N^S \equiv \dfrac{Z_S}{Z_N}$, **Bayes factor of the signal hypothesis.**

Evidence and model comparison

One can also compare the **evidences for different models** of the signal.

$$Z_{\text{spin}} \equiv \int d\lambda \mathscr{L}(d,\lambda)\pi_{\text{spin}}(\lambda)$$

$$Z_{\text{no-spin}} \equiv \int d\lambda \mathscr{L}(d,\lambda)\pi_{\text{no-spin}}(\lambda)$$

$$BF^{\text{spin}}_{\text{no-spin}} \equiv \frac{Z_{\text{spin}}}{Z_{\text{no-spin}}}$$

Bayes Factor of the system being spinning

The proper way to compare different models or assumptions A and B is to include our prior belief in the models, π_A and π_B, this defines the **odds:**

$$\mathcal{O}^A_B \equiv \frac{Z_A}{Z_B}\frac{\pi_A}{\pi_B},$$

Marginal distributions

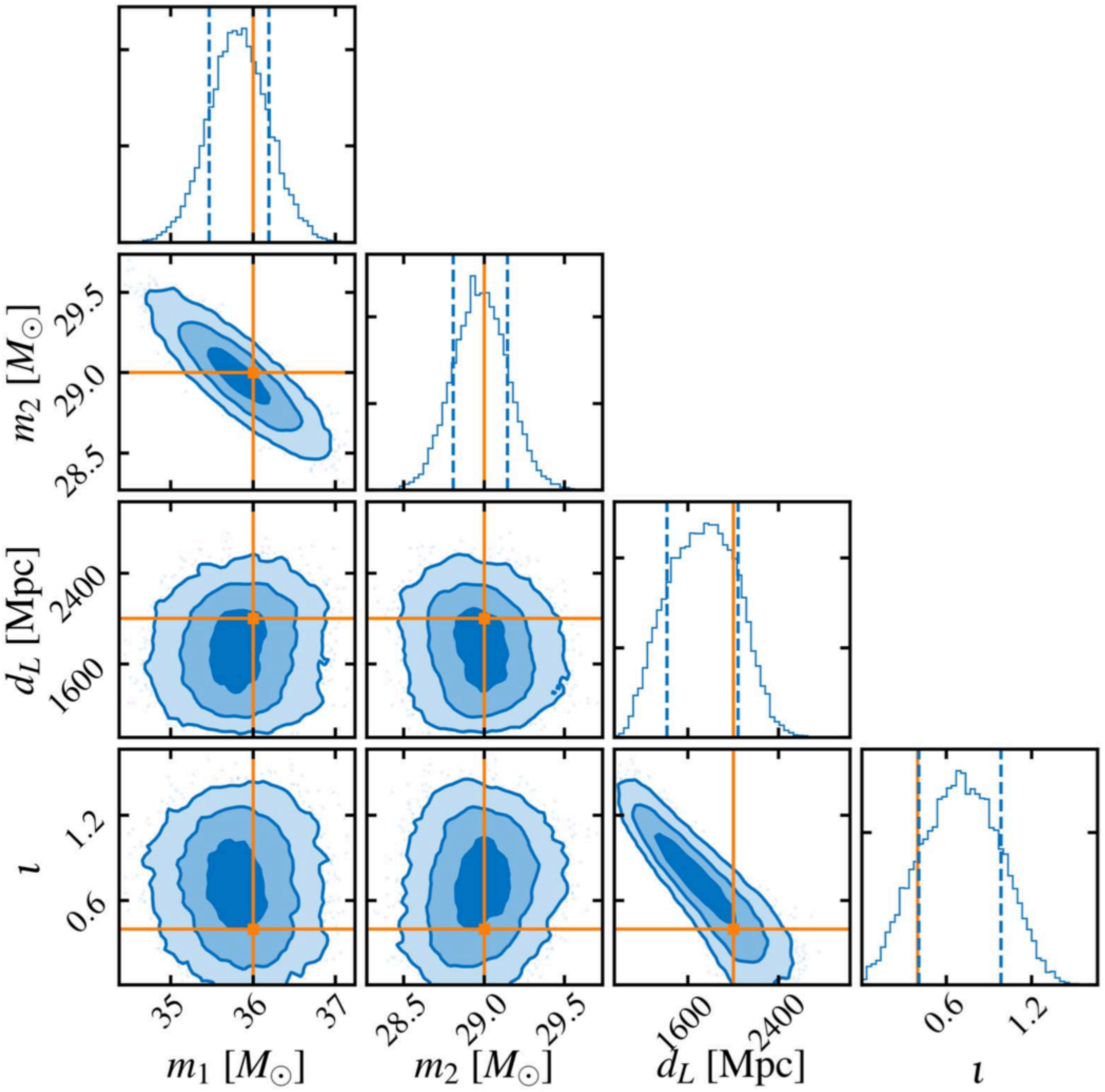
The **posterior probability distribution** $p(\boldsymbol{\lambda}, d)$ is a **multi-dimensional** probability distribution, which in general will have a complicated structure in the multidimensional parameter space.

We are often interested in information about a particular parameter, and we compute the **marginal distribution** for a parameter λ_i:

$$p(\lambda_i, d) = \int \left(\prod_{k \neq i} d\lambda_k \right) p(\boldsymbol{\lambda}, d)$$

We **marginalise** the posterior distribution over all the other parameters. This takes into account the uncertainty on each parameter.

It is also useful to compute 2D-marginal distributions, to visualise **correlations**.



from Ashton+2019

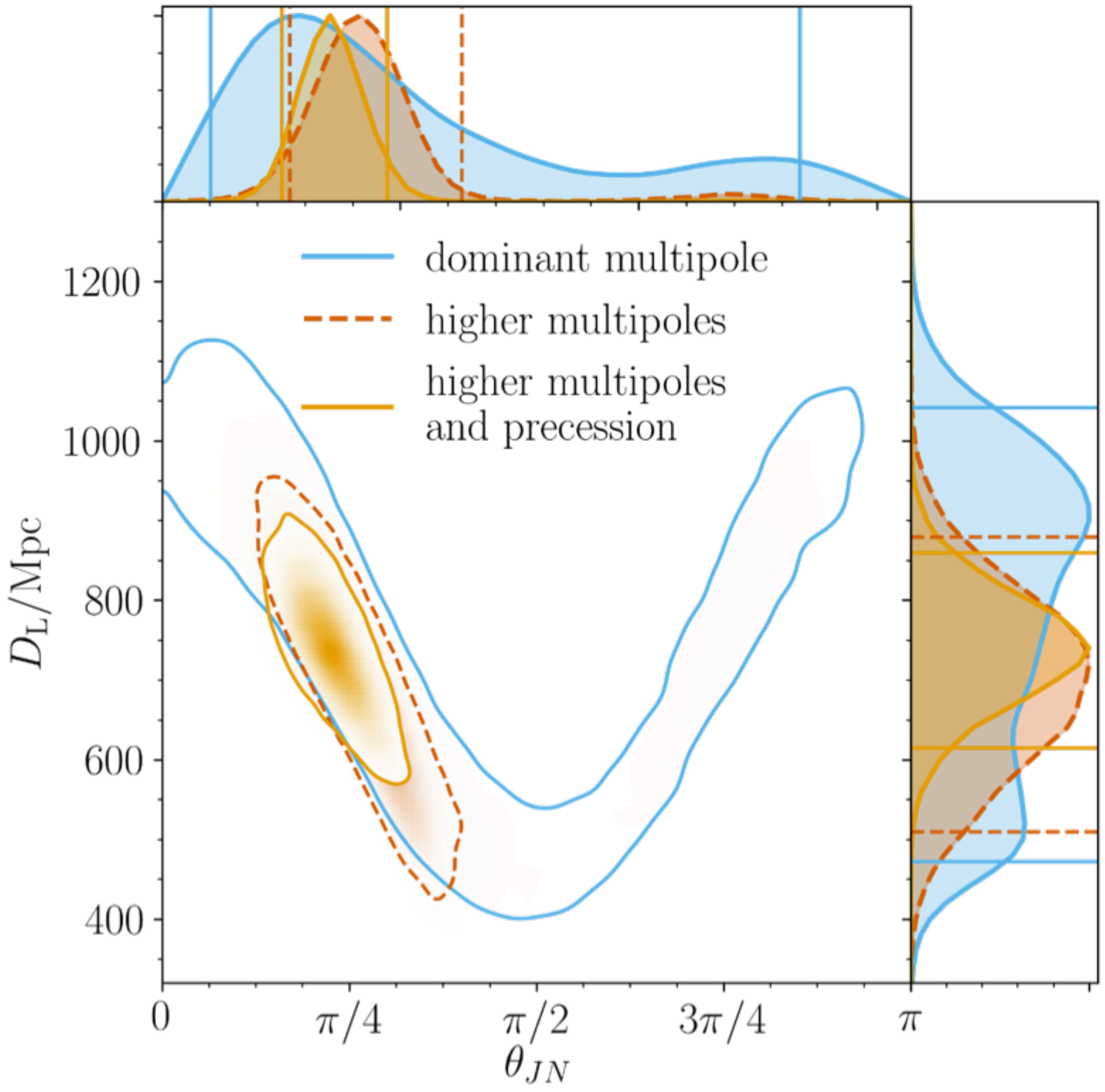
Parameter degeneracies

Some parameter can be **partially degenerated**, introducing large correlations between the marginal distributions.

For example, consider the quadrupole-formula signal:

$$h_+(t) \propto \frac{1+\cos^2 \iota}{D_L}$$

Improving accuracy of **waveform models** and adding more physical effects (higher spherical harmonics, precession) can help to reduce some degeneracies.



GW190412, from LVK2020

Point estimates and credible intervals

From the **marginalised** posterior distributions we can compute point estimates

$$\mu = \int \lambda_i p(\lambda_i, d) d\lambda_i$$, **mean**

$$\int_{\infty}^{m} p(\lambda_i, d) d\lambda_i = 0.5$$, **median** m

It is common to quantify the uncertainty on the parameter values in terms of the 90% **credible intervals** (a, b)**:**

$$\int_{a}^{b} p(\lambda_i, d) d\lambda_i = 0.9$$, with $p(\lambda_i, d) \geq p(\lambda_i', d)$

for $\lambda_i \in [a, b]$ and $\lambda_i' \notin [a, b]$ (**highest posterior density interval**)

TABLE II. Inferred parameter values for GW190412 and their 90% credible intervals, obtained using precessing models including higher multipoles.

Parameter[a]	EOBNR PHM	Phenom PHM	Combined
$m_1/M_\odot$	$31.7^{+3.6}_{-3.5}$	$28.1^{+4.8}_{-4.3}$	$30.1^{+4.6}_{-5.3}$
$m_2/M_\odot$	$8.0^{+0.9}_{-0.7}$	$8.8^{+1.5}_{-1.1}$	$8.3^{+1.6}_{-0.9}$
$M/M_\odot$	$39.7^{+3.0}_{-2.8}$	$36.9^{+3.7}_{-2.9}$	$38.4^{+3.8}_{-3.9}$
$\mathcal{M}/M_\odot$	$13.3^{+0.3}_{-0.3}$	$13.2^{+0.5}_{-0.3}$	$13.3^{+0.4}_{-0.4}$
q	$0.25^{+0.06}_{-0.04}$	$0.31^{+0.12}_{-0.07}$	$0.28^{+0.12}_{-0.07}$
$M_\mathrm{f}/M_\odot$	$38.6^{+3.1}_{-2.8}$	$35.7^{+3.8}_{-3.0}$	$37.3^{+3.8}_{-4.0}$
χ_f	$0.68^{+0.04}_{-0.04}$	$0.67^{+0.07}_{-0.07}$	$0.67^{+0.06}_{-0.05}$
$m_1^\mathrm{det}/M_\odot$	$36.5^{+4.2}_{-4.2}$	$32.3^{+5.7}_{-5.2}$	$34.6^{+5.4}_{-6.4}$
$m_2^\mathrm{det}/M_\odot$	$9.2^{+0.9}_{-0.7}$	$10.1^{+1.6}_{-1.2}$	$9.6^{+1.7}_{-1.0}$
$M^\mathrm{det}/M_\odot$	$45.7^{+3.5}_{-3.3}$	$42.5^{+4.4}_{-3.7}$	$44.2^{+4.4}_{-4.7}$
$\mathcal{M}^\mathrm{det}/M_\odot$	$15.3^{+0.1}_{-0.2}$	$15.2^{+0.3}_{-0.2}$	$15.2^{+0.3}_{-0.1}$
χ_eff	$0.28^{+0.06}_{-0.08}$	$0.22^{+0.08}_{-0.11}$	$0.25^{+0.08}_{-0.11}$
χ_p	$0.31^{+0.14}_{-0.15}$	$0.31^{+0.24}_{-0.17}$	$0.31^{+0.19}_{-0.16}$
χ_1	$0.46^{+0.12}_{-0.15}$	$0.41^{+0.22}_{-0.24}$	$0.44^{+0.16}_{-0.22}$
$D_\mathrm{L}/\mathrm{Mpc}$	740^{+120}_{-130}	740^{+150}_{-190}	740^{+130}_{-160}
z	$0.15^{+0.02}_{-0.02}$	$0.15^{+0.03}_{-0.04}$	$0.15^{+0.03}_{-0.03}$
$\hat{\theta}_{JN}$	$0.71^{+0.23}_{-0.21}$	$0.71^{+0.39}_{-0.27}$	$0.71^{+0.31}_{-0.24}$

Linear signal approximation

$p(\boldsymbol{\lambda}, d) = \frac{\mathscr{L}(d, \boldsymbol{\lambda})}{Z(d)}\pi(\boldsymbol{\lambda})$ Hard to compute, analytically or numerically (high dimensional parameter space)

Linear signal approximation: expand $\hat{h}(\boldsymbol{\lambda})$ linearly around **maximum likelihood estimator**

$$\hat{h}(\boldsymbol{\lambda}_{\text{MLE}} + \Delta\boldsymbol{\lambda}) = \hat{h}(\boldsymbol{\lambda}_{\text{MLE}}) + \sum_j \frac{\partial \hat{h}(\boldsymbol{\lambda})}{\partial \lambda_j}\Delta\lambda_j + \ldots \text{ with } \Delta\boldsymbol{\lambda} = \boldsymbol{\lambda} - \boldsymbol{\lambda}_{\text{MLE}}$$

$$\begin{aligned}(d - \hat{h}(\boldsymbol{\lambda}) \,|\, d - \hat{h}(\boldsymbol{\lambda})) \approx\ & (d - \hat{h}(\boldsymbol{\lambda}_{\text{MLE}}) \,|\, d - \hat{h}(\boldsymbol{\lambda}_{\text{MLE}})) \\ & - \sum_j 2(d - \hat{h}(\boldsymbol{\lambda}_{\text{MLE}}) \,|\, \partial_j \hat{h}(\boldsymbol{\lambda}))\Delta\lambda_j \\ & + \sum_{ij} (\partial_i \hat{h}(\boldsymbol{\lambda}) \,|\, \partial_j \hat{h}(\boldsymbol{\lambda}))\Delta\lambda_i \Delta\lambda_j\end{aligned}$$

This is a constant that we can reabsorbe in the normalisation factor

Linear signal approximation

Assuming $d(t) - \hat{h}(t, \boldsymbol{\lambda}_{\mathrm{MLE}}) \approx n(t)$, the expected value of the second term is 0:

$$\langle (d - \hat{h}(\boldsymbol{\lambda}_{\mathrm{MLE}}) \,|\, \partial_j \hat{h}(\boldsymbol{\lambda})) \rangle_n = (\langle n \rangle \,|\, \partial_j \hat{h}(\boldsymbol{\lambda})) = 0$$

Defining the **Fisher matrix** $\Gamma_{ij} = (\partial_i \hat{h} \,|\, \partial_j \hat{h})$ and assuming **uniform priors:**

$$p(\boldsymbol{\lambda}, d)_{\mathrm{LSA}} \propto \mathscr{L}(d, \boldsymbol{\lambda})_{\mathrm{LSA}} \propto \exp\left[-\frac{1}{2} \Gamma_{ij} \Delta\lambda_i \Delta\lambda_j \right]$$

This is a **multivariate Gaussian distribution** for $\boldsymbol{\lambda}$ with **mean** $\boldsymbol{\mu} = \boldsymbol{\lambda}_{\mathrm{MSE}}$ and covariance matrix:

$$\Sigma_{ij} \equiv (\Gamma_{ij})^{-1}$$

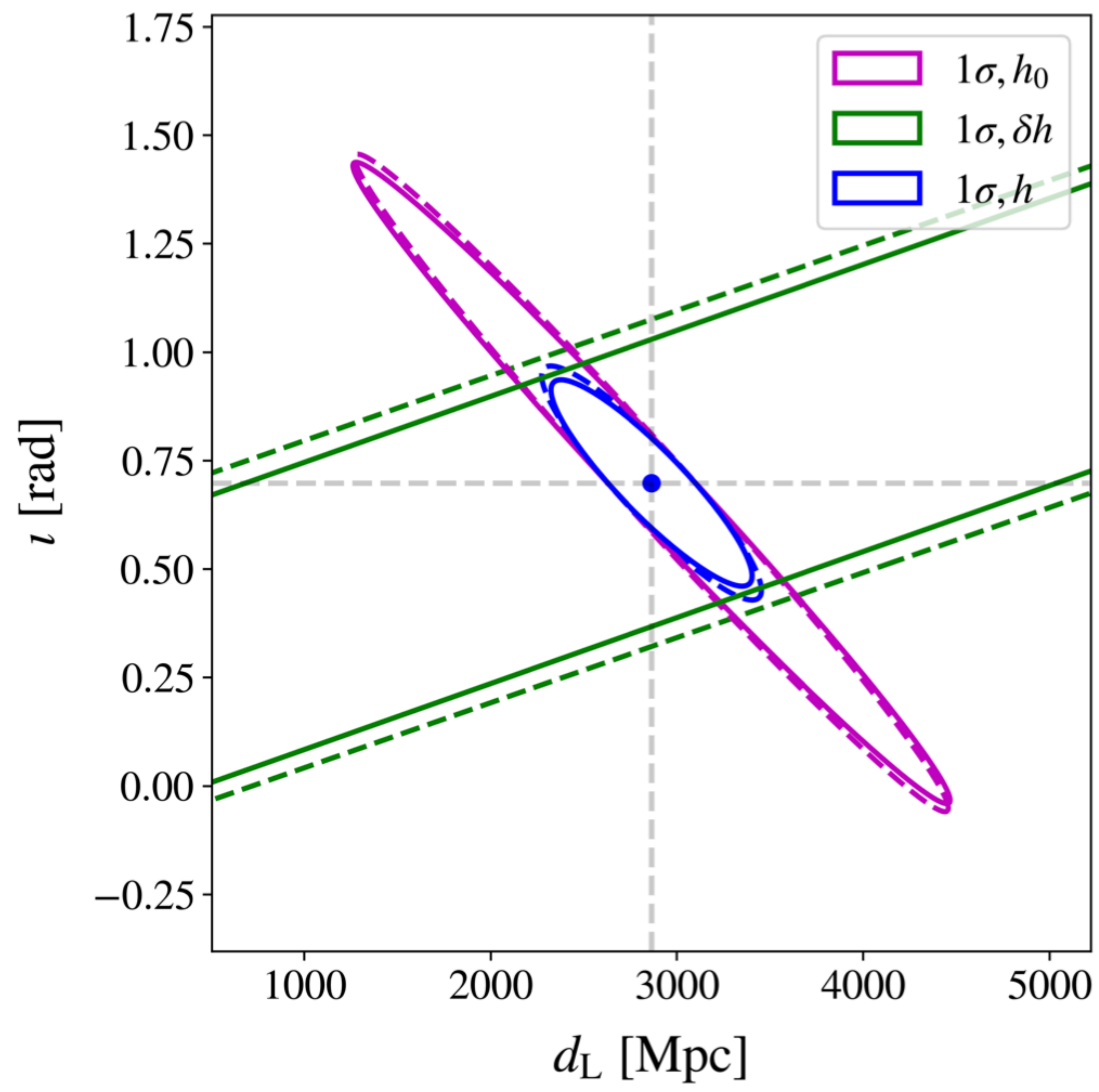
Linear signal approximation

The **Fisher matrix approximation** (valid for large SNR) is usually employed for estimating **how accurate we can measure parameters** for a given waveform model and a given detector (network)

$$\sigma_i^2 = \Sigma_{ii}$$

and for inspecting **linear correlations**:

$$\sigma_{ij} = \frac{\Sigma_{ij}}{\sigma_i \sigma_j}$$



Fisher study of the effect of incorporating non-linear memory effects for improving accuracy in measuring distance, from Gasparotto+2023

Sampling methods

In the general case, we **cannot compute exactly the posterior distribution**, but we can numerically **sample** $\mathscr{L}(d, \boldsymbol{\lambda})\pi(\boldsymbol{\lambda})$ across **parameter space** to draw samples $\{\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \ldots, \boldsymbol{\lambda}_M\}$ from the posterior $p(\boldsymbol{\lambda}, d)$.

With **enough samples**, we can then approximate any integral involving the posterior:

$$\int f(\boldsymbol{\lambda})p(\boldsymbol{\lambda}, d)d\boldsymbol{\lambda} \approx \frac{1}{M}\sum_i f(\boldsymbol{\lambda}_i)$$

and compute **marginal distributions**, **point estimates** and **credible intervals** from the samples.

Sampling methods

Typically **stochastic methods** are employed, since we cannot explore the high-dimensional parameter space with gridded approaches.

Two main techniques are commonly employed in GW astronomy: **Markov-Chain Monte-Carlo methods (MCMC) and Nested Sampling.**

The python parameter estimation software **Bilby** is the standard tool used by the LVK collaboration for estimating parameters from detected GW signals.

There are new emergent approaches that use **neural networks** to learn the posterior distributions, and employ **importance sampling** techniques to get accurate results: **DINGO** (developed by MPI-IS and this institute)

dingo-gw/**dingo**

Dingo: Deep inference for gravitational-wave observations

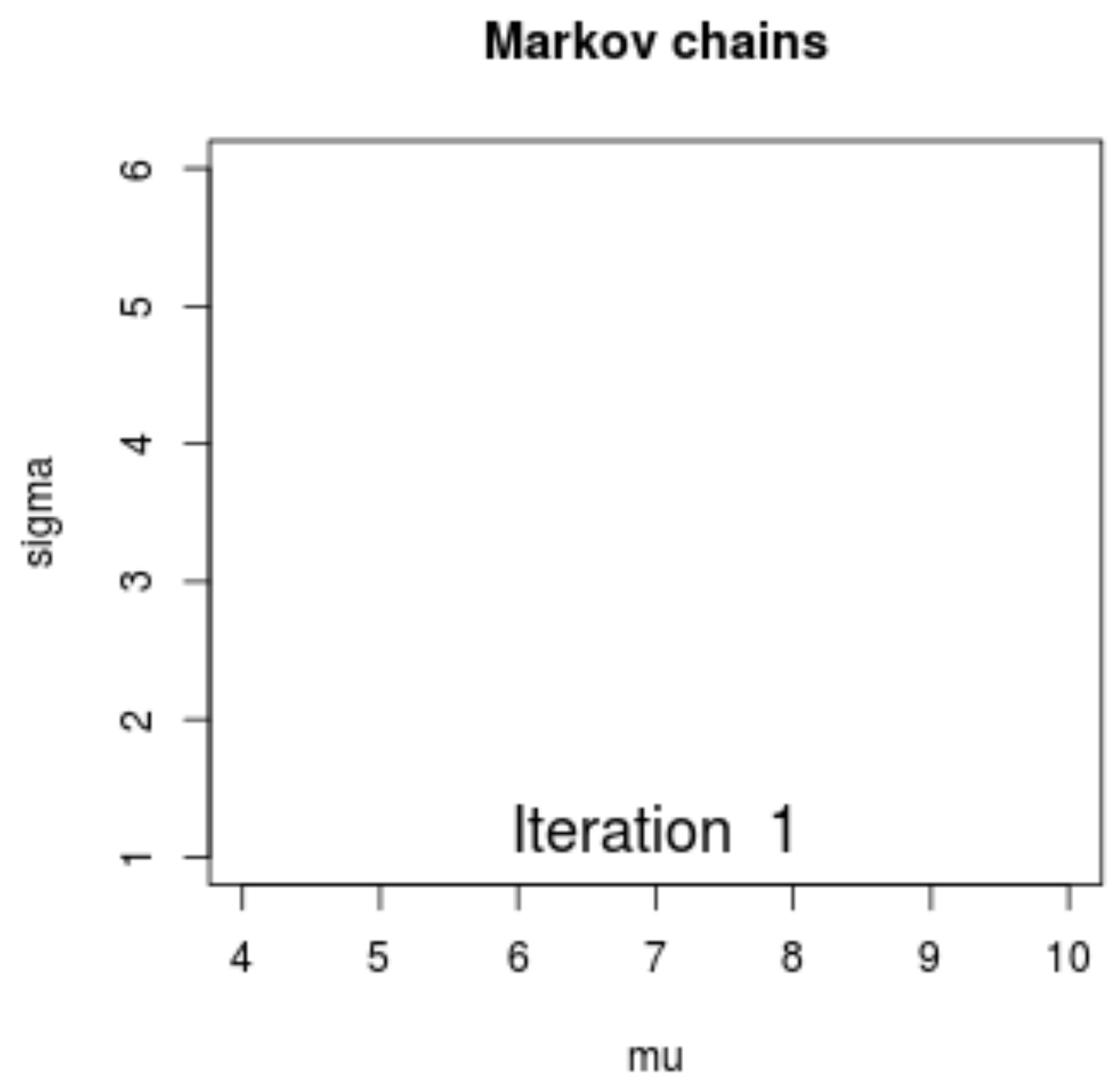
Markov chain Monte-Carlo

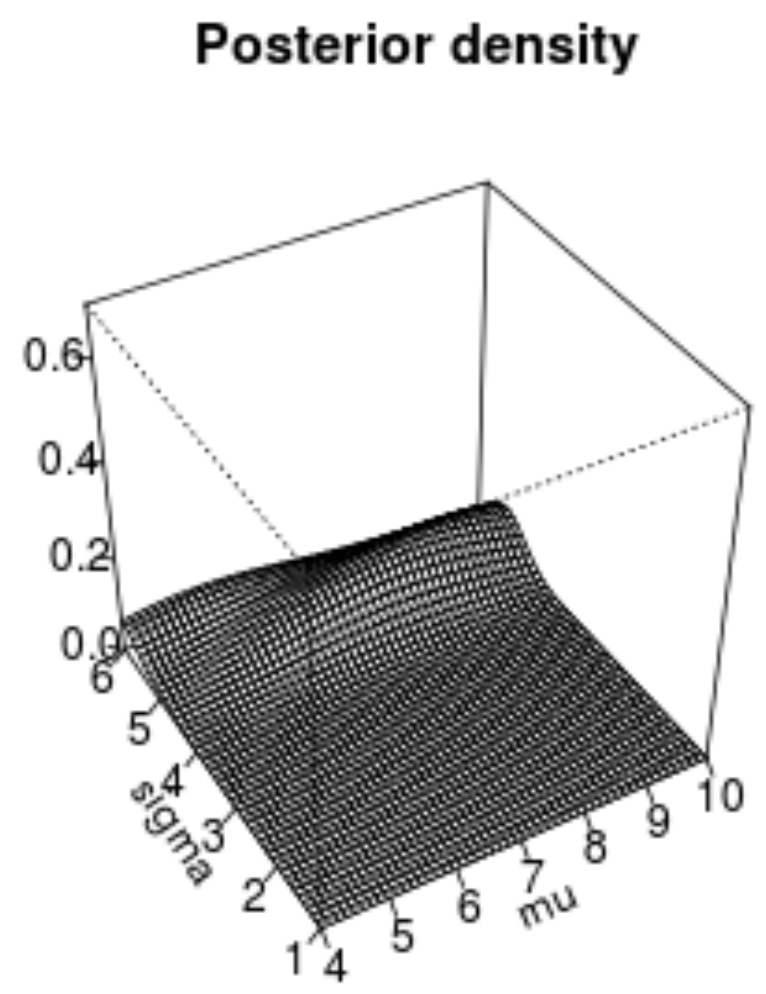
The idea is to generate samples using **Markov chains,** a type of random process where the probability of each event only depends on the previous event.

A **proposal distribution** is defined $Q(\lambda, \lambda')$ for generating a new sample λ' based on the last sample in the chain λ.

The new sample is accepted with probability $\min(1,\alpha)$ where:

$$\alpha = \frac{Q(\lambda, \lambda')p(\lambda', d)}{Q(\lambda', \lambda)p(\lambda, d)}$$





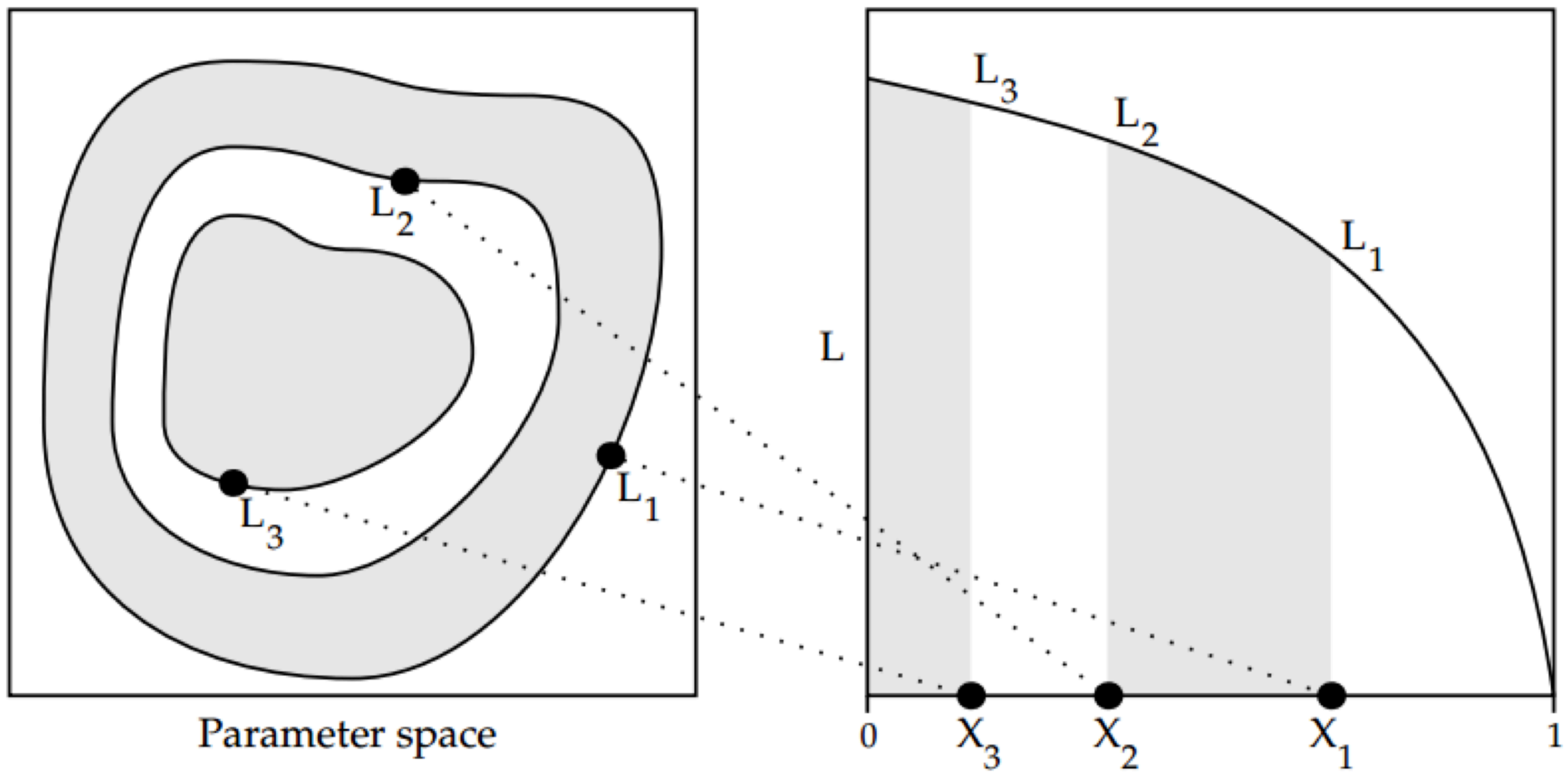
Nested sampling

Nested sampling is a technique for evaluating the **evidence** $Z(d)$ mapping the multidimensional problem into a 1D-problem (using **Lebesgue integration**):

$$Z(d) = \int d\lambda \mathscr{L}(d,\lambda)\pi(\lambda)$$

$$= \int_0^1 dX \mathscr{L}(X)$$

Where: $X(\mathscr{L}^*) = \int_{\mathscr{L}(d,\lambda)>\mathscr{L}^*} d\lambda \pi(\lambda)$



Evidence can be computed numerically:

$$Z = \sum_i^M \frac{1}{2}(X_{i-1} - X_{i+1}L_i)$$

Challenges

Even with **highly optimised** stochastic sampling techniques, obtaining a few thousand well-converged samples requires $\sim O(10^{7-8})$ **likelihood evaluations**, and therefore, **waveform evaluations**.

Typically, parameter estimation for just **one event** requires several days using the most accurate waveform models.

It is crucial to develop **more accurate** but also **more efficient** waveform models for describing CBC systems.

Summary of the lectures

Gravitational wave signals are **deeply buried into instrumental noise** for current detectors.

Modelling **noise** as **Gaussian** and **stationary** allows to define an **optimal detection statistic: matched filtering**, which needs the computation of the **PSD** of the noise and **theoretical models** for the signals.

GWs can be **searched** and **detected** using **templates banks** and performing several consistency tests, establishing strong conditions for the **detection significance** (robust against non-Gaussianities and non-stationarities).

The **noise model** also allows to perform **Bayesian inference** of the parameters of the signal, typically using **stochastic sampling methods.**

Continuous **improvements** are needed in **algorithms** and **waveform models** to be able to analise future data from future detectors.