
Do black holes have singularities?
A discussion of Kerr's paper
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# Introduction

- Kerr rejects the "singularity theorems" of [Penrose, 1965] and [Hawking, 1972]
- Consistent math ≠ physics
- Source of disagreement: how to define singularities and interpret results, not math details
- Geodesic incompleteness ≠ curvature divergence
- He shows "counterexample" in Kerr metric
- Is this argument new?
- "Although the singularity theorems do not prove that the singularities of classical general relativity must involve unboundedly large curvature, they strongly suggest the occurrence in cosmology and gravitational collapse of conditions in which quantum or other effects that will invalidate classical GR play a dominant role" [Wald, 1984]
- Purely classical argument: singularities surely eliminated with quantum gravity, but there are no real singularities even in GR!
- "When a theory predicts singularities, the theory is wrong."

## Do Black Holes have Singularities?

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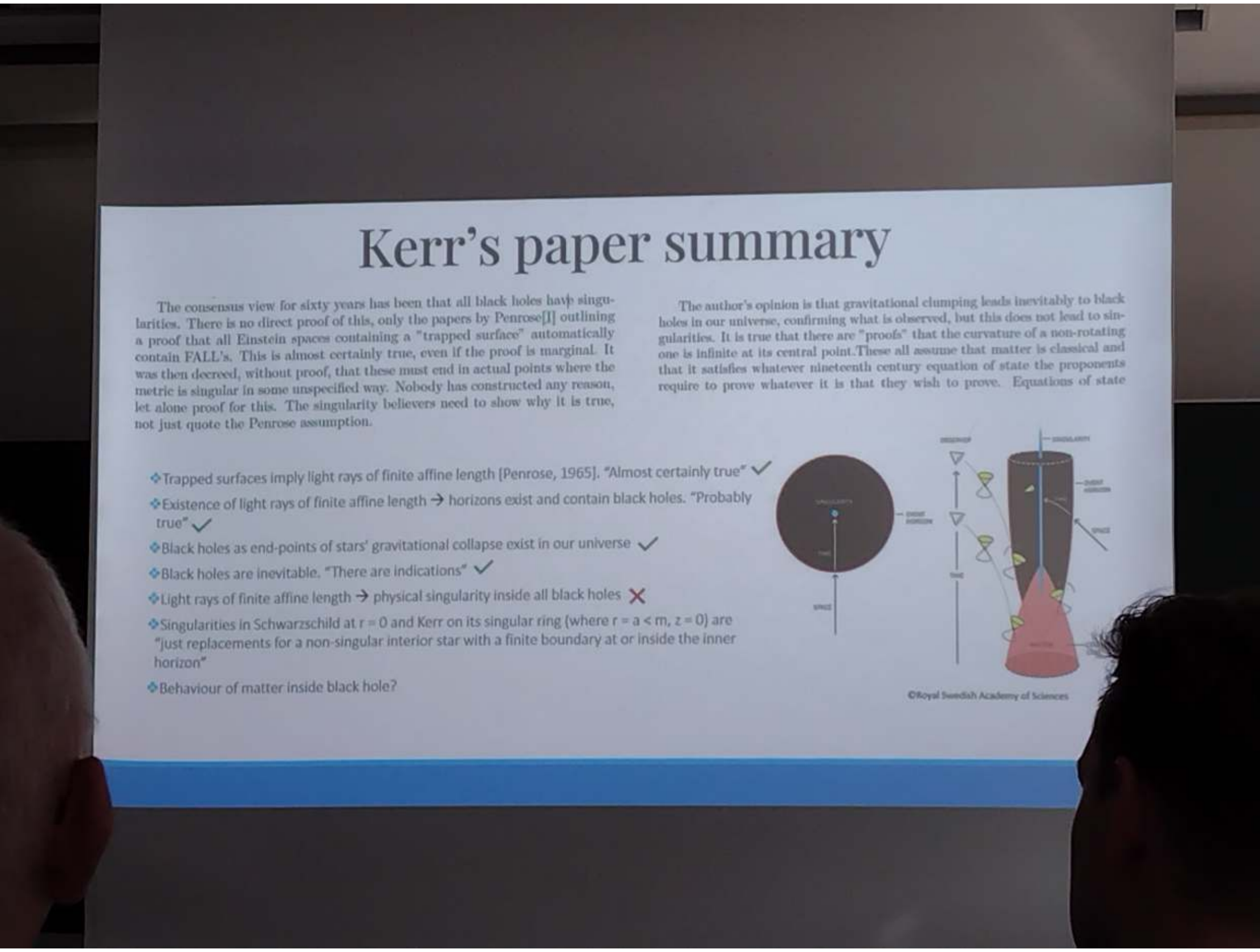
**Abstract**

There is no proof that black holes contain singularities when they are generated by real physical bodies. Roger Penrose[1] claimed sixty years ago that trapped surfaces inevitably lead to light rays of finite affine length (FALL's). Penrose and Stephen Hawking[2] then asserted that these must end in actual singularities. When they could not prove this they deemed it to be self evident. It is shown that there are counterexamples through every point in the Kerr metric. These are asymptotic[to at least
horizon and do not end in singularities.


Kerr's paper summary
The consensus view for sixty years has been that all black holes havþ singularities. There is no direct proof of this, only the papers by Penrose[1] outlining a proof that all Einstein spaces containing a "trapped surface" automatically contain FALL's. This is almost certainly true, even if the proof is marginal. It was then decreed, without proof, that these must end in actual points where the metric is singular in some unspecified way. Nobody has constructed any reason, let alone proof for this. The singularity believers need to show why it is true, not just quote the Penrose assumption.
The author's opinion is that gravitational clumping leads inevitably to black holes in our universe, confirming what is observed, but this does not lead to singularities. It is true that there are "proofs" that the curvature of a non-rotating one is infinite at its central point.These all assume that matter is classical and that it satisfies whatever nineteenth century equation of state the proponents require to prove whatever it is that they wish to prove. Equations of state
Trapped surfaces imply light rays of finite affine length [Penrose, 1965]. "Almost certainly true" ✓
Existence of light rays of finite affine length → horizons exist and contain black holes. "Probably true" ✓
Black holes as end-points of stars' gravitational collapse exist in our universe ✓
Black holes are inevitable. "There are indications" ✓
Light rays of finite affine length → physical singularity inside all black holes ✗
Singularities in Schwarzschild at r = 0 and Kerr on its singular ring (where r = a < m, z = 0) are "just replacements for a non-singular interior star with a finite boundary at or inside the inner horizon"
Behaviour of matter inside black hole?
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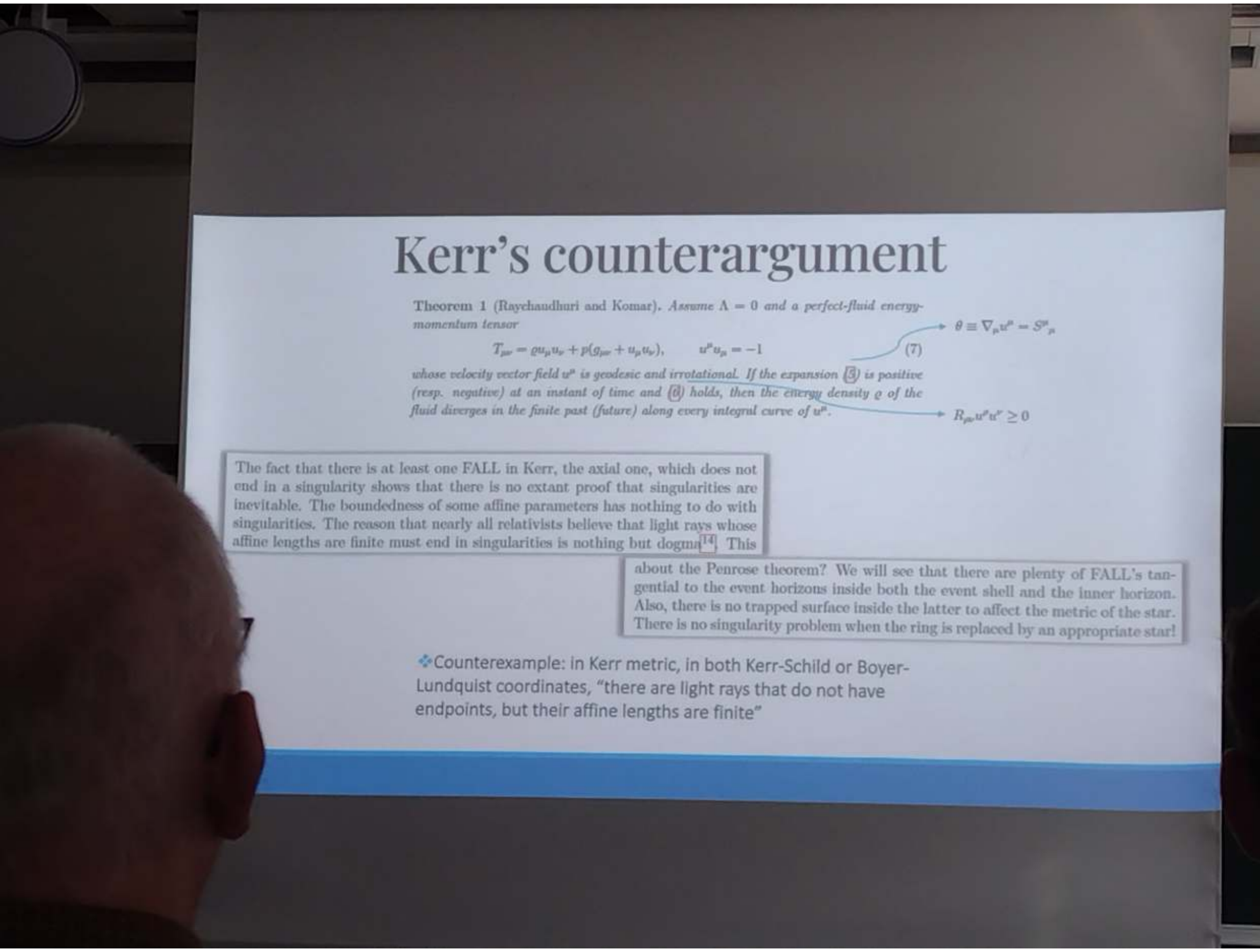
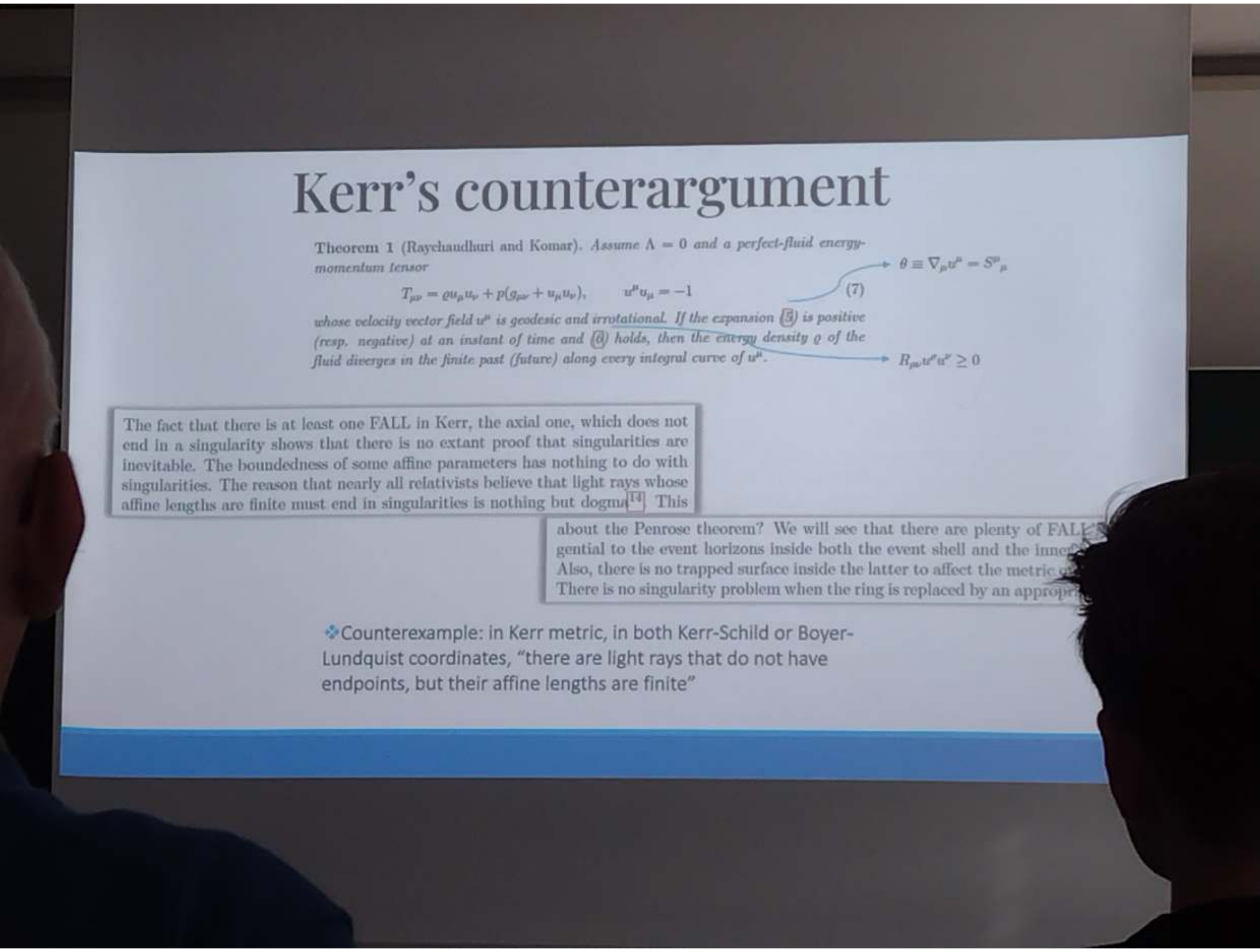

Kerr's counterargument
Theorem 1 (Raychaudhuri and Komar). Assume Λ = 0 and a perfect-fluid energy-momentum tensor
T_{μν} = ϱu_μu_ν + p(g_{μν} + u_μu_ν), u^μu_μ = −1 (7)
whose velocity vector field u^μ is geodesic and irrotational. If the expansion (5) is positive (resp. negative) at an instant of time and (6) holds, then the energy density ϱ of the fluid diverges in the finite past (future) along every integral curve of u^μ.
θ ≡ ∇_μu^μ = S^μ_μ
R_{μν}u^μu^ν ≥ 0
The fact that there is at least one FALL in Kerr, the axial one, which does not end in a singularity shows that there is no extant proof that singularities are inevitable. The boundedness of some affine parameters has nothing to do with singularities. The reason that nearly all relativists believe that light rays whose affine lengths are finite must end in singularities is nothing but dogma[14]. This
about the Penrose theorem? We will see that there are plenty of FALL's tangential to the event horizons inside both the event shell and the inner horizon. Also, there is no trapped surface inside the latter to affect the metric of the star. There is no singularity problem when the ring is replaced by an appropriate star!
Counterexample: in Kerr metric, in both Kerr-Schild or Boyer-Lundquist coordinates, "there are light rays that do not have endpoints, but their affine lengths are finite"



Kerr's counterargument
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$T_{\mu\nu} = \varrho u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu), \qquad u^\mu u_\mu = -1$ (7)
*whose velocity vector field $u^\mu$ is geodesic and irrotational. If the expansion (5) is positive (resp. negative) at an instant of time and (6) holds, then the energy density $\varrho$ of the fluid diverges in the finite past (future) along every integral curve of $u^\mu$.*
$\theta \equiv \nabla_\mu u^\mu = S^\mu{}_\mu$
$R_{\mu\nu} u^\mu u^\nu \geq 0$
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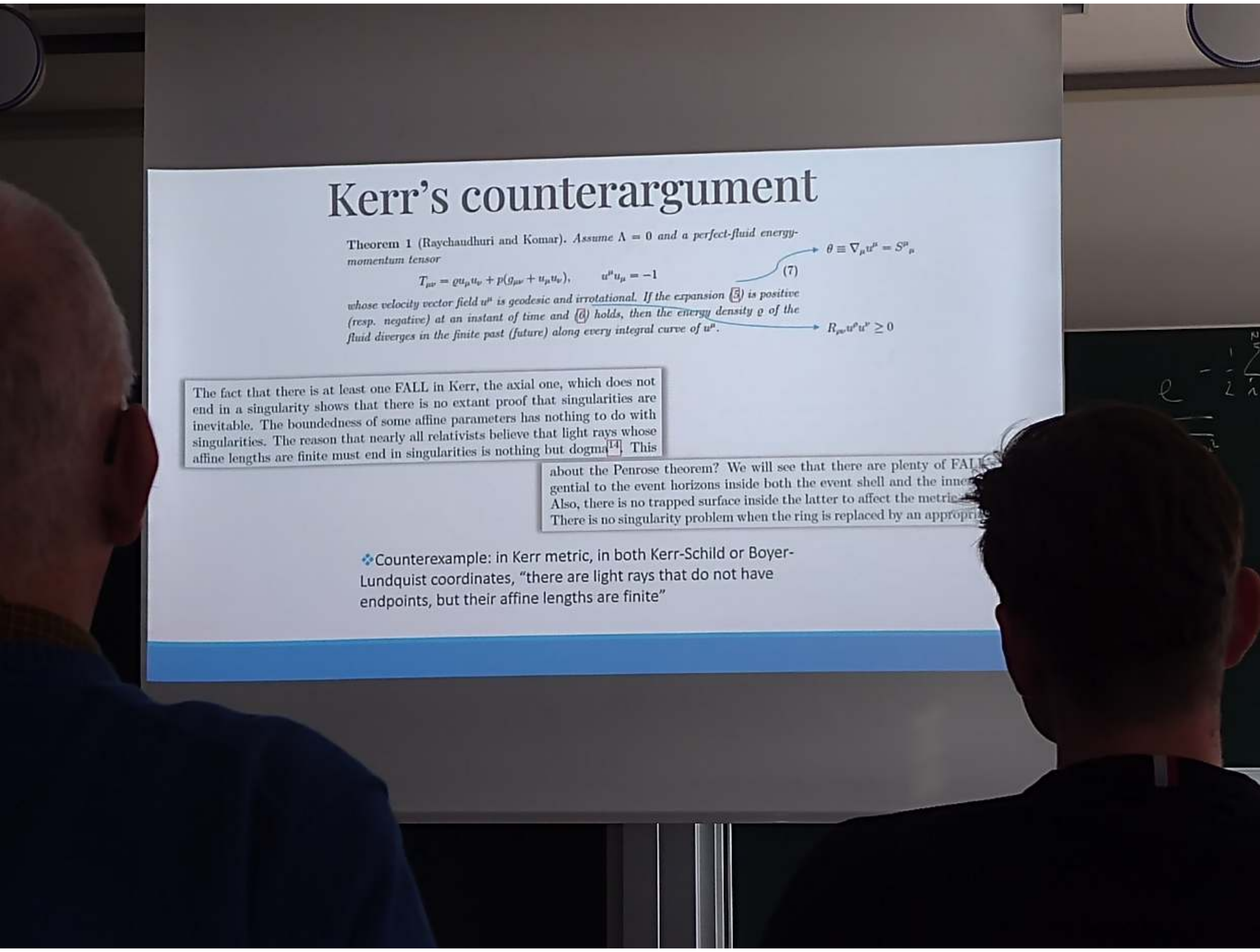
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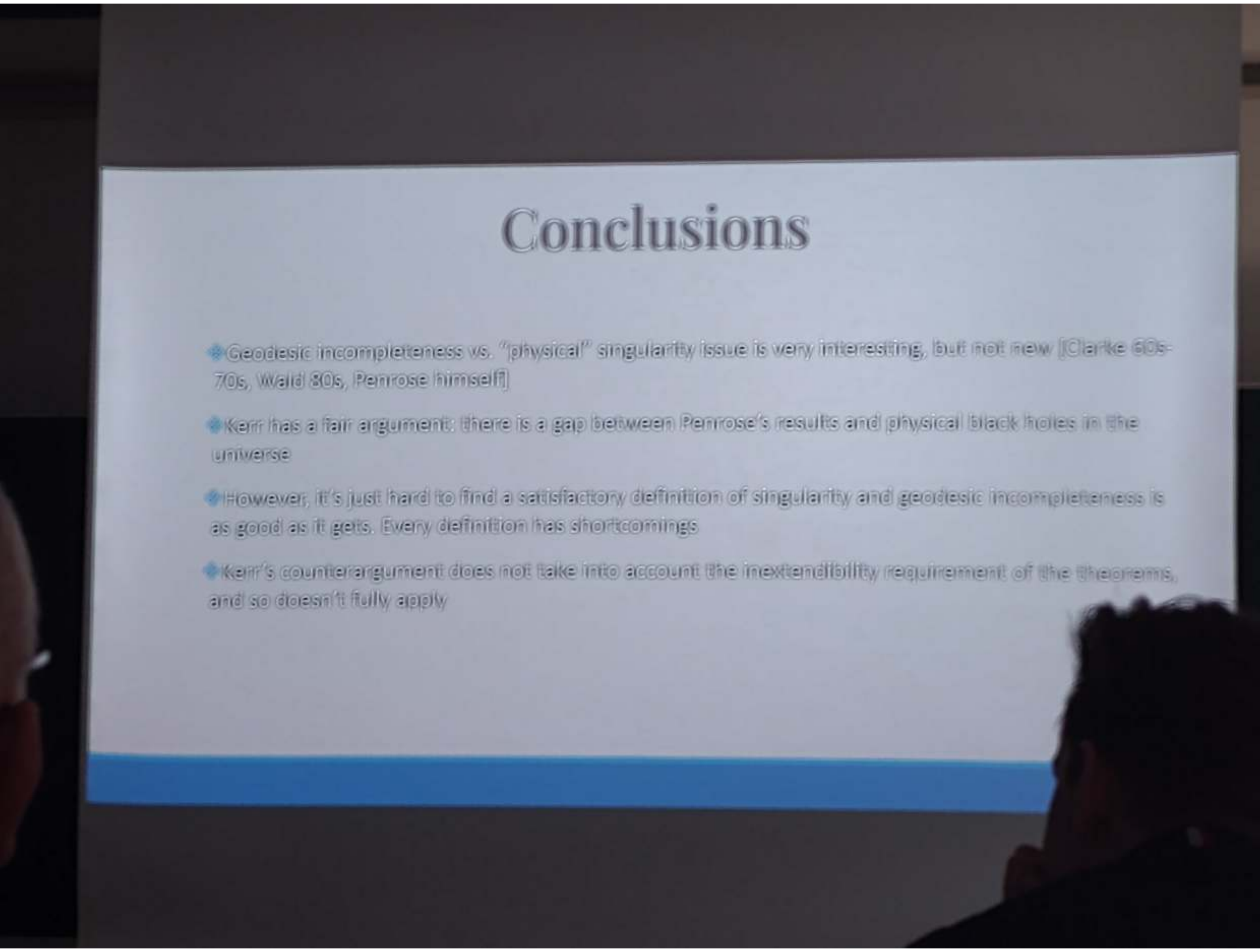
Conclusions
Geodesic incompleteness vs. "physical" singularity issue is very interesting, but not new [Clarke 60s-70s, Wald 80s, Penrose himself]
Kerr has a fair argument: there is a gap between Penrose's results and physical black holes in the universe
However, it's just hard to find a satisfactory definition of singularity and geodesic incompleteness is as good as it gets. Every definition has shortcomings
Kerr's counterargument does not take into account the inextendibility requirement of the theorems, and so doesn't fully apply